JUAS 2020 – Tutorial 1

$\mu = \mu_0\, \mu_r$
$\mu_0 = 4\pi \cdot 10^{-7}$ Vs/(Am)
$\varepsilon = \varepsilon_0\, \varepsilon_r$
$\varepsilon_0 = 8.854 \cdot 10^{-12}$ As/(Vm)
$c_0 = 2.998 \cdot 10^{8}$ m/s

F. Caspers, M. Bozzolan, M. Wendt

1.) Transmission-lines

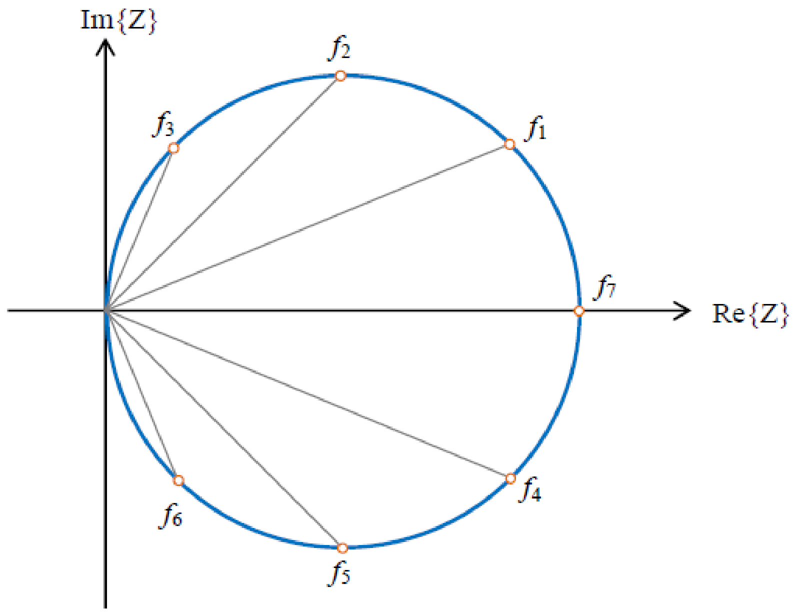
Given is a coaxial transmission-line with an inner diameter of the outer conductor of 100 mm, the dielectric is air (so-called "air-line").

Questions:

1. What is the outer diameter of the inner conduction to achieve a characteristic impedance of 50 Ω?
2. With which velocity is a wave travelling in this line?
3. Specify the capacitance and inductance per meter length of this transmission-line?
4. Instead of an air dielectric this transmission line is now homogeneously filled with Teflon ($\varepsilon_r = 2$). Determine the phase velocity, characteristic impedance, as well as capacitance and inductance per meter length?

2.) Impedances in the complex plane (2)

The impedance of a resonant circuit is a function of frequency. For a given resonator the impedance was measured at 7 different frequencies, f_1…f_7. The result is shown in the complex *Z*-plane:



	f_1	f_2	f_3	f_4	f_5	f_6	f_7
f / MHz	105.11	105.05	104.94	105.29	105.35	105.46	105.20
Z / kΩ	$200.0\, e^{j30°}$	$162.6\, e^{j45°}$	$115.0\, e^{j60°}$	$200.0\, e^{-j30°}$	$162.6\, e^{-j45°}$	$115.0\, e^{-j60°}$	$230.0\, e^{j0°}$

Questions:

1. Determine the resonant frequency.
2. Determine the 3-dB bandwidth (*BW*) of this resonator.
   (Hint: The bandwidth of a resonator is defined as the frequency difference between the upper and lower 3-dB frequency points.)
3. Sketch the equivalent circuit for the measured resonator.
4. Determine *R*.
5. Draw the locus of admittance of this circuit in the *Y*-plane, and indicate lower and upper 3-dB points.
6. Determine the Q-value, as well as *L* and *C* for this circuit.

3.) Multiple choice questions

1. How will the resonant frequency f_{res} of the E_{010} (TM_{010}) mode of a pill box cavity change if height of the cavity is doubled? (check 1)

- ○ The f_{res} decreases by a factor 2.
- ○ The f_{res} decreases by a factor $\sqrt{2}$.
- ○ The f_{res} increases by a factor 2.
- ○ The f_{res} increases by a factor $\sqrt{2}$.
- ○ The f_{res} will not change.

2. A critically coupled aluminum pill-box cavity is driven by an RF generator. The same pill-box cavity is now made out of copper, again with the generator operating at critical coupling, such that the gap voltage remains the same. $\sigma_{Al} = 3.8 \cdot 10^7\ S/m$, $\sigma_{Cu} = 5.8 \cdot 10^7\ S/m$. What happens with the dissipated power in the cavity? (check 1)

- ○ The power dissipation decreases
- ○ The power dissipation increases
- ○ The power dissipation will not change

3. Calculate the thickness of a copper wall of 5 times the penetrations depth for 50 Hz signals. $\sigma_{Cu} = 5.8 \cdot 10^7\ S/m$, $\mu = \mu_0 \mu_r$ with $\mu_0 = 4\pi \cdot 10^{-7}\ Vs/Am$ (check 1)

- ○ 46.7 mm
- ○ 4.67 mm
- ○ 0.46 mm
- ○ 0.046 mm

4. A rectangular waveguide has a width (long side!) of *a* = 10 cm. (check 2)

- ○ The mode TE_{10} or H_{10} has a cutoff frequency of 3 GHz.
- ○ The mode TE_{10} or H_{10} has a cutoff frequency of 1.5 GHz.
- ○ The electric field is parallel to the side with the larger dimension.
- ☒ The electric field is orthogonal to the side with the larger dimension.

5. Which mode is the fundamental mode (lowest cut-off frequency) in a cylindrical waveguide of circular cross-section *without* inner conductor? (check 1)

- *TE*
- *TEM*
- *TM*

6. Adding capacitive loading to a cavity (check 1)

- lowers the resonance frequency
- does not affect the resonance frequency
- increases the resonance frequency

7. When you cover the antenna of your mobile with your hand, the attenuation caused is in the order of 20 dB. Human tissue is a rather good absorber, so you can neglect reflections for this calculation. How many percent of the mobile's output power stay in the hand? (check 1)

- 9 %
- 99 %
- 99.9 %
- 99.99 %

4.) Impedances in the complex plane

Questions:

1. Plot the following impedances in the *Z*-plane, use the plot axes on the next page:

$Z = (3 + 4\,j)\ \Omega$	$\lvert Z\rvert = 2$, $\arg(Z) = \pi/4$	Z = short circuit
$Z = 2\ \Omega$	$\lvert Z\rvert = 1$, $\arg(Z) = -\pi/2$	$Y = Z^{-1} = (0.16 + 0.12j)\ \Omega^{-1}$
$Z = (1 - 4\,j)\ \Omega$	$\lvert Z\rvert = 5$, $\arg(Z) = 53°$	

2. Qualitatively, how would an inductor look like, plotted from DC to some arbitrary frequency, in the *Z*-plane? Hint: $Z_L = j\omega L$
3. How would a capacitor look like? Hint: $Z_C = 1/(j\omega C)$
4. The input impedance of a RLC circuit has been plotted in the *Z*-plane (blue circle). Mark the points in the diagram describing:
   a. Impedance at the resonant frequency
   b. DC impedance
   c. 3-dB bandwidth
   d. Impedance at $f \rightarrow \infty$

5.) Waves of a transmission line Z = 50 Ω

Problem: Convert the circuit-based formats, voltage *V* and current *I* into the
equivalent wave-based formats, forward wave *a* and backward wave *b*
and vice versa using the relations:

$a = \frac{V + I\,Z}{2}$	$V = a + b$
$b = \frac{V - I\,Z}{2}$	$I\,Z = a - b$

Questions:

1. In a 50 Ω system, a directional coupler measured forward and reflected waves a and b at a certain plane as: a = 100 ∠0° and b = 60 ∠45°.
    - Calculate the corresponding voltage V and current I
    - Sketch the "phasors" of V, I Z, a and b.

2. At some plane in the 50 Ω system, a voltage of V = 100 ∠0° V and a current of I = 1.0 ∠-45° A are measured.
    - Calculate the corresponding forward and backward waves a and b.
    - Sketch the "phasors" of V, I Z, a and b.

6.) Scaling laws

A cavity shall be scaled from existing designs for a frequency f_x = 318.32 MHz and C_x = 10 pF.
There are three test designs, with the following parameters:

Cavity	f_{res} / MHz	*C* / pF	*Q*	Diameter / mm
A	100	7.957	10000	600
B	500	3.18	5000	200
C	3000	1.061	2000	25

Questions:

1. Which cavity is suitable as reference design?
2. Calculate the diameter of the new design.
3. Calculate the expected *Q* factor for the new design, provided it will be built out of the same

7.) Thermal expansion and scaling laws

An accelerator cavity heats up under high RF power load.
The cavity used is constructed from a material having a:

thermal expansion coefficient:	$\Delta l/l$ = 20e-6/^{0}C (per degree Centigrade)
thermal resistivity coefficient:	$\Delta\rho/\rho$ = 4e-3/^{0}C (per degree Centigrade)

At room temperature the cavities resonance frequency is f_1 = 100 MHz and has a 3-dB bandwidth of BW_1 = 100 kHz.
Under RF power the cavity temperature increases by 100 ^{0}C (subscripts 2 apply).

Questions:

Determine

- the ratio λ_2/λ_1
- the ratio L_2/L_1
- the ratio C_2/C_1
- the ratio Q_2/Q_1 (hint: the skin depth δ is proportional to $\sqrt{\rho/f}$
- the resonance frequency f_2 under load
- and the 3-dB bandwidth BW_2 of the resonance under load