JUAS 2020 – Tutorial 2

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1) Design of a pillbox cavity

Problem: Design a simple "Pillbox" cavity with the following parameters

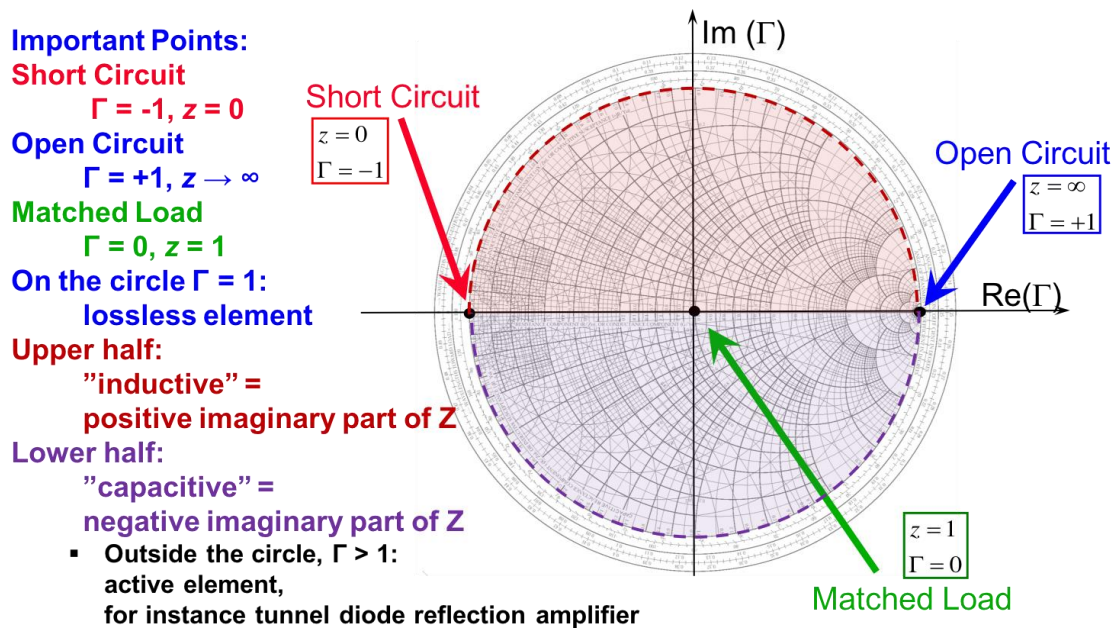|	
Frequency:	f = 299.98 MHz (λ = 1.00 m)
Wall material:	Copper (equivalent skin depth δ = 3.8 µm)
Axial length:	h = 0.2 m

For this example, we ignore beam ports, i.e. vacuum chamber stubs required for the beam passage, so that all analytical formulas describing the pillbox cavity apply.
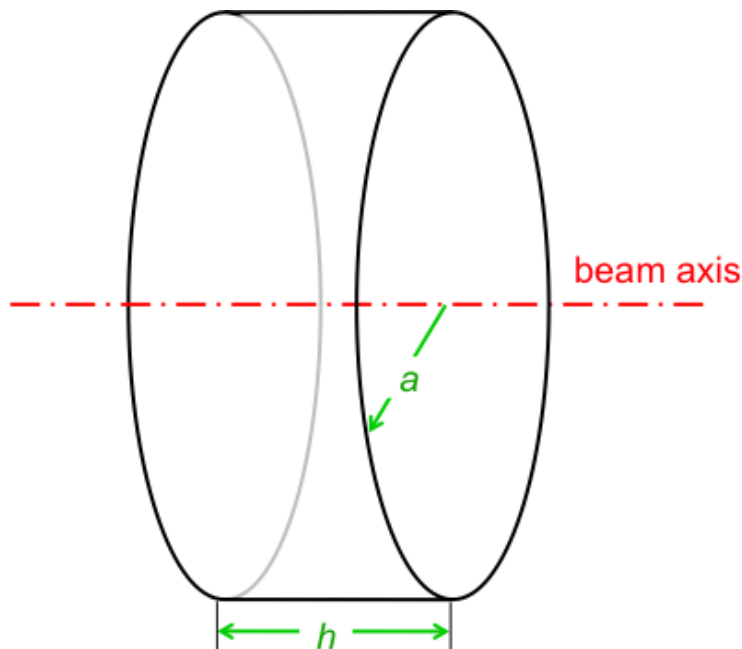
Questions:

1. Find from the analytical formulas:
    *   Cavity radius a
    *   Cavity quality factor Q
    *   "geometry factor", also known as "characteristic impedance" R/Q
2. Find the equivalent circuit of the cavity.
3. Calculate the 3-dB bandwidth of the intrinsic (not connector to any generator) cavity.
4. Calculate the necessary RF power (RMS) for a gap peak voltage of $\hat{V} = 100\ kV$, assuming critical coupling.

5. The cavity is critically coupled, fed by an amplifier, designed for a load impedance of 50 Ω. Determine:

- The peak voltage at the amplifier output.
- k of the input coupler.

Important Points:
Short Circuit
    $\Gamma = -1$, $z = 0$
Open Circuit
    $\Gamma = +1$, $z \rightarrow \infty$
Matched Load
    $\Gamma = 0$, $z = 1$
On the circle $\Gamma = 1$:
    lossless element
Upper half:
    "inductive" =
    positive imaginary part of Z
Lower half:
    "capacitive" =
    negative imaginary part of Z
- **Outside the circle, $\Gamma > 1$:**
  active element,
 **for instance tunnel diode reflection amplifier**
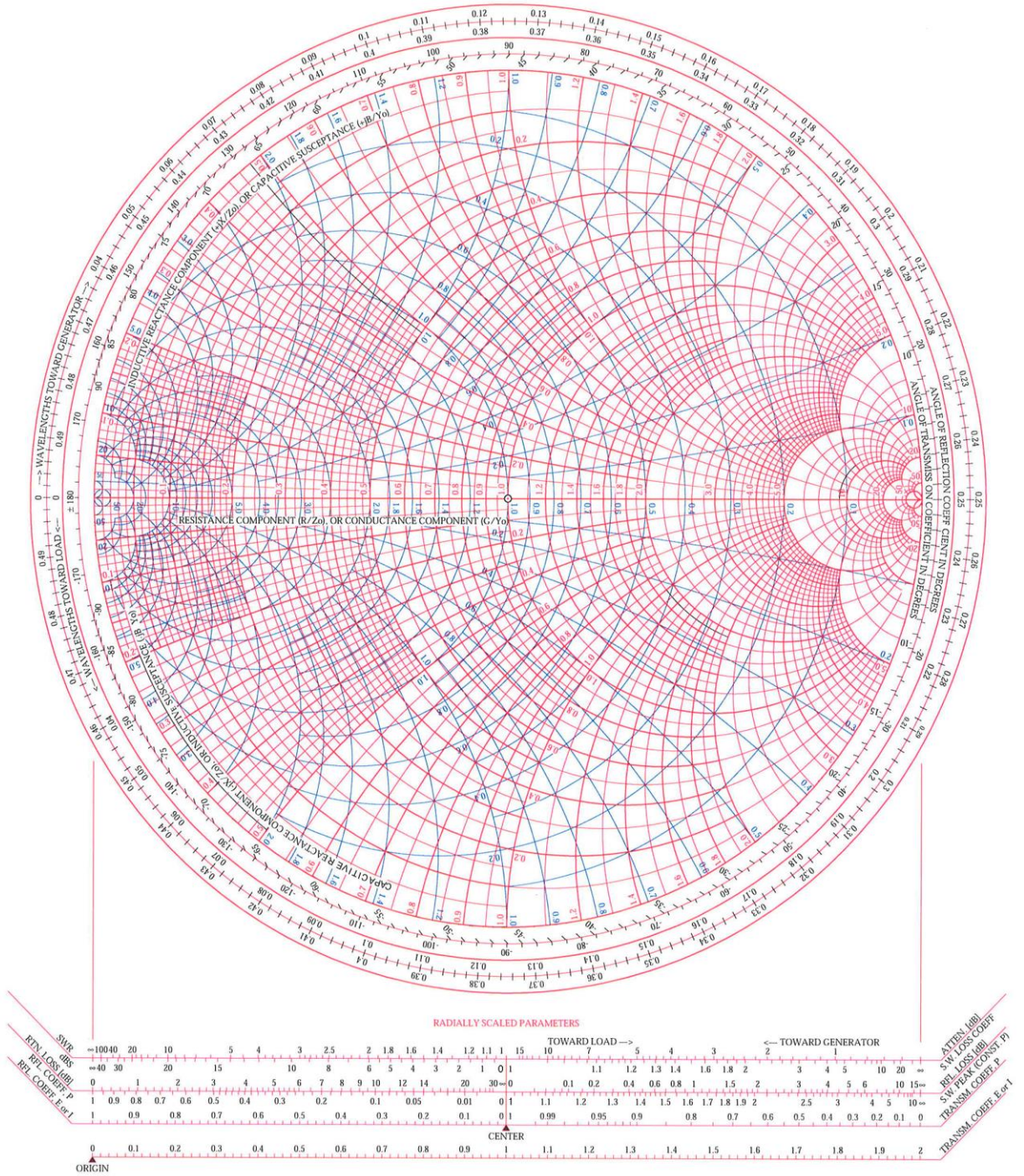

2.1.) Smith Chart (1)

1. Mark the reflection factors Γ of points A to F in the Smith Chart and find approximate values for the corresponding (normalized) impedances z:

Point	Reflection factor Γ	Normalized impedance z
A	1 \0°	∞
B	1 \45°	0 + J2.4
C	1 \90°	0 + J1.0
D	1 \180°	0
E	1 \-90°	0 - j1
F	0.5	3.0

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES

RADIALLY SCALED PARAMETERS

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RADIALLY SCALED PARAMETERS
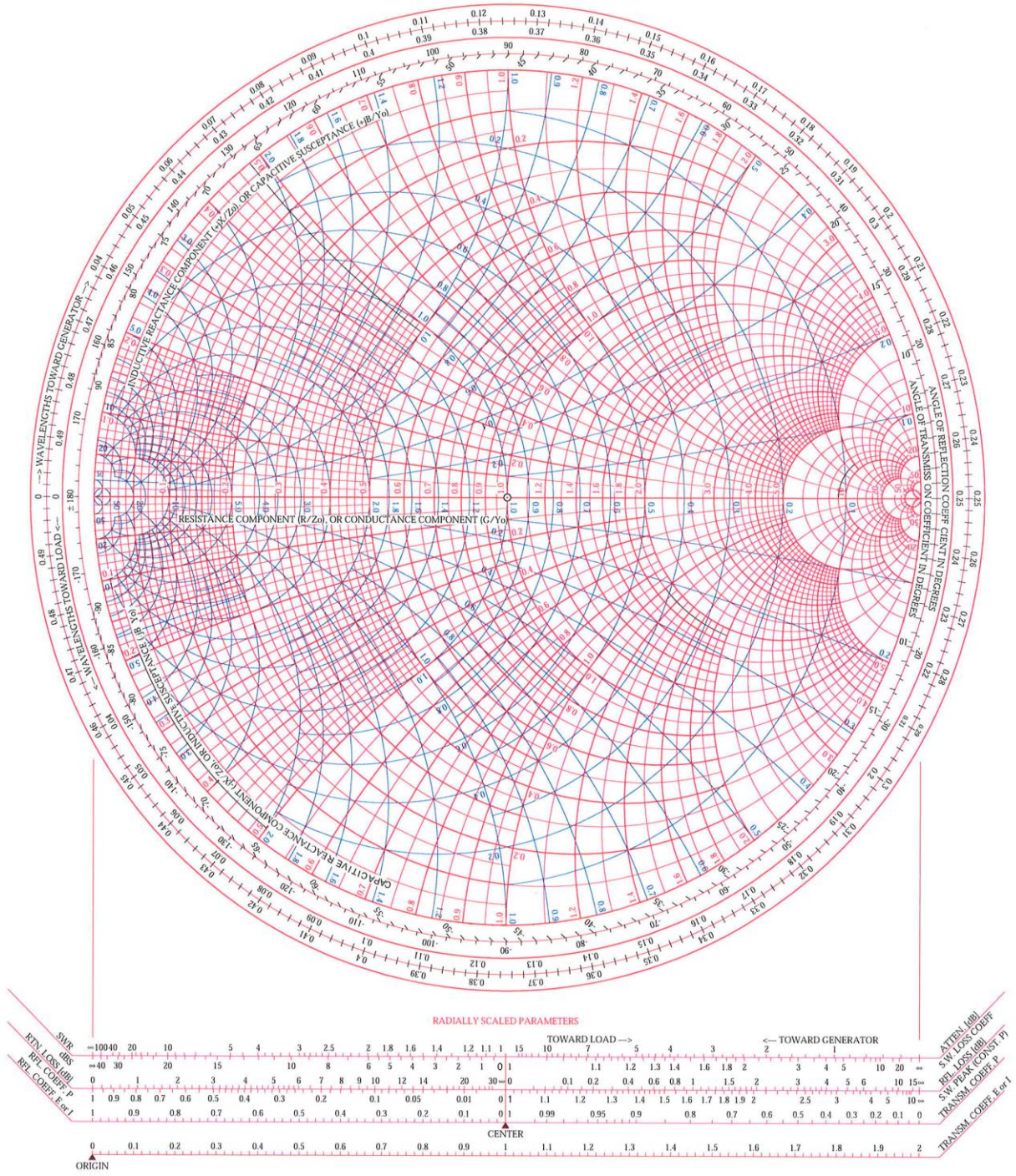
2.2) Smith Chart (2)

1. Plot the following **normalized** impedances z into the Smith Chart:

Point	a	b	c	d
z	0.6 + j0	0.6 - j0.6	0.6 - j0.8	0.6 - j1.0

2. Plot the following impedances Z into the Smith Chart,
   assuming a reference impedance of $Z_0 = 50\ \Omega$

Point	A	B	C	D
Z	50 + j0	20 – j15	10 + j25	0 – j50

2.3.) Impedances in the complex plane and in the Smith chart

Plot the following impedances as points (marks)

- in a "normal" Cartesian coordinate system (complex plane)
- as reflection factors in the *Smith* chart (the reflection factor coordinates are given for convenience)

X_{rect}	X_{polar}	Γ_{rect}	Γ_{polar}
0.05	$0.05 \angle 0^0$	-0.904	$0.904 \angle 180^0$
0.5	$0.5 \angle 0^0$	-0.333	$0.333 \angle 180^0$
1	$1.0 \angle 0^0$	0	0
2	$2.0 \angle 0^0$	0.333	$0.333 \angle 0^0$
20	$20 \angle 0^0$	0.904	$0.904 \angle 0^0$
0.8	$0.8 \angle 0^0$	-0.111	$0.111 \angle 180^0$
0.8 + j0.6	$1.00 \angle 36.9^0$	0 + j0.333	$0.333 \angle 90^0$
0.8 + j1.0	$1.28 \angle 51.3^0$	0.159 + j0.472	$0.459 \angle 72.3^0$
0.8 + j1.5	$1.70 \angle 61.9^0$	0.344 + j0.546	$0.645 \angle 57.8^0$
0.8 + j2.0	$2.15 \angle 68.2^0$	0.502 + j0.552	$0.747 \angle 47.7^0$
0.8 – j0.6	$1.00 \angle -36.9^0$	0 - j0.333	$0.333 \angle -90^0$

Convince yourself with a few examples that $\Gamma(1/X) = -\Gamma(X)$

2.4) Smith Chart (3)

The locus of impedance of a parallel RLC resonant circuit is given in the complex *z*-plane (*z*-pane = normalized *Z*-plane, normalization to 50 Ω; *z* = *Z* / 50 Ω).
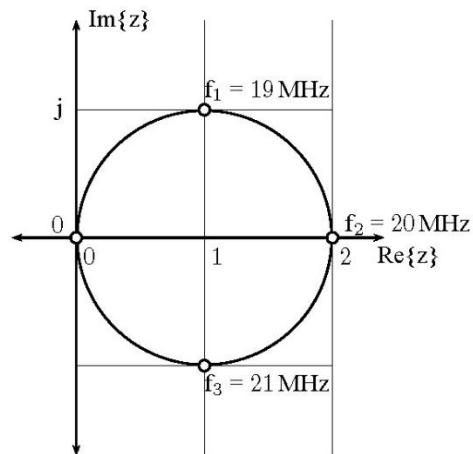
Questions:

1. Transform this locus of impedance into the Smith Chart
2. Mark the resonance frequency, both, in the *z*-plane and in the Smith Chart.
3. Mark the 3-dB points (for the unloaded Q), both, in the *z*-plane and in the Smith Chart.

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES

RADIALLY SCALED PARAMETERS
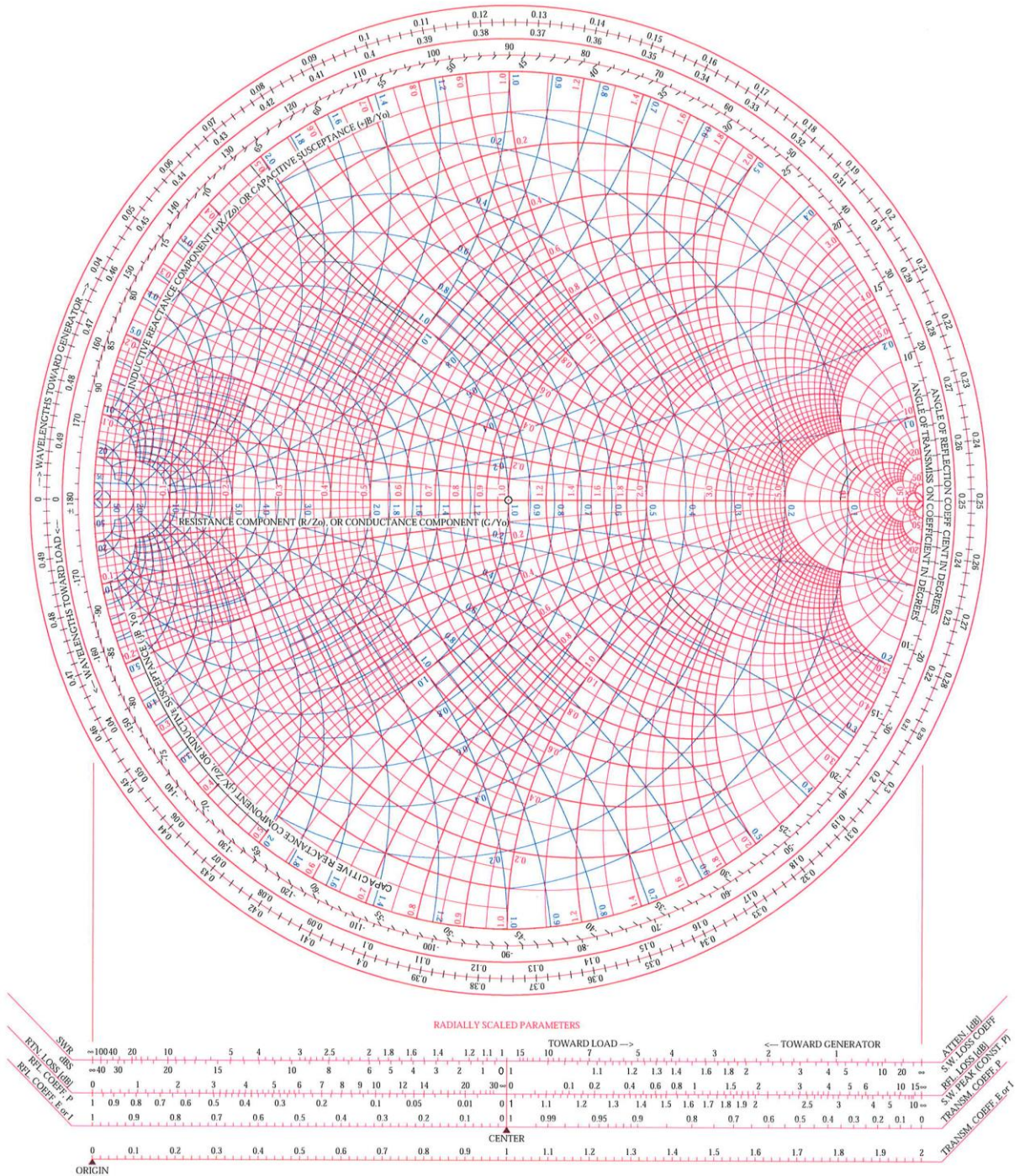
3.) Various questions

1. What is the difference between a *Stripline* and a *Microstripline*?

2. Name 3 disadvantages of *Microstriplines* compared to *Striplines*.

3. A RF signal needs to be guided from a power amplifier on the surface to the cavity of a particle accelerator in the tunnel below ground.
   The distance is *l* = 100 m.   The signal has parameters: *f* = 50 MHz,   *P* = 100 kW
   a. Would you use waveguide or coaxial transmission line? Why?
   b. What would you use if the signal would have a frequency of 500 MHz? Why?

4. Why are some accelerator cavities (for frequencies in the MHz range) loaded with ferrite? Explain how the resonant frequency of those cavities can be tuned without moving parts.

5. After deploying a new accelerating cavity, a RF-engineer starts pounding on it with a hammer. What is he doing?

4.) "Pillbox" cavity characteristics

The following data was measured on a "pillbox" cavity":

Inductance:	*L* = 15.915 nH
Capacitance:	*C* = 1.5915 pF
3-dB bandwidth:	*BW* = 50 kHz

Questions:
Determine

- the frequency at resonance
- the characteristic impedance R/Q
- the quality factor Q
- the time constant τ
- the peak induced voltage immediately after the passage of a short particle bunch with charge $q = 15.916 \cdot 10^{-9}\ As$
- the remnant cavity voltage 10 μs after the passage of the bunch

5.) Gap-width optimization of a cavity

The following parameters of a 100 MHz cavity have been evaluated by a numerical simulation software as function of the gap-width g:

characteristic impedance R/Q and quality factor Q

The cavity is connected to an amplifier delivering 1 kW of RF power.
The beam has a relative velocity of $\beta = 0.15$.

Questions:

Calculate for each gap-width:

- shunt impedance R
- intrinsic cavity voltage V_{cav} for 100 kW power
- angle θ of the beam passage through the gap
- transit time factor T
- beam voltage V_{beam} maximally seen by the beam taking the transit time factor T into account

G [mm]	R/Q [Ω]	Q	R [Ω]	V_{cav} [V]	θ	T	V_{beam} [V]
100	100	5000					
200	150	7000					
300	200	9000					

6.) Higher-order mode of a cavity

An RF cavity has an unwanted higher-order mode (HOM) at 600 MHz, with shunt impedance R = 6 MΩ, a 3-dB bandwidth of BW = 15 kHz, and a transit time factor $T \sim 1$.

The beam consists of very short bunches, following each other at intervals of 20 μs. The circulating beam current is 0.1 A. (Reminder: current I = charge per time)

Questions:

Calculate:

- Q, R/Q, and C at the HOM frequency
- HOM voltage induced by a single bunch
- Time constant τ of the cavity
- HOM voltage at the arrival of the next bunch
- Total HOM voltage in steady state, after the passage of an infinite number of bunches, assuming the HOM resonance is an exact multiple of the beam revolution frequency and in sync with the beam.