

# JUAS 2020 – Tutorial 2

F. Caspers, M. Bozzolan, M. Wendt

$$\mu = \mu_0 \mu_r$$

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/(Am)}$$

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

$$\varepsilon_0 = 8.854 \cdot 10^{-12} \text{ As/(Vm)}$$

$$c_0 = 2.998 \cdot 10^8 \text{ m/s}$$

## 1) Design of a pillbox cavity

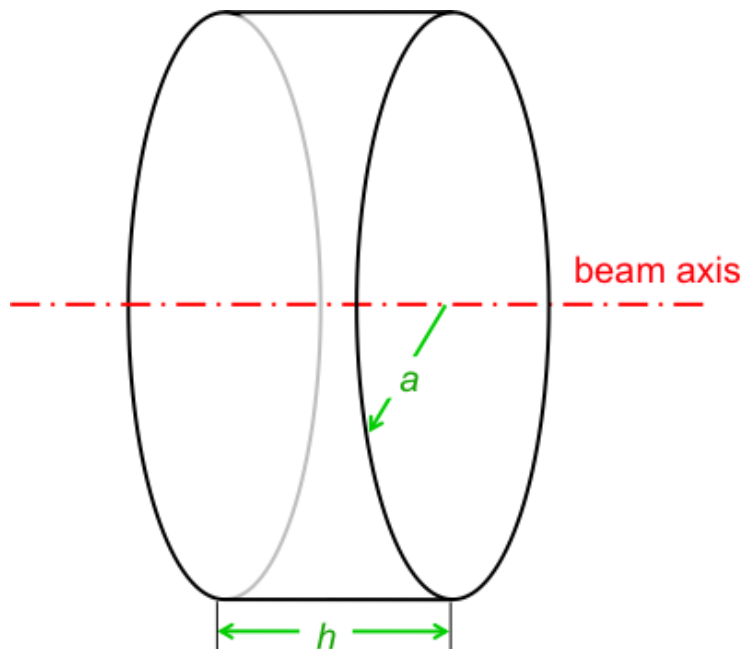
Problem: Design a simple “Pillbox” cavity with the following parameters

Frequency:  $f = 299.98 \text{ MHz}$  ( $\lambda = 1.00 \text{ m}$ )

Wall material: Copper (equivalent skin depth  $\delta = 3.8 \text{ }\mu\text{m}$ )

Axial length:  $h = 0.2 \text{ m}$

For this example, we ignore beam ports, i.e. vacuum chamber stubs required for the beam passage, so that all analytical formulas describing the pillbox cavity apply.



## Questions:

1. Find from the analytical formulas:

- Cavity radius  $a$

$$a = \frac{c_0}{2.61 f} = 0.383 \text{ m}$$

see script page 78

- Cavity quality factor  $Q$

$$Q = \frac{a}{\delta} \frac{1}{1 + a/h} = 34416$$

see script page 78

The approximation cannot be used as  $a/h = 0.526 > 0.5$

The skin depth can also be computed based on the conductivity of copper:

$$\sigma_{Cu} = 5.8 \cdot 10^7 \text{ S/m}$$

$$\delta = \sqrt{\frac{2}{2\pi f \sigma_{Cu} \mu_0}} = 3.82 \mu m$$

see script page 74

- “geometry factor”, also known as “characteristic impedance”  $R/Q$

$$\frac{R}{Q} = \frac{4\eta}{\chi_{01}^3 \pi J_1^2(\chi_{01})} \frac{\sin^2\left(\frac{\chi_{01} h}{2a}\right)}{\frac{h}{a}}$$

with:  $\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\mu_0^2 \epsilon_0^2} = 377 \Omega$  (characteristic impedance of the free space)

$$\chi_{01} = 2.4048 \text{ (1st zero of the Bessel function of 0th order)}$$

$$J_1(\chi_{01}) = 0.5192$$

Follows:

$$\frac{R}{Q} = 128 \frac{\sin^2\left(1.2024 \frac{h}{a}\right)}{\frac{h}{a}} = 84.61$$

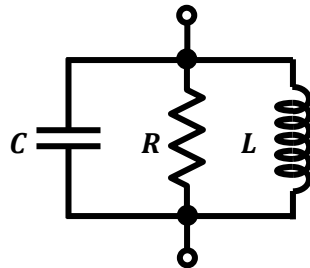
The approximation for small  $x$  cannot be applied:  $\sin(x) = x$  for  $x \ll 1$

see script page 79

Is the cavity completely determined?

Yes, 3 parameters are given, therefore the cavity is fully defined.

- Find the equivalent circuit of the cavity.



$$R = \frac{R}{Q} Q = 2.91 M\Omega$$

$$C = \frac{1}{2\pi f R/Q} = 6.27 pF$$

$$L = \frac{R/Q}{2\pi f} = 44.92 nH$$

see script pages 52, 53 and 56

- Calculate the 3-dB bandwidth of the intrinsic (not connector to any generator) cavity.

$$\Delta f = \frac{f}{Q} = 8.711 kHz$$

see script page 59

- Calculate the necessary RF power (RMS) for a gap peak voltage of  $\hat{V} = 100 kV$ , assuming critical coupling.

$$P = \frac{\hat{V}^2}{2R} = 1.717 kW$$

see script page 71

5. The cavity is critically coupled, fed by an amplifier, designed for a load impedance of  $50 \Omega$ . Determine:

- The peak voltage at the amplifier output.

$$v_{peak} = \sqrt{2Z_0 P} = 414 \text{ V}$$

see script page 56

- The necessary transformer ratio  $k$  of the input coupler.

$$k = \sqrt{\frac{R}{Z_0}} = 241$$

see script page 56

**Important Points:**

**Short Circuit**

$$\Gamma = -1, z = 0$$

**Open Circuit**

$$\Gamma = +1, z \rightarrow \infty$$

**Matched Load**

$$\Gamma = 0, z = 1$$

**On the circle  $\Gamma = 1$ :**

lossless element

**Upper half:**

"inductive" =

positive imaginary part of  $Z$

**Lower half:**

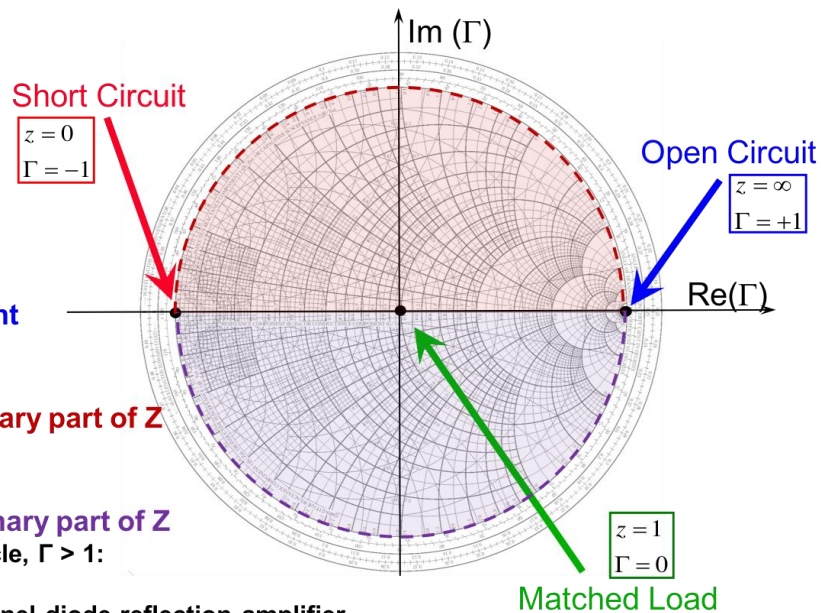
"capacitive" =

negative imaginary part of  $Z$

- Outside the circle,  $\Gamma > 1$ :

active element,

for instance tunnel diode reflection amplifier

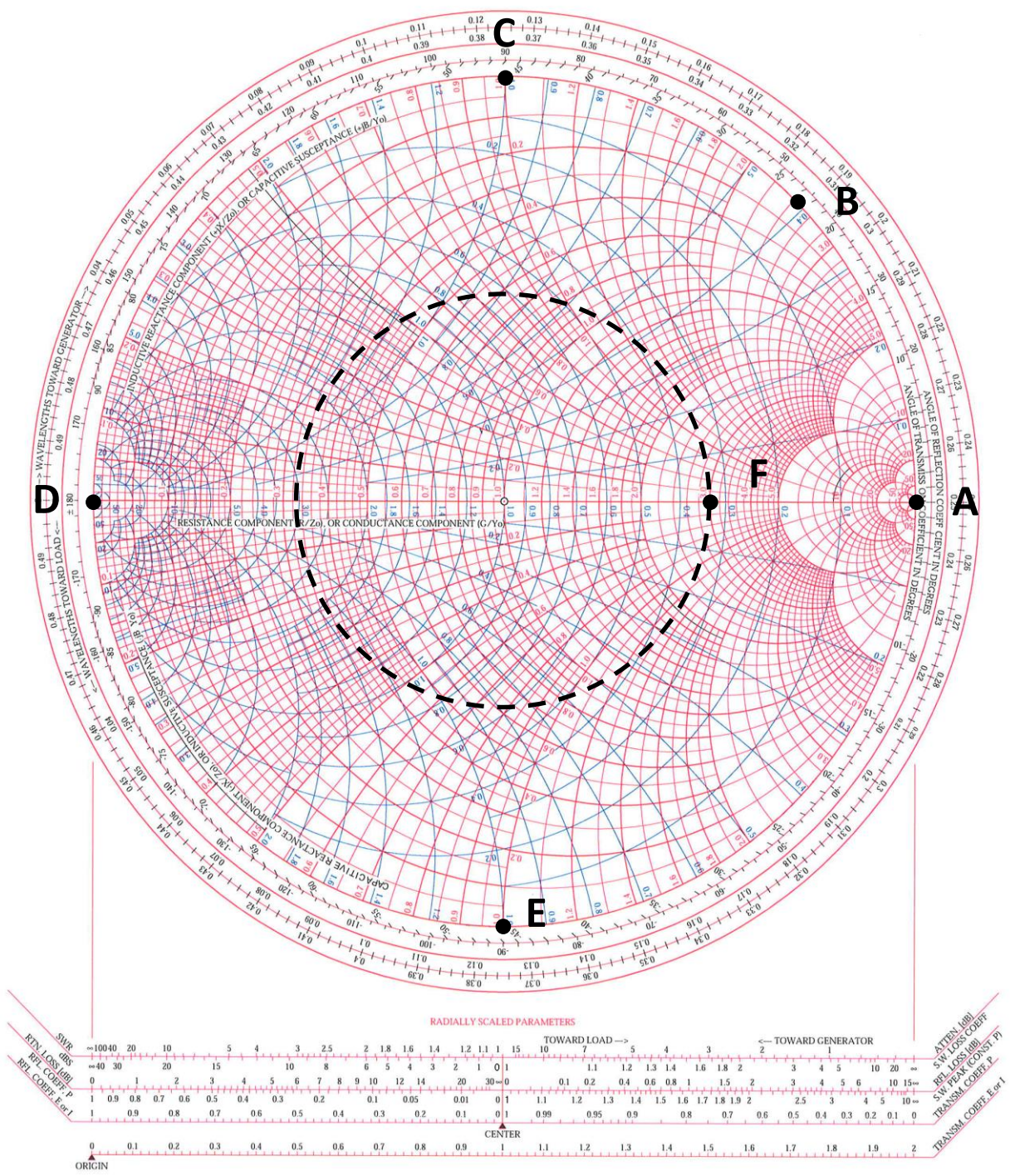


## 2.1.) Smith Chart (1)

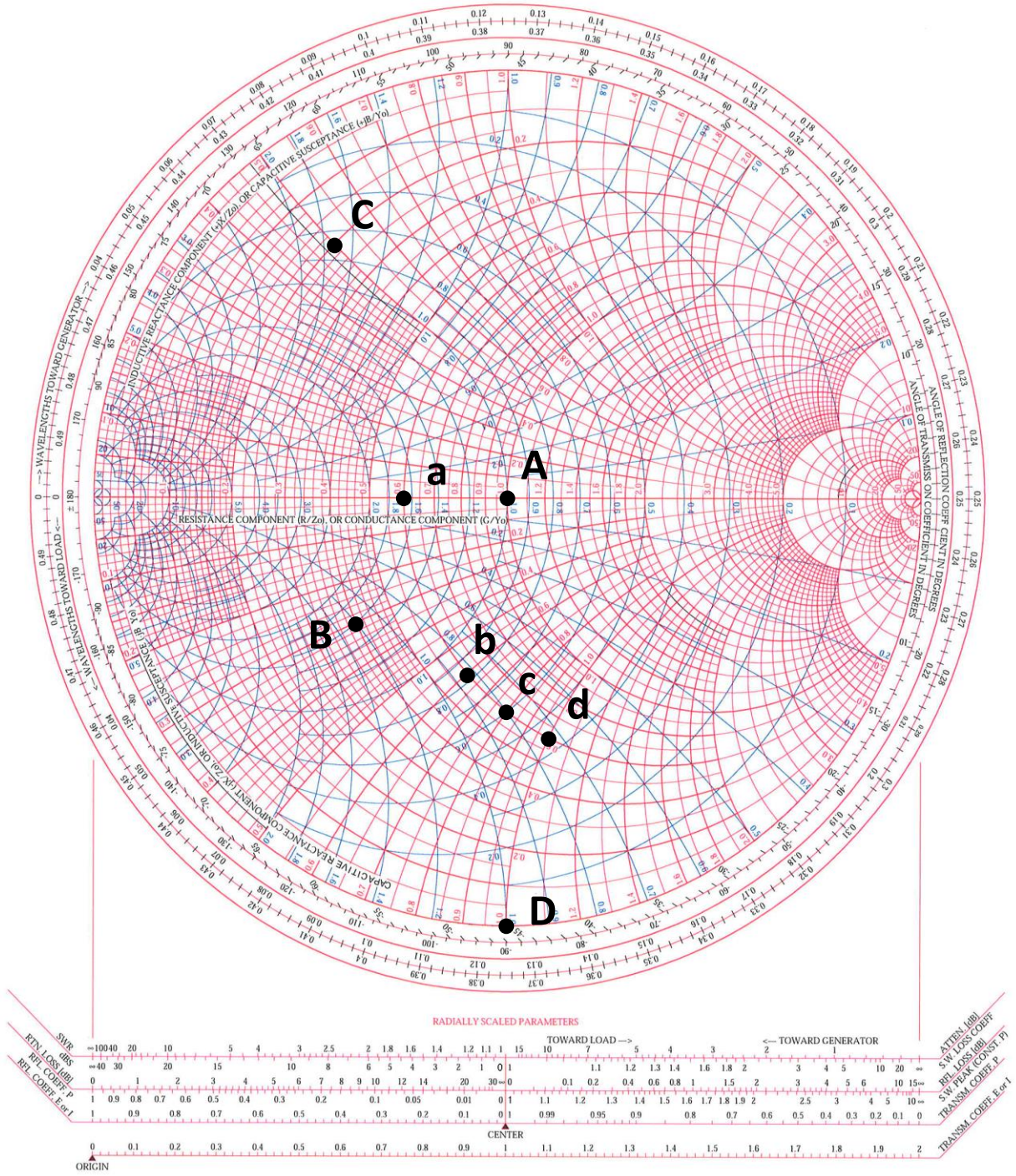
- Mark the reflection factors  $\Gamma$  of points A to F in the Smith Chart and find approximate values for the corresponding (normalized) impedances  $z$ :

| Point | Reflection factor $\Gamma$ | Normalized impedance $z$ |
|-------|----------------------------|--------------------------|
| A     | $1 \angle 0^\circ$         | $\infty$                 |
| B     | $1 \angle 45^\circ$        | $0 + j2.4$               |
| C     | $1 \angle 90^\circ$        | $0 + j1.0$               |
| D     | $1 \angle 180^\circ$       | $0$                      |
| E     | $1 \angle -90^\circ$       | $0 - j1$                 |
| F     | $0.5$                      | $3.0$                    |

# NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



# NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



## 2.2) Smith Chart (2)

- Plot the following **normalized** impedances  $z$  into the Smith Chart:

| Point | a          | b            | c            | d            |
|-------|------------|--------------|--------------|--------------|
| $z$   | $0.6 + j0$ | $0.6 - j0.6$ | $0.6 - j0.8$ | $0.6 - j1.0$ |

- Plot the following impedances  $Z$  into the Smith Chart, assuming a reference impedance of  $Z_0 = 50 \Omega$

| Point | A         | B          | C          | D         |
|-------|-----------|------------|------------|-----------|
| $Z$   | $50 + j0$ | $20 - j15$ | $10 + j25$ | $0 - j50$ |

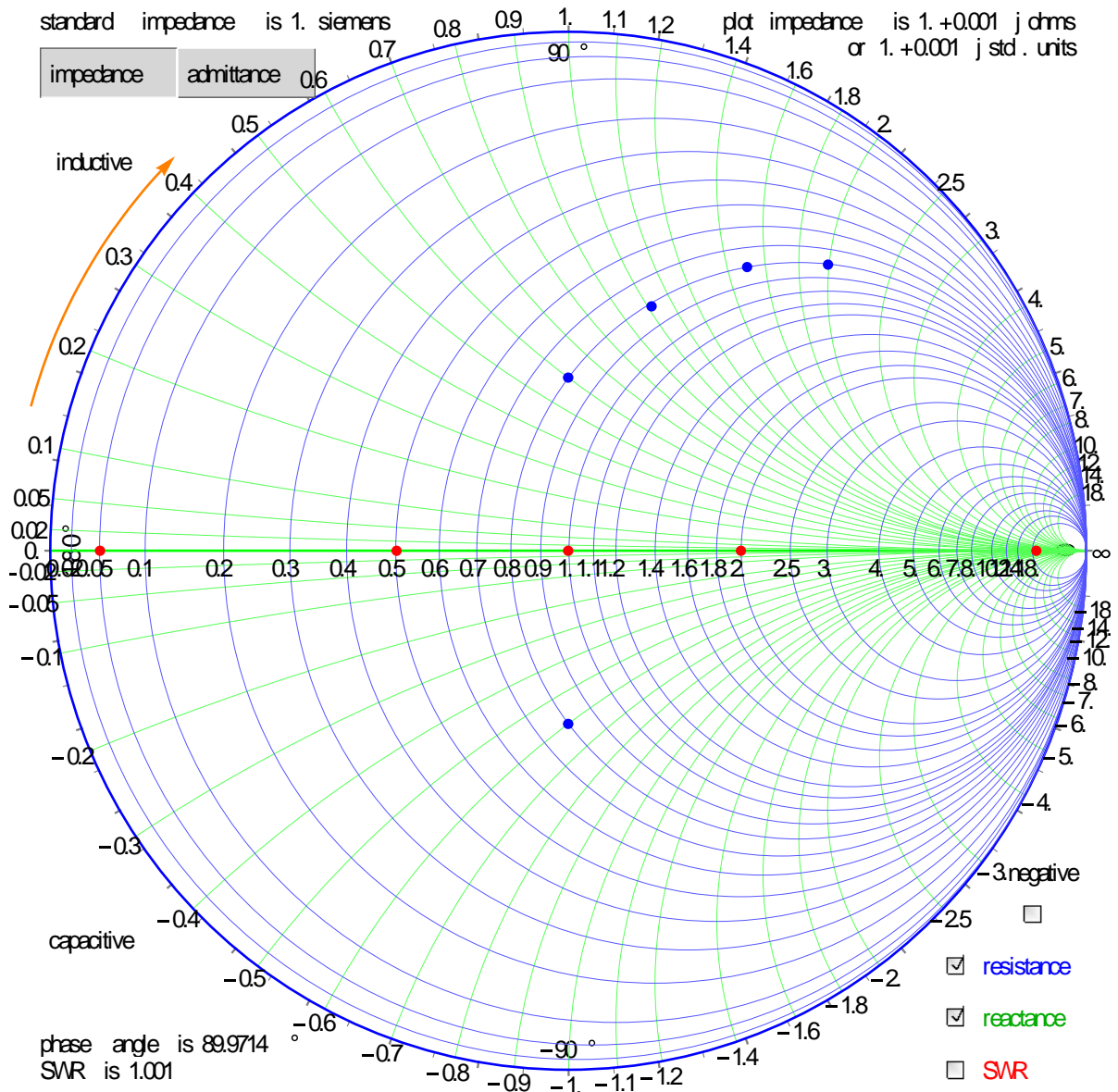
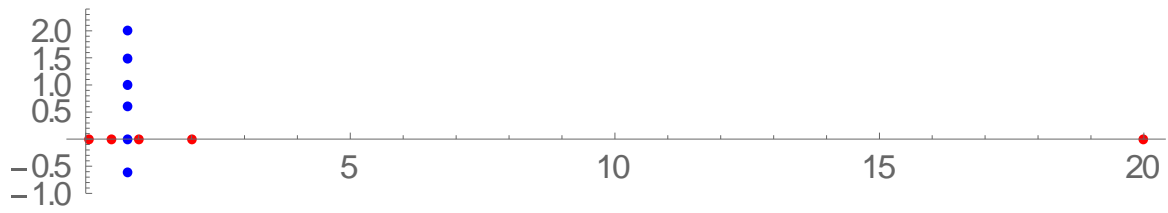
## 2.3.) Impedances in the complex plane and in the Smith chart

Plot the following impedances as points (marks)

- in a “normal” Cartesian coordinate system (complex plane)
- as reflection factors in the *Smith* chart (the reflection factor coordinates are given for convenience)

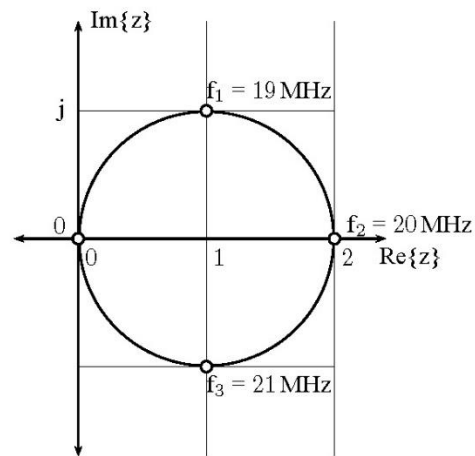
| $X_{rect}$   | $X_{polar}$               | $\Gamma_{rect}$  | $\Gamma_{polar}$          |
|--------------|---------------------------|------------------|---------------------------|
| 0.05         | $0.05 \angle 0^\circ$     | -0.904           | $0.904 \angle 180^\circ$  |
| 0.5          | $0.5 \angle 0^\circ$      | -0.333           | $0.333 \angle 180^\circ$  |
| 1            | $1.0 \angle 0^\circ$      | 0                | 0                         |
| 2            | $2.0 \angle 0^\circ$      | 0.333            | $0.333 \angle 0^\circ$    |
| 20           | $20 \angle 0^\circ$       | 0.904            | $0.904 \angle 0^\circ$    |
| 0.8          | $0.8 \angle 0^\circ$      | -0.111           | $0.111 \angle 180^\circ$  |
| $0.8 + j0.6$ | $1.00 \angle 36.9^\circ$  | $0 + j0.333$     | $0.333 \angle 90^\circ$   |
| $0.8 + j1.0$ | $1.28 \angle 51.3^\circ$  | $0.159 + j0.472$ | $0.459 \angle 72.3^\circ$ |
| $0.8 + j1.5$ | $1.70 \angle 61.9^\circ$  | $0.344 + j0.546$ | $0.645 \angle 57.8^\circ$ |
| $0.8 + j2.0$ | $2.15 \angle 68.2^\circ$  | $0.502 + j0.552$ | $0.747 \angle 47.7^\circ$ |
| $0.8 - j0.6$ | $1.00 \angle -36.9^\circ$ | $0 - j0.333$     | $0.333 \angle -90^\circ$  |

Convince yourself with a few examples that  $\Gamma(1/X) = -\Gamma(X)$



## 2.4) Smith Chart (3)

The locus of impedance of a parallel RLC resonant circuit is given in the complex z-plane (z-plane = normalized Z-plane, normalization to  $50 \Omega$ ;  $z = Z / 50 \Omega$ ).

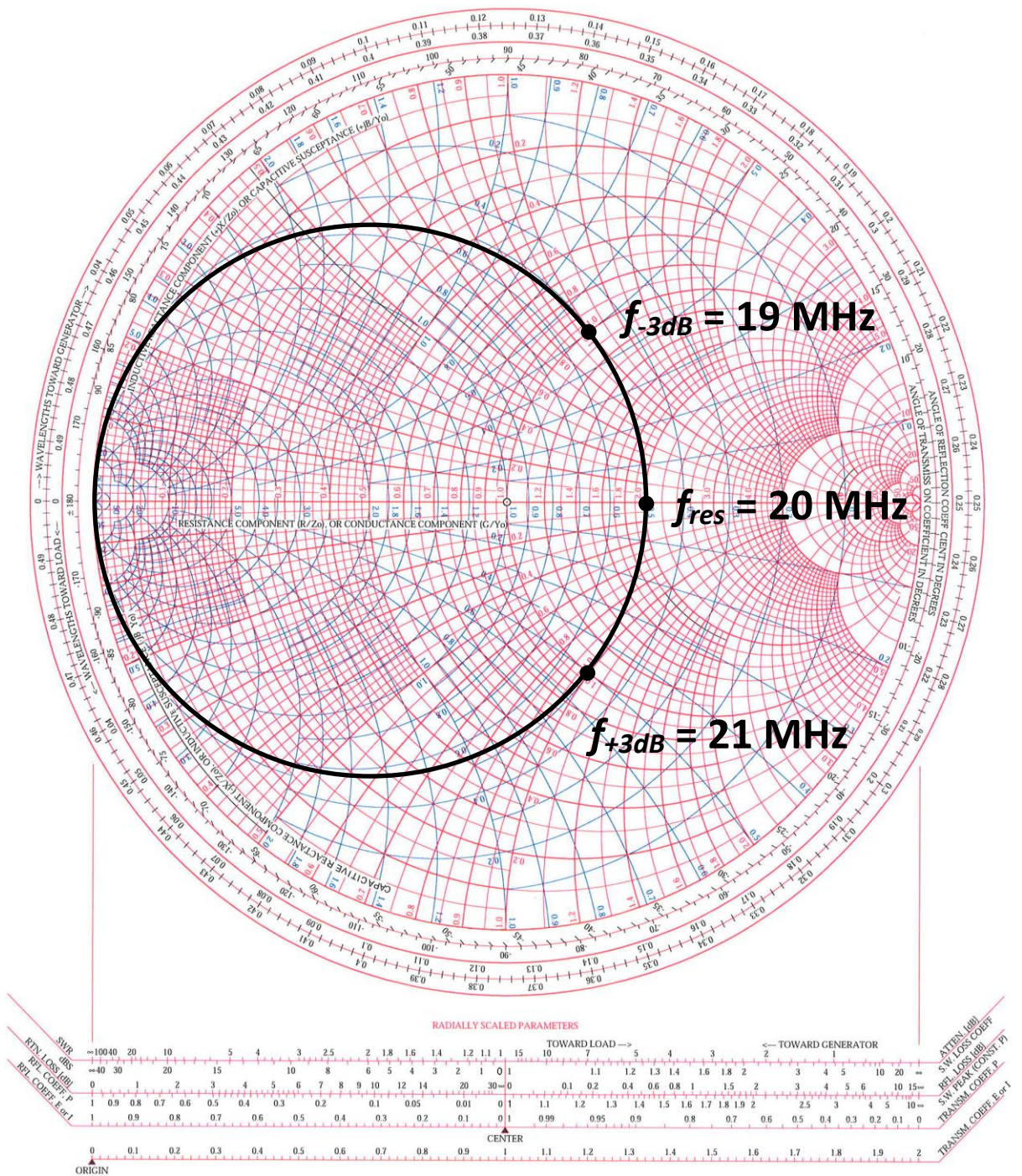


### Questions:

1. Transform this locus of impedance into the Smith Chart
2. Mark the resonance frequency, both, in the z-plane and in the Smith Chart.
3. Mark the 3-dB points (for the unloaded Q), both, in the z-plane and in the Smith Chart.



### NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



### 3.) Various questions

1. What is the difference between a *Stripline* and a *Microstripline*?

Striplines have a ground plane above and below the central conductor. Microstriplines have a groundplane only below the conductor, the other side is left open.

2. Name 3 disadvantages of *Microstriplines* compared to *Striplines*.

The open structure of a Microstripline resembles an antenna structure. RF power is radiated into the environment and electromagnetic interference is easily picked up. Crosstalk and mutual coupling become an issue if several Microstriplines are placed close to each other. The asymmetric, open structure causes a frequency-dependent characteristic impedance and a considerable dispersion.

3. A RF signal needs to be guided from a power amplifier on the surface to the cavity of a particle accelerator in the tunnel below ground.

The distance is  $l = 100$  m. The signal has parameters:  $f = 50$  MHz,  $P = 100$  kW

- a. Would you use waveguide or coaxial transmission line? Why?
- b. What would you use if the signal would have a frequency of 500 MHz? Why?

While a waveguide is in general more power efficient (less attenuation), the dimensions of this structure would be impractically large for 50 MHz. The free space wavelength at 50 MHz is  $\lambda = \frac{c}{f} = 6$  m. So, the waveguide needs to be at least 3 m wide. Coaxial transmission line does not have a lower cut-off frequency and would be the better choice.

4. Why are some accelerator cavities (for frequencies in the MHz range) loaded with ferrite?

Explain how the resonant frequency of those cavities can be tuned without moving parts.

Looking at the RLC equivalent circuit of the cavity, the ferrite increases the inductance part L and thus lowers the resonant frequency compared to air filled cavities. This allows to reduce the resonant frequency or alternatively the size of the structure.

Placing the ferrite in a static magnetic bias field, its  $\mu$  can be varied which leads to change in L and in the resonant frequency. So, by changing the bias current, the cavity can be electronically tuned.

5. After deploying a new accelerating cavity, a RF-engineer starts pounding on it with a hammer. What is he doing?

He is in fact fine tuning the resonant frequency of the cavity. This punch-tuning or dimple tuning procedure was used for example in the CERN Linear Injector for LEP. The structure was specially designed so it could be slightly deformed at certain points with a hammer.

## 4.) “Pillbox” cavity characteristics

The following data was measured on a “pillbox” cavity”:

|                 |                         |
|-----------------|-------------------------|
| Inductance:     | $L = 15.915 \text{ nH}$ |
| Capacitance:    | $C = 1.5915 \text{ pF}$ |
| 3-dB bandwidth: | $BW = 50 \text{ kHz}$   |

### Questions:

Determine

- the frequency at resonance

$$f_{res} = \frac{1}{2\pi\sqrt{LC}} = 1 \text{ GHz}$$

see script page 53

- the characteristic impedance  $R/Q$

$$\frac{R}{Q} = \sqrt{\frac{L}{C}} = 100$$

see script page 56

- the quality factor  $Q$

$$Q = \frac{f_{res}}{\Delta f} = 20000$$

see script page 55 and 59

- the time constant  $\tau$

$$\tau = \frac{Q}{\pi f_{res}} = 6.37 \mu\text{s}$$

see script page 66

- the peak induced voltage immediately after the passage of a short particle bunch with charge  $q = 15.916 \cdot 10^{-9} \text{ As}$

$$v_{step} = \left| 0 - \frac{q}{C} \right| = 10 \text{ kV}$$

see script page 68

- the remnant cavity voltage  $10 \mu\text{s}$  after the passage of the bunch

$$v_{end} = v_{step} e^{-\frac{t}{\tau}} = 2.08 \text{ kV}$$

see script page 68

## 5.) Gap-width optimization of a cavity

The following parameters of a 100 MHz cavity have been evaluated by a numerical simulation software as function of the gap-width  $g$ :

characteristic impedance  $R/Q$  and quality factor  $Q$

The cavity is connected to an amplifier delivering 1 kW of RF power.

The beam has a relative velocity of  $\beta = 0.15$ .

## Questions:

Calculate for each gap-width:

- shunt impedance  $R$

$$R = \frac{R}{Q} Q$$

- intrinsic cavity voltage  $V_{cav}$  for 100 kW power

$$V_{cav} = \sqrt{2RP}$$

see script page 56 and 71

- angle  $\theta$  of the beam passage through the gap

Velocity  $v$  of the beam through the cavity gap  $g$ :

$$v = \beta c = \frac{g}{t}$$

Time  $t$  through the gap:

$$t = \frac{g}{\beta c} = \frac{g}{\beta \lambda f}$$

The total phase angle accounts for  $\theta = 2\pi$

The phase  $\theta$  is given by multiplying both sides of the equation by  $2\pi/f$  (remember:  $ft = 1$ )

$$2\pi f t = 2\pi \frac{gf}{\beta \lambda f} \Rightarrow 2\pi = \theta = 2\pi \frac{g}{\beta \lambda} = \frac{2\pi f g}{\beta c}$$

- transit time factor  $T$

The transit time factor  $T$  is related to the phase angle  $\theta$ :

$$\frac{g\omega}{2\beta c} = \frac{2\pi f g}{2\beta c} = \frac{\theta}{2}$$

$$T = \frac{\sin \theta/2}{\theta/2}$$

see script page 101

- beam voltage  $V_{beam}$  maximally seen by the beam taking the transit time factor  $T$  into account

$$V_{beam} = V_{cav} T$$

| $G$ [mm] | $R/Q$ [ $\Omega$ ] | $Q$  | $R$ [ $\Omega$ ] | $V_{cav}$ [V] | $\theta$      | $T$  | $V_{beam}$ [V] |
|----------|--------------------|------|------------------|---------------|---------------|------|----------------|
| 100      | 100                | 5000 | 500000           | 316228        | $80.1^\circ$  | 0.92 | 291125         |
| 200      | 150                | 7000 | 1050000          | 458258        | $160.1^\circ$ | 0.7  | 323048         |
| 300      | 200                | 9000 | 1800000          | 600000        | $240.2^\circ$ | 0.41 | 247719         |

## 6.) Higher-order mode of a cavity

An RF cavity has an unwanted higher-order mode (HOM) at 600 MHz, with shunt impedance  $R = 6 \text{ M}\Omega$ , a 3-dB bandwidth of  $BW = 15 \text{ kHz}$ , and a transit time factor  $T \sim 1$ .

The beam consists of very short bunches, following each other at intervals of  $20 \mu\text{s}$ . The circulating beam current is  $0.1 \text{ A}$ . (Reminder: current  $I =$  charge per time)

## Questions:

Calculate:

- Q, R/Q, and C at the HOM frequency

$$Q_{HOM} = \frac{f_{HOM}}{\Delta f_{HOM}} = 40000$$

see script page 59

$$\frac{R}{Q} = \frac{R_{HOM}}{Q_{HOM}} = 150 \Omega$$

$$C_{HOM} = \frac{1}{2\pi f_{HOM} R/Q} = 1.77 \text{ pF}$$

see script page 56

- HOM voltage induced by a single bunch  
Assuming a single bunch in the ring:

$$q_{bunch} = i_{beam} t_{bunch}$$

$$\Delta V = \frac{q_{bunch}}{C_{HOM}} = 1.13 \text{ MV}$$

see script page 66

- Time constant  $\tau$  of the cavity

$$\tau = 2R_{HOM}C_{HOM} = 21.22 \mu\text{s}$$

see script page 66

- HOM voltage at the arrival of the next bunch

$$V_{end} e^{-\frac{t}{\tau}} = V_{step} \Rightarrow V_{next} = \Delta V e^{-\frac{t}{\tau}} = \Delta V e^{-\frac{t_{bunch}}{\tau}} = 440.7 \text{ kV}$$

see script page 68

- Total HOM voltage in steady state, after the passage of an infinite number of bunches, assuming the HOM resonance is an exact multiple of the beam revolution frequency and in sync with the beam.

$$V_{end} = \frac{q_{bunch}}{C_{HOM}(1 - e^{-t_{bunch}/\tau})} = 1.853 \mu\text{s}$$