Longitudinal beam dynamics examination

(1h30 – Free access to lecture notes and paper documents)

This exam is composed of two independent exercises totalling 120 points. The marks will be normalized to 20

Exercise A: Answer by "TRUE" or "FALSE"

We consider a synchrotron.

1) If the beam is kept on the same orbit when energy is changed, a decrease in magnetic field leads to a momentum decrease and a revolution frequency decrease above transition.

   TRUE, in fact this is also TRUE below transition (see slides 50 and 54).

2) If the RF cavity is turned off (i.e. V_{RF}=0), a decrease in orbit radius leads to a decrease in revolution frequency.

   FALSE, see slide 51: with $V_{RF} = 0$, df/f= - dR/R.

3) If the RF cavity is turned off (i.e. V_{RF}=0) and the momentum compaction factor is negative, an increase in magnetic field leads to a decrease in orbit radius.

   FALSE, see slide 51: with V_{RF}=0, $\frac{dB}{B} = -\gamma_{tr}{}^2 \frac{dR}{R}$. A negative momentum compaction factor means that $\gamma_{tr}{}^2 < 0$ and the magnetic field change is of the sign as the radius change.

4) Above transition, if the magnetic field is kept constant and the momentum compaction factor is positive, a momentum increase leads to an increase in revolution frequency.

   FALSE, see slide 52. Above transition and with $\gamma_{tr}{}^2 > 0$, a momentum increase leads to a decrease of revolution frequency as $\left(1 - \frac{\gamma_{tr}{}^2}{\gamma^2}\right) > 0$

5) If the RF voltage is strictly positive $V_{RF} > 0$, this always means that the RF bucket is accelerating.

   FALSE, even if the RF voltage is strictly positive, $\dot{B} = 0$ leads the synchronous phase to be 0, as the energy gain of the synchronous particle is $e\hat{V}_{RF}\sin\phi_s = 0$. Therefore the bucket can be stationary (i.e. non accelerating), even if the RF voltage is strictly positive.

6) There can be more bunches in a synchrotron than the harmonic number.

FALSE, the maximum number of bunches is the harmonic number, which represents the number of stable points.

7) An increase in particle momentum in a synchrotron always leads to higher velocity and higher revolution frequency.

FALSE, an increase in momentum in a synchrotron always leads to an increase of velocity (even infinitesimal when the particle is ultrarelativistic), but the revolution frequency would decrease above transition.

8) A decrease in particle velocity in a synchrotron always leads to a shorter orbit.

FALSE, as the momentum compaction factor can be negative (slide 42).

9) Transition crossing can be avoided in synchrotrons.

TRUE, for given injection and extraction energies of a synchrotron, one can avoid transition energy at design by optimising the optics such that the sign of the slip factor remains the same or even designing the optics such that the momentum compaction factor is negative.

10) Particles performing large amplitude oscillations in the synchrotron phase space always have a smaller tune than particles performing small amplitude oscillations.

TRUE, see slide 91.

11) The synchrotron tune depends on the transit time factor inside the RF cavity.

TRUE, as the synchrotron tune depends on the effective RF voltage (slide 79) and the effective RF voltage depends on the transit time factor (slide 36).

12) In a synchrotron, the rate of change of magnetic field $\dot{B}$ is limited by the maximum RF voltage.

TRUE, see slide 64.

13) One needs at least two RF systems to perform bunch rotations.

FALSE, one needs an abrupt increase of RF voltage. Double RF systems are needed for “bunch splitting” or “bunch flattening”.

14) The particle motion in the (phi, phi_dot) phase space is counterclockwise in proton synchrotrons.

FALSE, it is clockwise above transition.

15) The magnetic field in dipoles of a synchrotron always needs to be increased when increasing the momentum of the beam.

TRUE, see slide 63 $(\Delta p)_{turn} = e\rho\dot{B}T_{rev}$ or simply from $B\rho = \frac{p}{e}$.

16) When transitioning from a stationary bucket to an accelerating bucket at constant RF voltage, the height of the bucket in synchrotron phase space (phi, phi_dot) always decreases.

TRUE, see G(phi_s) on slide 87 that is less than 1 as soon as phi_s is not zero.

17) Particle motion outside the separatrix in phase space can be stable.

FALSE, by definition of the separatrix.

18) If one wants to decelerate a proton beam, the stability condition becomes eta*cos(phi_s)<0.

FALSE, see slide 77. For stability of synchrotron oscillations (accelerating or decelerating), the condition remains that $\eta \cos\phi_s > 0$

19) If one does not want to accelerate the beam in a synchrotron, there is no need of an RF cavity to keep the particle inside the bucket.

FALSE, without an RF voltage, there is no bucket.

20) When crossing transition energy, the sign of the phase of the RF system needs to be changed.

FALSE, the phase of the RF system needs to be changed from ϕ_s to $\pi - \phi_s$.

Exercise B: The Electron Ion Collider project → 60 pts

In January 2020, Brookhaven National Laboratory (NY, USA) was selected to host the Electron Ion Collider (EIC) project. In this future machine, an electron beam and an ion beam will be accelerated and brought into collision. The main parameters of the machines for both beams are described below:

	protons	electrons
Circumference	3834 m	
Total beam energy in collision	275 GeV	10 GeV
Number of dipoles	204	
Magnetic field	3.45 T	
Bending radius		381 m
Momentum compaction factor	1.9 10^{-3}	1.04 10^{-3}
RF frequency	563 MHz	563 MHz

1. For the proton beam, compute the relativistic factor gamma and the bending radius needed in the dipoles (6 pts).

$\gamma = \frac{E_{tot}}{E_0} = \frac{E_{tot}}{m_p c^2} = \frac{275\ 10^9 * e}{m_p c^2} = 293$ (or using also the rest energy in GeV $\frac{275\ GeV}{0.938\ GeV}$).

$\rho = \frac{p}{eB} = \frac{\beta E_{tot}}{ceB} = \sqrt{1 - \frac{1}{\gamma^2}} \frac{E_{tot}}{ceB} = \sqrt{1 - \frac{1}{293^2}} \frac{275\ 10^9}{3\ 10^8\ 3.45} = 265$ m

2. For the electron beam, compute the relativistic factor gamma and the magnetic field in the dipoles (6 pts).

$\gamma = \frac{E_{tot}}{E_0} = \frac{E_{tot}}{m_e c^2} = \frac{10\ 10^9 * e}{m_e c^2} = 19600$ (or using also the rest energy in MeV $\frac{275000\ MeV}{0.511\ MeV}$).

$B = \frac{p}{e\rho} = \frac{\beta E_{tot}}{ce\rho} = \sqrt{1 - \frac{1}{\gamma^2}} \frac{E_{tot}}{ce\rho} = \sqrt{1 - \frac{1}{19600^2}} \frac{10\ 10^9}{3\ 10^8\ 381} = 0.0874\ T$

3. Could both electrons and protons be stored in the same vacuum chamber with the same magnet system? (4 pts)

Since the magnetic field and bending radius is very different for both beams, one needs separate magnets and vacuum chambers.

4.
    a. Define the transition energy and describe what should be done when the beam crosses transition. (4 pts)

    Transition energy is the energy at which the sign of the slippage factor changes: before transition, an increase in momentum yields to an increase of revolution frequency. Above transition, an increase of momentum yields to a decrease of revolution frequency. When crossing transition, the phase of the RF voltage should be changed from ϕ_s to $\pi - \phi_s$.

    b. Compute transition energy for both electron and proton rings. (6 pts)

$$E_{tr} = \gamma_{tr} E_0 = \sqrt{\frac{1}{\alpha_p}} E_0$$

$$E_{tr} = \sqrt{\frac{1}{1.9\ 10^{-3}}} 0.938\ GeV = 21.5\ GeV \text{ for protons}$$

$$E_{tr} = \sqrt{\frac{1}{1.04\ 10^{-3}}} 0.511\ MeV = 15.8\ MeV \text{ for protons}$$

c. Assuming that the electron beam is injected at collision energy, but that the proton beam is injected at a total energy of 27 GeV, compare (1) the electron beam energy to transition energy, and (2) the range of proton beam energy to transition energy. (6 pts)

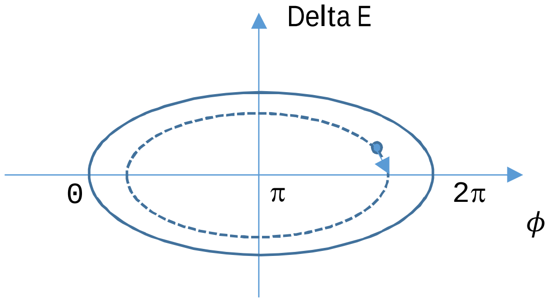
Electron beam: $E_{coll} > E_{tr}$

Proton beam: $E_{coll} > E_{inj} > E_{tr}$

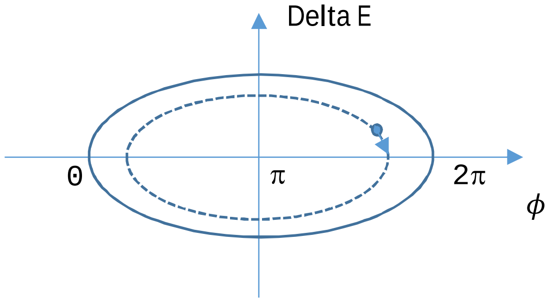
d. Draw a qualitative sketch of the phase space separatrix in coordinates $(\phi, \Delta E)$, for the proton beam at injection energy, during acceleration and at top energy (12 pts).

Each time indicate the direction of rotation of particles with a momentum offset in phase space and compute the synchronous phase and indicate it on the graph, assuming the magnetic field ramp rate dB/dt is 0.1% of the maximum rate allowed by an effective RF voltage $\hat{V}_{RF}$=24 MV. (10 pts)

Injection energy (above transition) → synchronous phase = π



Top energy (above transition) → synchronous phase = π



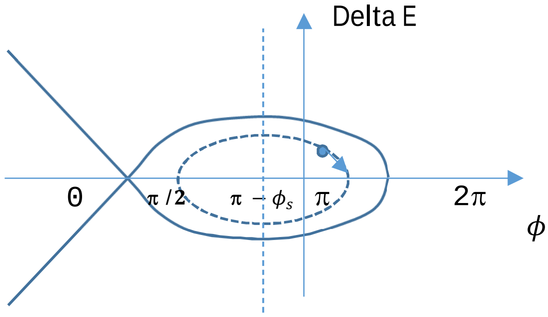
Ramp (above transition) → synchronous phase = π - $\boldsymbol{\phi_s}$

With $\phi_s = \arcsin\left(2\pi\rho R \frac{\dot{B}}{\hat{V}_{RF}}\right)$

The maximum magnetic field rate allowed by an effective RF voltage of 24 MV is given by $\sin(\phi_s) = 1$, i.e. $2\pi\rho R \frac{\dot{B}_{max}}{\hat{V}_{RF}} = 1$, i.e. $\dot{B}_{max} = \frac{\hat{V}_{RF}}{2\pi\rho R} = \frac{24\ 10^6}{265 * 3834} = 23.6\ T/s$

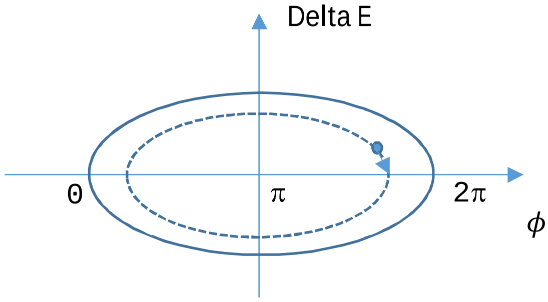
Since the ramp rate is 0.1% of this maximum ramp rate, the actual ramp rate is $\dot{B} = 0.02\ T/s$.

The synchronous phase is $\phi_s = \arcsin\left(2\pi\rho R \frac{\dot{B}}{\hat{V}_{RF}}\right) = \arcsin\left(\frac{\dot{B}}{\dot{B}_{max}}\right) =$ arcsin(0.001) = 1 mrad (modulo π) = 0.0573 degrees (modulo 180 degrees). However, since we are above transition, we can conclude that the synchronous phase is in the interval]π/2, π[and therefore the synchronous phase is $\phi_s = 179.94$ degrees.



e. Draw a qualitative sketch of the phase space separatrix for the electron beam in coordinates $(\phi, \Delta E)$, the synchronous phase and the direction of rotation of particles with a momentum offset in phase space. (6 pts)

Collision energy (above transition) → synchronous phase = π



Physical constants:

- Elementary charge: $e = 1.60\ 10^{-19}$ C
- Electron mass: $m_e = 9.11\ 10^{-31}$ kg
- Proton mass: $m_p = 1.67\ 10^{-27}$ kg
- Speed of light: $c = 3.00\ 10^{8}$ m/s
- Vacuum permittivity: $\varepsilon_0 = 8.85\ 10^{-12}$ F/m