Cyclotrons

Chapter 3 Cyclotron Design

- Isochronism
- Maximal energy (Bmax, R, stability)
- **Simulation // tracking**
  (numerical integration, realistic cyclo simulation)

Design Strategy for K=10 MeV cyclo

Design Strategy for K=250 MeV cyclo

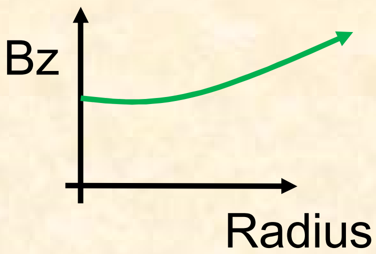
Design Strategy for a research facility (E/A vs I)

Cyclotron Summary : without formulas

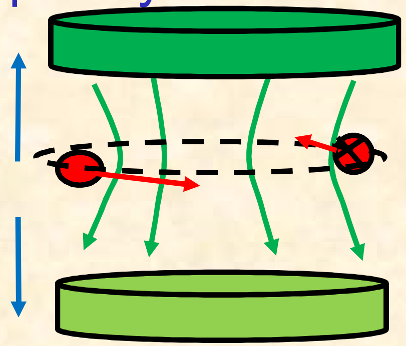
Longitudinal dynamics particles synchronous with RF

Isochronous cyclotron = constant revolution frequency

Bz=Bz(R)



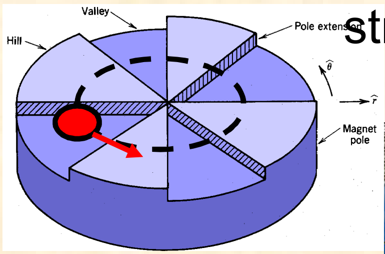
radial component
Br defocusing



Transverse dynamics : vertical defocusing forces have to be compensated

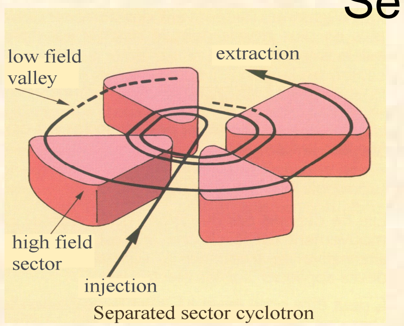
Azimutal(θ) Field modulation → vertical focusing

straight sectors in one magnet

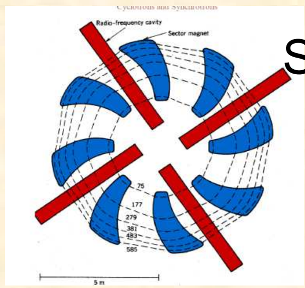


Spiralled sectors

Separated sectors



Separated spiralled sectors



Cyclotron Summary : with formulas

Isochronous cyclotron = constant revolution frequency

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m} = const$$

field index n <0

$$B_z \sim B_0 \, (r/R_0)^{-n}$$

$$<R> = \frac{B\rho}{<B_z>} = \frac{\gamma\, mv}{q<B_z>}$$

$$\omega_{rev} h = \omega_{RF}$$

$$E/A = Kb \, . \, (Q/A)^2$$

Vertical stability in isochronous cyclotron $B_z = F(R,\theta)$

requires Azimuthal Field Modulation (N sectors)

$$\ddot{z} + [\nu_z \omega_{revolution}]^2 z = 0$$

$$\nu_z^2 = n + \ldots < 0$$

$z(t) \sim z_0 \exp(-i \; \nu_z \, \omega \, t)$: vertical tune ν_z; real for stability

$$\nu_z^2 = n + \frac{N^2}{N^2 - 1} F_l \, (1 + 2 \tan^2 \varepsilon) > 0$$

Cyclotron Design

1) Particle kind : proton, heavy ion ? **Q/A**

2) Max Kinetic Energy of reference ions : *E*max

3) Magnet (Bmax, size ,sectors, hill/valley gap) : Compute a field Map

4) Number of cavities (Ngap) : energy gain per turn

Then, let's start the SIMULATIONs

Multi_particlelcode in « realistic » magnetic field
- compute reference orbit
- simulate injection
- simulate extraction

Analytical B Field + RF Kick
Computed field map B + RF Kick
Computed field map B + E

IS IT OK ?, IF NOT restart at 1)

Magnet design

How to adjust B(R) : <B>~ R^{-n}

- Pole Gap evolution <B (R)> : $n(R) = 1 - \gamma^2$
- Correction coils (trim coils)

FLUTTER $= \dfrac{\langle (B - \langle B \rangle)^2 \rangle}{\langle B \rangle^2}$

$$FLUTTER \approx \frac{(B_{hill} - B_{val})^2}{8 \langle B \rangle^2}$$

Sufficient FLUTTER F for axial stability($\nu_z^2 > 0$)

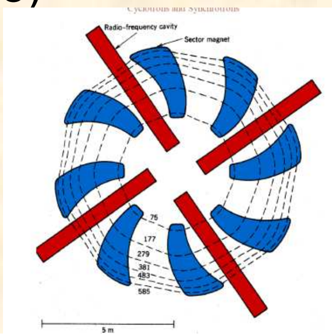
- Valley // hill B field
- sector angle
- spiral angle ε

$$\nu_z^2 = n + \frac{N^2}{N^2 - 1} F (1 + 2 \tan^2 \varepsilon) > 0$$

Space for injection beam line and RF

- Azimutally Varying Field or Separated Sectors : $B=B(R, \theta)$
- large Number of sectors N (4, 6, 8)





Isochronism : Field B=f(r)

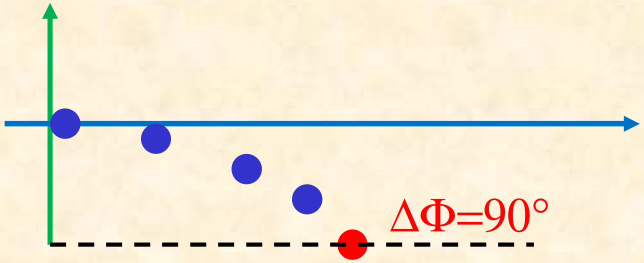
Bz(R) adjusted to get h ωrev = ωrf

$$\gamma(R) = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1-(R\omega_{rev})^2/c^2}}$$

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\,m}$$

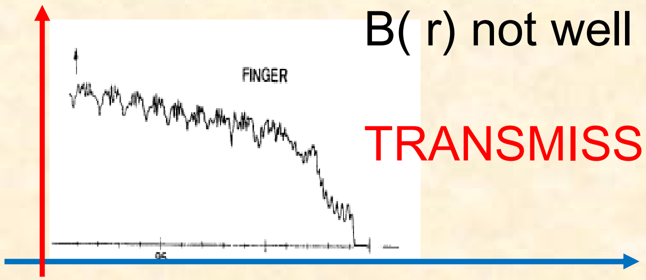
$$< B_z(R) > = < B_{z0} > / \sqrt{1-(R\omega_{rev})^2/c^2}$$

ΔΦ= ΔT/ ωrf



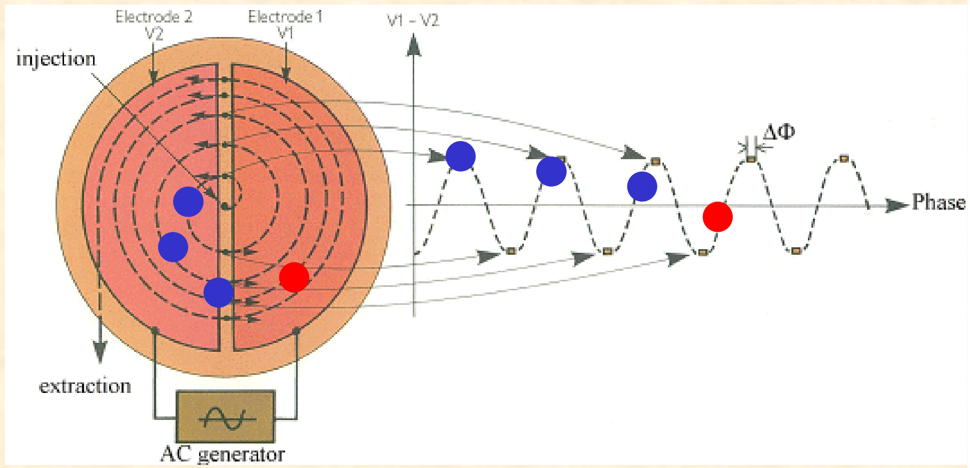
B(Radius) obtained -with correction coils
or -with pole shapes

I=f(R)



B(r) not well tuned

TRANSMISSION=0%



Max Energy for Cyclotrons : R x B

Heavy Ion
A= nucleon number
Q= charge number

$$\omega_{rev} = \frac{qB}{\gamma m} \approx \frac{qB}{m}$$

Max Kinetic Energy

$(\gamma-1)mc^2 \approx \frac{1}{2}\, m\, v^2$

$= \frac{1}{2}\, m\, (R_{extraction} . \omega_{rev})^2$

For ions: $m = A\, m_0 = A. [1.6\ 10^{-27}\ kg]$ $q = Q\, e_0$

-

ex : $^{12}C^{4+}$ A=12 Q=4

$$[E/A]_{max}\ (MeV/nucleon) = K_b \left\{\frac{Q}{A}\right\}^2$$

with $K_b \approx 48.2\, (\langle B \rangle . R_{ext})^2$

<B> limitation and size limitation (: Rextraction) for *Emax*

Cyclotron simulation :
Particle Tracking with a computer code

SIMULATION : tracking ions (M,Q,v_0)

Multi-particle-code
in « realistic » magnetic field

In cylindrical coodinates

~~TRANSPORT (1rst order)~~
~~MAD (3rd order)~~
~~CPO~~

~~Cyclotron Magnets are too complex~~

$$\mathbf{r} = r \cdot \mathbf{e}_r + z \cdot \mathbf{e}_z$$

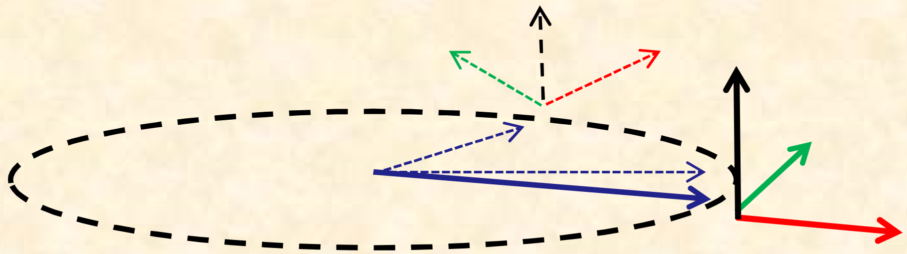
Comoving Frame : er = f(t)

$$d\mathbf{e}_r = \mathbf{e}_\theta \cdot d\theta \qquad d\mathbf{e}_z = 0 \qquad d\mathbf{e}_\theta = -\mathbf{e}_r \cdot d\theta$$

Velocity : $\dot{\mathbf{r}} = \frac{d\mathbf{r}}{dt}$?

$$\dot{\mathbf{r}} = \dot{r} \cdot \mathbf{e}_r + \dot{z} \cdot \mathbf{e}_z + r \cdot \dot{\mathbf{e}}_r + z \cdot \dot{\mathbf{e}}_z$$

$$\frac{d}{dt}\left[m\gamma \dot{\mathbf{r}} \right] = q \cdot (\mathbf{E} + \dot{\mathbf{r}} \times \mathbf{B})$$



Unit vectors are evolving in time !!!

Cyclotron simulation :
Particle Tracking with a computer code

SIMULATION : tracking ions (M,Q,v_0)   In cylindrical coodinates

Let's track one particle   Start $\theta=\theta_0$ (At t=0)

$r = r0$ $pr = pr\_0$

$z = z0$ $pz = pz\_0$

$p\theta = p\theta\_0$

What is the particle position at $\theta=\theta_0+\Delta\theta$ (At $t=0+\Delta\theta$ [$dt/d\theta$])

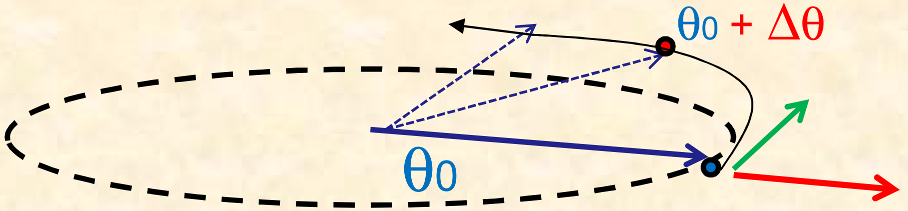
$r(\theta_0+\Delta\theta) = r0+\Delta\theta\ [dr/d\theta]$ (first order extrapolation= euler algorithm)

$z = z0 + \Delta\theta\ [dz/d\theta]$

$pr = pr\_0 + \Delta\theta\ [d\,pr/d\theta]$

$pz = pz\_0 + \Delta\theta\ [d\,pz/d\theta]$

$p\theta = p\theta\_0 + \Delta\theta\ [d\,p\theta/d\theta]$



[$d\,r/d\theta$] =

……

[$d\,pr/d\theta$] = cylindrical equation of motion = f [B(r, θ z)]

Cyclotrons simulation: cylindrical equation

$$\frac{d\mathbf{p}}{dt} = q\cdot(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \dot{r} & \dot{z} & r\,\dot{\theta} \\ B_r & B_z & B_\theta \end{vmatrix} =$$

$$= (\dot{z}\cdot B_\theta - r\,\dot{\theta}\cdot B_z)\cdot\mathbf{e}_r + (r\,\dot{\theta}\cdot B_x - \dot{r}\cdot B_\theta)\cdot\mathbf{e}_z + (\dot{r}\cdot B_z - \dot{z}\,B_r)\cdot\mathbf{e}_\theta$$

Evolution in time t is not convenient, evolution in θ is better !!!

$$\frac{d}{dt} = \frac{d\theta}{dt}\frac{d}{d\theta} = \dot{\theta}\frac{d}{d\theta}$$

$$\frac{d\mathbf{p}}{dt} = \dot{\theta}\frac{d\mathbf{p}}{d\theta} = q\cdot(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

$$\frac{d}{d\theta}\left[m\gamma\dot{r}\right] = \frac{d}{d\theta}[p_r] = m\gamma r\dot{\theta} + \frac{q}{\dot{\theta}}(\dot{z}\cdot B_\theta - r\,\dot{\theta}\cdot B_z)$$

$$\frac{d}{d\theta}\left[m\gamma\dot{z}\right] = \frac{d}{d\theta}[pz] = \frac{q}{\dot{\theta}}(r\,\dot{\theta}\cdot B_x - \dot{r}\cdot B_\theta)$$

$$\frac{d}{d\theta}\left[m\gamma\, r\,\dot{\theta}\right] = \frac{d}{d\theta}[p_\theta] = \frac{q}{\dot{\theta}}....$$

$$\frac{dr}{rd\theta} = \frac{\dot{r}}{r\dot{\theta}} = \frac{p_r}{p_\theta} \qquad \frac{dz}{rd\theta} = \frac{\dot{z}}{r\dot{\theta}} = \frac{p_z}{p_\theta}$$

Cyclotrons simulation: trajectory in (r, z) = f(θ)

The integration of particle's equation can be obtained with numerical methods

The equations to be solved is a set of Ordinary Differential Equations (ODE).

START at $\theta=0$ $(r_0, z_0, P_r, P_z, P_\theta)$: what are (r,z) at $\theta=0+\Delta\theta$?

At first order , we can compute (r,z) and (pr,pz)

$$\frac{d}{d\theta}[pr] = m\gamma r\dot{\theta} + \frac{q}{\dot{\theta}}(\dot{z}\cdot B_\theta - r\,\dot{\theta}\cdot B_z)$$

$$p_r(\theta_0 + d\theta) = p_r(\theta_0) + \frac{dp_r(\theta_0)}{d\theta}d\theta + 0(d\theta^2) + ...$$

$$r(\theta = \theta_0 + d\theta) = r_0 + \frac{dr}{d\theta}d\theta + ...$$

$$t = t_0 + \frac{dt}{d\theta}d\theta + ...$$

$$\frac{dr}{r\,d\theta} = \frac{dr/dt}{r\,d\theta/dt} = \frac{v_r}{v_\theta} = \frac{p_r}{p_\theta} \qquad \frac{dr}{d\theta} = r.\frac{p_r}{p_\theta}$$

$$\frac{dt}{d\theta} = \frac{1}{d\theta/dt} = \frac{m\gamma r}{P_\theta}$$

This the EULER method ! = 1rst order expansion

Cyclotrons simulation : the algorithm

Loop j=1, Nparticles

INITIAL position and momentum : $\theta = 0$ r, z pr, pz, pθ

Loop i=1,Nstep // **step in $\Delta\theta$**

$$B_r = BR(r,z,\theta) \qquad B_z = BZ(r,z,\theta) \qquad B_\theta = B\theta(r,z,\theta)$$

FIELD MAP

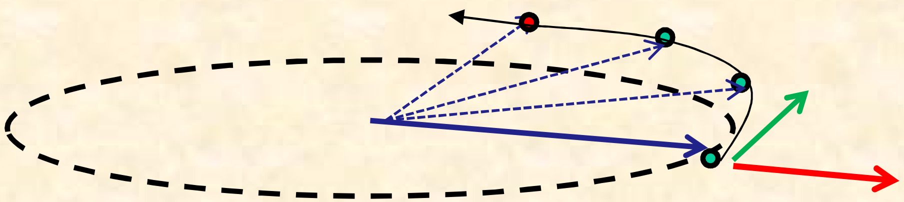
$$\theta = \theta_0 + \Delta\theta$$

$$r(\theta = \theta + d\theta) = r + \frac{dr}{d\theta}\Delta\theta \qquad p_r(\theta) = pr_0 + \frac{dp_r}{d\theta}\Delta\theta$$

$$z(\theta = \theta + d\theta) = z + \frac{dz}{d\theta}\Delta\theta \qquad p_z(\theta) = \ldots\ldots$$

$$\frac{d}{d\theta}[pr] = m\gamma r\dot{\theta} + \frac{q}{\dot{\theta}}(\dot{z}\cdot B_\theta - r\dot{\theta}\cdot B_z)$$

Endloop I // **end $\Delta\theta$ loop**



Endloop J // Nparticle **loop**

Euler algorithm (second order accurate in $d\theta$)

Numerical integration of the equations of motion

Euler algorithm (second order accurate in $\Delta\theta$) is not the best !!

RK4 (runge kutta order 4) is better (4th order accurate in $\Delta\theta$)
See a Numerical analysis Lecture

SPECIAL ATTENTION to FIELD INTERPOLATION
between the points of the field map B (ri, θi, zi)

How to simulate a cyclotron in 4 steps

- 1) **Define the basic parameters of the cyclotron (B,R,F) :** ***$B\rho$ =< B> Rextraction***

START The simulation in the “middle” of cycloton
With a defined magnetic structure

- 2) **Find the closed orbit** (1 particle)
without acceleration at R=Rref

- 3) **Find a matched beam** in the cyclotron (multiparticles)
backward tracking toward injection

- 4) Forward tracking (multiparticles) toward extraction
- Extraction (multiparticles) : (deflector, precession, resonance)

Iterative process

Basic parameters R,<B>,Sectors,Flutter (1/4)

Ex: cyclotron design 20 MeV/A for carbon ion 4+

What is the Max energy (MeV/A) : 20 MeV/A for carbon 4+

What are the ions (Q/A) = 4+/12 ⇨ Bρextraction= 2 T.m

ION (M,Q)
FINAL ENERGY

Bρextraction= 2 T.m γ= 1.02 ***Bρ =< B> Rextraction***

Reasonnable Fied
<B> =1.5 T

Rextraction= Bρextrac /<**B**> =1.4 m

ION (M,Q)
Source Voltage
(30kV -100kVolts ?)

Bρinjection= 0.04 T.m

Rinjection= Bρinjection /<B>=0.04m

Vertical stability

B(r,θ) Nsector , Hill//Valley gap

$$\nu_z^2 = n + \frac{N^2}{N^2 - 1} F_l (1 + 2\tan^2 \varepsilon) > 0$$

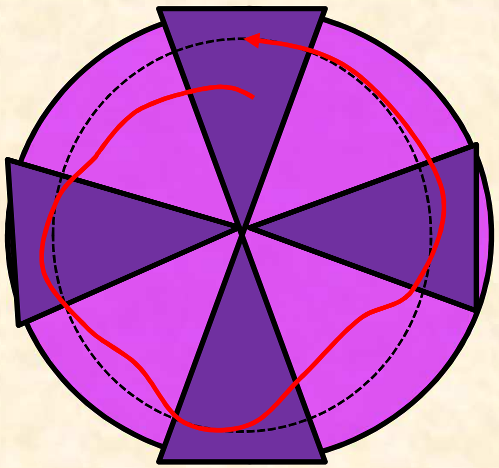
let's take 4sectors & gap hill =12 cm //gap valley =30 cm //…ε=0

Find the closed orbit at R=Rref (2/4)

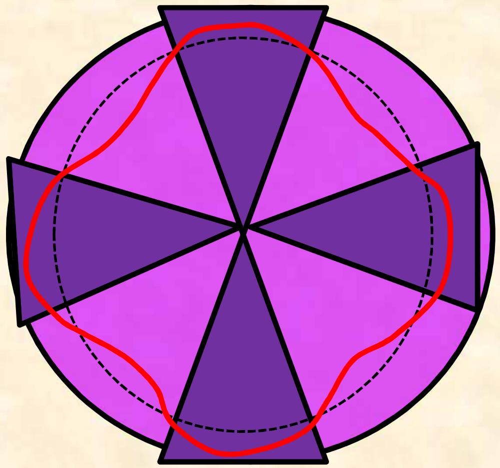
Dynamic of 1 particle in the middle of the cyclotron
Without acceleration

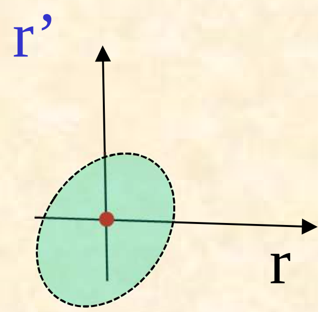
- Choose reference particle (1 particle) (M,Q, $B\rho_0$)
- **Choose a field Level $B(r,\theta)$ =k. FIELD MAP**

Find the reference radius Rref ?

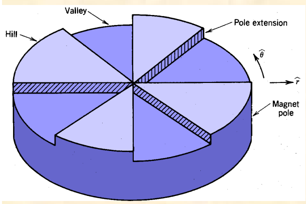


Trajectory is not a closed orbit
Not a good starting point





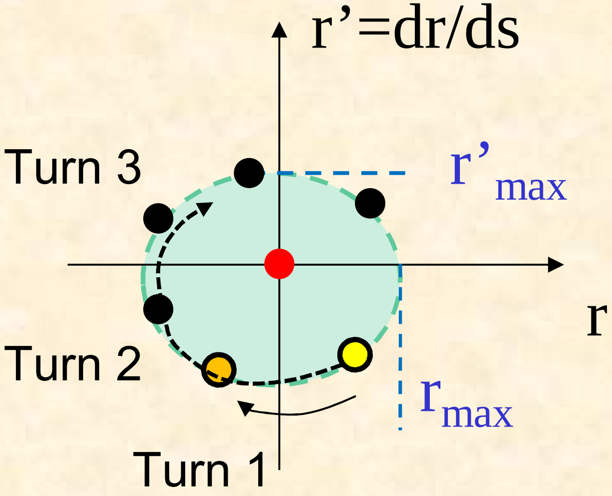
Trajectory is a closed orbit :OK



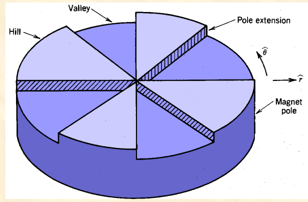
$\langle Rref \rangle = \langle B(r,\theta) \rangle / B\rho$

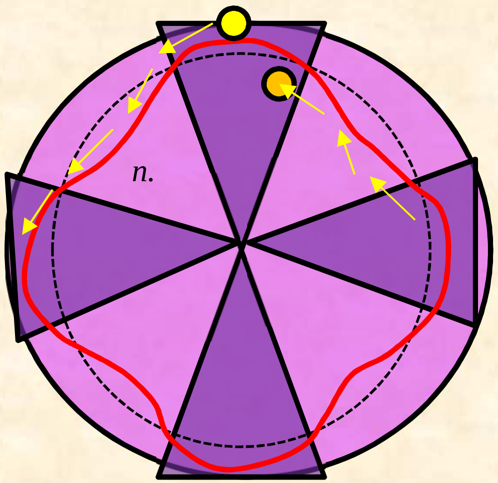
Find a matched beam in the cyclotron (3/4)

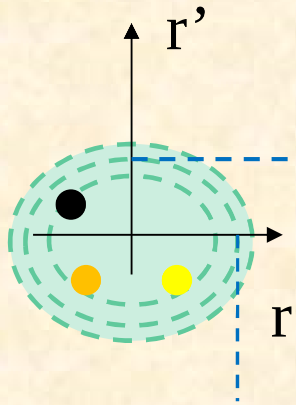
Around the reference trajectory, send a particle for many turns



Hill-Valley is a periodic lattice







A particle trajectory follows an ellipse

Beam matching =

Choose a beam ellipse with

$\Delta r' / \Delta r = r'_{max} / r_{max}$

$r(t) = r_0 + r_{max} \cos(\nu_r \omega_0 t)$

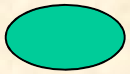
$r'(t) = r'_{max} \sin(\nu_r \omega_0 t)$

This ellipse occupy the minimal size in the cyclotron

Mismatched beam recall (3/4)

Because of each individual trajectory over N turn

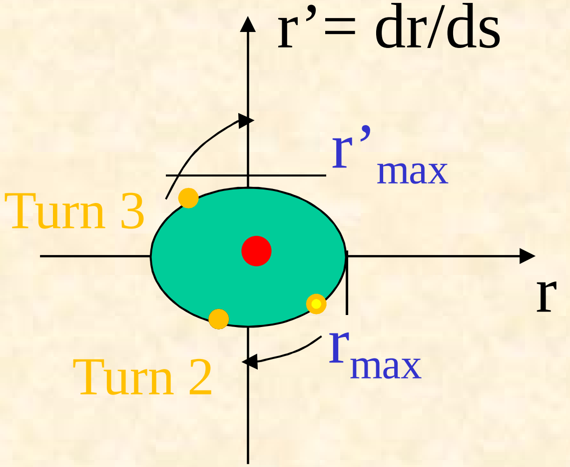
$$\begin{cases} r(t) = r_0 + r_{max} \cos(\nu_r \, \omega_0 t) & \text{(without acceleration)} \\ r'(t) = r'_{max} \sin(\nu_r \, \omega_0 t) \end{cases}$$

it exist an optimal ellipse 

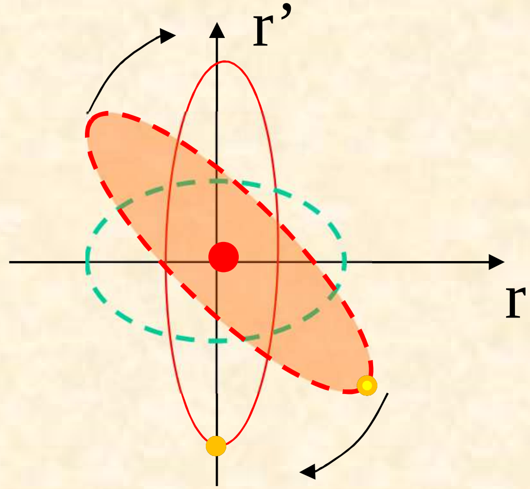
for a given beam Emittance : $\varepsilon = \pi \, \Delta r_{max} . \, \Delta r'_{max}$

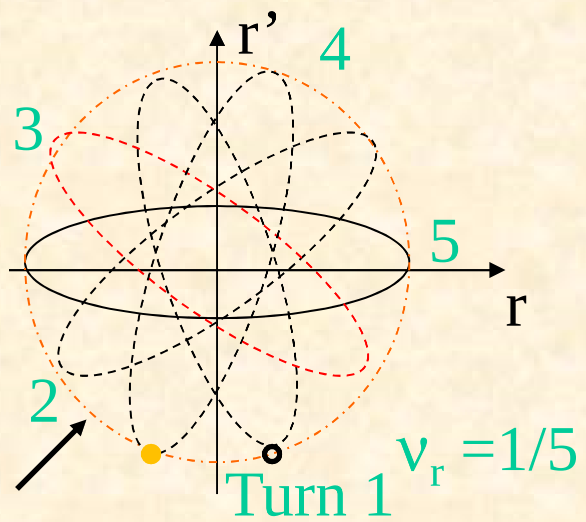
Betatron oscillation with mismatched beam

MisMatched beam



Matched beam





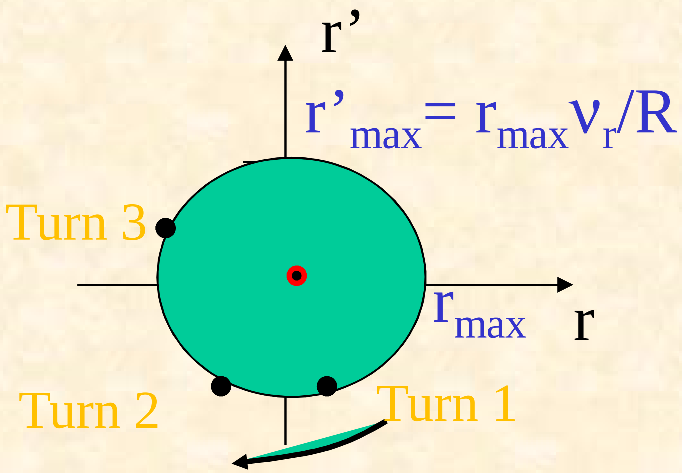
Larger acceptance required for the cyclo : not good

Matched beam recall (3/4)

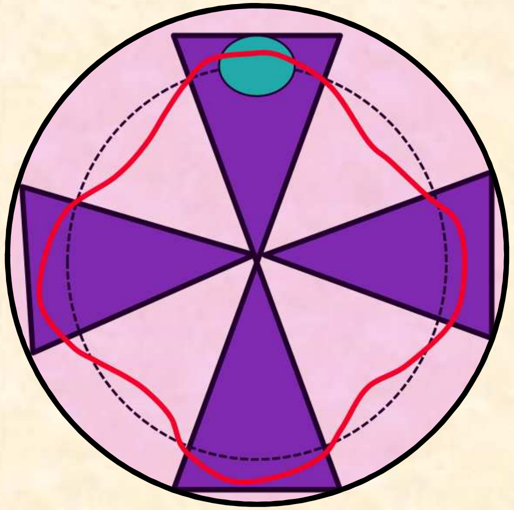
$$\begin{cases} r(t) = r_0 + r_{max}\cos(\nu_r\omega_0 t) \\ r'(t) = dr/ds = dr/R\omega_0 dt = -(r_{max}\nu_r/R)\sin(\nu_r\omega_0 t) \end{cases}$$

the Matched ellipse $|r'_{max}| = |r_{max}\,\nu_r / R|$

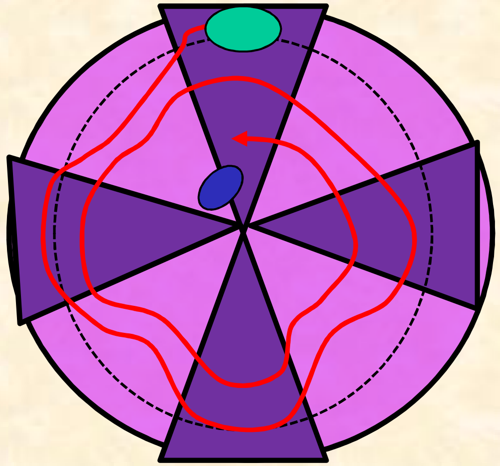
- ⇨ Initial beam conditions depend of the tune (ν_r) of the cyclotron at the matching point.
  - ⇨ Betatron oscillation disappears
  - ⇨ Matched beam
  - ⇨ Minimum of acceptance



backward tracking toward injection (3/4)

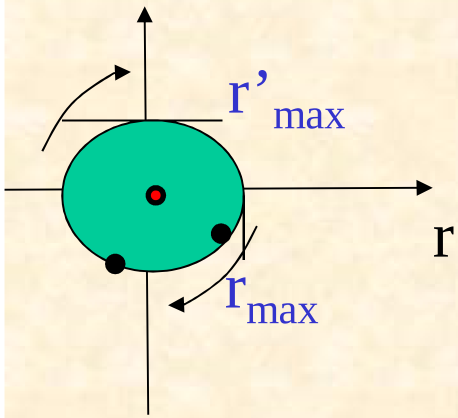


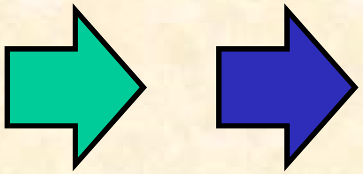
Central Trajectory is a closed orbit :OK

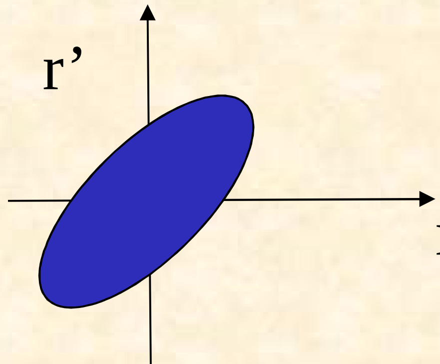


turn on RF : backward toward injection
Adjust Vrf, central field.....

Beam matched : OK





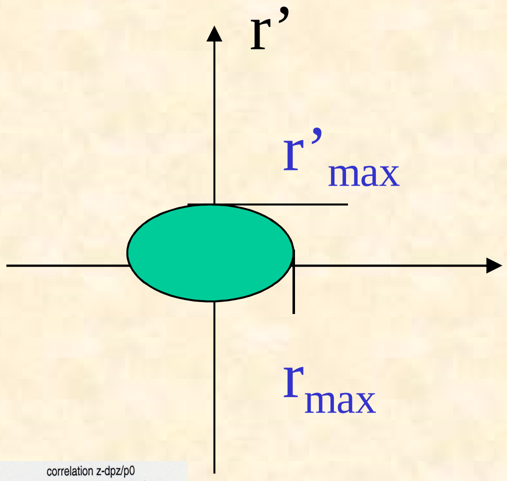


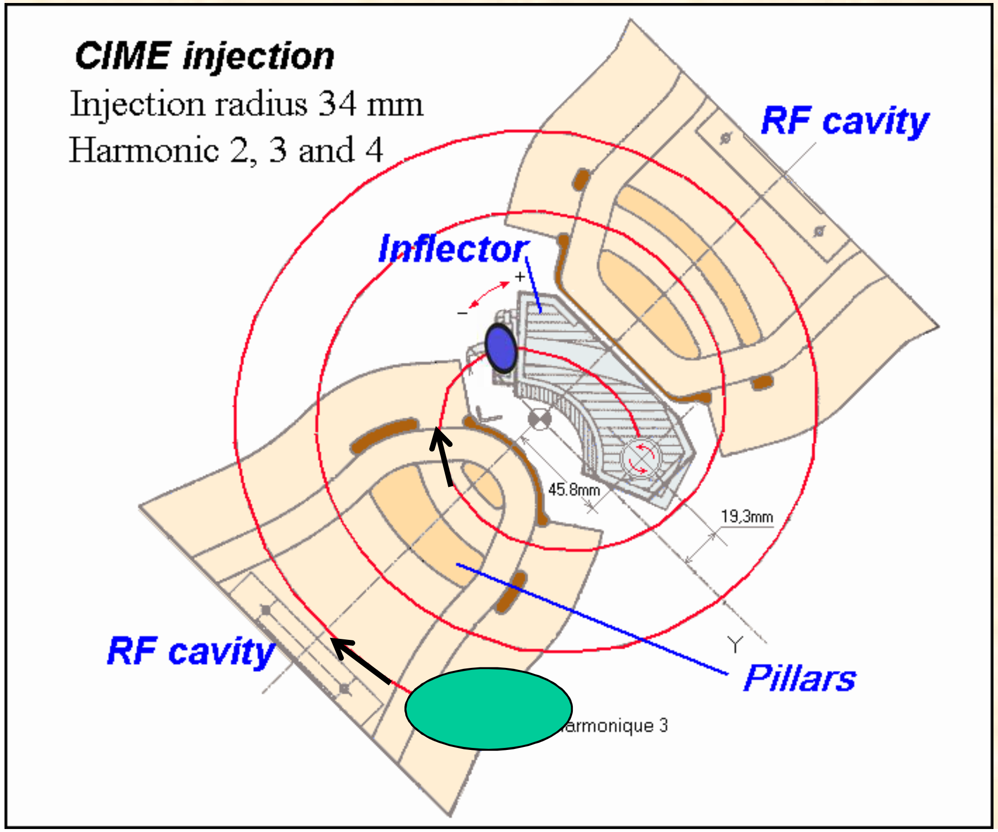
Corresponding optimal beam at injection radius

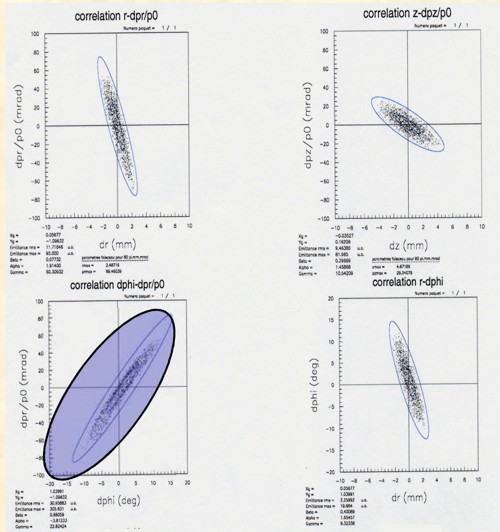
backward tracking toward injection (3/4)

Start with matched beam in the cyclotron (multiparticles) at large radius
Then Adjust Vrf, central field to reach injection Radius

Find the optimal beam at injection radius





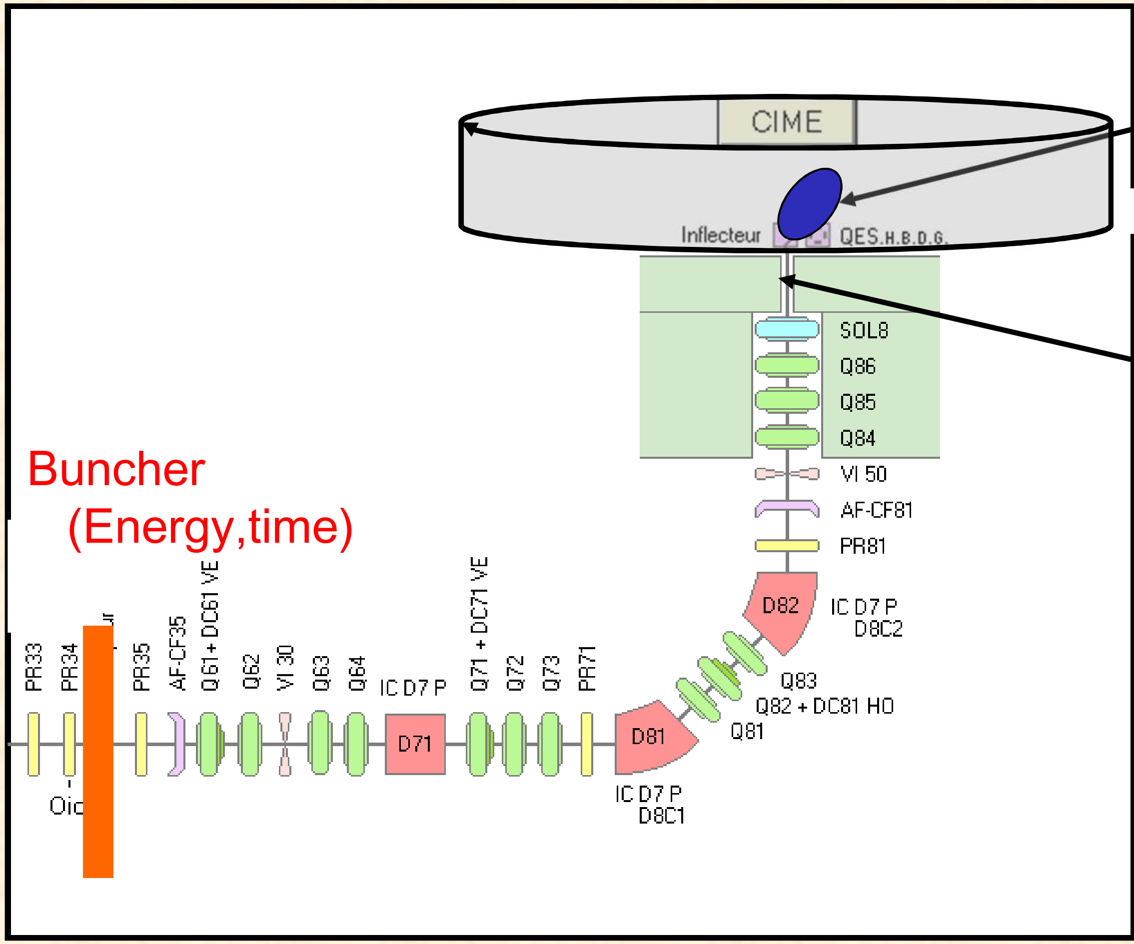


Beam Obtained With backward tracking

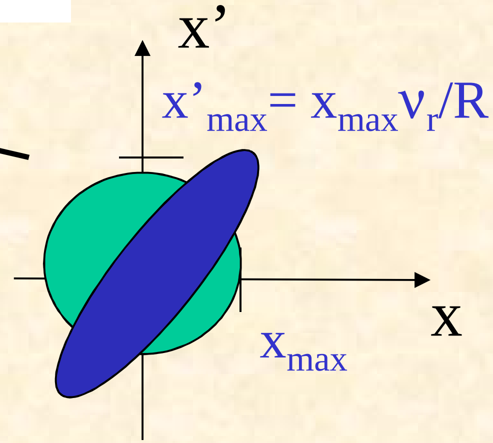
(r, r', z, z', E, ϕ)

Simulate the injection beam line

to get the perfect beam at injection



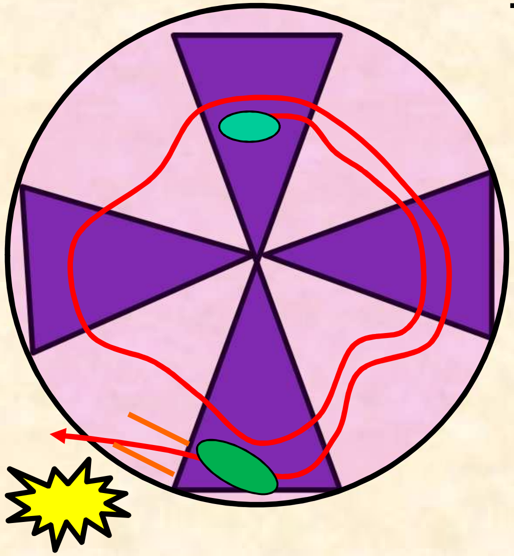
Matching at this point



Classical transport line problems :

Adjust quad to get desired beam at injection (r,r') (z,z') (t,E)

Forward tracking up to extraction (4/4)

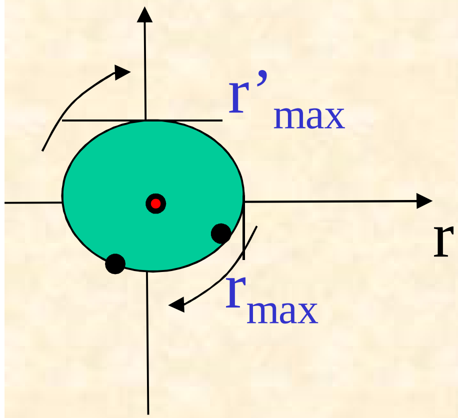


turn on RF : Forward toward extraction

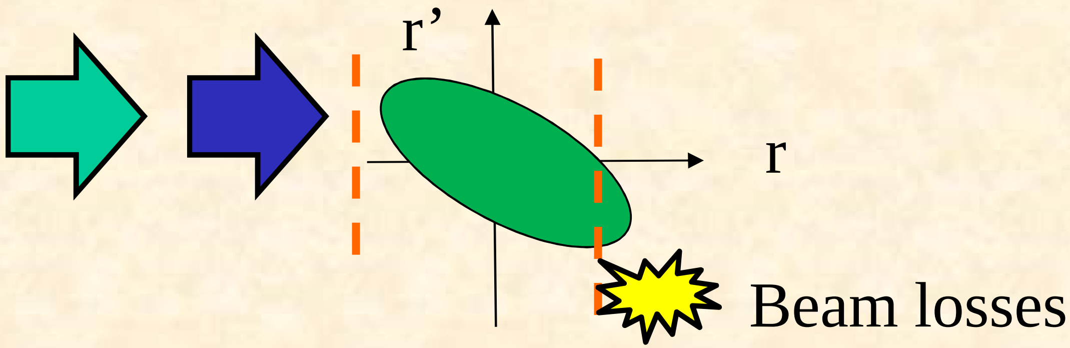
tune the isochronism $\langle B(r)\rangle = \langle B\rangle \gamma(r)$

Extraction
- design the extraction (deflector+..)
- turn separation (RF +precession? + magnetic bump?)
- beam losses ?

Beam matched



beam at extraction radius : Watch the beam losses

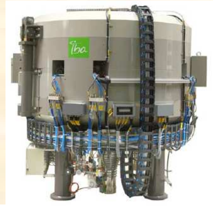


THE CYCLOTRON IS COMPUTED, Let's construct it !

Cyclotron Design strategies

Radio-Isotopes production

cost & reliability

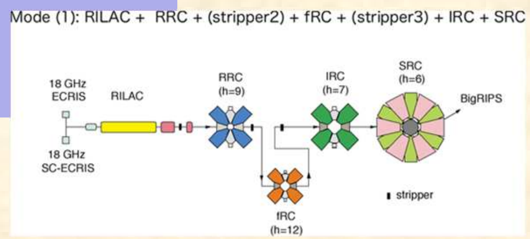


Medical applications : Cancer treatment

cost & reliability

Nuclear physics& Research facility

performance , intensity,..



Strategy for Radio-Isotopes production medical applications

10 MeV Protons / 5 MeV Deutons : @ low cost

Bρmax =0.458 T.m = <B> Rextraction

Rextract = 0.34 m

<B> = 1.35Tesla [hill =1.8 T // valley =0.5 T]

AVF with 4 straight sectors (sufficient z-focusing)

I_{beam} ~ 0.1 -0.05 mA

Rf Dees : 2 (so 4 gaps)

2 possibilities for extraction

Extraction By stripping :
external target (18F, radiotracer)

No Extraction :
internal target (in yoke)

A « low energy » industrial Cyclotron
Cyclone 10/5 : 2 particles : ^{1}H & ^{2}D

Kb=10 MeV

Fixed energy ;

4 straight sectors 50°

fixed **Frf =42Mhz**

<B> =1.35 Tesla

Harmonic h=2(p) ,4 (D)

Internal source

Rextraction=0.33m

Bρmax=0.33x 1.35=0.45 T.m

$$\left[\frac{E}{A}\right]_{max} (MeV\ /\ nucleon) = K_b \left\{\frac{Q}{A}\right\}^2$$

Eprotons=10 MeV protons= $^{1}H^{1+}$ A=1 Q=1

(E/A=Kb*1^2 =10MeV/A)

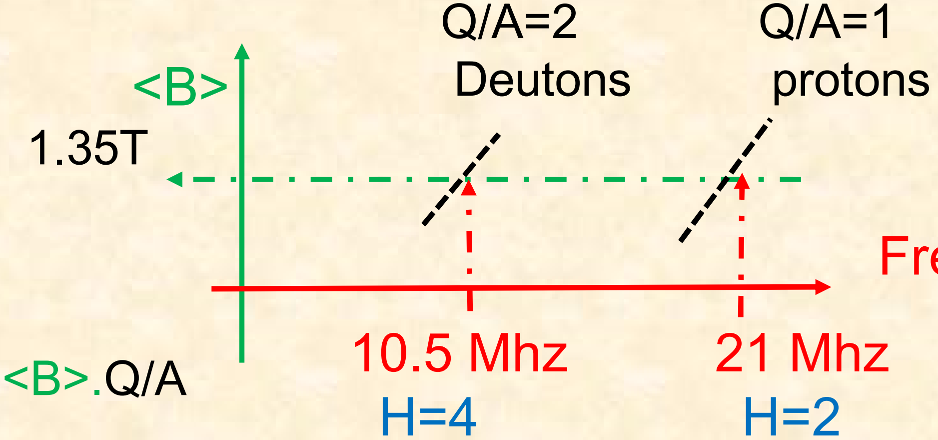
RF Harmonic =2 Frev=42 Mhz/ h= 21 Mhz

EDeutons=5 MeV Deutons= $^{2}H^{1+}$ A=2 Q=1

(E/A=Kb*0.5^2= 2.5 MeV/A)

RF Harmonic =4

$$F_{rev} \propto \frac{Q.B_{cyclo}}{A}$$



Frev~ <B>.Q/A

Strategy for cancer treatment
proton therapy (>80 facilities in the world)

-250 MeV Protons

Accel VARIAN Isochronous cyclo

Superconducting <B> = 2.2 Tesla

Rextrac~1.2m

-230 MeV Protons

IBA Synchro cyclotron

Superconducting <B> = 5. Tesla

Rextract ~0.6 m

Very compact

Hill/valley not needed

Strategy for a Cyclotron in a research facility

High energy

$E/A_{max} = Kb\,.(Q/A)^2$

High Kb ~ $(R.B)^2$

R
Large magnet (Radius)

High Bz (superconducting)

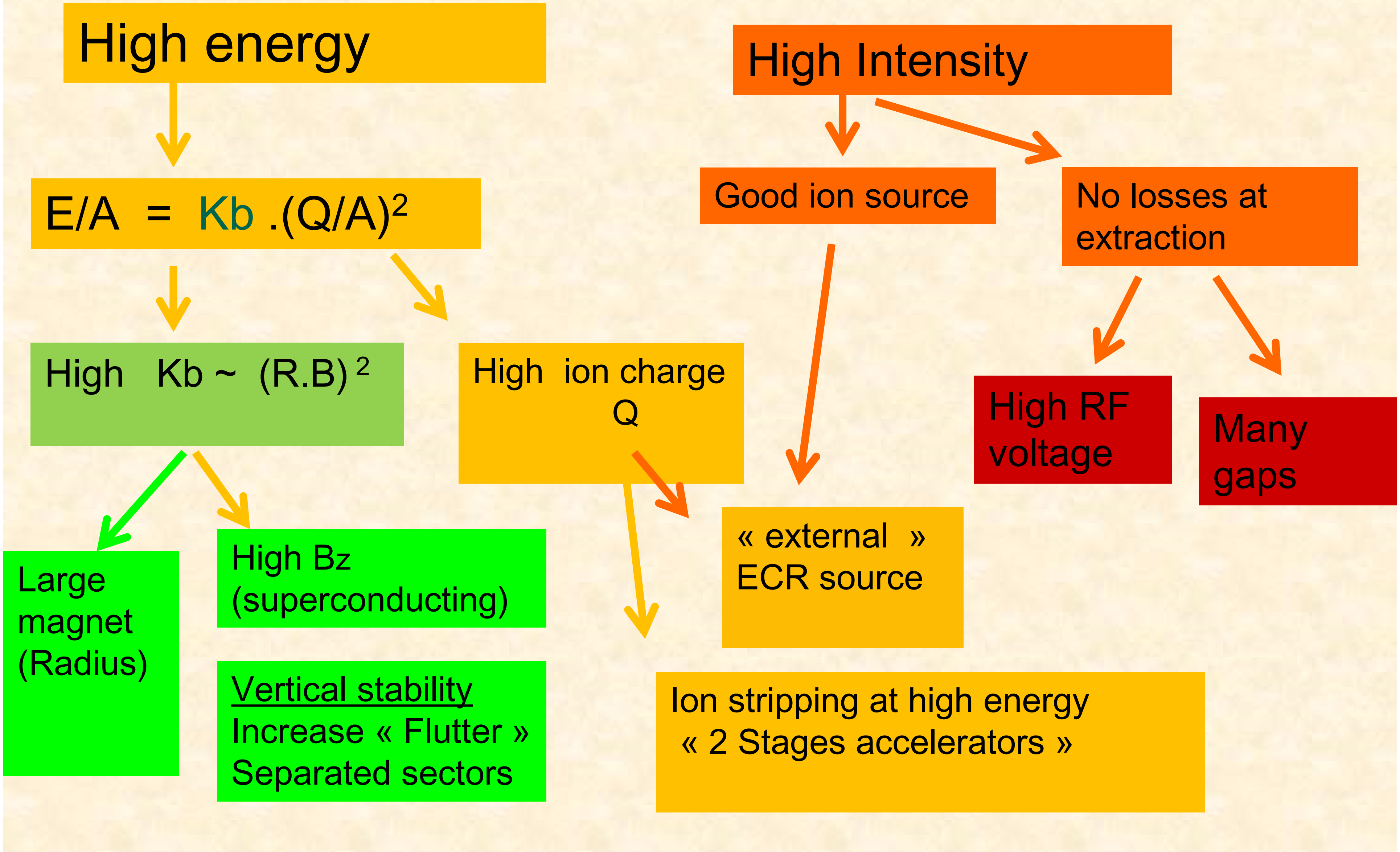
Vertical stability
Increase « Flutter »
Separated sectors

High ion charge Q

« external » ECR source

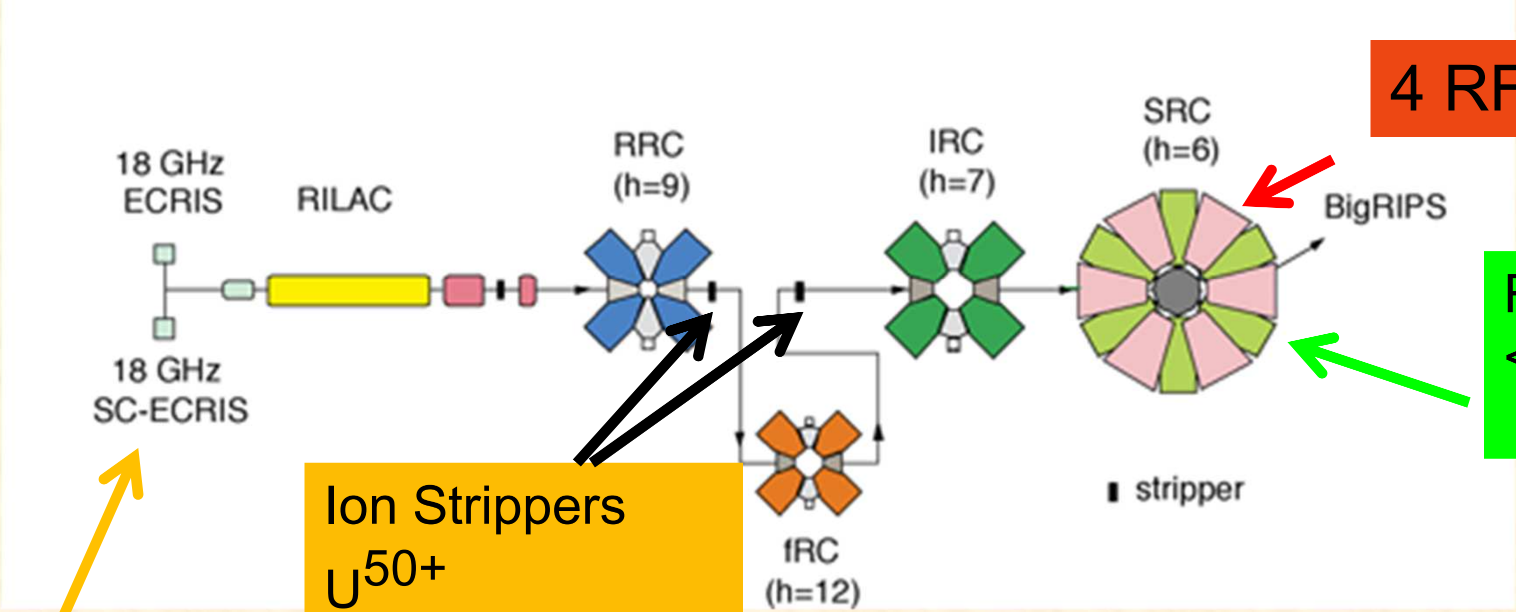
Ion stripping at high energy
« 2 Stages accelerators »

Strategy for a Cyclotron in a research facility



RIBF (Japan) : SRC (K=2600 MeV) –the bigest cyclo
Uranium beam $^{238}U^{88+}$ @345 MeV/A cw

Mode (1): RILAC + RRC + (stripper2) + fRC + (stripper3) + IRC + SRC



4 RF cavities

Radius=6m
<B>=3.8T

Ion Strippers
U^{50+}
U^{88+}

ECR source
U^{30+}

5 Stages accelerators

1 LINAC
+ 4 Cyclotrons

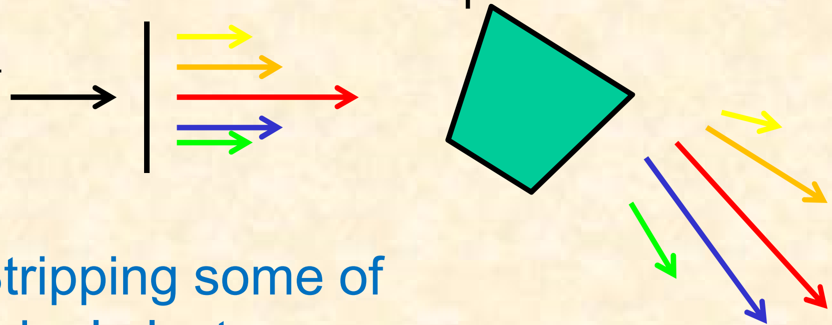
Ion Stripping at high energy

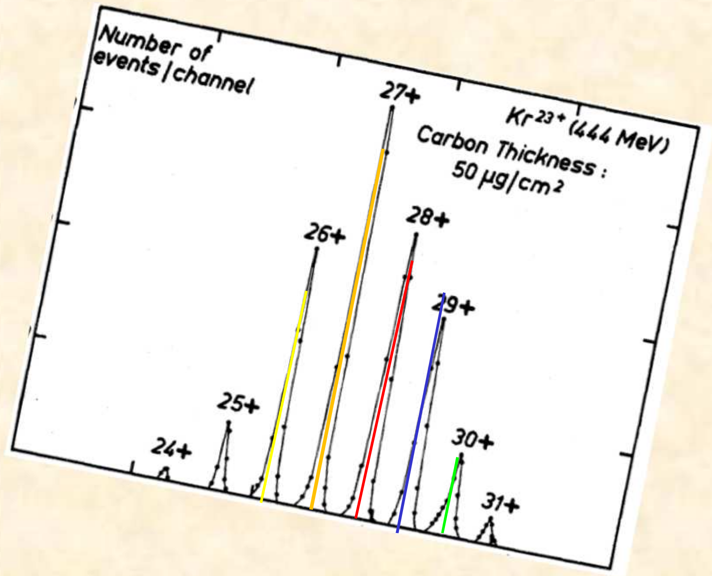
Heavy ions are not fully stripped by ion sources :

Incoming Ions

Magnetic spectrometer

A^{Q_1+}





Stripping some of residual electrons

final Ions A^{Q_2+}

$Q_2 > Q_1$

$B\rho_2 < B\rho_1$

$$B\rho = \frac{P}{q} = \frac{\gamma\, m.v}{q}$$

Ion Stripping help to increase the maximal energy of a given cyclotron….

$$[E/A]\max = Kb\left[\frac{Q}{A}\right]^2$$

- End Chapter 3

important facts for cyclotron :

1) Simulations are done with realistic magnetic field (not transport matrices)
2) Magnetic structure should provide the vertical stability (field index n compensated by sectors)
3) The Beam matching at injection for better transverse acceptance

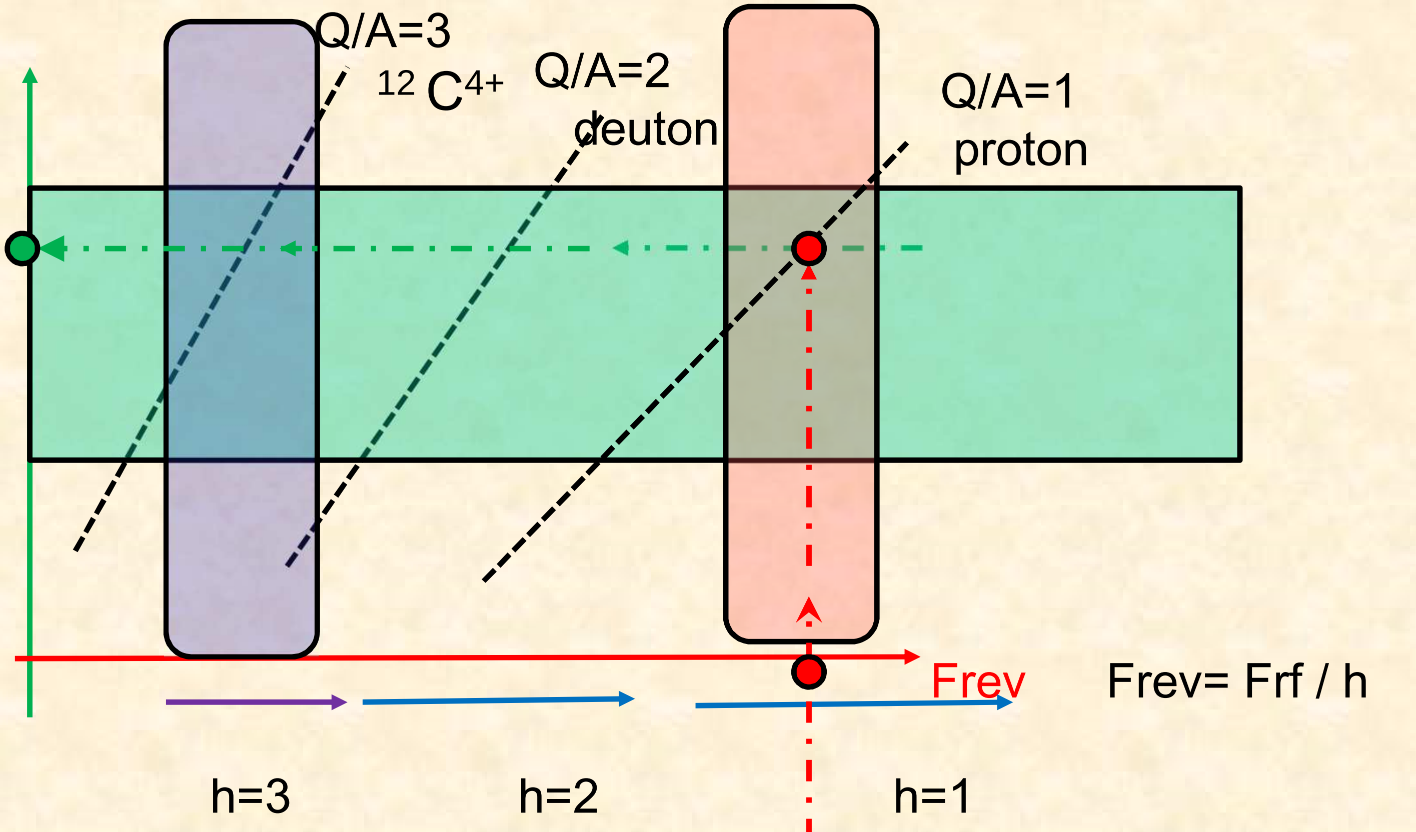
Additive slides …..

Diagram for
The variable energy cyclotrons

$$F_{rev} \propto \frac{Q.B_{cyclo}}{A} \propto h \cdot F_{RF}$$

A= nucleon number
Q= charge number

$$\omega_{rev} = \frac{qB}{\gamma\, m}$$



2) Compute
<B>
For a given
(Q,A)

1) Select the energy (Frev)
Select the ions (Q,A)
Adjust <B>

Bρ # <B> Rextract

E/A (MeV/A) # K (Q/A) ^{2}

Coupling of 2 Cyclotrons : velocity matching

Two cyclotrons can be used to reach higher energy :

- Harmonic&Radius of the 2 cyclotrons have to be matched

$$\frac{v}{2\pi} = \left[\frac{F_{HF}.R_{ejec}}{h}\right]_{cycloA} = \left[\frac{F_{HF}.R_{inj}}{h}\right]_{cycloB}$$

The velocity of extraction CycloA

= velocity of injection CycloB

- Ion stripping can be used, to increase Q before injection into the second cyclo

large Q ⇨ large *Emax*

$$\left[\frac{E}{A}\right]_{max} = K_b \left\{\frac{Q}{A}\right\}^2$$