JUAS 2020 – Exercise Solution

$\mu = \mu_0\ \mu_r$

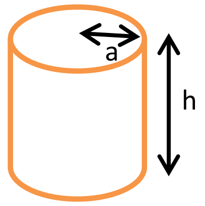
$\mu_0 = 4\pi \cdot 10^{-7}$ Vs/(Am)

$\varepsilon = \varepsilon_0\ \varepsilon_r$

$\varepsilon_0 = 8.854 \cdot 10^{-12}$ As/(Vm)

$c_0 = 2.998 \cdot 10^{8}$ m/s

1.) Cavities



Design a pillbox cavity. The E_{010} = TM_{010} mode shall resonate at f_{res} = 2.95 GHz. The aspect ratio shall be a/h = 0.5.

1. What is the radius *a* of the cavity?

$$a = 0.383\ \lambda = \frac{0.383\ c}{f} = 38.92\ mm$$

see script page 51

2. What is the height *h* of the cavity?

$$h = 2\ a = 77.85\ mm$$

3. The 3 dB bandwidth of the unloaded resonance shall be 150 kHz, how big does the unloaded Q_0 of the cavity need to be?

$$Q_0 = \frac{f_{res}}{f_{3dB}} = 19667$$

see script page 55 and 59

4. What is the maximum tolerable surface resistance ρ of the cavity walls to get Q_0 = 20 000.

$$\delta = \frac{a}{Q_0}\left[1 + \frac{a}{h}\right]^{-1} = 1.23\ \mu m = \sqrt{\frac{2}{\omega \sigma \mu}}$$

see script page 78

$$\sigma = \frac{2}{\delta^2\ 2\ \pi\ f_{res}\ \mu_0} = 51 \cdot 10^6\ S/m$$

see script page 75

$$\rho = \frac{1}{\sigma} = 19.6 \cdot 10^{-9}\ \Omega m$$

Note: the resistivity ρ is inverse proportional to the conductivity σ

5. What is the R/Q for this cavity geometry?

$$\frac{R}{Q} = 128 \frac{\sin^2\left(1.2024\ {}^{h}/_{a}\right)}{{}^{h}/_{a}} = 28.9$$

see script page 78

Note: We cannot use the approximate equation as the argument x≈2.4 in sin(x) has a large value and therefore sin(x) ≠ x

Keep in mind, the R/Q is purely defined by the geometry of the cavity,
while the unloaded Q_0 depends on the losses defined by the material properties.

6. Derive the equivalent circuit parameters R, L and C for the cavity with
   Q_0 = 20 000 and R/Q = 30

$$\frac{R}{Q} = \omega_{res} L = \frac{1}{\omega_{res} C}$$

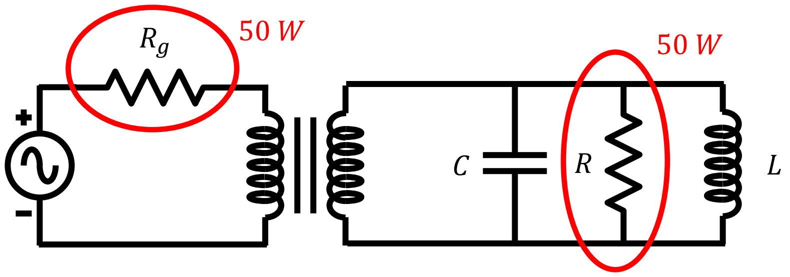
$$L = \frac{R/Q}{2\,\pi f_{res}} = 1.62\,nH$$

$$C = \frac{1}{2\,\pi f_{res}\, R/Q} = 1.8\,pF$$

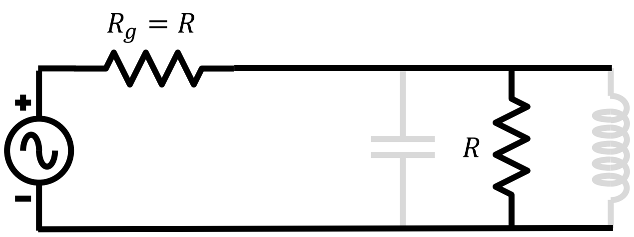
$$R = \frac{R}{Q} Q_0 = 600\,k\Omega$$

see script page 56

7. The cavity is critically coupled to an RF power amplifier and driven by 50 W of input power on its resonant frequency. The loaded Q_L is 10 000. What is the stored energy W_{CAV} in the cavity?
   Equivalent circuit:



At critical coupling at resonance this simplifies to:



The transformed source impedance of the generator is equal to the shunt impedance of the resonator $R_g = R$, therefore the total impedance is $R/2$,
and the power is $P = 2 \cdot 50\,W = 100\,W$.

$$W_{cav} = \frac{Q_L\,P}{2\,\pi f_{res}} = 54\,\mu Ws$$

or

$$W_{cav} = \frac{1}{2} C \hat{V}^2 = 54\,\mu Ws \quad \text{with: } \hat{V} = \sqrt{2PR}$$

see script page 56

8. Determine the peak gap voltage V_{gap}?
Note: Two ways to compute the gap voltage, both using the equations on page 56:

$$V_{gap} = V = \sqrt{P\,\frac{2\,R}{2}} = 5.47\;kV$$

Note: The cavity is critically coupled, i.e. the transformed source impedance (see schematic page 66) equals the cavity equals resistance R. As both are parallel the total resistance is R/2

$$V_{gap} = V_C = \sqrt{\frac{2\,W_{cav}}{C}} = 5.47\;kV$$

see script page 56

9. Operating the cavity in air, is "Kilpatrick" voltage breakdown a problem?
gap = h = 78 mm, V_{gap} = 5.5 kV, seems to be no problem

see script page 110

Note: The gap with is h ≈ 78 mm, and the voltage is ~5.5 kV, operating in air.
The graphics shows this operating point is safe, no breakdowns to be expected

2.) Smith chart

Given in the table below are several impedances $\mathbf{Z_N}$ or reflection coefficients $\mathbf{\Gamma_N}$, measured at specific frequencies.
They have been normalized already with the characteristic impedance Z_C = 50 Ω in the following way:
$Z_N = Z/Z_C$

Point no.	**P_1**	**P_2**	**P_3**	**P_4**
Z_N or Γ_N	$\mathbf{Z_N}$ = 1	$\mathbf{Z_N}$ = 0.5 + 0.5j	$\lvert\mathbf{\Gamma_N}\rvert$ = 0.45 arc($\mathbf{\Gamma_N}$) = -117°	$\mathbf{Z_N}$ = 0
f [GHz]	3.000	2.997	3.003	0.01

Point no.	**P_5**	**P_6**	**P_7**	**P_8**
Z_N or Γ_N	$\mathbf{Z_N}$ = 0.2 + 0.4j	$\mathbf{Z_N}$ = 0.2 - 0.4j	$\lvert\mathbf{\Gamma_N}\rvert$ = 1 arc($\mathbf{\Gamma_N}$) = 0°	$\lvert\mathbf{\Gamma_N}\rvert$ = 0.49 arc($\mathbf{\Gamma_N}$) = 52°
f [GHz]	2.994	3.006	-	-

1. Mark all the points in the attached smith chart (including each point no.)
Use your compass and the rulers at the bottom of the smith chart for the polar coordinates

see script page 127 ff

2. Design a matching circuit. The point P_2 in the smith chart shall be matched to $Z_C = 50\ \Omega$. Use only **one 50 Ω transmission line** and **one series capacitor.** Draw the solution in the smith chart. Numerical answers are not needed.

see script page 134 and 136

3. Draw a circle through the points P1 – P6. This is the result of a S_{11} measurement of a microwave cavity with a network analyser.
   What is the resonant frequency f_{res} of the cavity? (Look it up in the table)

see script page 147

Note: At resonance the impedance is purely real (no imaginary components)

f_{res} = 3 GHz (P_1)

4. What is the unloaded Q_0 of the cavity?

$$Q_L = \frac{f_{res}}{f_{3dB}} = \frac{3\ GHz}{(3.006 - 2.994)\ GHz} = 250$$

$$Q_0 = 2\ Q_L = 500$$

see script page 147

Note: At critical coupling the unloaded Q_0 is twice the loaded

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES