JUAS 2020 – Tutorial 1

$\mu = \mu_0\ \mu_r$

$\mu_0 = 4\pi \cdot 10^{-7}$ Vs/(Am)

$\varepsilon = \varepsilon_0\ \varepsilon_r$

$\varepsilon_0 = 8.854 \cdot 10^{-12}$ As/(Vm)

$c_0 = 2.998 \cdot 10^{8}$ m/s

F. Caspers, M. Bozzolan, M. Wendt

1.) Transmission-lines

Given is a coaxial transmission-line with an inner diameter of the outer conductor of 100 mm, the dielectric is air (so-called "air-line").

Questions:

1. What is the outer diameter of the inner conductor to achieve a characteristic impedance of 50 Ω?

   $$Z_0 = \sqrt{\frac{\mu_r}{\varepsilon_r}}\, 60\Omega \ln\left(\frac{R}{r}\right) = 60\Omega \ln\left(\frac{D}{d}\right)$$

   $$d = \frac{D}{\exp\left(\frac{50}{60}\right)} = \frac{D}{2.3} = 43.46\ mm$$

   see script page 10

2. With which velocity is a wave travelling in this line?

   $v = c = 2.998 \cdot 10^8\ m/s$

3. Specify the capacitance and inductance per meter length of this transmission-line?

   $$C' = \frac{1}{vZ_0} = 66.71\ pF/m$$

   $$L' = \frac{Z_0}{v} = 166.78\ nH/m$$

   see script page 10

4. Instead of an air dielectric this transmission line is now homogeneously filled with Teflon ($\varepsilon_r = 2$). Determine the phase velocity, characteristic impedance, as well as capacitance and inductance per meter length?

   $$v = \frac{c}{\sqrt{\varepsilon_r \mu_r}} = 2.12 \cdot 10^8\ m/s$$

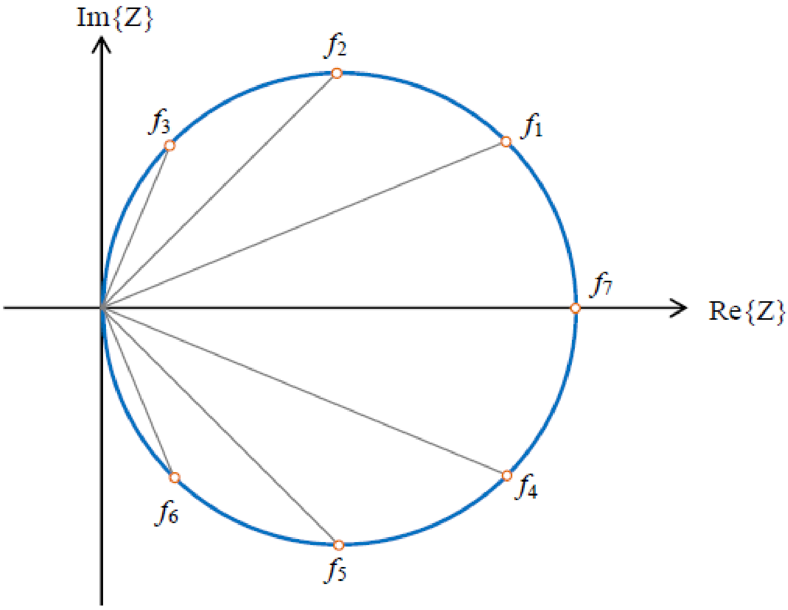
   $$Z_0 = \frac{60\Omega}{\sqrt{\varepsilon_r}} \ln\left(\frac{D}{d}\right) = 35.36\ \Omega$$

   $$C' = \frac{1}{vZ_0} = 133.43\ pF/m$$

   $$L' = \frac{Z_0}{v} = 166.78\ nH/m$$

2.) Impedances in the complex plane (2)

The impedance of a resonant circuit is a function of frequency. For a given resonator the impedance was measured at 7 different frequencies, $f_1 \ldots f_7$. The result is shown in the complex Z-plane:



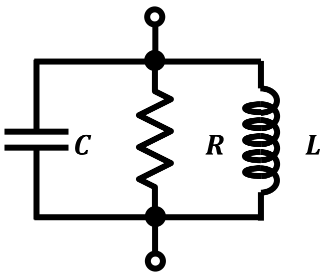
	f_1	f_2	f_3	f_4	f_5	f_6	f_7
f / MHz	105.11	105.05	104.94	105.29	105.35	105.46	105.20
Z / kΩ	$200.0\ e^{j30°}$	$162.6\ e^{j45°}$	$115.0\ e^{j60°}$	$200.0\ e^{-j30°}$	$162.6\ e^{-j45°}$	$115.0\ e^{-j60°}$	$230.0\ e^{j0°}$

Questions:

1. Determine the resonant frequency.
   $f_{res} = f_7 = 105.20\ MHz$
2. Determine the 3-dB bandwidth (BW) of this resonator.
   (Hint: The bandwidth of a resonator is defined as the frequency difference between the upper and lower 3-dB frequency points.)
   $BW = f_5 - f_2 = 300\ kHz$

In order to evaluate the properties of a resonator,
it is common to model it as equivalent circuit with lumped RLC elements.

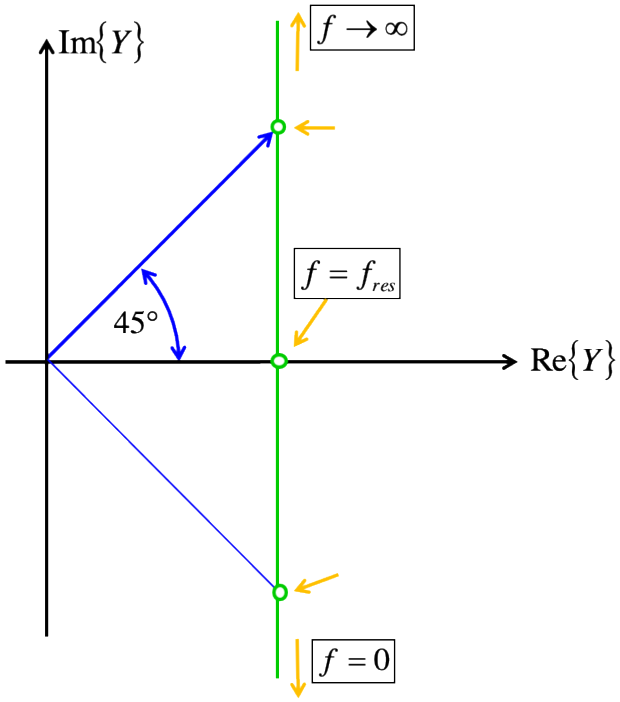
3. Sketch the equivalent circuit for the measured resonator.



4. Determine R.

$R = 230\ k\Omega$ (at $f_{res} = f_7$)

5. Draw the locus of admittance of this circuit in the Y-plane, and indicate lower and upper 3-dB points.



Straight line parallel to the $Im\{Y\}$ axes, crossing $Re\{Y\}$ at 1/230 mS. The 3-dB points are located on the locus as points crossing with lines from the origin under $\pm 45^0$.

see script page 58

6. Determine the Q-value, as well as L and C for this circuit.

$$Q = \frac{f_{res}}{BW} = 350.667$$

$$L = \frac{R}{2\pi f_{res} Q} = 992\ nH$$

$$C = \frac{1}{(2\pi f_{res})^2 L} = 2.31\ pF$$

3.) Multiple choice questions

1. How will the resonant frequency f_{res} of the E_{010} (TM_{010}) mode of a pill box cavity change if height of the cavity is doubled? (check 1)

- ○ The f_{res} decreases by a factor 2.
- ○ The f_{res} decreases by a factor $\sqrt{2}$.
- ○ The f_{res} increases by a factor 2.
- ○ The f_{res} increases by a factor $\sqrt{2}$.
- ☒ The f_{res} will not change.

2. A critically coupled aluminum pill-box cavity is driven by an RF generator. The same pill-box cavity is now made out of copper, again with the generator operating at critical coupling, such that the gap voltage remains the same. $\sigma_{Al} = 3.8 \cdot 10^7\ S/m$, $\sigma_{Cu} = 5.8 \cdot 10^7\ S/m$. What happens with the dissipated power in the cavity? (check 1)

- ☒ The power dissipation decreases
- ○ The power dissipation increases
- ○ The power dissipation will not change

3. Calculate the thickness of a copper wall of 5 times the penetrations depth for 50 Hz signals. $\sigma_{Cu} = 5.8 \cdot 10^7\ S/m\,, \mu = \mu_0\mu_r$ with $\mu_0 = 4\pi \cdot 10^{-7}\ Vs/Am$ (check 1)

- ☒ 46.7 mm
- ○ 4.67 mm
- ○ 0.46 mm
- ○ 0.046 mm

4. A rectangular waveguide has a width (long side!) of *a* = 10 cm. (check 2)

- ○ The mode TE_{10} or H_{10} has a cutoff frequency of 3 GHz.
- ☒ The mode TE_{10} or H_{10} has a cutoff frequency of 1.5 GHz.
- ○ The electric field is parallel to the side with the larger dimension.
- ☒ The electric field is orthogonal to the side with the larger dimension.

5. Which mode is the fundamental mode (lowest cut-off frequency) in a cylindrical waveguide of circular cross-section *without* inner conductor? (check 1)

- ☒ *TE*
- ○ *TEM*
- ○ *TM*

6. Adding capacitive loading to a cavity (check 1)

- ☒ lowers the resonance frequency
- ○ does not affect the resonance frequency
- ○ increases the resonance frequency

7. When you cover the antenna of your mobile with your hand, the attenuation caused is in the order of 20 dB. Human tissue is a rather good absorber, so you can neglect reflections for this calculation. How many percent of the mobile's output power stay in the hand? (check 1)

- ○ 9 %
- ☒ 99 %
- ○ 99.9 %
- ○ 99.99 %

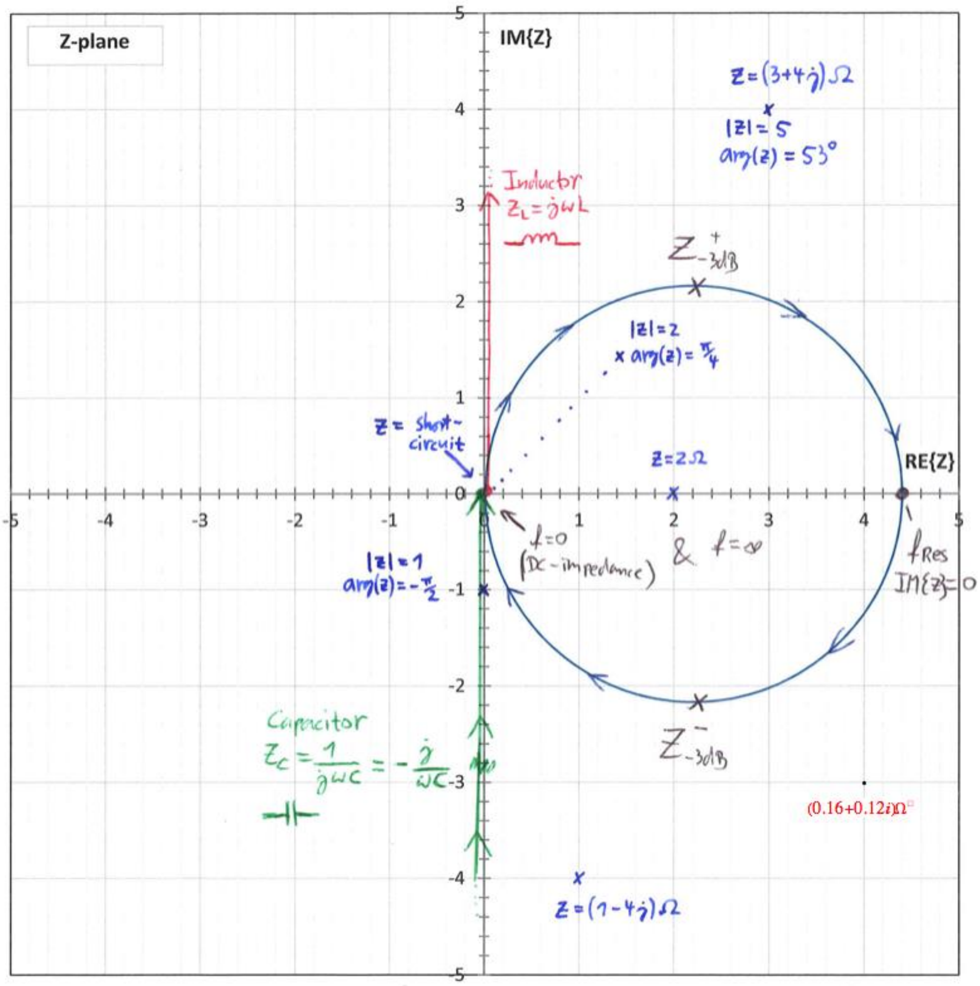
4.) Impedances in the complex plane

Questions:

1. Plot the following impedances in the *Z*-plane, use the plot axes on the next page:

Z = (3 + 4 j) Ω	|*Z*| = 2, arg(*Z*) = π/4	*Z* = short circuit
Z = 2 Ω	|*Z*| = 1, arg(*Z*) = -π/2	$Y = Z^{-1}$ = (0.16 + 0.12j) Ω^{-1}
Z = (1 – 4 j) Ω	|*Z*| = 5, arg(*Z*) = 53°	

2. Qualitatively, how would an inductor look like, plotted from DC to some arbitrary frequency, in the *Z*-plane? Hint: $Z_L = j\omega L$
3. How would a capacitor look like? Hint: $Z_C = 1/(j\omega C)$
4. The input impedance of a RLC circuit has been plotted in the *Z*-plane (blue circle). Mark the points in the diagram describing:
    a. Impedance at the resonant frequency
    b. DC impedance
    c. 3-dB bandwidth
    d. Impedance at *f* → ∞



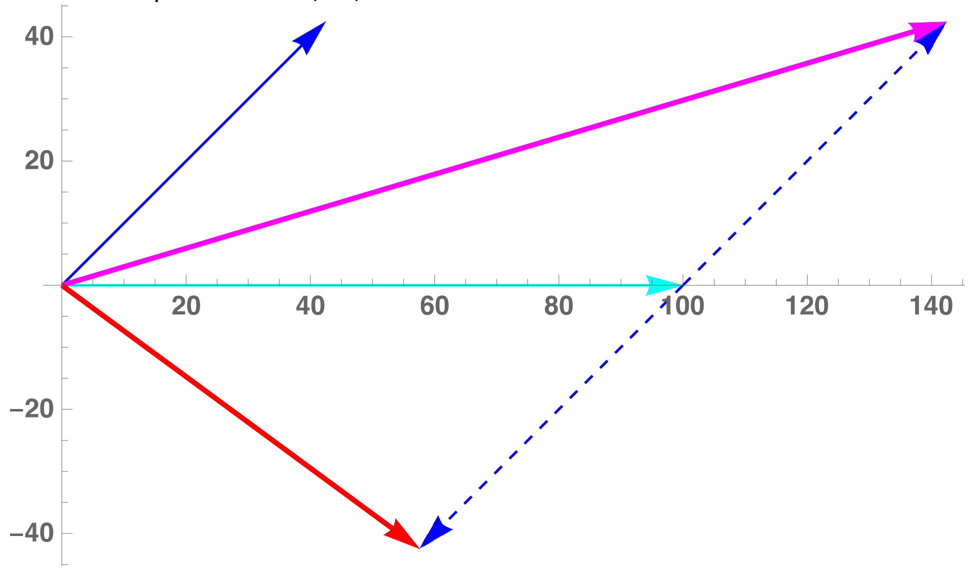
5.) Waves of a transmission line Z = 50 Ω

Problem: Convert the circuit-based formats, voltage *V* and current *I* into the equivalent wave-based formats, forward wave *a* and backward wave *b* and vice versa using the relations:

$a = \frac{V + I\,Z}{2}$	$V = a + b$
$b = \frac{V - I\,Z}{2}$	$I\,Z = a - b$

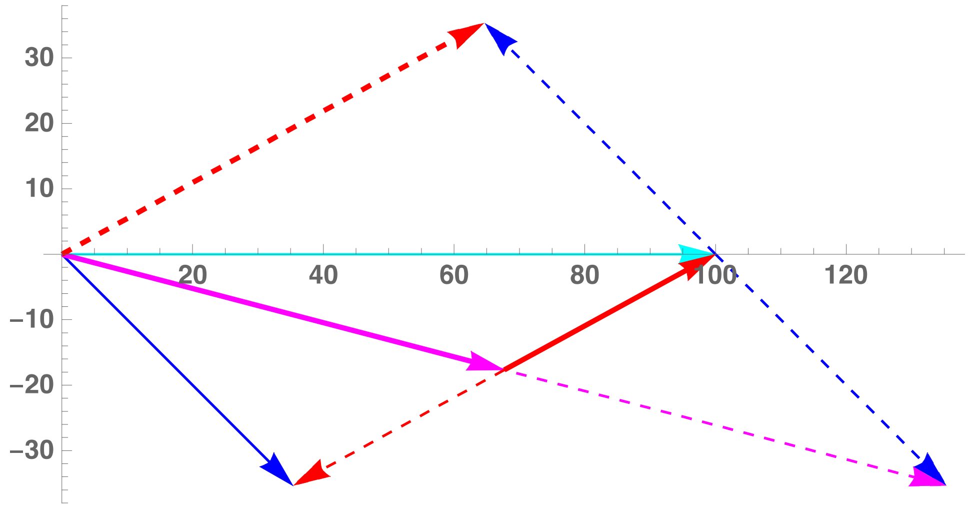
Questions:

1. In a 50 Ω system, a directional coupler measured forward and reflected waves a and b at a certain plane as: a = 100 ∠0° and b = 60 ∠45°.
   - Calculate the corresponding voltage V and current I
     $V = a + b = (100 + 42.43 + j42.43)\ V = (142.43 + j42.43)V = 148.61\ V e^{j16.59°}$
     $I = \frac{a - b}{Z} = \frac{(100 - 42.43 - j42.43)V}{50\ \Omega} = (1.15 - j0.849)\ A = 1.43\ A e^{-j36.39°}$
   - Sketch the "phasors" of V, I Z, a and b.



2. At some plane in the 50 Ω system, a voltage of V = 100 ∠0° V and a current of I = 1.0 ∠-45° A are measured.
   - Calculate the corresponding forward and backward waves a and b.
     $a = \frac{V + IZ}{2} = \frac{100\ V + (0.7 - j0.7)A\ 50\ \Omega}{2} = (67.68 - j17.68)\ = 69.95\ V e^{-j14.6°}$
     $b = \frac{V - IZ}{2} = \frac{100\ V - (0.7 - j0.7)A\ 50\ \Omega}{2} = (32.32 - j17.68)\ = 36.84\ V e^{j28.68°}$

- Sketch the "phasors" of V, I Z, a and b.



6.) Scaling laws

A cavity shall be scaled from existing designs for a frequency f_x = 318.32 MHz and C_x = 10 pF. There are three test designs, with the following parameters:

Cavity	f_{res} / MHz	*C* / pF	*Q*	Diameter / mm
A	100	7.957	10000	600
B	500	3.18	5000	200
C	3000	1.061	2000	25

Questions:

1. Which cavity is suitable as reference design?
   For the optimal candidate design, the parameter r/Q has to agree with the one calculated for the actual cavity. Using:
   $$ {}^{r}/_{Q} = \frac{1}{\omega C} $$
   we find $r/Q = 50$ for the scaled cavity,
   and $r/Q = 200(\text{cavity A}), 100\ (\text{cavity B}), 50\ (\text{cavityC})$ for the three design candidates. Therefore, **cavity C** is chosen for scaling.

2. Calculate the diameter of the new design.
   All physical dimensions of the cavity scale proportional to $\lambda \propto 1/f$.
   $$ D_x = D\frac{f}{f_x} = 0.236\,m $$
3. Calculate the expected *Q* factor for the new design, provided it will be built out of the same material as the reference design.
   Using the 3rd scaling law:
   $$ Q\frac{\delta}{\lambda} = const. $$
   we find $Q \propto \lambda/\delta$, with the skin depth δ decreasing linear with the frequency f:

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}} \propto \frac{1}{\sqrt{f}} \propto \sqrt{\lambda}$$

This results in:

$$Q \propto \sqrt{\lambda} \propto \frac{1}{\sqrt{f}} \quad \Rightarrow \quad Q_x = Q\sqrt{\frac{f}{f_x}} = 6140$$

7.) Thermal expansion and scaling laws

An accelerator cavity heats up under high RF power load.
The cavity used is constructed from a material having a:

thermal expansion coefficient:	$\Delta l/l$ = 20e-6/^{0}C (per degree Centigrade)
thermal resistivity coefficient:	$\Delta\rho/\rho$ = 4e-3/^{0}C (per degree Centigrade)

At room temperature the cavities resonance frequency is f_1 = 100 MHz and has a 3-dB bandwidth of BW_1 = 100 kHz.
Under RF power the cavity temperature increases by 100 ^{0}C (subscripts 2 apply).

Questions:

Determine

- the ratio λ_2/λ_1
  The wavelength λ scales proportional with the cavity dimension d, because λ is inverse proportional to f. Therefore:
  $$\frac{\lambda_2}{\lambda_1} = \frac{d_2}{d_1} = \frac{d_1\left(1 + \frac{\Delta l}{l}\Delta T\right)}{d_1} = 1 + \frac{\Delta l}{l}\Delta T = 1.002$$

- the ratio L_2/L_1
  The characteristic impedance $\sqrt{L/C} = R/Q$ stays constant, therefore:
  $$\frac{L_2}{L_1} = \frac{R/Q}{\omega_2} = \frac{\omega_1}{R/Q} = \frac{\omega_1}{\omega_2} = \frac{\lambda_2}{\lambda_1} = 1.002$$

- the ratio C_2/C_1
  Similar to the previous we get:
  $$\frac{C_2}{C_1} = \frac{1}{R/Q\omega_2} = \frac{R/Q\omega_1}{1} = \frac{\omega_1}{\omega_2} = \frac{\lambda_2}{\lambda_1} = 1.002$$

- the ratio Q_2/Q_1 (hint: the skin depth δ is proportional to $\sqrt{\rho/f}$
  The quality factor $Q \propto \lambda_0/\delta$, and the skin depth follows $\delta \propto \sqrt{\rho/f}$, therefore:

$$\frac{Q_2}{Q_1} = \frac{\text{const}\,\lambda_2}{\delta_2} = \frac{\delta_1}{\text{const}\,\lambda_1} = \frac{\lambda_2}{\lambda_1}\frac{\delta_1}{\delta_2} = \frac{\lambda_2}{\lambda_1}\sqrt{\frac{\rho_1}{\rho_2}\frac{f_2}{f_1}} = \frac{\lambda_2}{\lambda_1}\sqrt{\frac{\rho_1}{\rho_2}\frac{\lambda_1}{\lambda_2}} = \sqrt{\frac{\rho_1}{\rho_2}\frac{\lambda_2}{\lambda_1}} = 1.1844$$

with: $\rho_1/\rho_2 = 1 + \Delta\rho/\rho\,\Delta T = 1.4$

- the resonance frequency f_2 under load

$$\frac{f_2}{f_1} = \frac{\lambda_1}{\lambda_2} \quad \Rightarrow \quad f_2 = \frac{f_1}{\lambda_2/\lambda_1} = 99.8\,MHz$$

- and the 3-dB bandwidth BW_2 of the resonance under load

$$\Delta f_1 = \frac{f_1}{Q_1}, \Delta f_2 = \frac{f_2}{Q_2} \quad \Rightarrow \quad \frac{\Delta f_2}{\Delta f_1} = \frac{f_1}{Q_2}\frac{Q_1}{f_1}$$

$$\Delta f_2 = \frac{f_2}{f_1}\frac{1}{Q_2/Q_1}\Delta f_1 = 84.26\,kHz$$