Synchrotron Radiation

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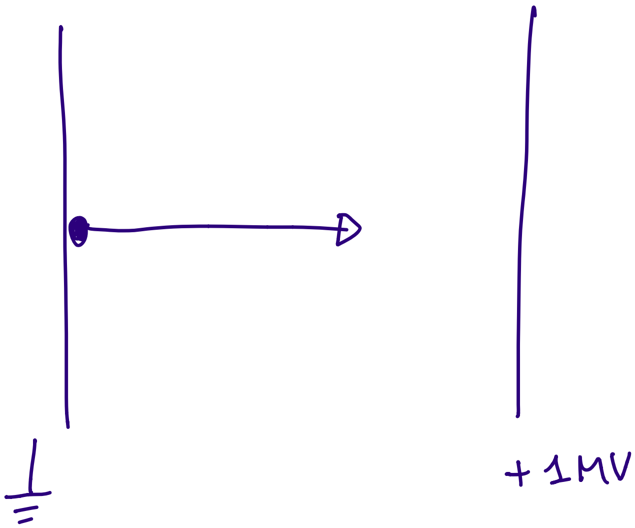
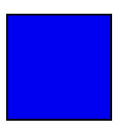
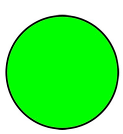
Basic Concepts

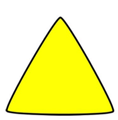
- Acceleration by DC Field

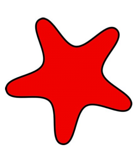


What is the total energy of the particle?

 $E = 1\ \text{MeV}$

 $E = 1\ \text{MeV} + 511\ \text{keV} = 1.511\ \text{MeV}$

 $E = \sqrt{1^2 + 0.511^2}\ \text{MeV} = 1.123\ \text{MeV}$

 This depends on the trajectory

Total energy: $E = mc^2 + E_{kin}$

Momentum: $E^2 = p^2c^2 + m^2c^4$

<u>Only</u> for ultra-relativistic particles:

$$E \approx E_{kin} \approx p/c$$

- Acceleration by AC fields

→ integrate field along the trajectory of the electron

$$E_{kin} = \int_0^t -e\,E(\vec{x}(t), t)\,dt$$

- Deflection in a Dipole Magnet

Particle flies in a circular orbit:

$$\rho = \frac{p}{eB}$$

- Particle emits radiation

→ For non-relativistic particles "cyclotron radiation"

$$P = \frac{\sigma_t B^2 v^2}{\mu_0 c} \quad \text{where } \sigma_t = \frac{8\pi}{3}\left(\frac{q^2}{4\pi\varepsilon_0 mc^2}\right)^2$$

$$= \frac{8\pi}{3} r_e^2$$

$$= 6.65 \cdot 10^{-29} m^2$$

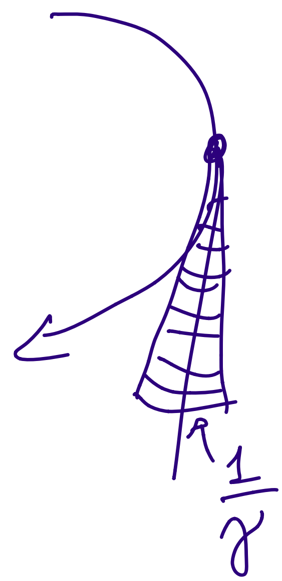
$$= 66.5 \, (fm)^2$$

Radiation is emitted at the revolution frequency

→ for relativistic particles
"synchrotron radiation"

- Radiation emission is Lorentz-boosted

- To calculate radiated power, need to consider retarded time



Radiation emitted at <u>much</u> higher frequencies
"critical frequency"

$$\omega_c = \frac{3}{2} \frac{c}{\rho} \gamma^3$$

corresponding photon energy

$$\varepsilon_c = \hbar \omega_c = \frac{3}{2} \frac{\hbar c}{\rho} \gamma^3$$

Total Radiated power

$$P = \frac{e^2 c}{6\pi \varepsilon_0} \cdot \frac{\beta^4 \gamma^4}{\rho^2}$$

Energy lost per turn

$$U_0 = \frac{e^2 \beta^4 \gamma^4}{3 \varepsilon_0 \rho}$$

time per turn

$$\frac{2\pi\rho}{c} \cdot \frac{e^2 c \beta^4 \gamma^4}{6\pi \varepsilon_0 \rho^2}$$

$$= \frac{e^2 \beta^4 \gamma^4}{3 \varepsilon_0 \rho}$$