Longitudinal beam dynamics examination

(1h30 – Free access to lecture notes and paper documents)

This exam is composed of two independent exercises totalling 120 points.

The marks will be normalized to 20.

Exercise A: Answer by "TRUE" or "FALSE" → 60 points

Rules:

- +2 for correct answer,
- -1 for incorrect answer,
- +1 bonus for correct justification

We consider a synchrotron.

1) If the beam is kept on the same orbit when energy is changed, a decrease in magnetic field leads to a momentum decrease and a revolution frequency decrease above transition.

2) If the RF cavity is turned off (i.e. $V_{RF}=0$), a decrease in orbit radius leads to a decrease in revolution frequency.

3) If the RF cavity is turned off (i.e. $V_{RF}=0$) and the momentum compaction factor is negative, an increase in magnetic field leads to a decrease in orbit radius.

4) Above transition, if the magnetic field is kept constant and the momentum compaction factor is positive, a momentum increase leads to an increase in revolution frequency.

5) If the RF voltage is strictly positive $V_{RF} > 0$, this always means that the RF bucket is accelerating.

6) There can be more bunches in a synchrotron than the harmonic number.

7) An increase in particle momentum in a synchrotron always leads to higher velocity and higher revolution frequency.

8) A decrease in particle velocity in a synchrotron always leads to a shorter orbit.

9) Transition crossing can be avoided in synchrotrons.

10) Particles performing large amplitude oscillations in the synchrotron phase space always have a smaller tune than particles performing small amplitude oscillations.

11) The synchrotron tune depends on the transit time factor inside the RF cavity.

12) In a synchrotron, the rate of change of magnetic field $\dot{B}$ is limited by the maximum RF voltage.

13) One needs at least two RF systems to perform bunch rotations.

14) The particle motion in the $(\phi, \dot{\phi})$ phase space is counterclockwise in proton synchrotrons.

15) The magnetic field in dipoles of a synchrotron always needs to be increased when increasing the momentum of the beam.

16) When transitioning from a stationary bucket to an accelerating bucket at constant RF voltage, the height of the bucket in synchrotron phase space $(\phi, \dot{\phi})$ always decreases.

17) Particle motion outside the separatrix in phase space can be stable.

18) If one wants to decelerate a proton beam, the stability condition becomes $\eta \cos \phi_s < 0$, with η the slippage factor defined in the same convention as in the course and ϕ_s the synchronous phase.

19) If one does not want to accelerate the beam in a synchrotron, there is no need of an RF cavity to keep the particle inside the bucket.

20) When crossing transition energy, the sign of the phase of the RF system needs to be changed.

Exercise B: The Electron Ion Collider project → 60 points

In January 2020, Brookhaven National Laboratory (NY, USA) was selected to host the Electron Ion Collider (EIC) project. In this future machine, an electron beam and an ion beam will be accelerated and brought into collision. The main parameters of the machines for both beams are described below:

	protons	electrons
Circumference	3834 m	
Total beam energy in collision	275 GeV	10 GeV
Number of dipoles	204	
Magnetic field	3.45 T	
Bending radius		381 m
Momentum compaction factor	1.9 10^{-3}	1.04 10^{-3}
RF frequency	563 MHz	563 MHz

1. For the proton beam, compute the relativistic factor gamma and the bending radius needed in the dipoles (6 pts).

2. For the electron beam, compute the relativistic factor gamma and the magnetic field in the dipoles (6 pts).

3. Could both electrons and protons be stored in the same vacuum chamber with the same magnet system? (4 pts)

4.
    a. Define the transition energy and describe what should be done when the beam crosses transition. (4 pts)

    b. Compute transition energy for both electron and proton rings. (6 pts)

    c. Assuming that the electron beam is injected at collision energy, but that the proton beam is injected at a total energy of 27 GeV, compare (1) the electron beam energy to transition energy, and (2) the range of proton beam energy to transition energy. (4 pts)

    d. Draw a qualitative sketch of the phase space separatrix in coordinates $(\phi, \Delta E)$, for the proton beam at injection energy, during acceleration and at top energy (12 pts).

       Each time indicate the direction of rotation of particles with a momentum offset in phase space and compute the synchronous phase and indicate it on the graph, assuming the magnetic field ramp rate $\dot{B}$ is 0.1% of the maximum rate allowed by an effective RF voltage $\hat{V}_{RF}$=24 MV. (12 pts)

    e. Draw a qualitative sketch of the phase space separatrix for the electron beam in coordinates $(\phi, \Delta E)$, the synchronous phase and the direction of rotation of particles with a momentum offset in phase space. (6 pts)

Physical constants:

- Elementary charge: $e = 1.60\ 10^{-19}$ C
- Electron mass: $m_e = 9.11\ 10^{-31}$ kg
  $m_e = 0.511$ MeV/c^2
- Proton mass: $m_p = 1.67\ 10^{-27}$ kg
  $m_p = 0.938$ GeV/c^2
- Speed of light: $c = 3.00\ 10^{8}$ m/s
- Vacuum permittivity: $\varepsilon_0 = 8.85\ 10^{-12}$ F/m