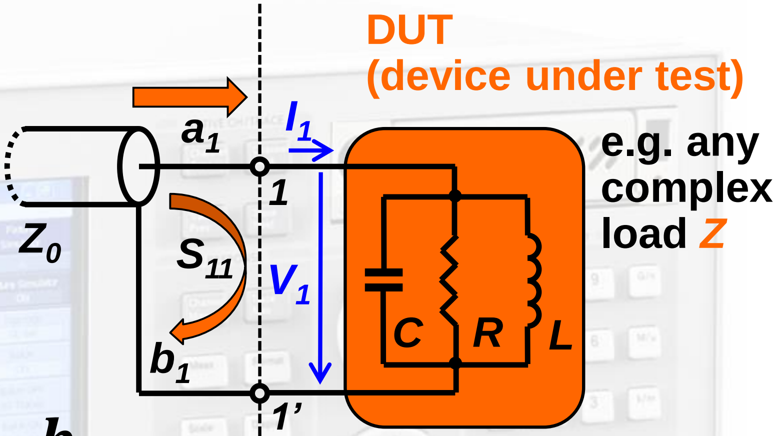
S-Parameters – Introduction (1)

- **Light falling on a car window:**
  - Some parts of the incident light is reflected (you see the mirror image)
  - Another part of the light is transmitted through the window (you can still see inside the car)
- **Optical reflection and transmission coefficients of the window glass define the ratio of reflected and transmitted light**
- **Similar:**
  Scattering (S-) parameters of an *n*-port electrical network (DUT) characterize reflected and transmitted (power) waves

S-Parameters – Introduction (2)

- **Electrical networks**
  - 1…*n*-ports circuits
  - Defined by voltages $V_n(\omega)$ or $v_n(t)$ and currents $I_n(\omega)$ or $i_n(t)$ at the ports
  - Characterized by circuit matrices, e.g. ABCD (chain), Z, Y, H, etc.
- **RF networks**
  - 1…n-port RF DUT circuit or subsystem, e.g. filter, amplifier, transmission-line, hybrid, circulator, resonator, etc.
  - Defined by incident $a_n(\omega, s)$ and reflected waves $b_n(\omega, s)$ at a reference plane *s* (physical position) at the ports
  - Characterized by a scattering parameter (S-parameter) matrix of the reflected and transmitted power waves
  - Normalized to a reference impedance $\sqrt{Z_0}$ of typically Z_0 = 50 Ω



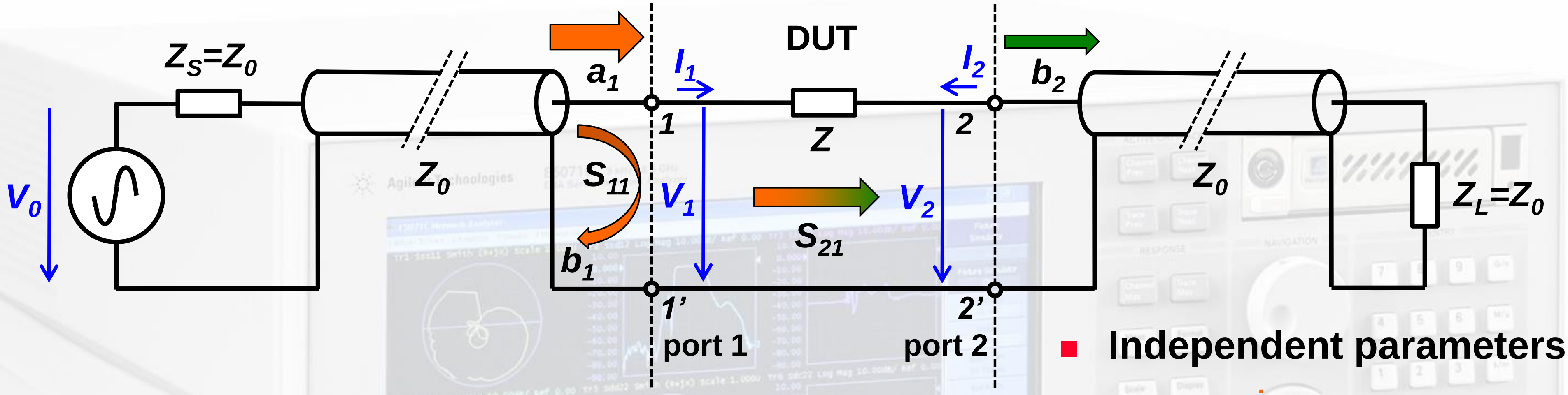
$$S_{11} = \frac{b_1}{a_1} = \Gamma$$

1-1' reference plane (port 1)

1-port DUT example

- S-Parameters allow to characterize the DUT with the measurement equipment to be located at some distance
- All high frequency effects of distributed elements are taken into account with respect to the reference plane

S-Parameters – Example: 2-port DUT



◆ **Analysis of the forward S-parameters:**

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \equiv \text{input reflection coefficient}$$

$$(Z_L = Z_0 \Rightarrow a_2 = 0)$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \equiv \text{forward transmission gain}$$

- **Examples of 2-ports DUT: filters, amplifiers, attenuators, transmission-lines (cables), etc.**
- **ALL ports ALWAYS need to be terminated in their characteristic impedance!**

- **Independent parameters:**

$$a_1 = \frac{V_1^{inc}}{\sqrt{Z_0}} = \frac{V_1 + I_1 Z_0}{2\sqrt{Z_0}}$$

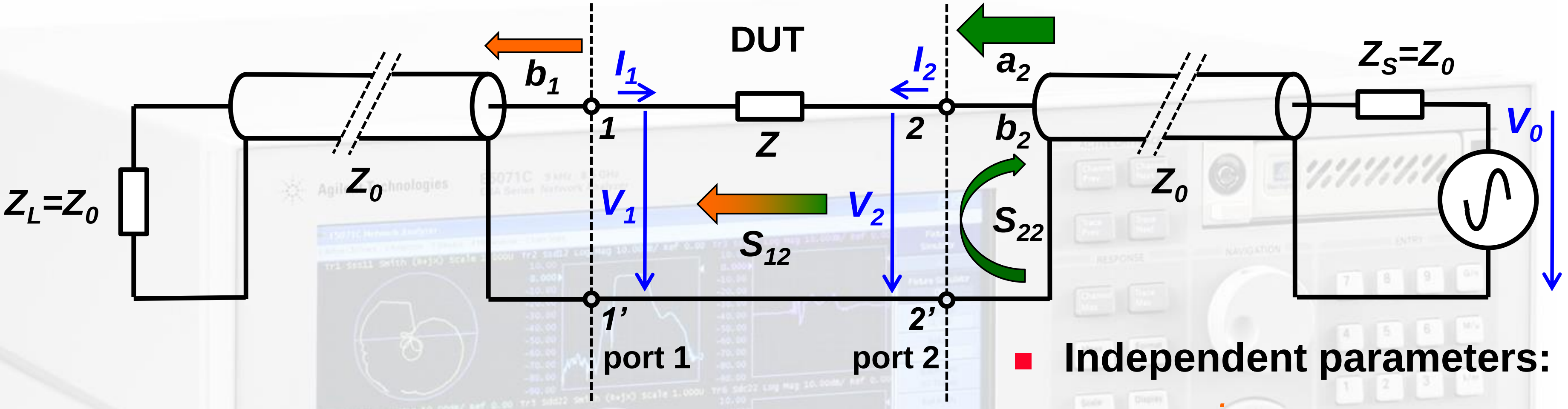
$$a_2 = \frac{V_2^{inc}}{\sqrt{Z_0}} = \frac{V_2 + I_2 Z_0}{2\sqrt{Z_0}}$$

- **Dependent parameters:**

$$b_1 = \frac{V_1^{refl}}{\sqrt{Z_0}} = \frac{V_1 - I_1 Z_0}{2\sqrt{Z_0}}$$

$$b_2 = \frac{V_2^{refl}}{\sqrt{Z_0}} = \frac{V_2 - I_2 Z_0}{2\sqrt{Z_0}}$$

S-Parameters – Example: 2-port DUT



- **Independent parameters:**

$$a_1 = \frac{V_1^{inc}}{\sqrt{Z_0}} = \frac{V_1 + I_1 Z_0}{2\sqrt{Z_0}}$$

$$a_2 = \frac{V_2^{inc}}{\sqrt{Z_0}} = \frac{V_2 + I_2 Z_0}{2\sqrt{Z_0}}$$

- **Dependent parameters:**

$$b_1 = \frac{V_1^{refl}}{\sqrt{Z_0}} = \frac{V_1 - I_1 Z_0}{2\sqrt{Z_0}}$$

$$b_2 = \frac{V_2^{refl}}{\sqrt{Z_0}} = \frac{V_2 - I_2 Z_0}{2\sqrt{Z_0}}$$

◆ **Analysis of the reverse S-parameters:**

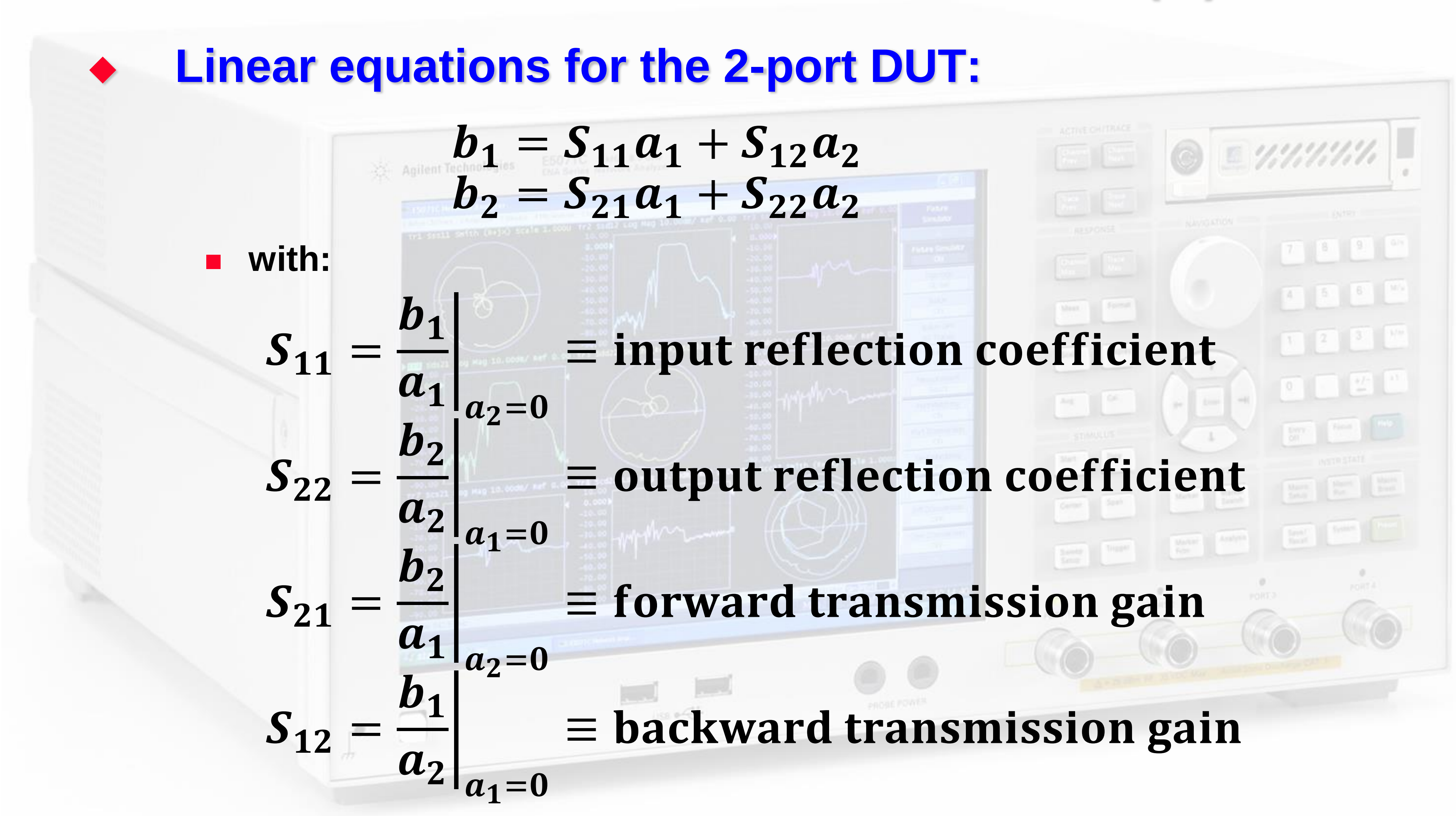
$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} \equiv \text{output reflection coefficient}$$

$$(Z_L = Z_0 \Rightarrow a_1 = 0)$$

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} \equiv \text{backward transmission gain}$$

- *n*-port DUTs still can be fully characterized with a 2-port VNA, but again: don’t forget to terminate unused ports!

S-Parameters – Definition (1)



- **Linear equations for the 2-port DUT:**

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

  - with:

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \equiv \textbf{input reflection coefficient}$$

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} \equiv \textbf{output reflection coefficient}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \equiv \textbf{forward transmission gain}$$

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} \equiv \textbf{backward transmission gain}$$

S-Parameters – Definition (2)

- **Reflection coefficient and impedance at the *n*th-port of a DUT:**

$$S_{nn} = \frac{b_n}{a_n} = \frac{\frac{V_n}{I_n} - Z_0}{\frac{V_n}{I_n} + Z_0} = \frac{Z_n - Z_0}{Z_n + Z_0} = \Gamma_n$$

$$Z_n = Z_0 \frac{1 + S_{nn}}{1 - S_{nn}} \text{ with } Z_n = \frac{\mathrm{V_n}}{\mathrm{I_n}} \text{ being the input impedance at the } n^{th} \text{port}$$

- **Power reflection and transmission for a *n*-port DUT**

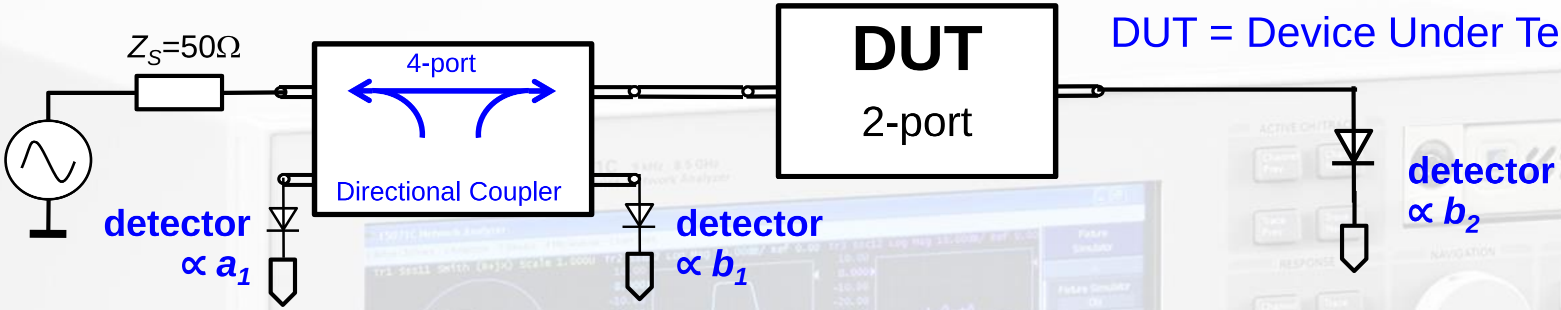
$$|S_{nn}|^2 = \frac{\text{power reflected from port } n}{\text{power incident on port } n}$$

$$|S_{nm}|^2 = \text{transmitted power between ports } n \text{ and } m$$

with all ports terminated in their characteristic impedance Z_0 and $Z_S = Z_0$

Here the US notion is used, where power = $|a|^2$.
European notation (often): power = $|a|^2/2$
These conventions have no impact on the S-parameters, they are only relevant for absolute power calculations

How to measure S-Parameters?



DUT = Device Under Test

- **Performed in the frequency domain**
  - **Single or swept frequency generator, stand-alone or as part of a VNA or SA**
  - **Requires a directional coupler and RF detector(s) or receiver(s)**
- **Evaluate S_{11} and S_{21} of a 2-port DUT**
  - **Ensure $a_2=0$, i.e. the detector at port 2 offers a well matched impedance**
  - **Measure incident wave a1 and reflected wave b1 at the directional coupler ports and compute for each frequency**

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$$

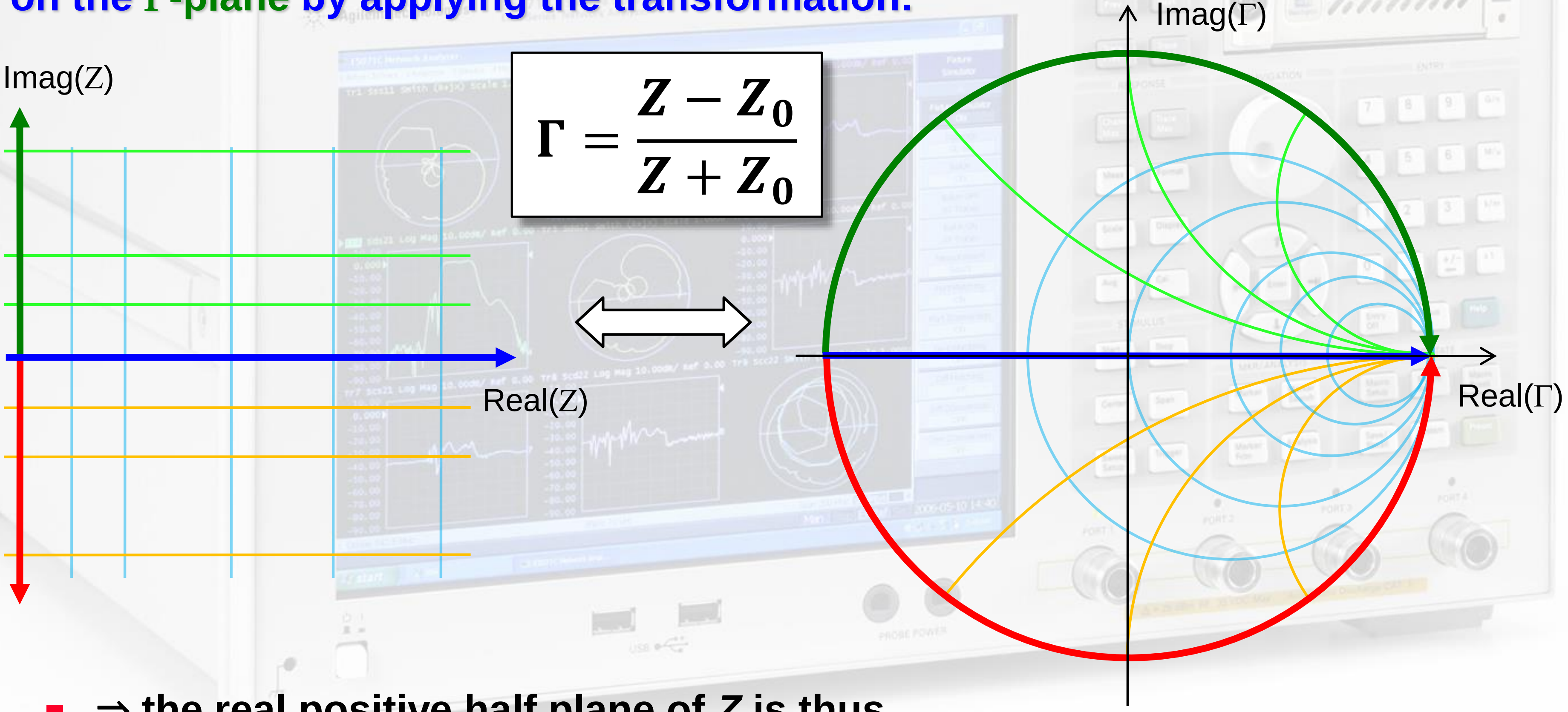
  - **Measure transmitted wave b_2 at DUT port 2 and compute**

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

- **Evaluate S_{22} and S_{12} of the 2-port DUT**
  - **Perform the same methodology as above by exchanging the measurement equipment on the DUT ports**

The *Smith* Chart (1)

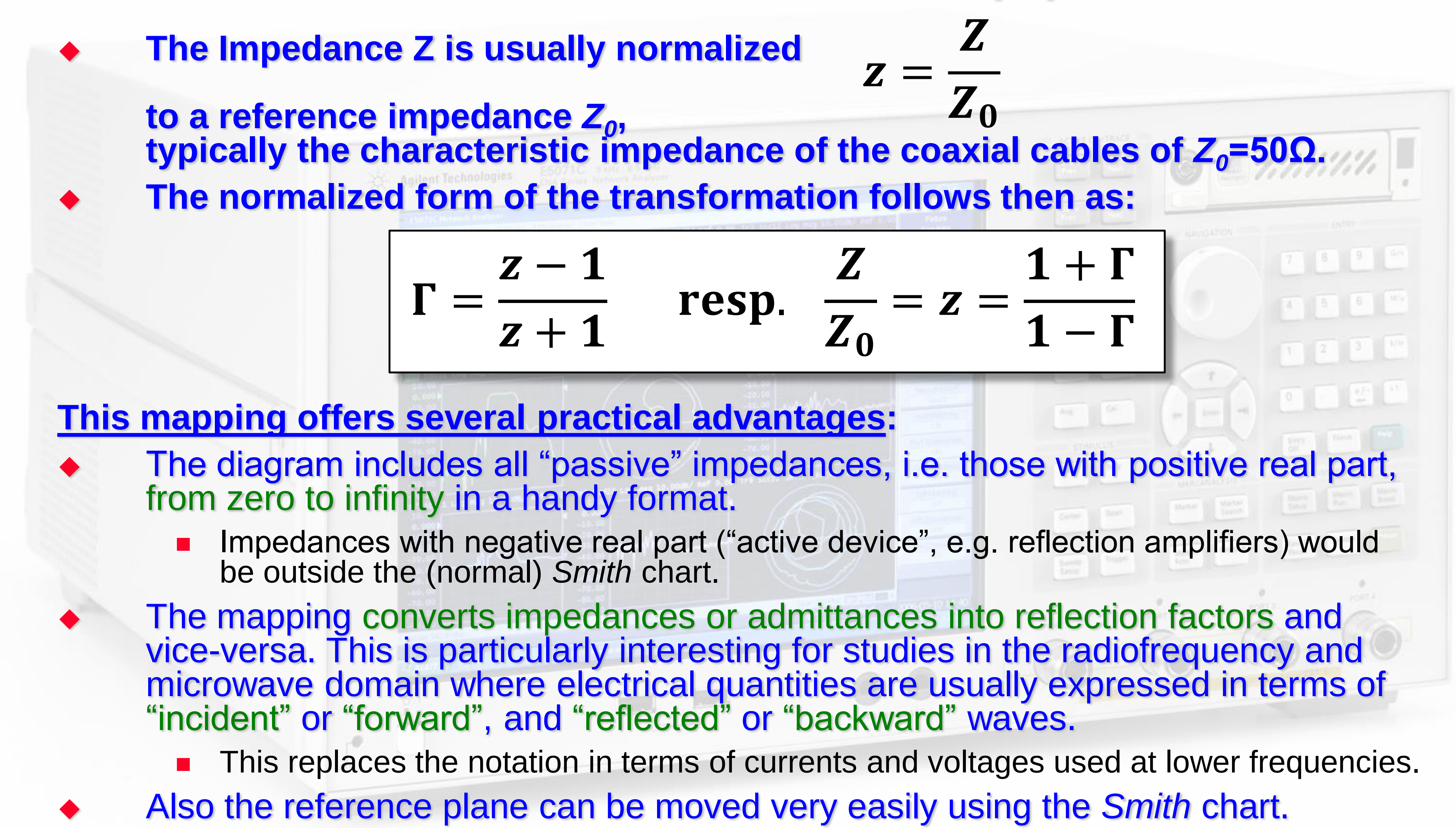
- The *Smith* Chart (in impedance coordinates) represents the complex Γ-plane (in polar coordinates) within the unit circle.
- It is a conformal mapping of the complex *Z*-plane on the Γ-plane by applying the transformation:



$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

- ⇒ the real positive half plane of *Z* is thus transformed (*Möbius*) into the interior of the unit circle!

The *Smith* Chart (2)



- **The Impedance Z is usually normalized**

  $$z = \frac{Z}{Z_0}$$

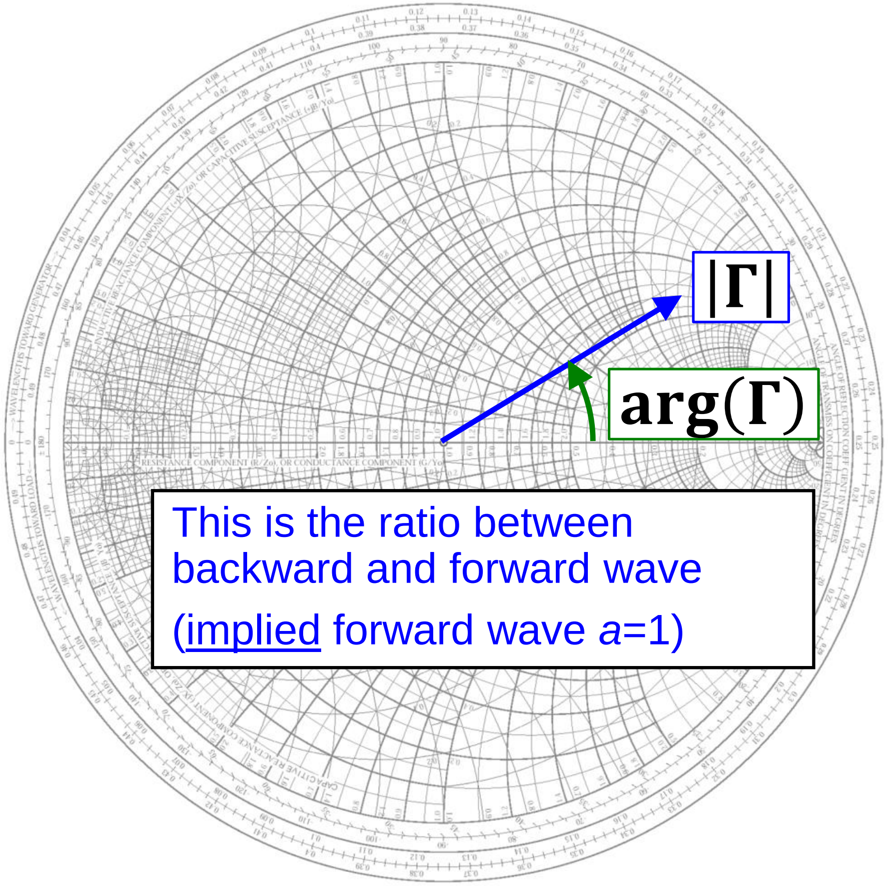
  to a reference impedance Z_0,
  typically the characteristic impedance of the coaxial cables of Z_0=50Ω.
- **The normalized form of the transformation follows then as:**

$$\boxed{\Gamma = \frac{z-1}{z+1} \quad \text{resp.} \quad \frac{Z}{Z_0} = z = \frac{1+\Gamma}{1-\Gamma}}$$

<u>This mapping offers several practical advantages</u>:

- The diagram includes all “passive” impedances, i.e. those with positive real part, from zero to infinity in a handy format.
  - Impedances with negative real part (“active device”, e.g. reflection amplifiers) would be outside the (normal) *Smith* chart.
- The mapping converts impedances or admittances into reflection factors and vice-versa. This is particularly interesting for studies in the radiofrequency and microwave domain where electrical quantities are usually expressed in terms of “incident” or “forward”, and “reflected” or “backward” waves.
  - This replaces the notation in terms of currents and voltages used at lower frequencies.
- Also the reference plane can be moved very easily using the *Smith* chart.

The *Smith* Chart (3)



This is the ratio between backward and forward wave

(implied forward wave *a*=1)

- **In the *Smith* chart, the complex reflection factor**

$$\Gamma = |\Gamma| e^{j\varphi} = \frac{b}{a}$$

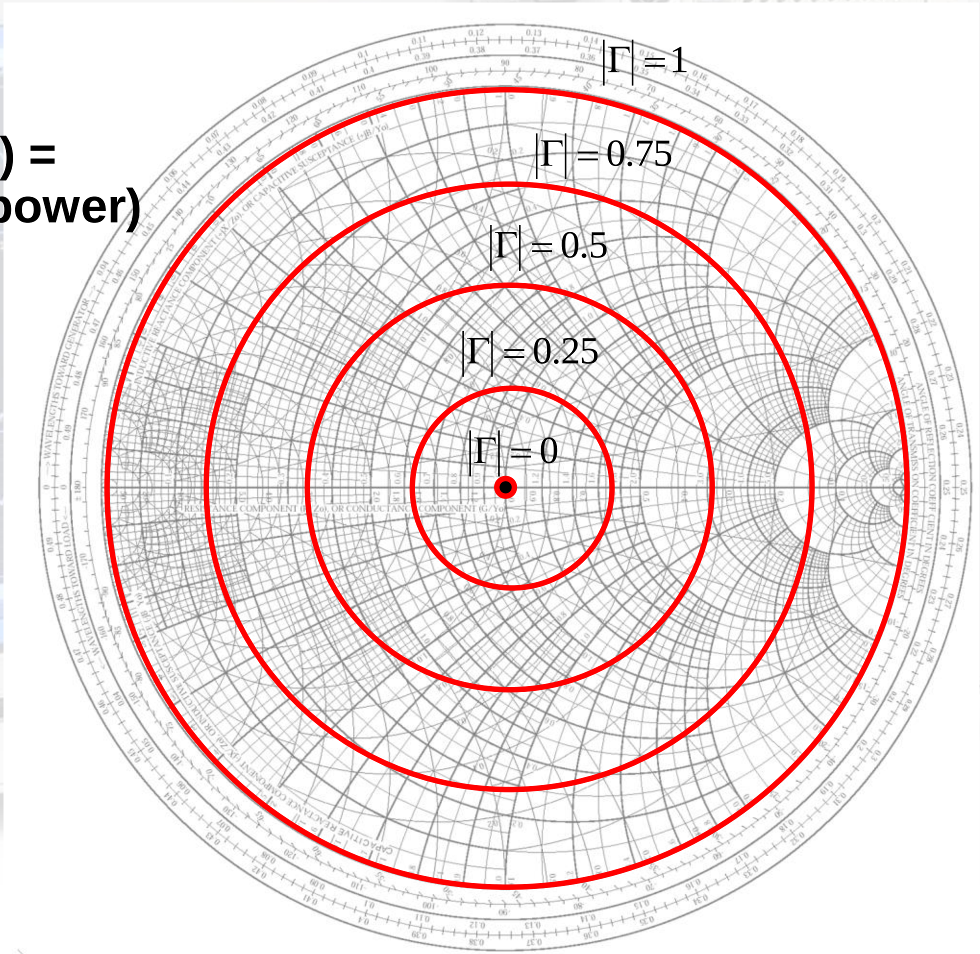
is expressed in linear cylindrical coordinates, representing the ratio of backward vs. forward traveling waves.

The *Smith* Chart (4)

- **The distance from the center of the directly proportional to the magnitude of the reflection factor |Γ|, and permits an easy visualization of the matching performance.**
  - **In particular, the perimeter of the diagram represents total reflection: |Γ|=1.**
  - **(power dissipated in the load) = (forward power) – (reflected power)**

$$\mathbf{P} = |a|^2 - |b|^2$$

$$= \underbrace{|a|^2}_{\text{available source power}}\underbrace{\left(1 - |\Gamma|^2\right)}_{\text{mismatch losses}}$$



The *Smith* Chart – “Important Points”

Important Points:

- **Short Circuit**
  $\Gamma = -1$, $z = 0$
- **Open Circuit**
  $\Gamma = +1$, $z \rightarrow \infty$
- **Matched Load**
  $\Gamma = 0$, $z = 1$
- **On the circle $\Gamma = 1$:**
  lossless element
- **Upper half:**
  ”inductive” =
  positive imaginary part of Z
- **Lower half:**
  ”capacitive” =
  negative imaginary part of Z
  - **Outside the circle, $\Gamma > 1$:**
    active element,
    for instance tunnel diode reflection amplifier