Cyclotrons Bz =f(R , θ)

Isochronism condition
(longitudinal)

Bz =B (R)

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m}$$

Bz should increase with Radius (B ~ r $^{-n}$ with n<0)

Unstable Vertical oscillations (*Br* defocus in z plane)

Additive Vertical focusing is needed

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \dot{r} & \dot{z} & r\dot{\theta} \\ B_r & B_z & B_\theta \end{vmatrix}$$

Bθ component needed (Fz =-q vr Bθ) : « AVF » Cyclo

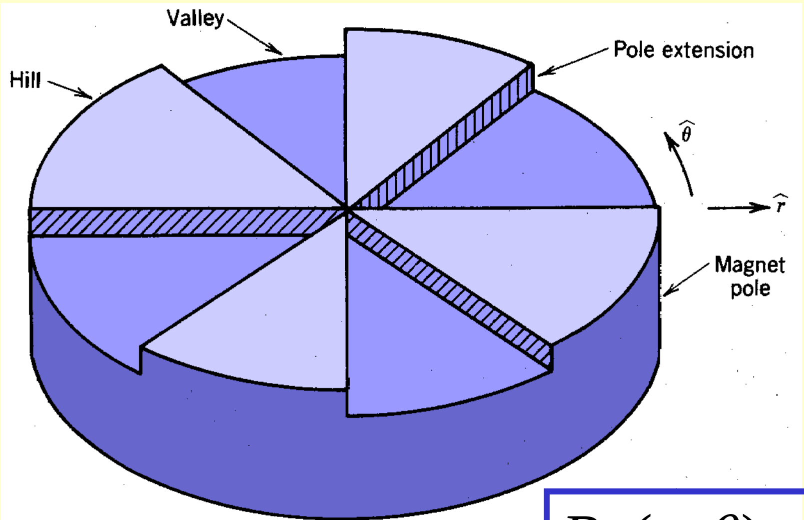
Azimuthally varying Field (AVF) $B_z = f(R, \theta)$

an additive focusing vertical force $v_r \cdot B_\theta$

B_θ created by:

- Succession of high field & low field regions : $Bz = f(R, \theta)$
  - Valley : large gap, weak field
  - Hill : small gap, strong field

FLUTTER function (definition)



N=4 sectors

$$F_l = \frac{< (B - < B >)^2 >}{\langle B \rangle^2}$$

1 turn

$$F_l = \frac{\sigma_B^{\ 2}}{\langle B \rangle^2}$$

$$B_z(r,\theta) = B_0.[1 + f\cos(N\theta)]$$

$\Rightarrow$ $Fl = f^2/2$

Azimuthally varying Field (AVF) cyclo

Vertical focusing $\langle F_z \rangle \propto v_r \cdot B_\theta$

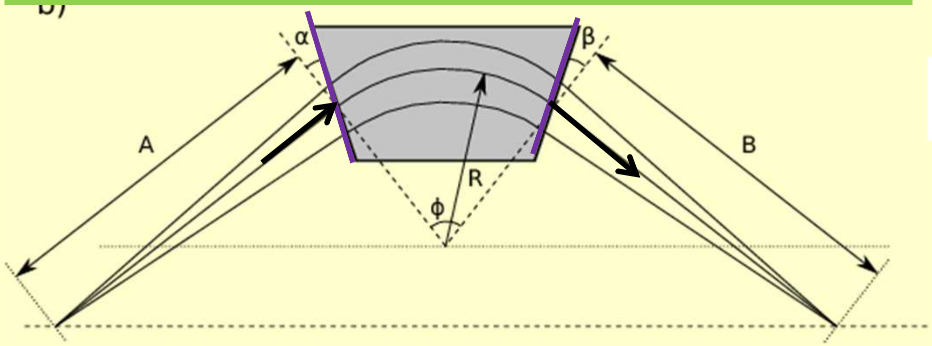
$V_r = dr/dt\, \overrightarrow{er}$ **created by :**

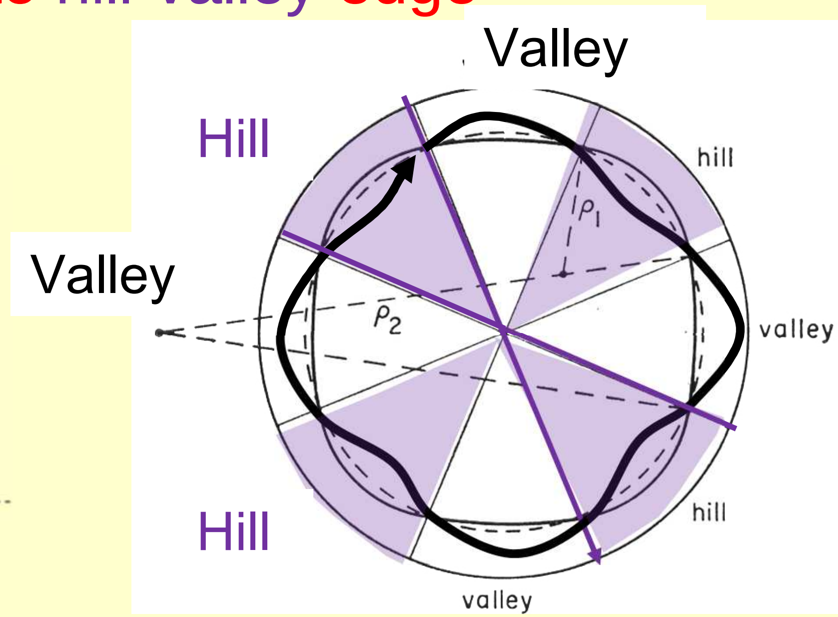
- Valley: weak field, large trajectory curvature
- **Hill : strong field**, small trajectory curvature

Trajectory is not a circle

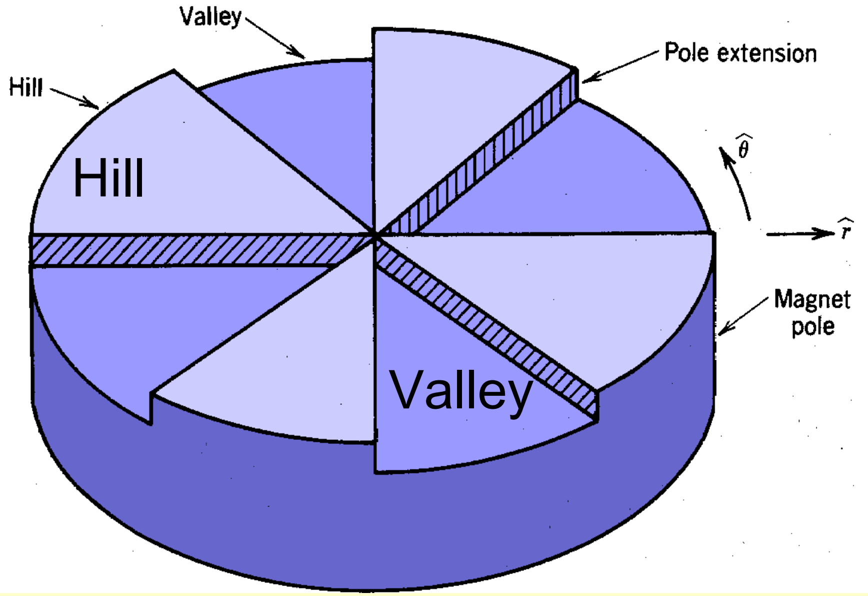
- Orbit not perpendicular to hill-valley edge

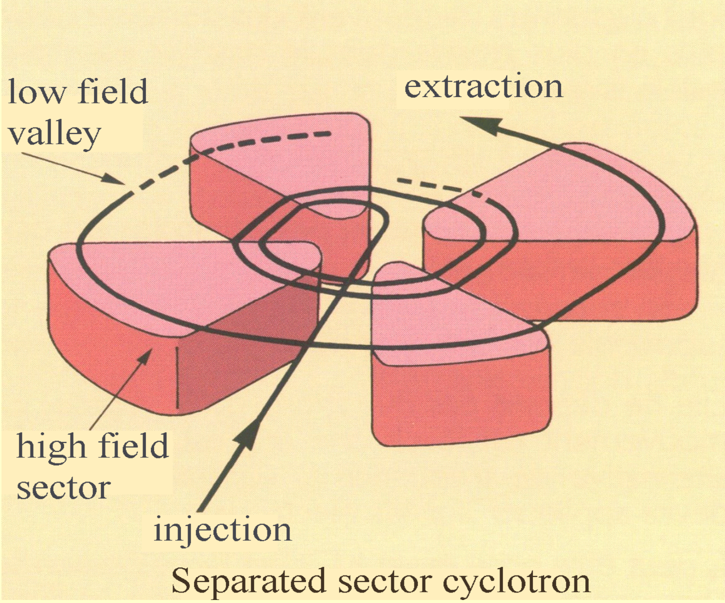
Like edge focusing in Dipole Magnet





Vertical focusing with sectors





Compact cyclo : pole oscillation in θ

$B_z = \langle B_0 \rangle [\,1 + f.\cos(N\theta)\,]$

$f < 1$ $f = 0.5.(B_{hill} - B_{valley}) / \langle B_0 \rangle$

FLUTTER function (definition)

$$F_l = \frac{\langle (B - \langle B \rangle)^2 \rangle}{\langle B \rangle^2}$$ <1turn>

Separated magnets
generate field oscillation in θ

$B_z = \langle B_0 \rangle [\,1 + \cos(N\theta)\,]$

Separated sector cyclotron

The FLUTTER is larger

Larger vertical focusing

Tutorial 1 :

Give the **Lorentz force** in a **cyclotron**
and explain the focusing and defocusing effect in Vertical plane of the **B**r (radial) and **B**θ (azimuthal) components

$$m\gamma \frac{d^2\mathbf{r}}{dt^2} = m\gamma \frac{d^2(r\mathbf{e_r} + z\mathbf{e_z})}{dt^2} = q(\mathbf{v}\times B) = ?$$

$$B_z = B_{0z} R^{-n}$$

$$m\gamma \frac{d^2 z}{dt^2} = F_z = ?$$

$$B_r = ?$$

« Curl *B* =0 »

Tutorial 1 : Give the Lorentz force in a cyclotron and explain the focusing and defocusing effect of the Br (radial) and Bθ (azimuthal) components

$$m\gamma \frac{d^2z}{dt^2} = F_z = q(\mathrm{v}\times B)_z = -q(\mathrm{v}_r B_\theta - \mathrm{v}_\theta B_r)$$

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \mathrm{v}_\mathrm{r} & \mathrm{v}_\mathrm{z} & \mathrm{v}_\theta \\ B_r & B_z & B_\theta \end{vmatrix}$$

$$\frac{d^2z}{dt^2} + \frac{q}{m\gamma}\left(\mathrm{v}_r B_\theta + R\omega_{rev} n.\frac{B_z}{R} z\right) = 0$$

$$\mathrm{v}_\mathrm{r} = \frac{dr}{dt} \qquad \mathrm{v}_\theta = R\frac{d\theta}{dt} = R\omega_{rev}$$

Isochronous cyclotron n <0

Bz (R) increases

Bz= f(R,θ) and « Curl *B* =0 »

Br= h(R,θ)= -z.n Bz /R. er

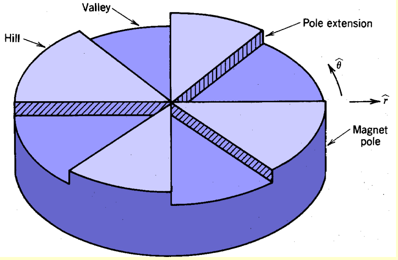
n<0 Br : Defocusing in vertical plan

B_θ =g(R,θ). eθ

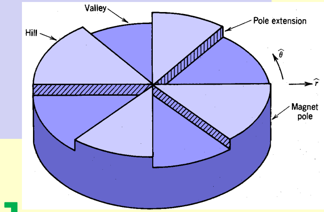
Induced by Sectors (AVF)

Focusing in vertical plan

it Compensates n<0



Tutorial 2 : Flutter **F** in AVF cyclo



The field of a cyclotron is $B_z(r,\theta) = B_0(r)\,[1 + f.\cos(4\,\theta)]$

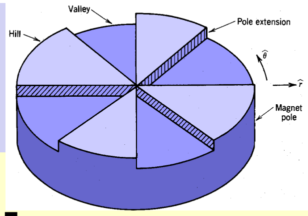
A: How many hills and valleys have such a cyclotron

B: Compute B_θ With *Curl B=0*

$$\nabla \times \mathbf{B} = \left[\mathbf{e}_r \frac{\partial}{r\partial \mathrm{r}} + \mathbf{e}_z \frac{\partial}{\partial \mathrm{z}} + \mathbf{e}_\theta \frac{\partial}{r\partial \theta} \right] \times \left[B_r \mathbf{e}_r + B_z \mathbf{e}_z + B_\theta \mathbf{e}_\theta \right]$$

C: Compute the flutter function « *Fl* »

Tutorial 2 : Flutter **F** in AVF cyclo



if the field of a cyclotron is $B_z(r,\theta) = B_0(r)\,[1 + f.\cos(4\theta)]$

4 valleys, 4 hills : 4 sectors

$B_{hill} = B_0(r)\,[1+ f]$ $\qquad B_{valley} = B_0(r)\,[1- f]$

$\langle B \rangle = B_0$

$$\nabla \times \mathbf{B} = \left[\mathbf{e}_r \frac{\partial}{\partial r} + \mathbf{e}_z \frac{\partial}{\partial z} + \mathbf{e}_\theta \frac{\partial}{r\partial \theta} \right] \times \left[B_r \mathbf{e}_r + B_z \mathbf{e}_z + B_\theta \mathbf{e}_\theta \right]$$

$$\boxed{\frac{\partial B_\theta}{\partial z} - \frac{\partial B_z}{r \partial \theta} = 0}$$

$B_\theta(z, r, \theta) = z.\, B_0(r)\,[-4 f. \sin(4\theta)]/r + K$

C: $$\boxed{\langle (B - \langle B \rangle)^2 \rangle = \frac{1}{2\pi} \int_0^{2\pi} B_0^{\,2}\,[f \cos(4\theta)]^2\, d\theta}$$

$$\boxed{F_l = \frac{\langle (B - \langle B \rangle)^2 \rangle}{\langle B \rangle^2}}$$

Flutter function $\qquad F_l = B_0^2\,(f.)^2 / 2B_0^2 = f^2/2$

Separated Sectors(ring) Cyclotron

Focusing condition limit: (n<0)

$$\nu_z^2 = n + \frac{N^2}{N^2 - 1} F_l + \ldots > 0$$

Increase the flutter F_l, using separated sectors where $B_{valley} = 0$

$$F_l = \frac{< (B - < B >)^2 >}{\langle B \rangle^2}$$

PSI= 590 MeV proton

γ=1.63

Separated sectors cyclotron needed at "High energies" ($n(R) = 1-\gamma^2 << 0$)

Vertical focusing and isochronism

2 conditions to fulfill

- Increase the vertical focusing force strength: $\nu_z^2 = n + \frac{N^2}{N^2-1} F_l + \ldots > 0$
- Keep the isochronism condition true: n<0

$$n = -\frac{R}{B_{0z}} \frac{\partial B_z}{\partial R} = 1 - \gamma^2 < 0$$

So we should have:

$$\frac{N^2}{N^2-1} F_l > \gamma^2 - 1$$

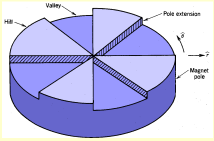
For High Energy cyclotron : 2 solutions for vertical stability

1) Larger Flutter (separated sectors) Fl

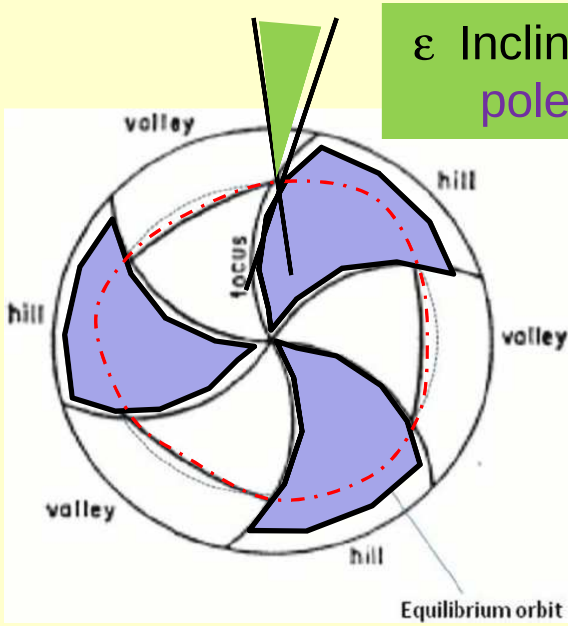
2) Other idea ??? Yes (spiralled sectors)

Increase N **sectors doesn't help Z stability but it reduces resonances**

Better vertical focusing : Spiralled sectors



AVF with straight sectors



ε Inclination angle
pole edge AND Trajectory

AVF cyclo with spiralled sectors

Larger vertical focusing

$B_z = \langle B_0 \rangle (1 + f(r).g(r, \theta))$

Spiral eq. $r = A.(\theta + 2\pi j / 2N)$

$\tan \varepsilon = 2r/A$

Additive vertical focusing : + FLUTTER $.(1 + 2 \tan^2 \varepsilon)$

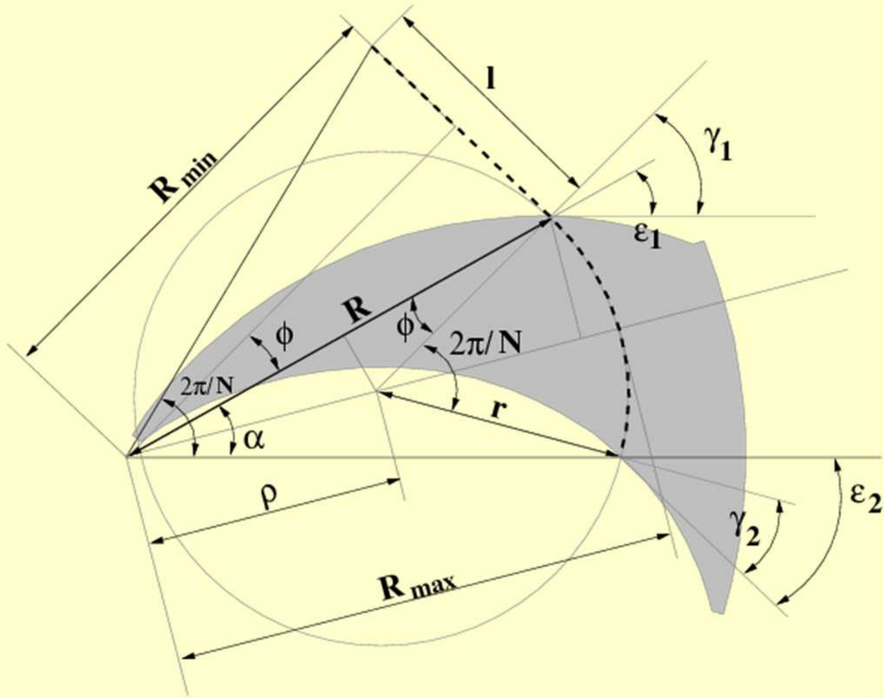
$$\nu_z^2 = n + \frac{N^2}{N^2 - 1} F_l \left(1 + 2 \tan^2 \varepsilon\right)$$

Spiralled sectors

By tilting the edges (ε angle) :

- The valley-hill transition became more focusing
- The hill-valley transition became less focusing

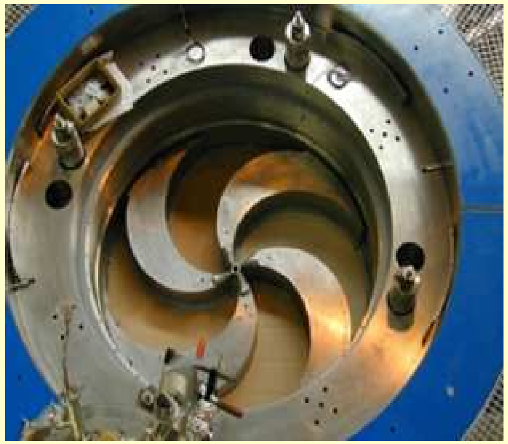
But by the strong focusing principle (larger betatron amplitude in focusing, smaller in defocusing), the net effect is focusing (cf F+D quadrupole).



$$\nu_z^2 = n + \frac{N^2}{N^2 - 1} F_l \left(1 + 2\tan^2 \varepsilon\right)$$

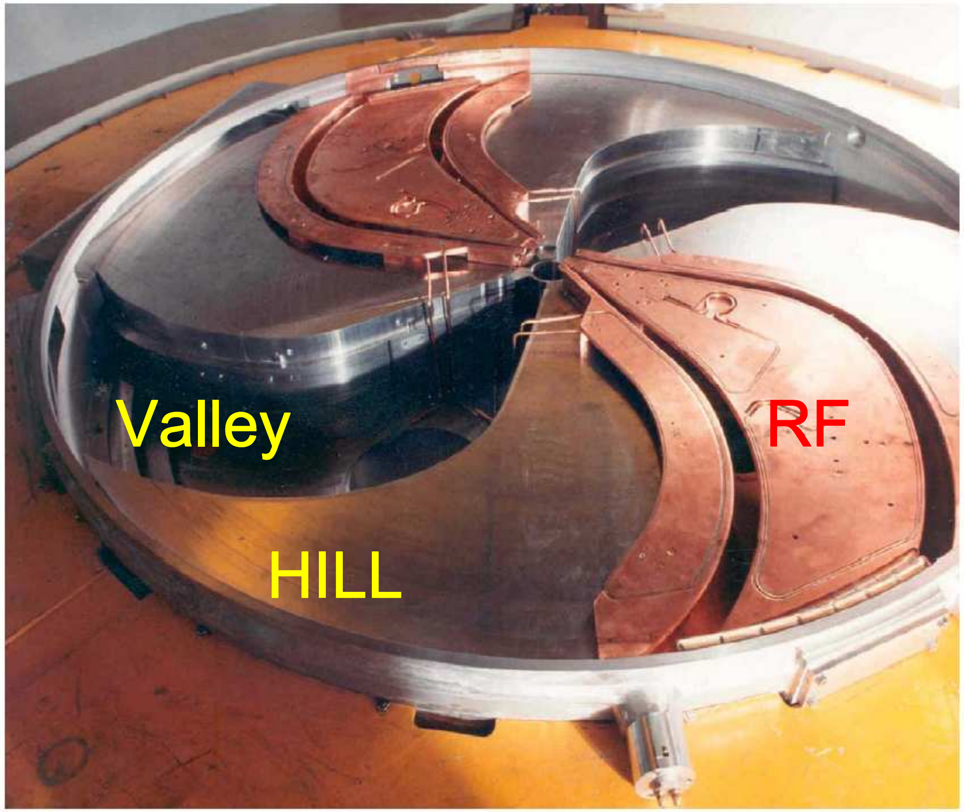
n<0

$F_l > 0$



Exemple : 235 MeV compact proton cyclo
4 spiralled sectors

C235 poles and valleys



-2 RF cavities (Dees)
Inserted in the valleys

4 Spiralled sectors:

Higher energy=
Higher axial focusing
required

Exemple : 235 MeV compact proton cyclo

C235 lower part

Without RF

4 Spiralled sectors:

Valley gap =60cm
Hill gap =10cm-1cm

BHill~ 2-3 Teslas
Bvalley~1 Tesla

<B> ~ 1.7 T in center
<B> ~ 2.1T extraction

Beam dynamics in the ISOCHRONOUS cyclotrons

B=Constant ≠Isochronism condition

A STRONG LIMITIATION in energy γ=1

to get the ions synchrone With RF

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m}$$

$B_z = B_0 .\ g(R)$

B_z increase with R (field index n < 0)

Unstable Vertical oscillations

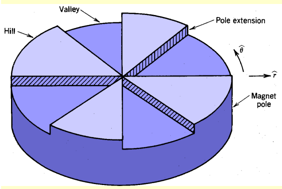
strong limitation in transmission

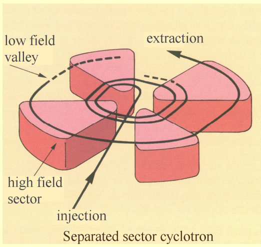
Additive Vertical focusing is needed : -N sectors (Hills//valleys)

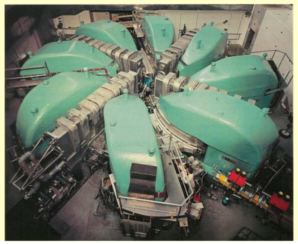
-separated straight sectors

-spiralled sectors

-separated spiralled sectors







$B_z = B_0 .\ g(R,\theta)$

4 techniques

One other possibility

SYNCHRO CYCLOTRON (NOT ISOCHRONOUS)

Acceleration condition with Bz uniform or decreasing (n ≥ 0)

ωrev =not constant

Not isochronous !!

But no vertical instabilities

$\nu_z^2 = n \geq 0$ stable oscillation

Revolution frequency evolves Frev(t)= Frev(Radius)

beam has to be synchrone With RF :

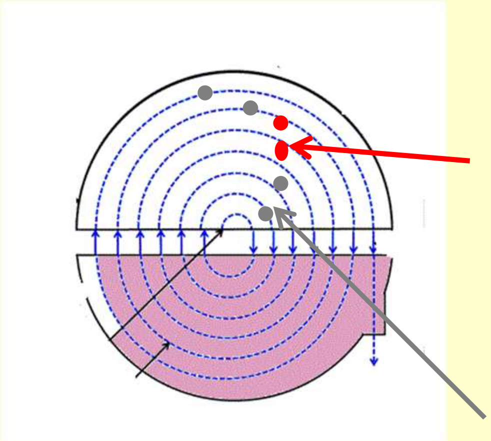
ωrev /h = ωRF synchronous

But ωrev /h = ωRF = f(time) =f(Radius) NOT constant

Revolution frequency is evolving during acceleration FRF(t)
(like a synchrotron)

Pulsed Machine ωRF (t) : SYNCHRO CYCLOTRON

Synchro-cyclotrons : RF cycled, but stable in z

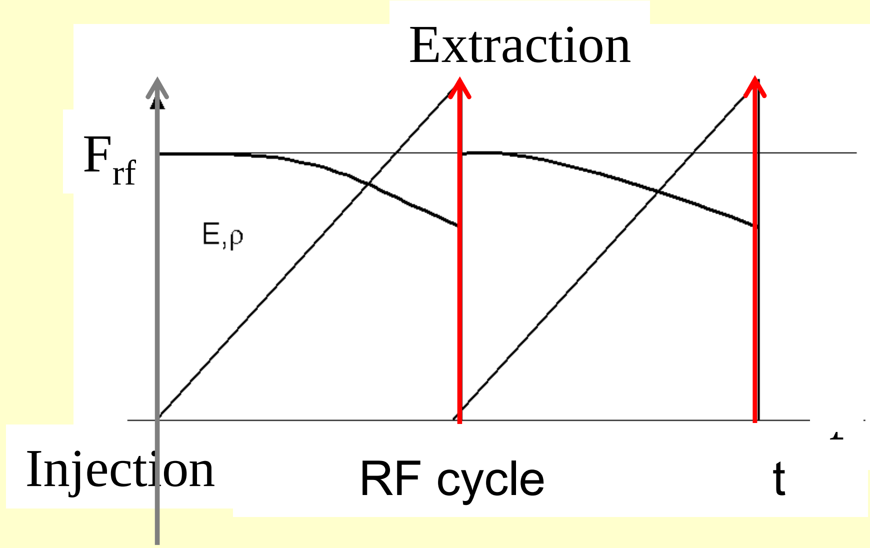


Pulsed beam

Accelerated bunches

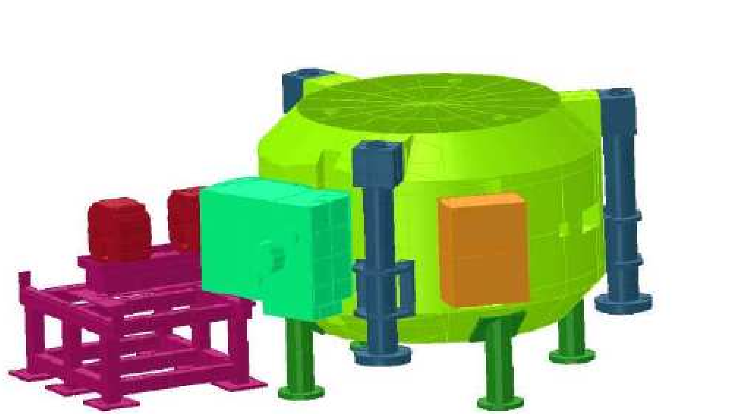
$F_{RF} = h \; F_{rev}$

Not accelerated



Less intensity (pulsed) avalaible (not cw)

Exemple : medical aplication
Superconducting synchrocyclo.



ProteusOne® (IBA) : 250 MeV proton

Bz = 5.7 - 5.0 Tesla (very compact)

Rextraction =0.6 m / harmonics=1

F_{RF}= 93 MHz -63 MHz (Rextraction)

Beam pulse : Every 1 ms

Few other slides
for questions

Summary N° 2 : without equations

Isochronous Cyclotron : *Bz = f(R,θ)* *complex magnet*

Frf = constant

Isochronism : ωRF = H ωrev = constant

(particle are synchronous with RF : FRF = const.)

with <Bz > = f(Radius) ~ γ (Radius) . B_0

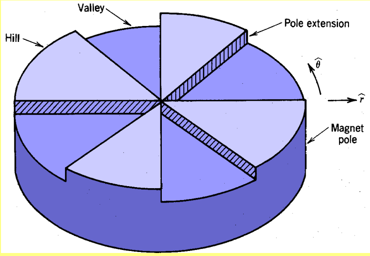
Defocusing Vertical force : Bz # γ (r) . B_0 => Fz = v_θ Br *ez*

unstable oscillation

Compensation with focusing force with θ field oscillations

Vertical stability can be obtained with azimutal field variations

$Bz = f(r , \theta)$



cyclotron pole

valley and hill

Summary N° 3 : without equations

Synchro-Cyclotron : *Bz # constant (uniform) =simpler magnet*
Frf =Not constant =Not CW

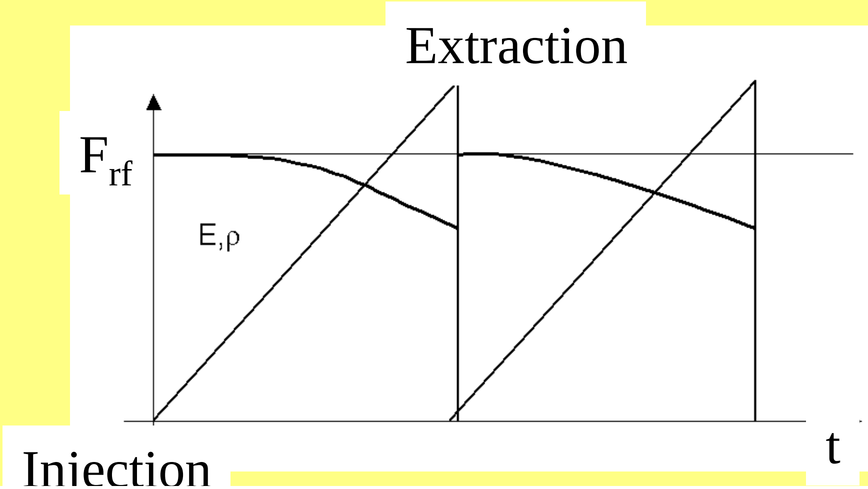
-Not Isochron : ωrev = NOT constant

$\omega_{rev} = q\, Bz / m\, \gamma\, (R)$

NO Defocusing Vertical force Br = 0

- Synchronicity with RF: $\omega_{rev} = q\, Bz / m\gamma = f(\text{Radius}) = \omega_{RF}(t)$

For longitudinal stability !
(RF to be varied)



CYCLOTRONS
The Family

$$\omega_{rev} = \frac{qB_z}{\gamma.m}$$

1

Conventional cyclotrons

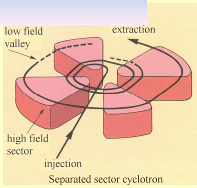
B*z* = uniform
F*rev* = evolves with γ !!!
F*RF* = constant

NOT USED anymore
Limited in energy :EK<1MeV

2: isochronous

Compact cyclotrons
1 magnet
with modulation

Separated sectors



B*z* = NOT uniform =f(radius)
F*revolution* = Constant
F*RF* = constant

Vertical focusing with
B*z* = f(r,θ)

Isochronous

$$\omega_{rev} / h = \omega_{RF}$$

3 : non isochronous

Synchro-cyclotrons

F*rev* = NOT Constant
F*RF* = NOT Constant = beam pulsed

Not Isochronous

Less intensity (pulsed) ≠ cw

$$\omega_{rev}(R) / h = \omega_{RF}(t)$$

Tutorial 3 : What is the field index n(R) ?

n(R) : it gives the radial evolution of *Bz*

$$n = -\frac{R}{B_0}\frac{\partial B_z}{\partial R}$$

Equivalent definition

$$B_z \sim B_0\,(r/R_0)^{-n}$$

The field index is not constant in a cyclotron $n = n\,(\text{radius})$

Isochronous cyclotron $n(r) < 0$: $\langle B_z(r,\theta)\rangle_{turn}$ increases with R

$$B_z(R) = \gamma(r)\;B_0 = k\,r^{-n}$$

$$\langle B_z(R)\rangle = \frac{B_0}{\sqrt{1-[R\;\omega/c]^2}} = B_0.R^{-n(R)}$$

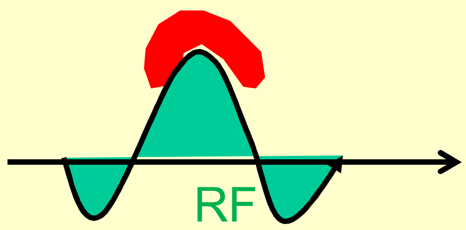
$$\gamma = \frac{1.}{\sqrt{1-[v/c]^2}} = \frac{1.}{\sqrt{1-[R\omega/c]^2}}$$

Dynamics in cyclotron

summary

$Qe_0 \hat{V} \cos\phi . N_{gap}$ — ***Energy gain per turn***

$\phi_0 \approx 0°$ — ***Central RF phase ,***
Ion bunches are centered at 0°



$\omega_{RF} = h\omega_{rev} = const$

RF synchronism = Isochronism
(h - harmonic number)

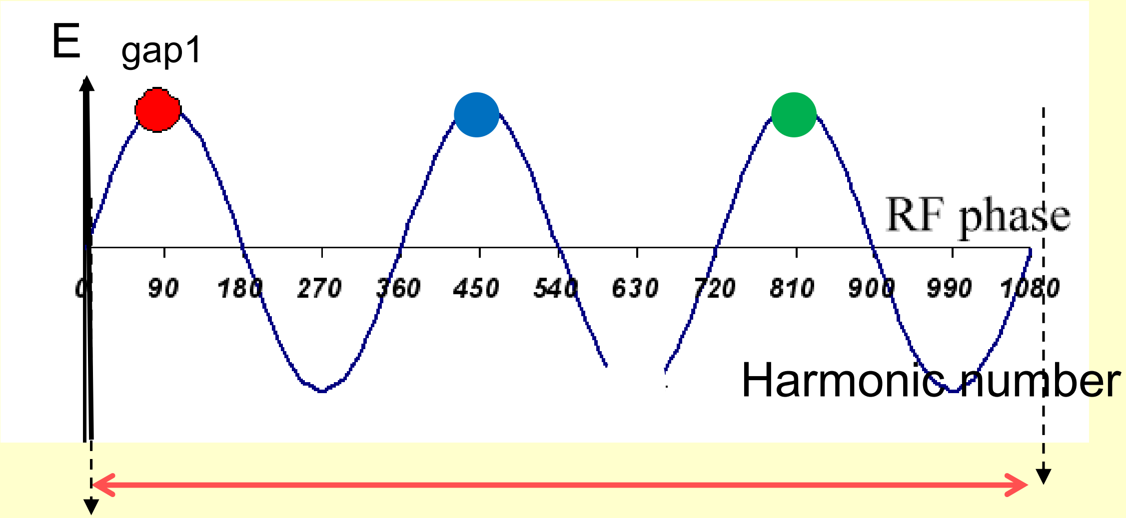
$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m} = const$$

$R = R(t) = R(N°turn\quad)$ — ***Orbit evolving***

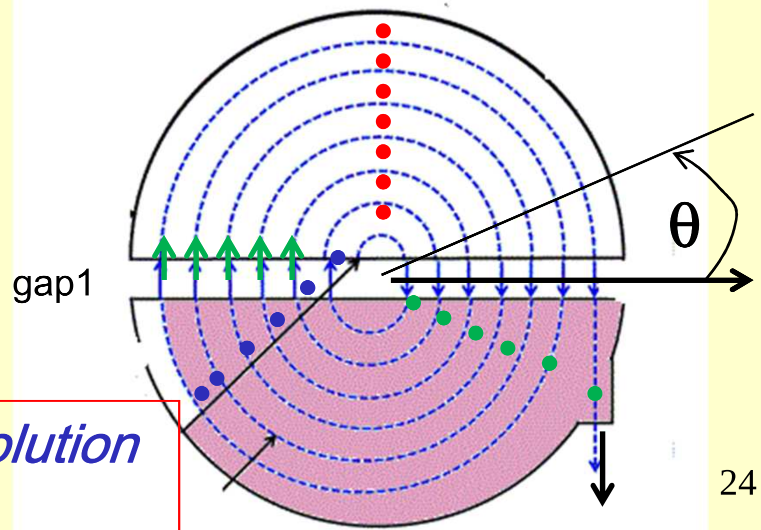
$B\rho(t) = P/q \Rightarrow \langle B \rangle = B\rho / R$ — ***Average Magnetic field***

Harmonic number h=FRF/ Frev

h=integer $\omega_{RF} = \mathbf{h}\ \omega_{rev}$



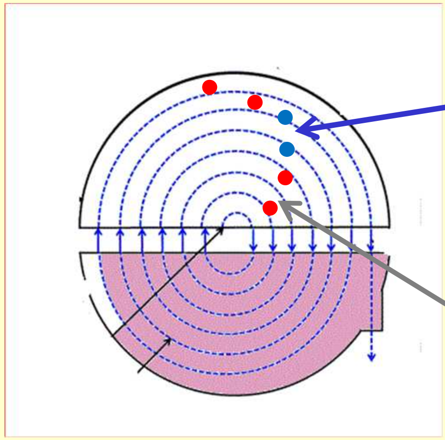
1 turn RF= h x 360°



h =1,2,3 Number RF oscillations per revolution

⇨ h *bunches by turn* $\omega_{rf} = \mathbf{h}\ \omega_{rev}$

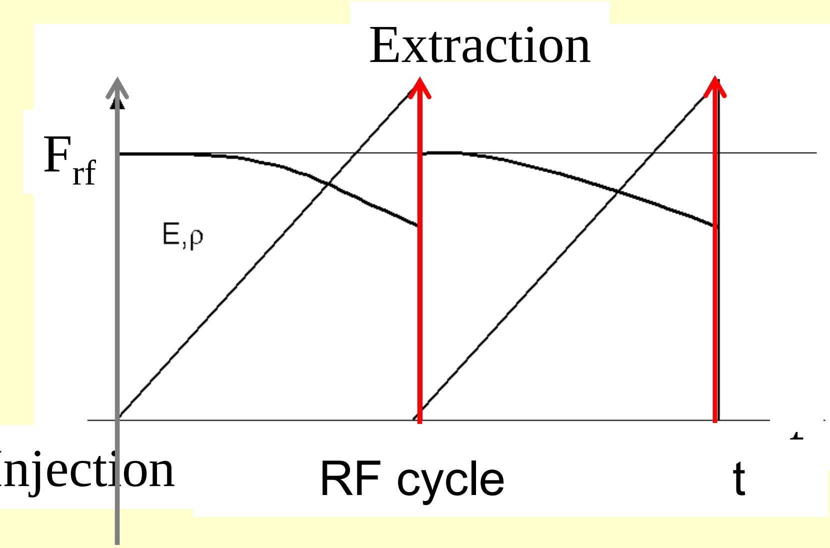
Synchro-cyclotrons : pulsed acclerator



Accelerated bunches

$F_{RF} = h\ F_{rev}$

Not accelerated (beam losses)



Less intensity (pulsed) avalaible (since not *cw*)

$$\omega_{rev} = \frac{\omega_{RF}}{h} = \frac{qB_z}{\gamma(R)\,m}$$

$$\omega RF\,(injection) = h\,\frac{qB}{\gamma m} \approx h\,\frac{qB}{m}$$

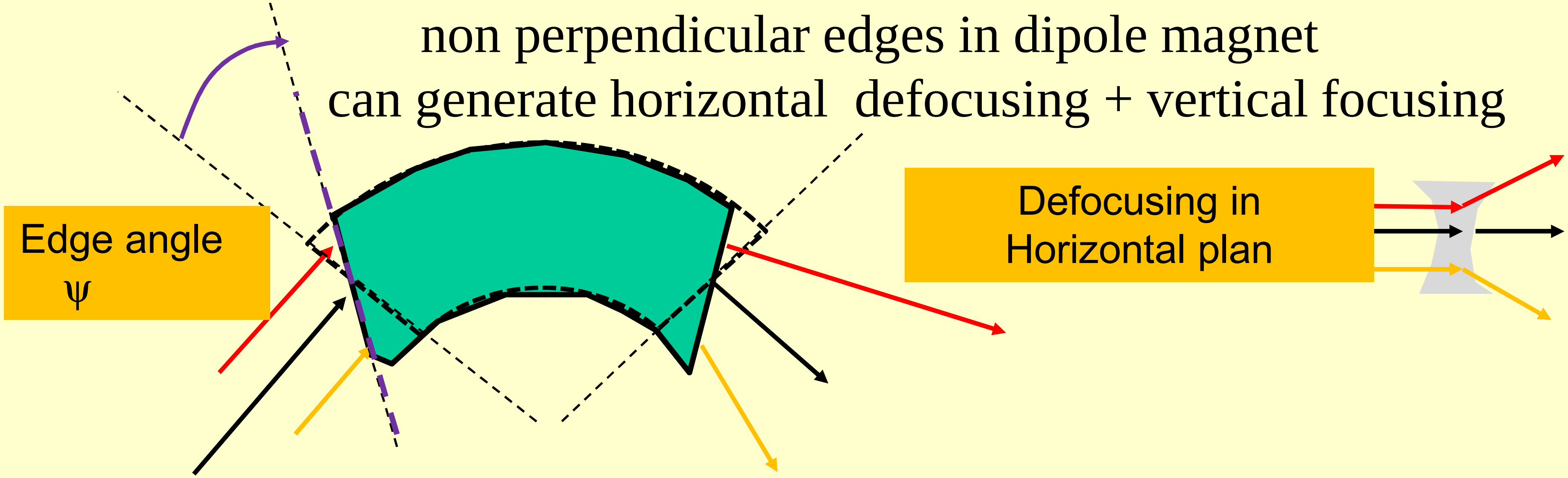
$$\omega RF\,(extraction) = h\,\frac{qB_0}{\gamma(R).m}$$

2-3 bunches are synchronized with RF

Other bunches are lost (not accelerated properly)

Edge focusing in dipole magnet recap

non perpendicular edges in dipole magnet
can generate horizontal defocusing + vertical focusing

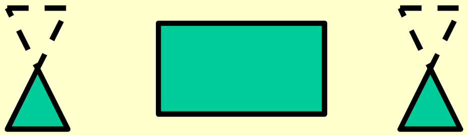


Non perpendicular edge in dipole magnet can provide

1) additive focusing in vertical + 2) defocusing in horizontal plane

The optical Transfer Matrix M is

Mdipole = M*edge*1. Mbody. M*edge2*



$$Mz_{edge}\begin{bmatrix} z \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{\tan\psi}{R} & 1 \end{bmatrix}\begin{bmatrix} z_0 \\ z_0' \end{bmatrix}$$

Equivalent effect in AVF Cyclo