Linear imperfections and correction

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  - K. Wille, The physics of Particle Accelerators, Oxford University Press, 2000.
  - S.Y. Lee, Accelerator Physics, 3rd edition, World Scientific, 2011.

Transverse beam dynamics reminder

Equations reminder

Lorentz equation $$\frac{d\mathbf{p}}{dt} = \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

E : total energy

T : kinetic energy $$E = \sqrt{p^2c^2 + m_0^2c^4} = T + m_0c^2 = T + E_0$$

p : momentum

β : reduced velocity $$\beta = \frac{v}{c}$$

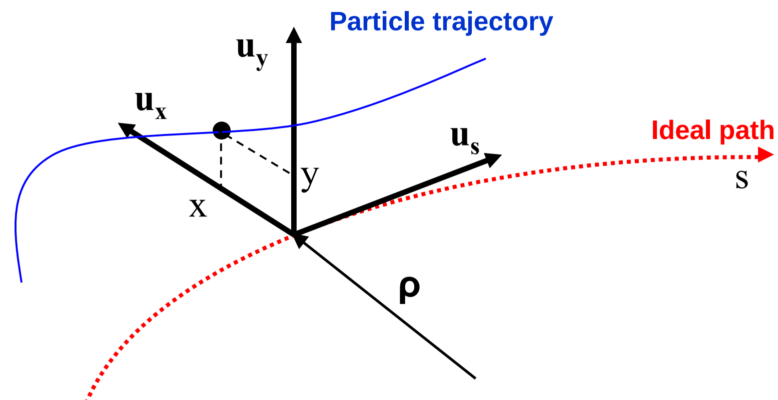
γ : reduced energy $$\gamma = \frac{E}{m_0c^2}$$

$\beta\gamma$: reduced momentum $$\beta\gamma = \frac{p}{m_0c}$$

Reference trajectory

- Cartesian coordinates not useful to describe motion in a circular accelerator (not true for linacs)
- A system following an ideal path along the accelerator is used (Frenet reference system)

$$(\mathbf{u_x}, \mathbf{u_y}, \mathbf{u_z}) \rightarrow (\mathbf{u_x}, \mathbf{u_y}, \mathbf{u_s})$$



  - The curvature vector is $\boldsymbol{\kappa} = -\frac{d^2\mathbf{s}}{ds^2}$
  - From Lorentz equation

$$\frac{d\mathbf{p}}{dt} = m_0\gamma\frac{d^2\mathbf{s}}{dt^2} = m_0\gamma v_s^2\frac{d^2\mathbf{s}}{ds^2} = -m_0\gamma v_s^2\boldsymbol{\kappa} = q[\mathbf{v}\times\mathbf{B}]$$

where we used the curvature vector definition and $\frac{d^2}{dt^2} = v_s^2\frac{d^2}{ds^2}$

- Using $m_0\gamma v_s = p_s = (p^2 - p_x^2 - p_y^2)^{1/2} \approx p$, the ideal path of the reference trajectory is defined by

$$\boldsymbol{\kappa}_0 = -\frac{q}{p}\left[\frac{\mathbf{v}}{v_s}\times\mathbf{B_0}\right]$$

Beam guidance

- Consider uniform magnetic field $\mathbf{B} = \{0, B_y, 0\}$ in a direction perpendicular to particle motion. From the reference trajectory equation, after developing the cross product and considering that the transverse velocities v_x , $v_y \ll v_s$, the radius of curvature ρ is given by

$$\frac{1}{\rho} = |k| = |\frac{q}{p}B|$$

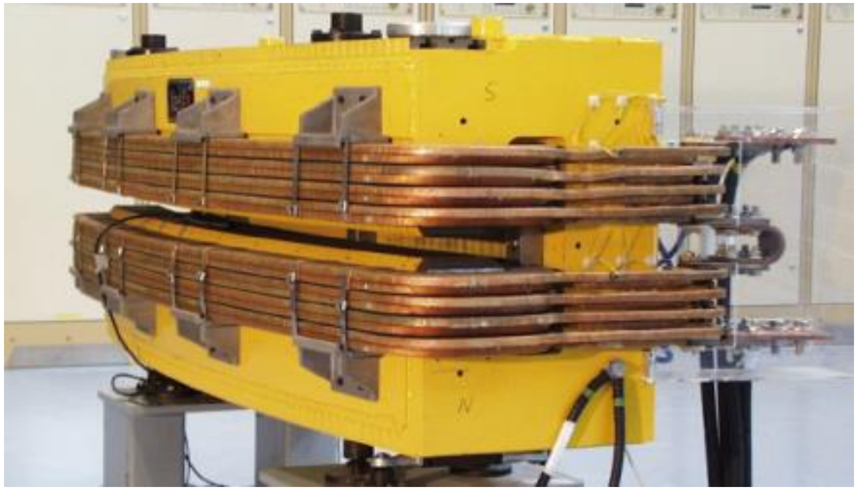
- We define the magnetic rigidity $|B\rho| = \frac{p}{q}$

- In more practical units $\beta E[GeV] = 0.2998|B\rho|[Tm]$

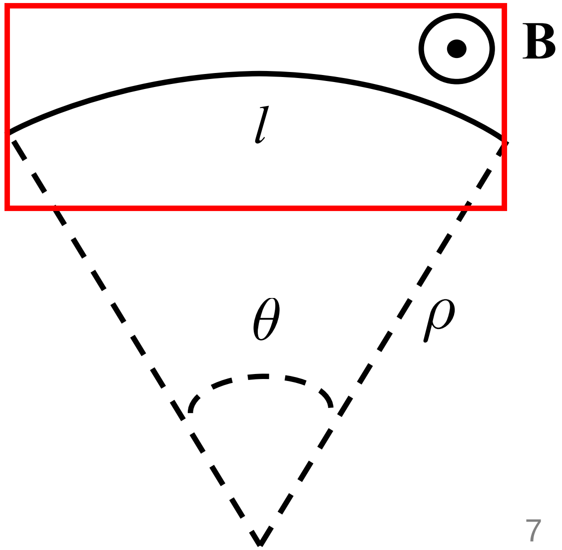
- For ions with charge multiplicity *Z* and atomic mass *A*, the energy per nucleon is

$$\beta \bar{E}[GeV/u] = 0.2998\frac{Z}{A}|B\rho|[Tm]$$

Dipoles



- Consider a ring for particles with energy E with N dipoles of length L (or effective length l, i.e. measured on beam path)
  - **Bending angle** $\theta = \frac{2\pi}{N}$
  - **Bending radius** $\rho = \frac{l}{\theta}$
  - ***Integrated dipole strength*** $Bl = \frac{2\pi}{N}\frac{p}{q}$



- Note:
  - By choosing a dipole field, the dipole length is imposed and vice versa
  - The higher the field, the shorter or smaller number of dipoles can be used
  - The ring circumference (cost) is influenced by the field choice

Beam focusing

- **Consider a particle in a dipole field**
  - In the horizontal plane
    - it performs harmonic oscillations

$$x = x_0 \cos(\omega t + \phi)$$

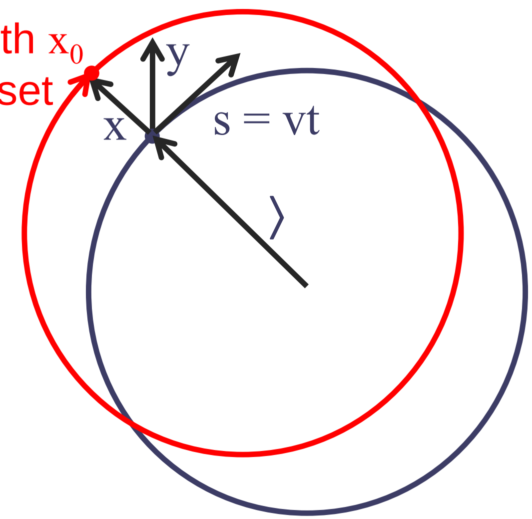
with frequency

$$\omega = \frac{v_s}{\rho}$$

    - the horizontal acceleration is described by

$$\frac{d^2x}{ds^2} = \frac{1}{v_s^2}\frac{d^2x}{dt^2} = -\frac{1}{\rho^2}x$$

    - there is a **weak focusing** effect in the horizontal plane



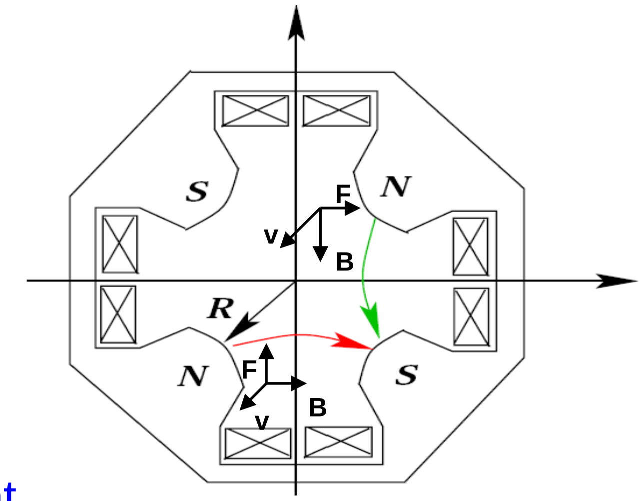
  - In the vertical plane, the only force present is gravity
    - Particles are displaced vertically following the usual law: $\Delta y = \frac{1}{2} a_g \Delta t^2$
    - With $a_g \approx 10$ m/s^2, the particle is displaced by **18 mm** (LHC dipole aperture) in **60 ms** (few hundred turns in LHC) → **need focusing!**

Quadrupoles

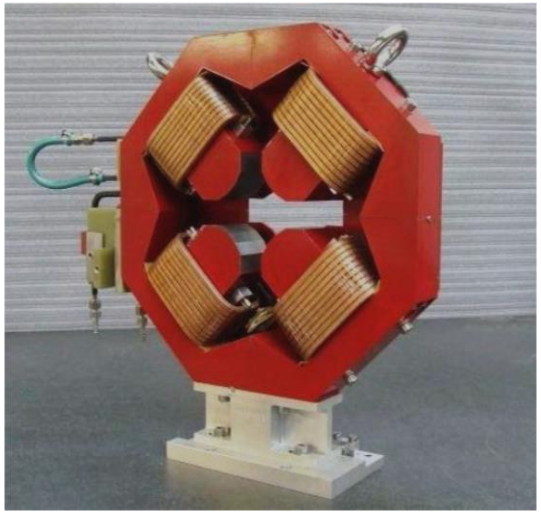
- **Quadrupoles are focusing in one plane and defocusing in the other**
  - The field is $(B_x, B_y) = G \cdot (y, x)$
  - The resulting force $(F_x, F_y) = k \cdot (-x, y)$ with the ***normalised gradient k*** defined as

$$k = \frac{G}{B\rho} = \frac{qG}{p} = \frac{qG}{\beta E}$$

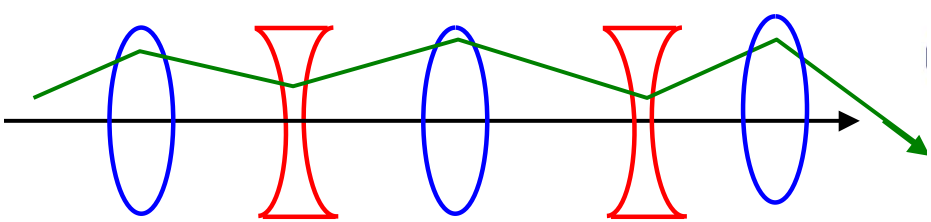
… G is the gradient



  - In practical units: $k[m^{-2}] = 0.2998 \frac{G[T/m]}{\beta E[GeV]}$
  - kL is the integrated norm. quadrupole strength



- Need to alternate focusing and defocusing to control the beam, i.e. **alternating gradient focusing**



Equations of motion – Linear fields

- Consider s-dependent fields from dipoles and normal quadrupoles

$$B_y = B_0(s) + G(s) \cdot x \qquad B_x = G(s) \cdot y$$

- The total momentum can be written $p = p_0 \left(1 + \frac{\Delta p}{p_0}\right)$
- With magnetic rigidity $B_0\rho = \frac{p_0}{q}$ and normalized gradient

$k(s) = \frac{G(s)}{B_0\rho}$ the equations of motion are

$$x'' + \left(k(s) + \frac{1}{\rho(s)^2}\right) x = \frac{1}{\rho(s)} \frac{\Delta p}{p}$$

$$y'' - k(s)\, y = 0$$

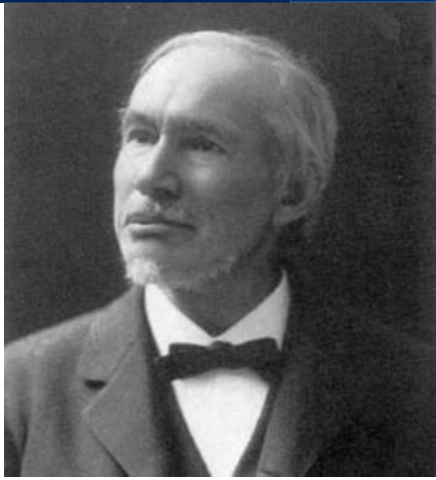
- Inhomogeneous equations with s-dependent coefficients
- The term $\frac{1}{\rho^2}$ corresponds to the dipole **weak focusing** and

$\frac{1}{\rho} \frac{\Delta p}{p}$ represents **off-momentum** particles

Hill's equations

- Solutions are combination of the homogeneous and inhomogeneous equations' solutions
- Consider particles with the design momentum. The Equations of motion become

$$\begin{aligned} x'' + K_x(s)\ x &= 0 \\ y'' + K_y(s)\ y &= 0 \end{aligned}$$



George Hill

with $K_x(s) = \left(k(s) + \frac{1}{\rho(s)^2}\right)$ and $K_y(s) = -k(s)$

- **Hill's equations of linear transverse particle motion**
  - Linear equations with s-dependent coefficients (harmonic oscillator with time dependent frequency)
  - In a ring (or in transport line with symmetries), the **coefficients are periodic** $K_x(s) = K_x(s+C)\ ,\quad K_y(s) = K_y(s+C)$
  - Not straightforward to derive analytical solutions for whole accelerator

Betatron motion

- The on-momentum linear betatron motion of a particle in both planes is described by (Floquet theorem)

$$u(s) = \sqrt{\epsilon\beta(s)}\cos(\psi(s)+\psi_0) \qquad u \mapsto \{x, y\}$$

with $\alpha,\ \beta,\ \gamma$ the twiss functions $\alpha(s) = -\frac{\beta(s)'}{2},\quad \gamma = \frac{1+\alpha(s)^2}{\beta(s)}$

ψ the betatron phase $\psi(s) = \int \frac{ds}{\beta(s)}$

and the beta function β is defined by the envelope equation

$$2\beta\beta'' - \beta'^2 + 4\beta^2 K = 4$$

- By differentiation, we have that the angle is

$$u'(s) = \sqrt{\frac{\epsilon}{\beta(s)}}\left(\sin(\psi(s)+\psi_0) + \alpha(s)\cos(\psi(s)+\psi_0)\right)$$

- From the position and angle equations it follows that

$$\cos(\psi(s)+\psi_0) = \frac{u}{\sqrt{\epsilon\beta(s)}}\ , \quad \sin(\psi(s)+\psi_0) = \sqrt{\frac{\beta(s)}{\epsilon}}u' + \frac{\alpha(s)}{\sqrt{\epsilon\beta(s)}}u$$

- Expand the trigonometric formulas and set $\psi(0)=0$ to get the transfer matrix from location 0 to s

$$\begin{pmatrix} u(s) \\ u'(s) \end{pmatrix} = \mathcal{M}_{0\to s} \begin{pmatrix} u_0 \\ u'_0 \end{pmatrix}$$

with

$$\mathcal{M}_{0\to s} = \begin{pmatrix} \sqrt{\frac{\beta(s)}{\beta_0}}(\cos\Delta\psi + \alpha_0\sin\Delta\psi) & \sqrt{\beta(s)\beta_0}\sin\Delta\psi \\ \frac{(\alpha_0-\alpha(s))\cos\Delta\psi-(1+\alpha_0\alpha(s))\sin\Delta\psi}{\sqrt{\beta(s)\beta_0}} & \sqrt{\frac{\beta_0}{\beta(s)}}(\cos\Delta\psi - \alpha(s)\sin\Delta\psi) \end{pmatrix}$$

and $\mu(s) = \Delta\psi = \int_0^s \frac{ds}{\beta(s)}$ the **phase advance**

Periodic transfer matrix

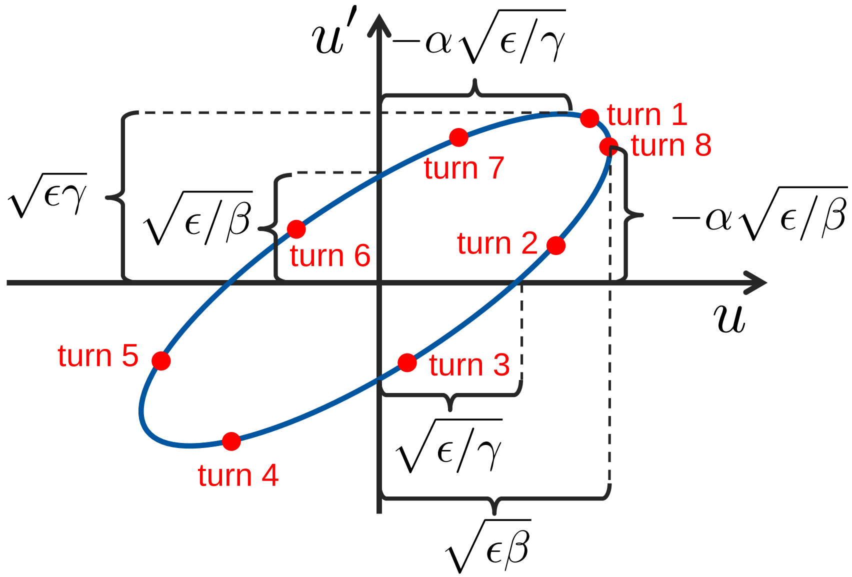
- **Consider a periodic cell of length *C***
  - The optics functions are $\beta_0 = \beta(C) = \beta\ , \quad \alpha_0 = \alpha(C) = \alpha$

    and the phase advance $\mu = \int_0^C \frac{ds}{\beta(s)}$
  - The transfer matrix is $\mathcal{M}_C = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$
  - The cell matrix can be also written as

$$\mathcal{M}_C = \mathcal{I}\cos\mu + \mathcal{J}\sin\mu$$

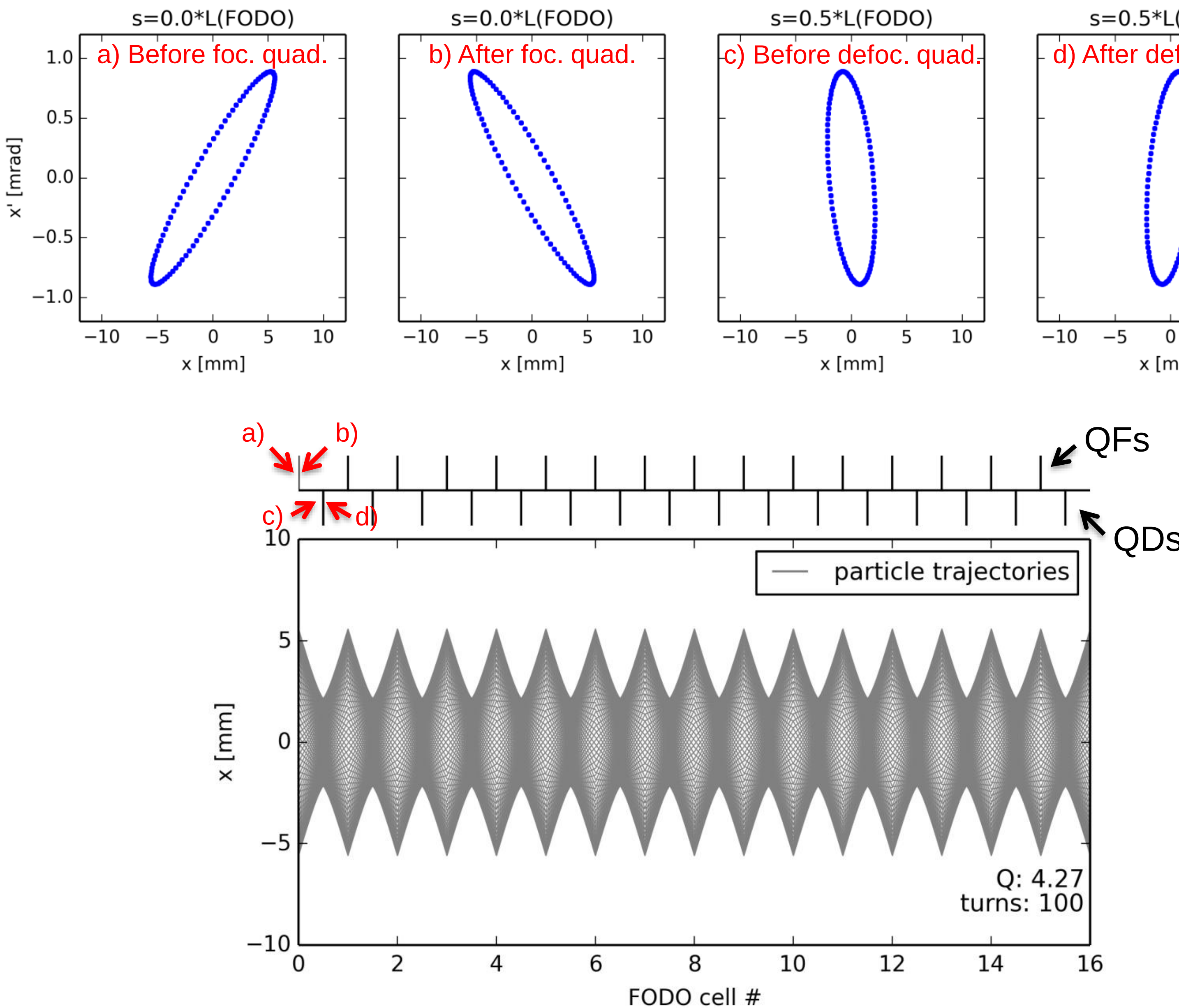
with $\mathcal{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and the **Twiss matrix** $\mathcal{J} = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix}$

Phase space ellipse



- The **phase space coordinates (u, u')** of a single particle at a given location s of the machine **lie on the phase space ellipse** when plotted for several turns
- The **values of the Twiss parameters** and therefore the orientation of the phase space ellipse **depend on the s location in the machine**
- The **Twiss parameters are periodic with the machine circumference.** Their values are derived from the transfer matrix and they are uniquely defined at any point in the machine

Illustration on a FODO lattice



Tune and working point

- In a ring, the **betatron tune** is defined from the 1-turn phase advance

$$Q_{x,y} = \frac{1}{2\pi}\oint \frac{ds}{\beta_{x,y}(s)} = \frac{\nu_{x,y}}{2\pi}$$

  i.e. number of betatron oscillations per turn

- The tune is defined by the quadrupole arrangement and strength around the machine
- The position of the tunes in a diagram of horizontal versus vertical tune is called **working point**
- The tunes are imposed by the choice of the quadrupole strengths
- One should try to avoid **resonance conditions**

Transverse linear imperfections and correction

Outline

- **Introduction**
- **Closed orbit distortion (steering error)**
  - Beam orbit stability
  - Imperfections leading to closed orbit distortion
  - Effect of single and multiple dipole kicks
  - Closed orbit correction methods
  - Dispersion and chromatic orbit
- **Optics function distortion (gradient error)**
  - Imperfections leading to optics distortion
  - Tune-shift and beta distortion due to gradient errors
  - Gradient error correction
- **Coupling error**
  - Coupling errors and their effect
  - Coupling correction
- **Chromaticity**

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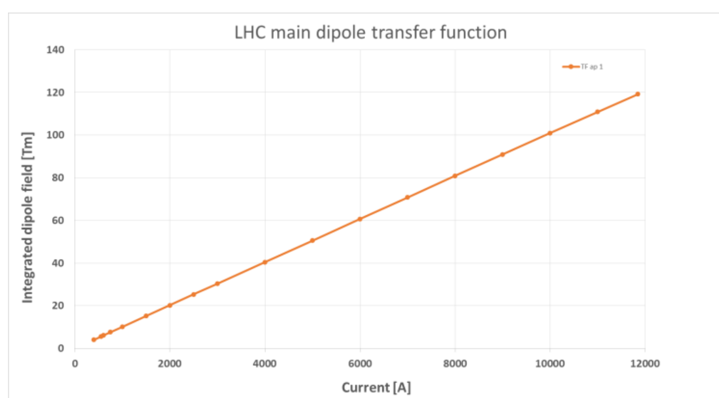
From model to reality - fields

- The physical units of the machine model defined by the accelerator physicist must be converted into magnetic fields and eventually into currents for the power converters that feed the magnet circuits.
- **Imperfections** (= errors) in the real accelerator optics can be introduced by uncertainties or errors on: beam momentum, magnet calibrations and power converter regulation.

Magnet strength → **Magnetic field (gradient)** → **Requested current** → **Actual magnet current**

↑ **Beam momentum** ↑ **Magnet calibration curve (transfer function)** ↑ **Power converter**

Example of the LHC main dipole calibration curve



From model to reality - alignment

- To ensure that the accelerator elements are in the correct position the alignment must be precise – to the level of micrometers for CLIC !
  - For CERN hadron machines we aim for accuracies of around 0.1 mm.
- The alignment process implies:
  - Precise measurements of the magnetic axis in the laboratory with reference to the element alignment markers used by the survey group.
  - Precise in-situ alignment (position and angle) of the element in the tunnel.
- **Alignment errors are a common source of imperfections**

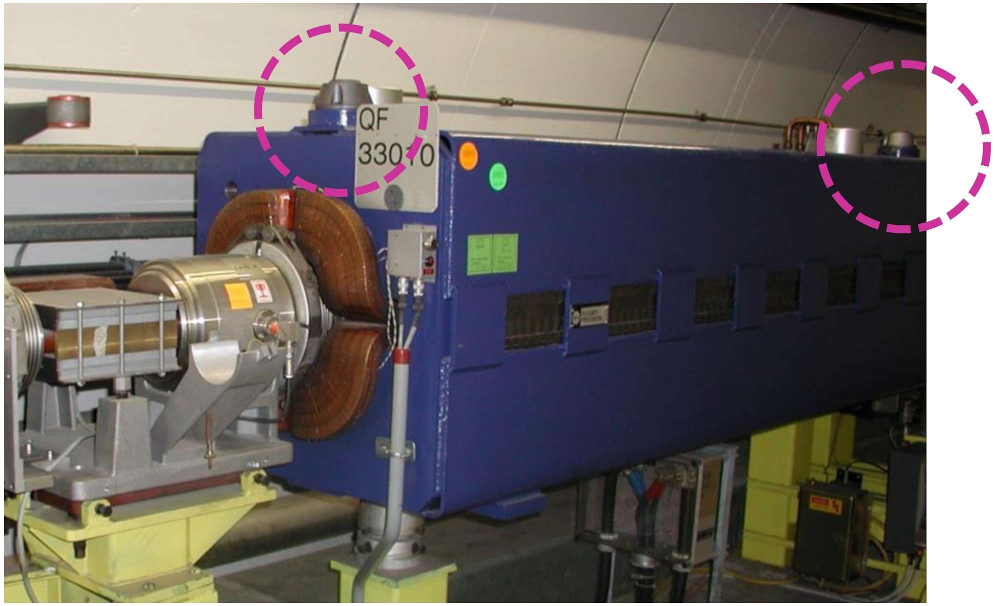


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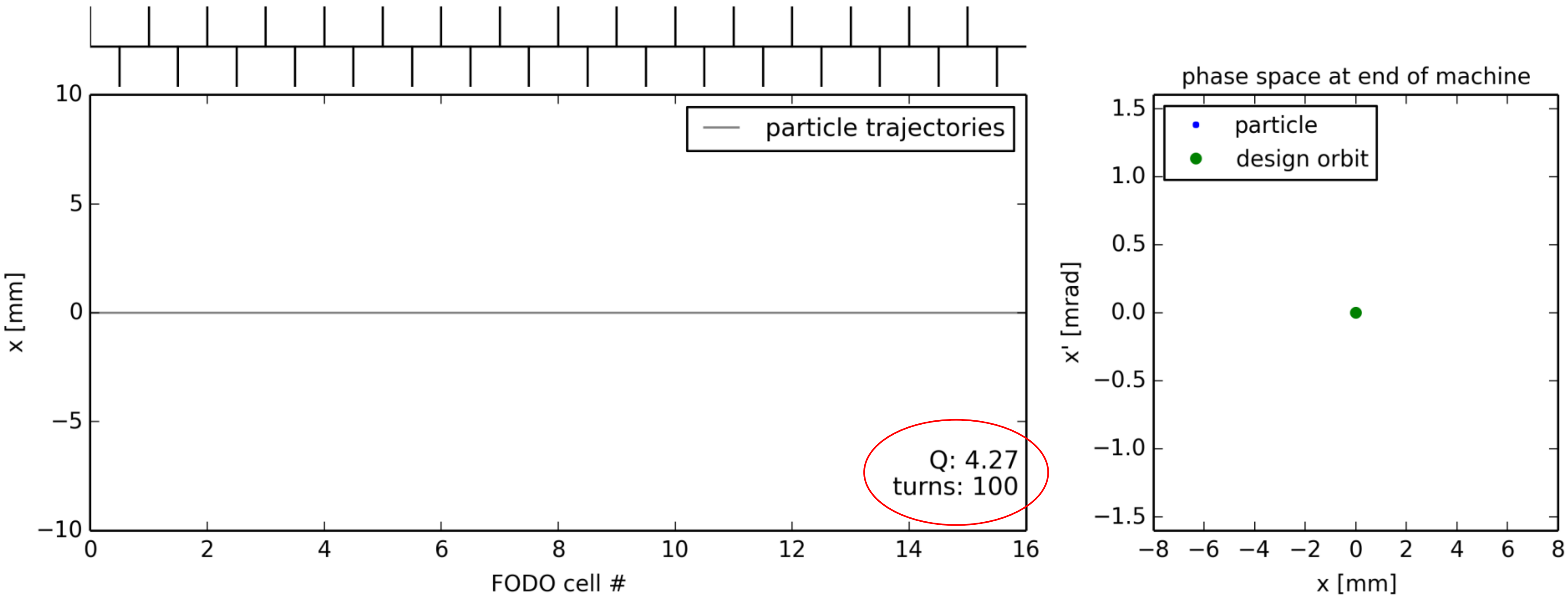
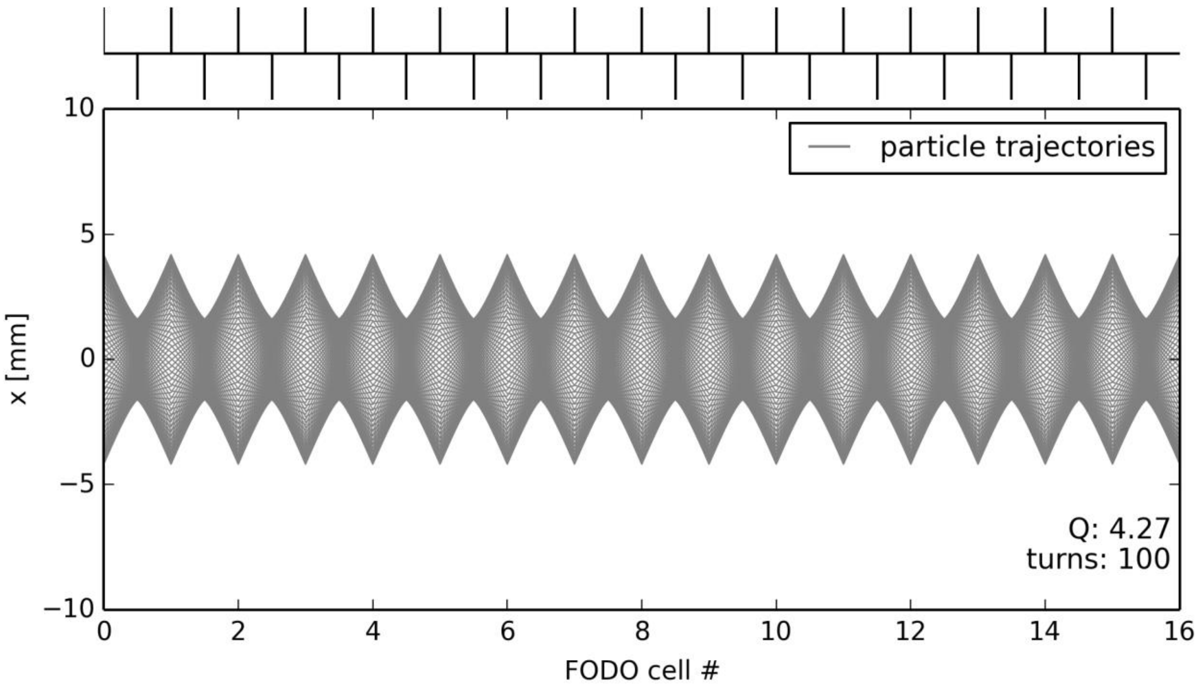
Illustration of closed orbit distortion

1. Ideal machine toy model (no errors)

a) Particle injected on the design (or reference) orbit … remains on the design orbit turn after turn



1. **Ideal machine toy model (no errors)**
   a) Particle injected on the design (or reference) orbit … remains on the design orbit turn after turn
   b) Particle injected with offset



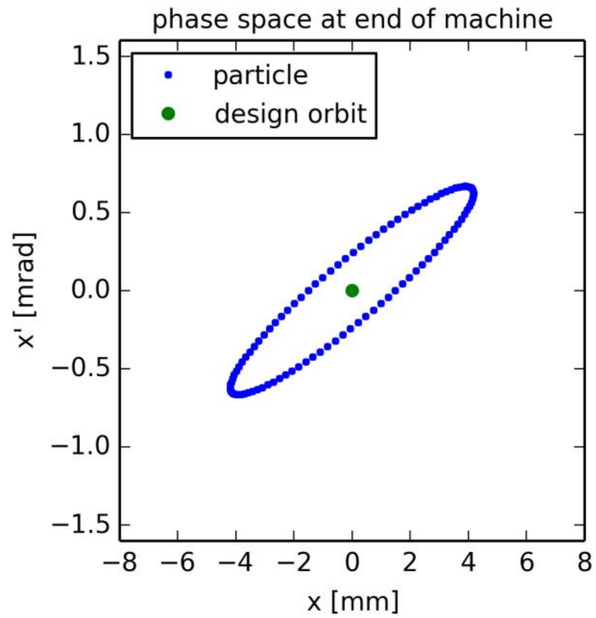
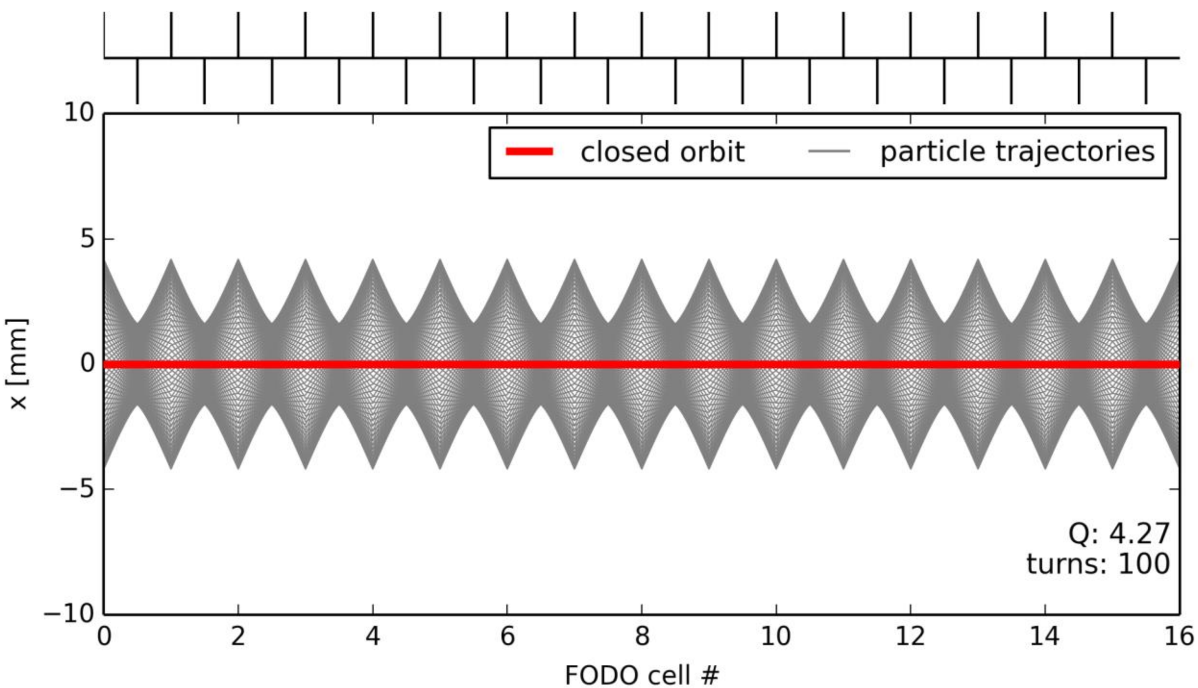


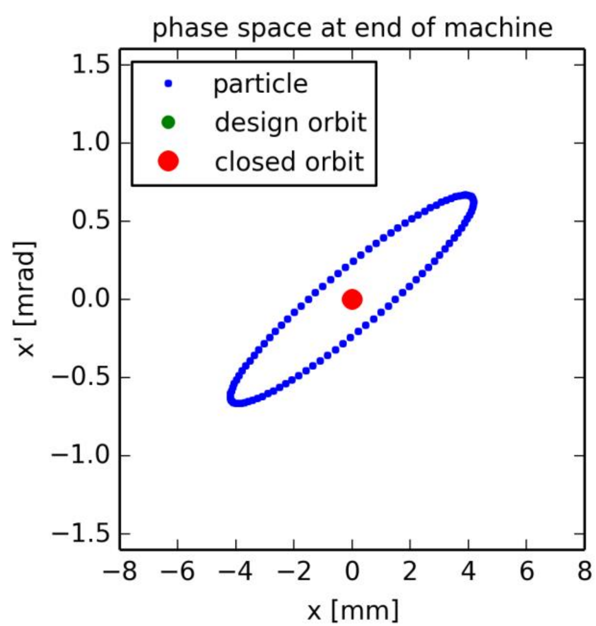
Illustration of closed orbit distortion

1. Ideal machine toy model (no errors)

a) Particle injected on the design (or reference) orbit … remains on the design orbit turn after turn

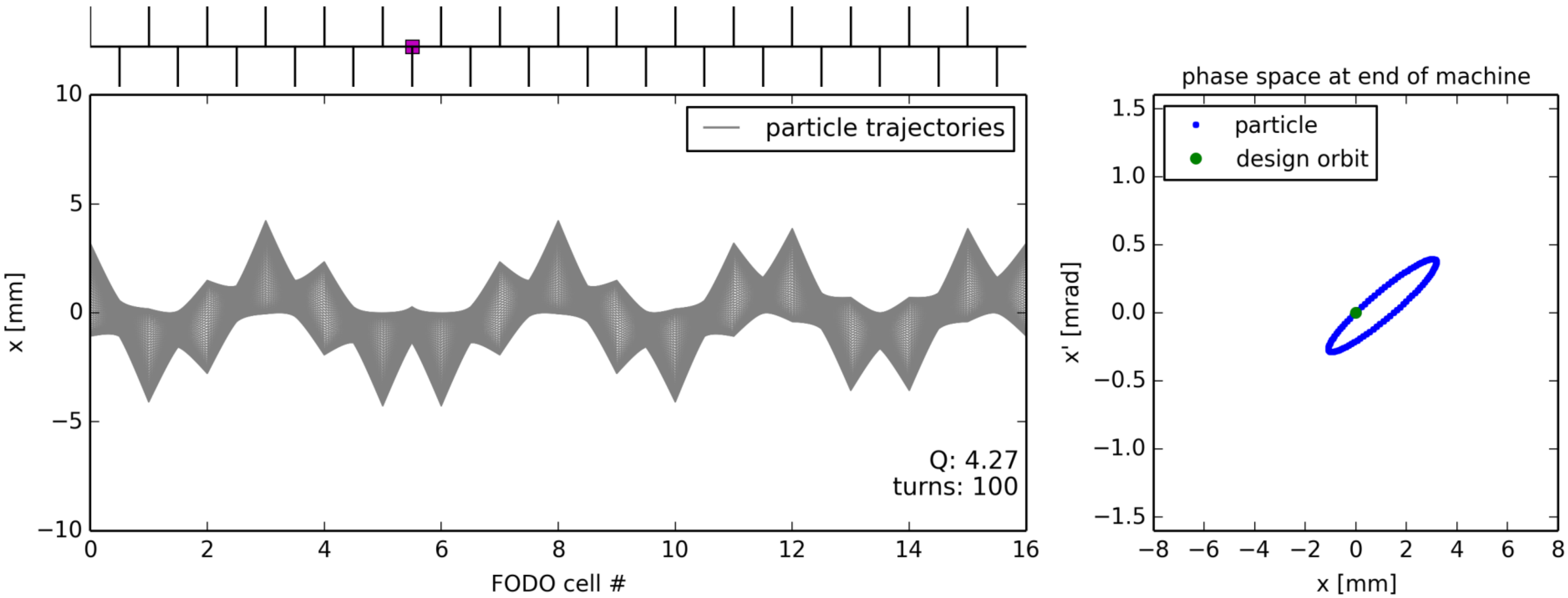
b) Particle injected with offset … performs betatron oscillations around the **closed orbit** which is the same as design orbit as long as there are no imperfections





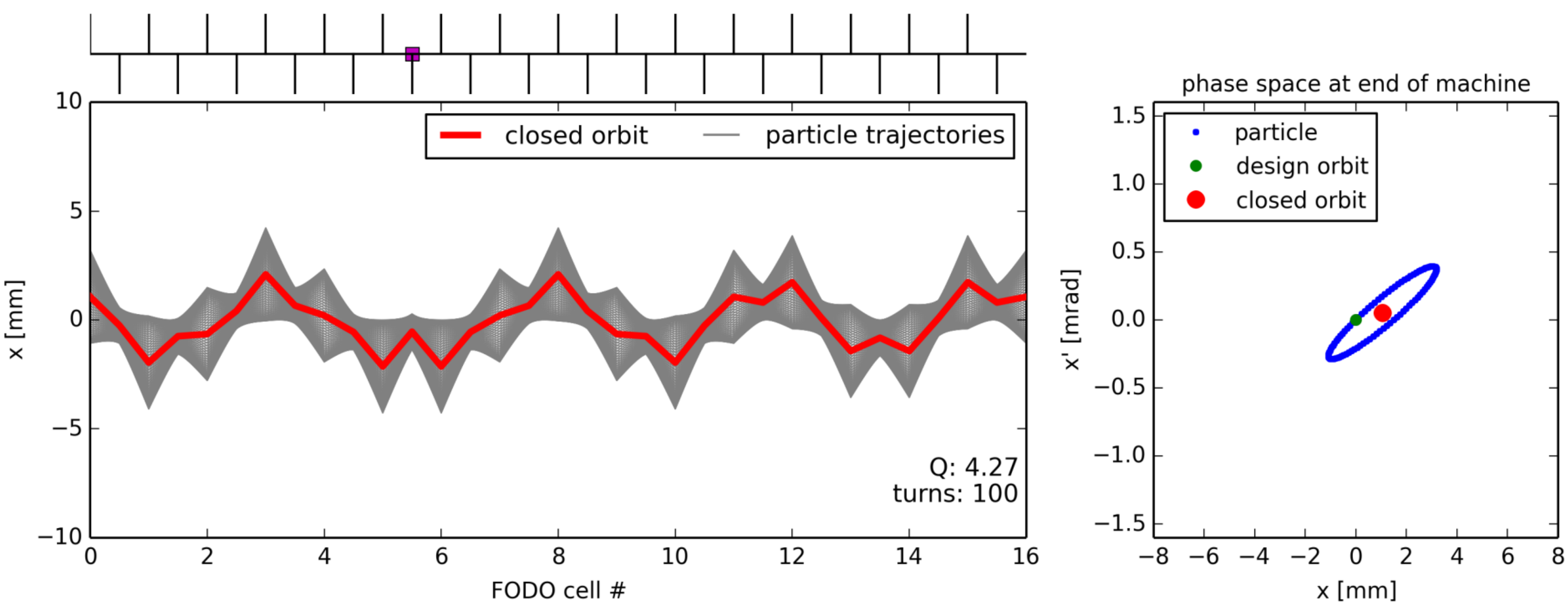
2. **Ideal machine toy model with dipole error (unintended deflection) somewhere in the lattice**

   a) Particle injected on the design orbit … receives dipole kick every turn



2. **Ideal machine toy model with dipole error (unintended deflection) somewhere in the lattice**

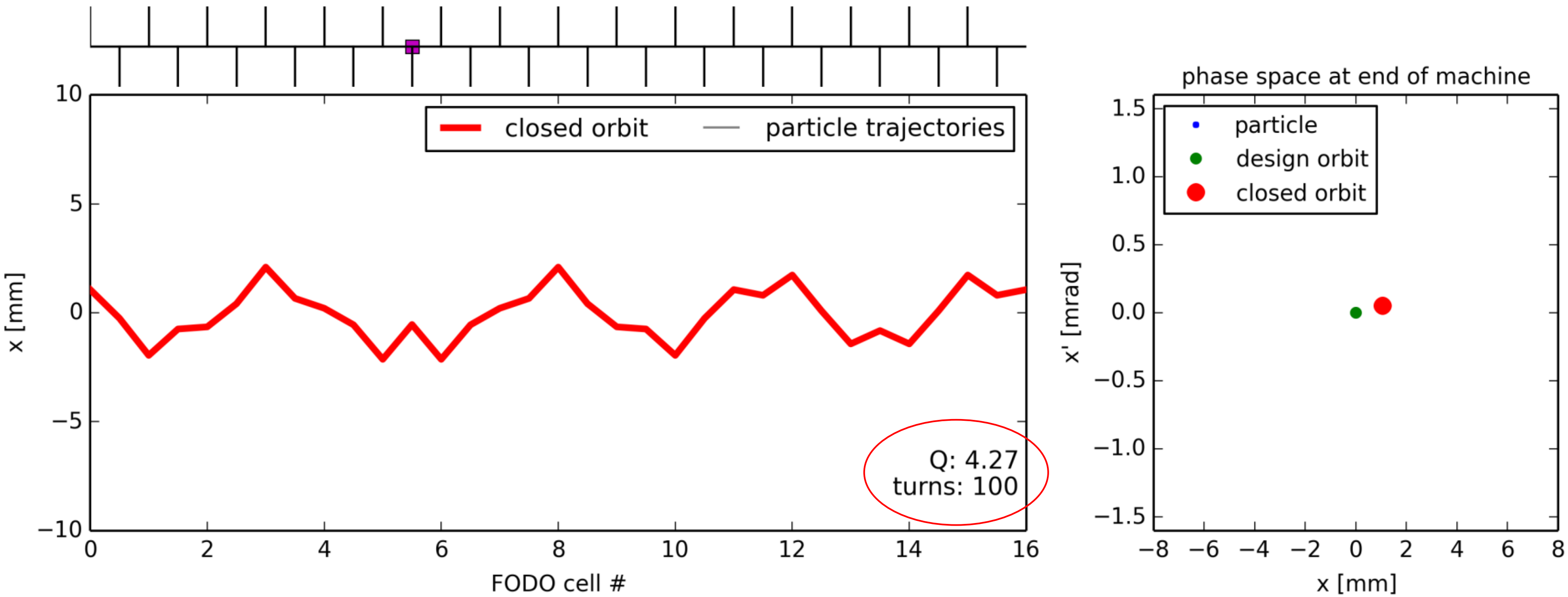
   a) <u>Particle injected on the design orbit … receives dipole kick every turn … and consequently performs betatron oscillation around a **distorted closed orbit**</u>



2. **Ideal machine toy model with dipole error (unintended deflection) somewhere in the lattice**

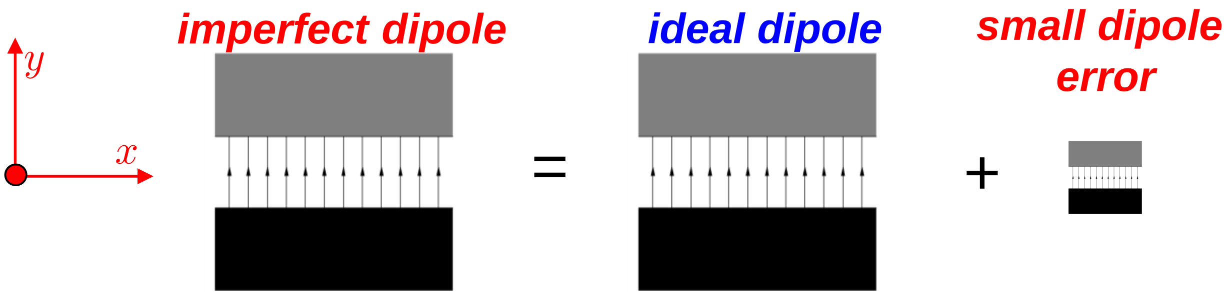
    a) Particle injected on the design orbit … receives dipole kick every turn … and consequently performs betatron oscillation around a **distorted closed orbit**

    b) Particle injected onto distorted closed orbit remains on closed orbit



Sources of unintended deflections

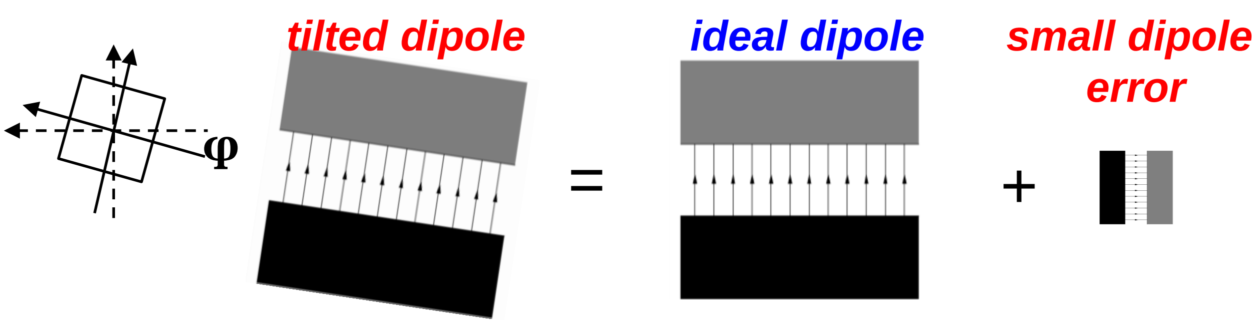
- **Field error (deflection error) of a dipole magnet**
  - This can be due to an **error** in the **magnet current** or in the **calibration table** (measurement accuracy etc.)
  - The imperfect dipole can be expressed as the ideal one + a small error



→ horizontal kick

$$\theta = \frac{\delta(Bl)}{B\rho}$$

- **A small rotation (misalignment) of a dipole magnet has the same effect, but (mostly) in the other plane**

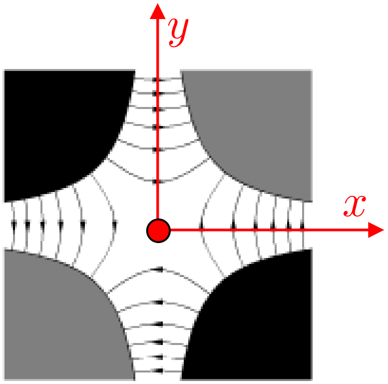


→ vertical kick

$$\theta = \frac{Bl \sin \phi}{B\rho}$$

Misalignments causing feed-down

- **Misalignment of a <u>quadrupole</u> magnet**
  - Equivalent to perfectly aligned quadrupole plus small dipole



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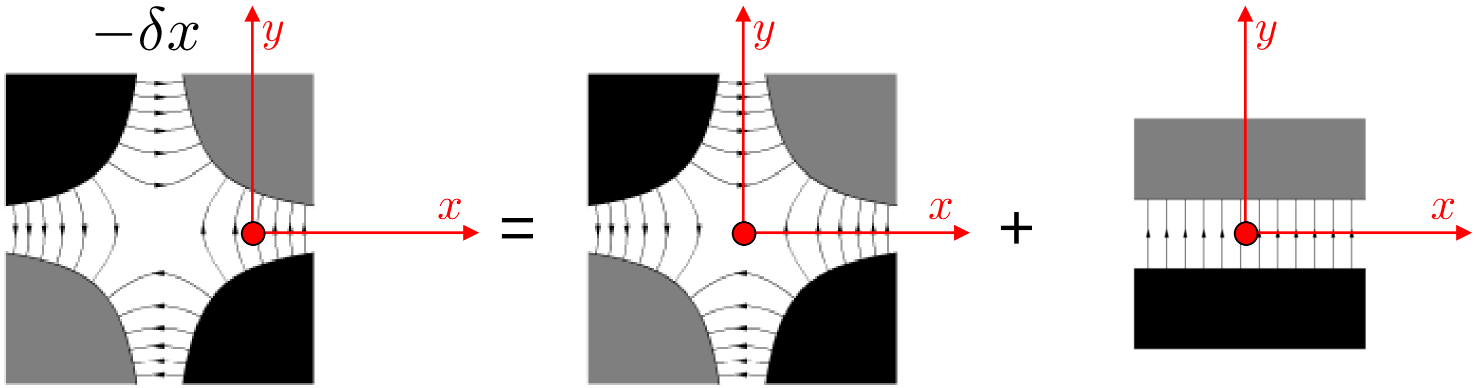

$$B_y = G(x+\delta x) = \underbrace{Gx}_{\text{quadrupole}} + \underbrace{G\delta x}_{\text{dipole}}$$

Misalignments causing feed-down

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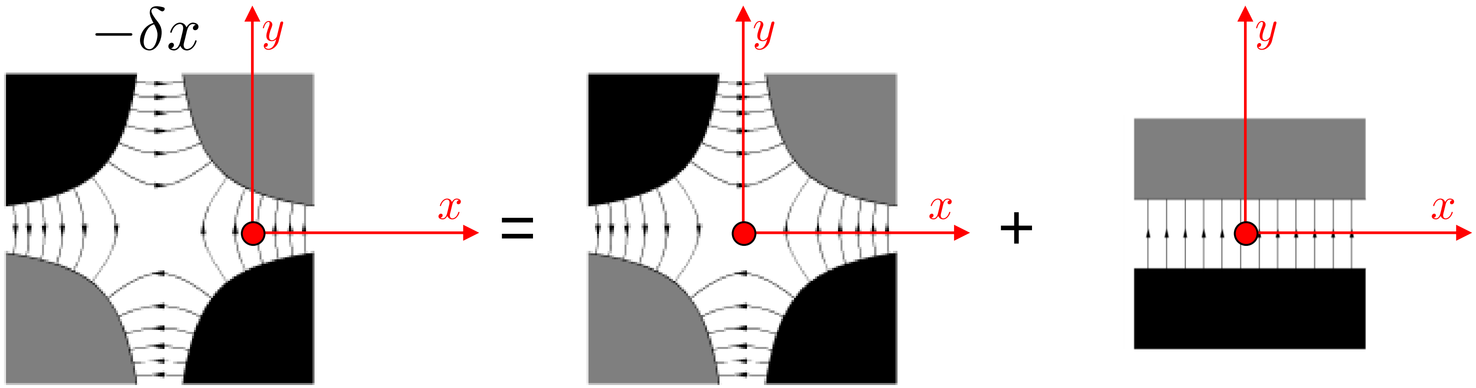
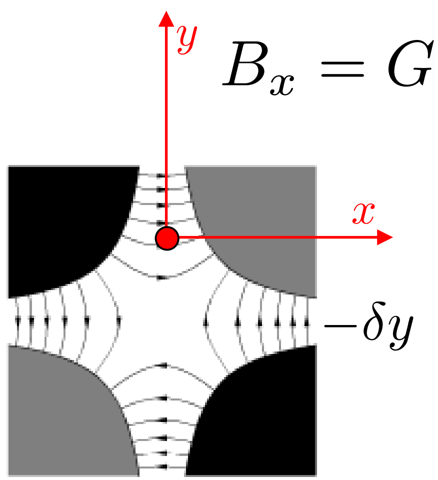


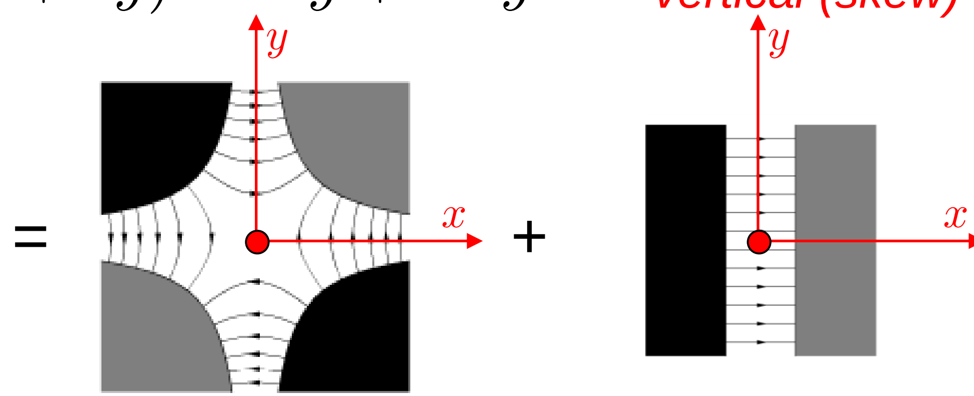
$$B_y = G(x+\delta x) = \underbrace{Gx}_{\text{quadrupole}} + \underbrace{G\delta x}_{\text{dipole}}$$

horizontal offset creates horizontal (normal) dipole

$$B_x = G(y+\delta y) = Gy + G\delta y$$

vertical offset creates vertical (skew) dipole





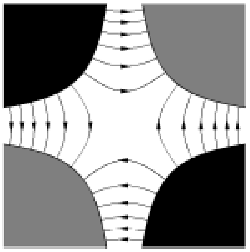
Multipole expansion

Multipole expansion of transverse magnetic field

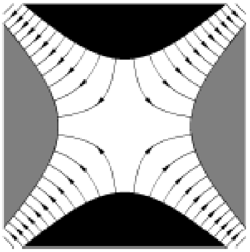
- Start from the general expression for the transverse magnetic flux in terms of multipole coefficients

$$\mathbf{B} = B_y + iB_x = \sum_{n=0}^{\infty}(B_n + iA_n)\cdot(x+iy)^n$$

e.g. normal quad



e.g. skew quad



Normal components

("upright" magnets)

$$B_n = \frac{1}{n!}\frac{\partial^n B_y}{\partial x^n}\bigg|_{(0,0)}$$

Skew components

(magnets rotated by $\frac{\pi}{2(n+1)}$)

$$A_n = \frac{1}{n!}\frac{\partial^n B_x}{\partial y^n}\bigg|_{(0,0)}$$

- In some cases it is more convenient to use "normalized" components:

Normalized normal components

$$k_n = \frac{1}{B_0\rho_0}\frac{\partial^n B_y}{\partial x^n}\bigg|_{(0,0)} = \frac{n!}{B_0\rho_0}B_n\bigg|_{(0,0)}$$

Normalized skew components

$$j_n = \frac{1}{B_0\rho_0}\frac{\partial^n B_x}{\partial y^n}\bigg|_{(0,0)} = \frac{n!}{B_0\rho_0}A_n\bigg|_{(0,0)}$$

so that:

$$B_y + iB_x = B_0\rho_0\sum_{n=0}^{M}(k_n + ij_n)\frac{(x+iy)^n}{n!}$$

Feed-down from multipoles

- Explicitly: the vertical field is the sum of all multipole components

$$B_y = \underbrace{B_0}_{\text{dipole}} + \underbrace{B_1 x - A_1 y}_{\text{quadrupole}} + \underbrace{B_2(x^2 - y^2) - 2A_2 xy}_{\text{sextupole}} + \underbrace{B_3(x^3 - 3xy^2) + A_3(y^3 - 3x^2 y)}_{\text{octupole}} + \ldots$$

- **Feed-down: lower order field components from misalignments**
  - Systematic horizontal offset in normal (skew) magnets creates normal (skew) feed-down components as seen with $\bar{x} = x + \delta x$ at y=0:

$$B_x(y\!=\!0) = A_n \bar{x}^n = A_n (x + \delta x)^n = A_n \left(x^n + n\delta x x^{n-1} + \tfrac{n(n-1)}{2}\delta x^2 x^{n-2} + \cdots + (\delta x)^n\right)$$

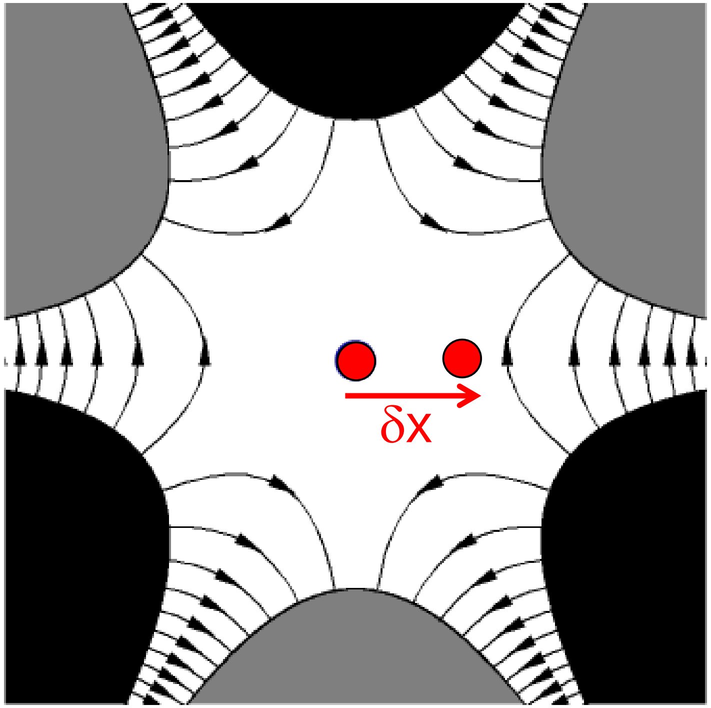
$$B_y(y\!=\!0) = B_n \bar{x}^n = B_n (x + \delta x)^n = B_n \Big(\underbrace{x^n}_{2(n+1)\text{-pole}} + \underbrace{n\delta x x^{n-1}}_{2n\text{-pole}} + \underbrace{\tfrac{n(n-1)}{2}\delta x^2 x^{n-2}}_{2(n-1)\text{-pole}} + \cdots + \underbrace{(\delta x)^n}_{\text{dipole}}\Big)$$

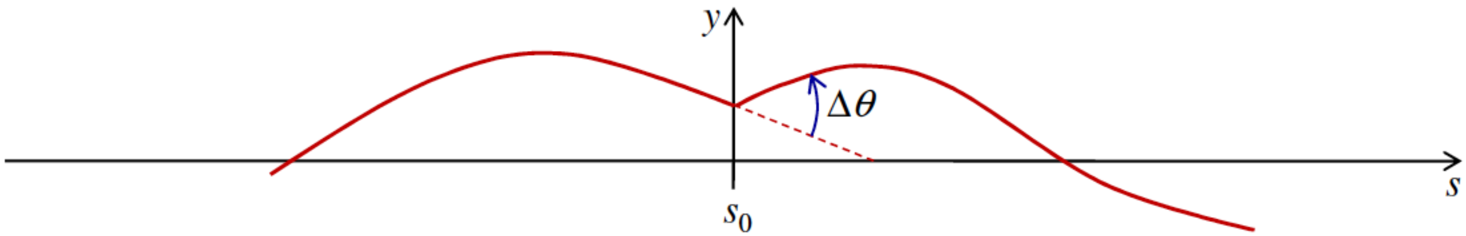
  - Systematic vertical offset in normal magnets results in alternating skew and normal feed-down components (and vice-versa for skew magnets), as can be worked out from

for n = even $\begin{cases} B_y(x\!=\!0) = i^n B_n \bar{y}^n \\ B_x(x\!=\!0) = i^n A_n \bar{y}^n \end{cases}$ for n = odd $\begin{cases} B_y(x\!=\!0) = i^{n+1} A_n \bar{y}^n \\ B_x(x\!=\!0) = i^{n-1} B_n \bar{y}^n \end{cases}$

Problem 1

Derive an expression for the resulting magnetic field when the closed orbit in a normal sextupole is displaced by **δx** from its center position. What are the resulting field components? Do the same for an octupole. What is the leading order multi-pole field error when displacing a general **2(n+1)**-pole magnet?





- Consider a single dipole kick $\theta = \delta u_0' = \delta u'(s_0) = \frac{\delta(Bl)}{B\rho}$ at s=s_0
- The coordinates before and after the kick are

$$\begin{pmatrix} u_0 \\ u_0' - \theta \end{pmatrix} = \mathcal{M} \begin{pmatrix} u_0 \\ u_0' \end{pmatrix}$$

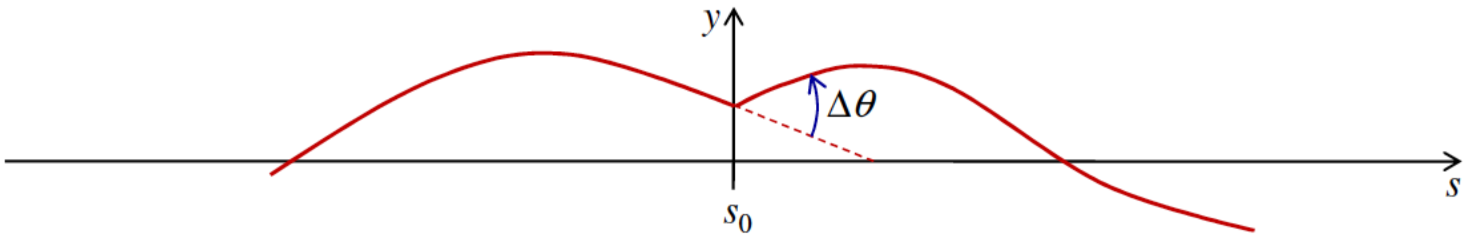
- Taking the solutions of Hill's equations (u and u') at the location of the kick, the orbit will close to itself only if

$$\sqrt{\epsilon\beta_0}\cos(\phi_0) = \sqrt{\epsilon\beta_0}\cos(\phi_0 + 2\pi Q)$$

$$\sqrt{\frac{\epsilon}{\beta_0}}(\sin(\phi_0) + \alpha_0\cos(\phi_0)) - \theta = \sqrt{\frac{\epsilon}{\beta_0}}(\sin(\phi_0 + 2\pi Q) + \alpha_0\cos(\phi_0 + 2\pi Q))$$

- This yields the following relations for the invariant and phase

$$\epsilon = \frac{\beta_0\theta^2}{4\sin^2(\pi Q)}, \quad \phi_0 = -\pi Q$$



- The initial conditions of the closed orbit at the location of the kick are therefore obtained as

$$u_0 = \theta \frac{\beta_0}{2\tan\pi Q} \quad \text{and} \quad u_0' = \frac{\theta}{2}\left(1 - \frac{\alpha_0}{\tan\pi Q}\right)$$

- For any location around the ring, the orbit distortion is written as

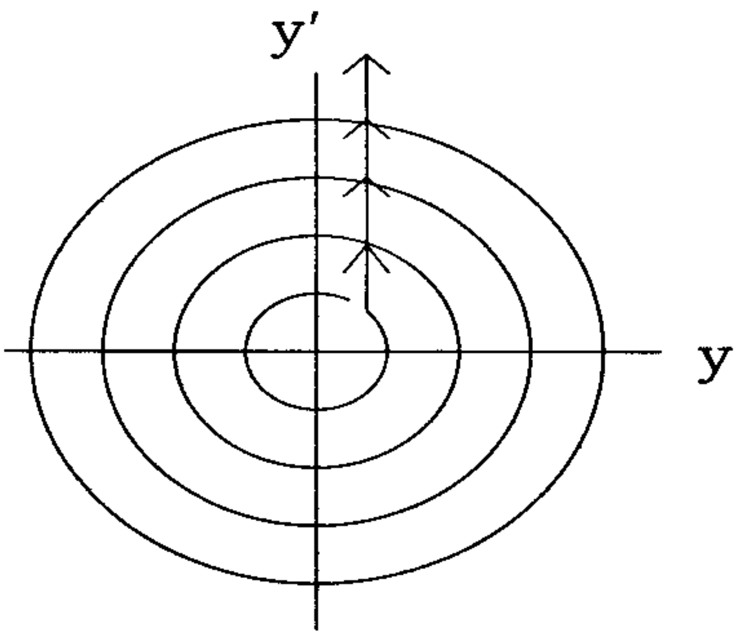
$$u(s) = \underbrace{\theta \frac{\sqrt{\beta(s)\beta_0}}{2\sin(\pi Q)}}_{\text{maximum distortion amplitude}} \cos(\pi Q - |\psi(s) - \psi_0|)$$

maximum distortion amplitude

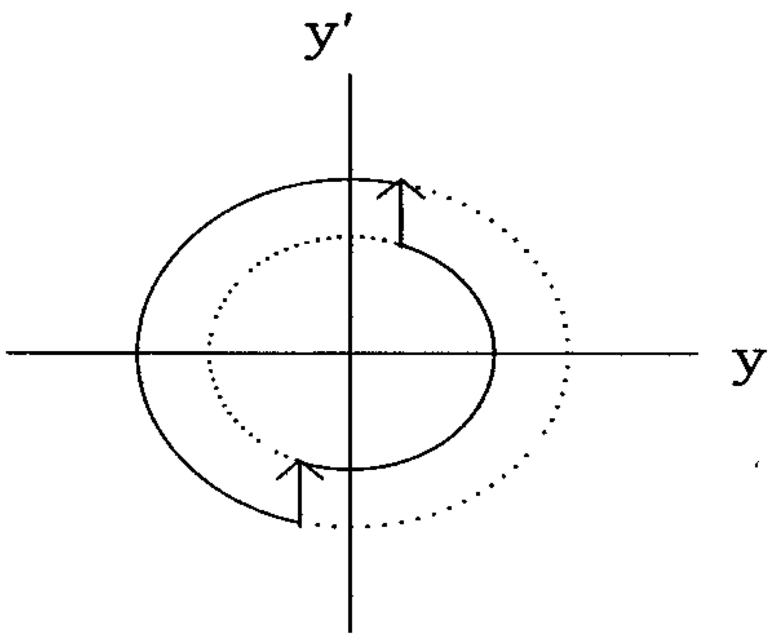
Integer and half integer resonance

$$u(s) = \theta \frac{\sqrt{\beta(s)\beta_0}}{2\sin(\pi Q)} \cos(\pi Q - |\psi(s) - \psi_0|)$$

- Dipole kicks add-up in consecutive turns for **Q = n**
- Integer tune excites orbit oscillations (resonance) → orbit becomes unstable

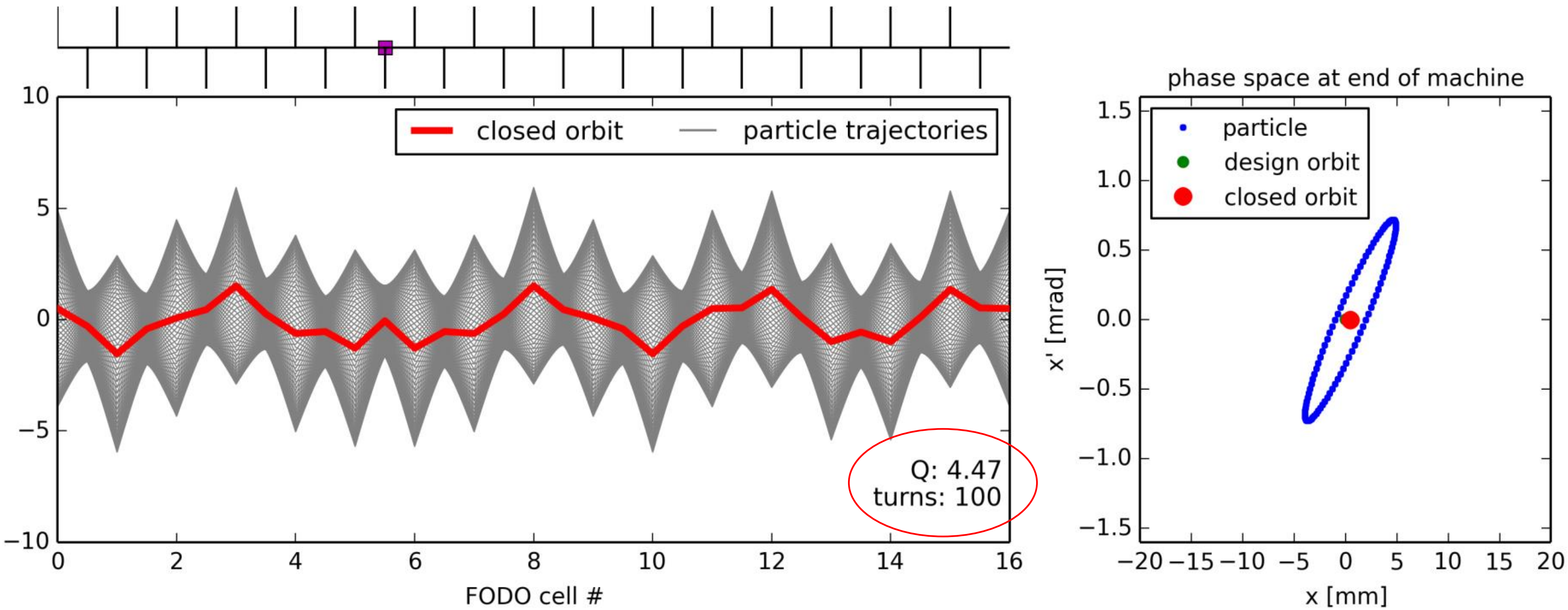


- Dipole kicks get cancelled in consecutive turns for **Q = n/2**
- Half-integer tune cancels orbit oscillations



Single dipole kick vs. tune

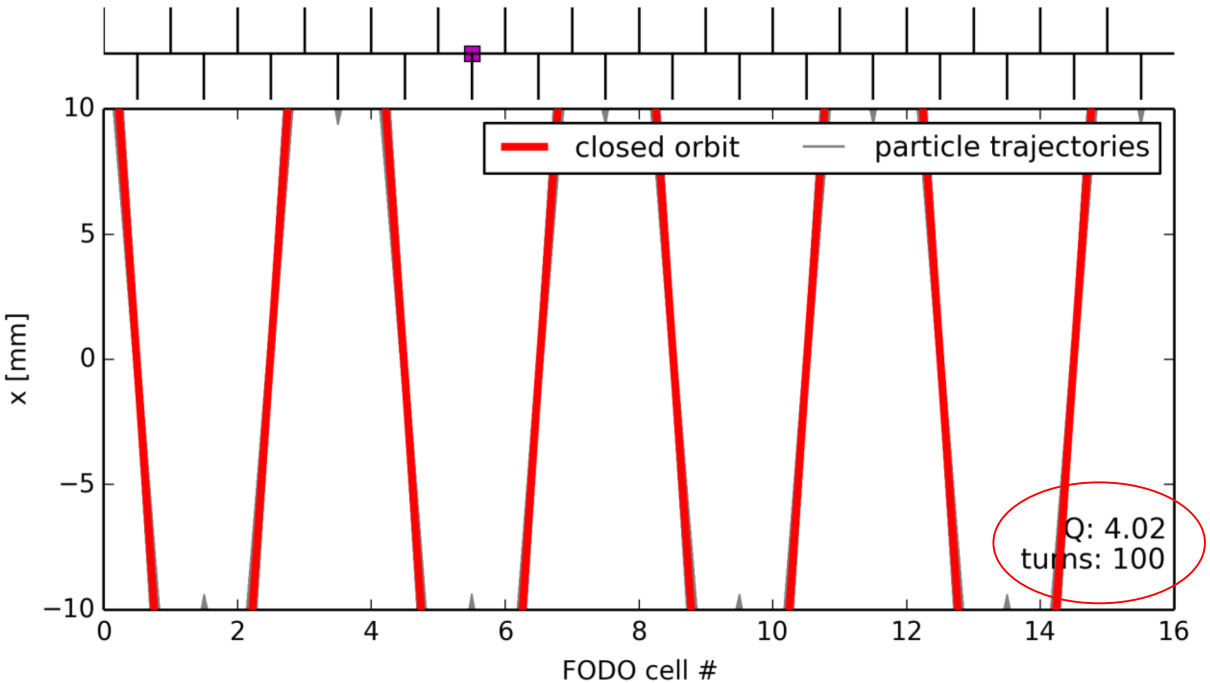
$$u(s) = \theta \frac{\sqrt{\beta(s)\beta_0}}{2\sin(\pi Q)} \cos(\pi Q - |\psi(s) - \psi_0|)$$

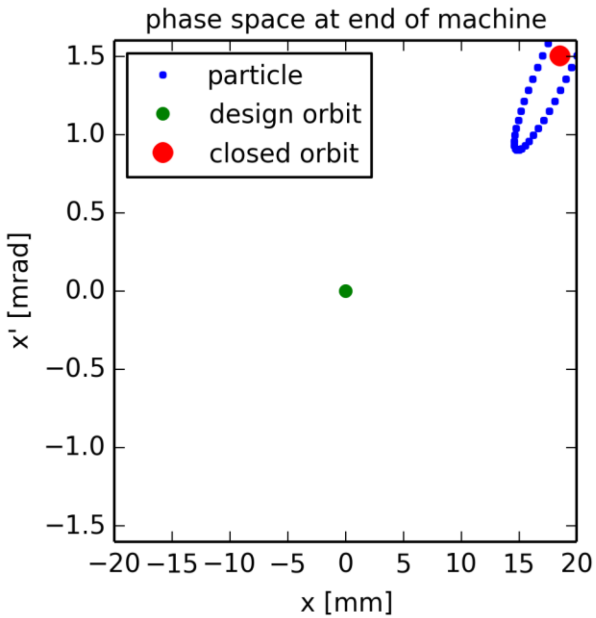


Single dipole kick vs. tune

- Closed orbit distortion is most critical for **tunes close to integer → closed orbit becomes unstable (but beam size not affected)**
- The closed orbit distortion propagates with the betatron phase advance (e.g. single kick induces 4 oscillations for a tune of Q=4.x)

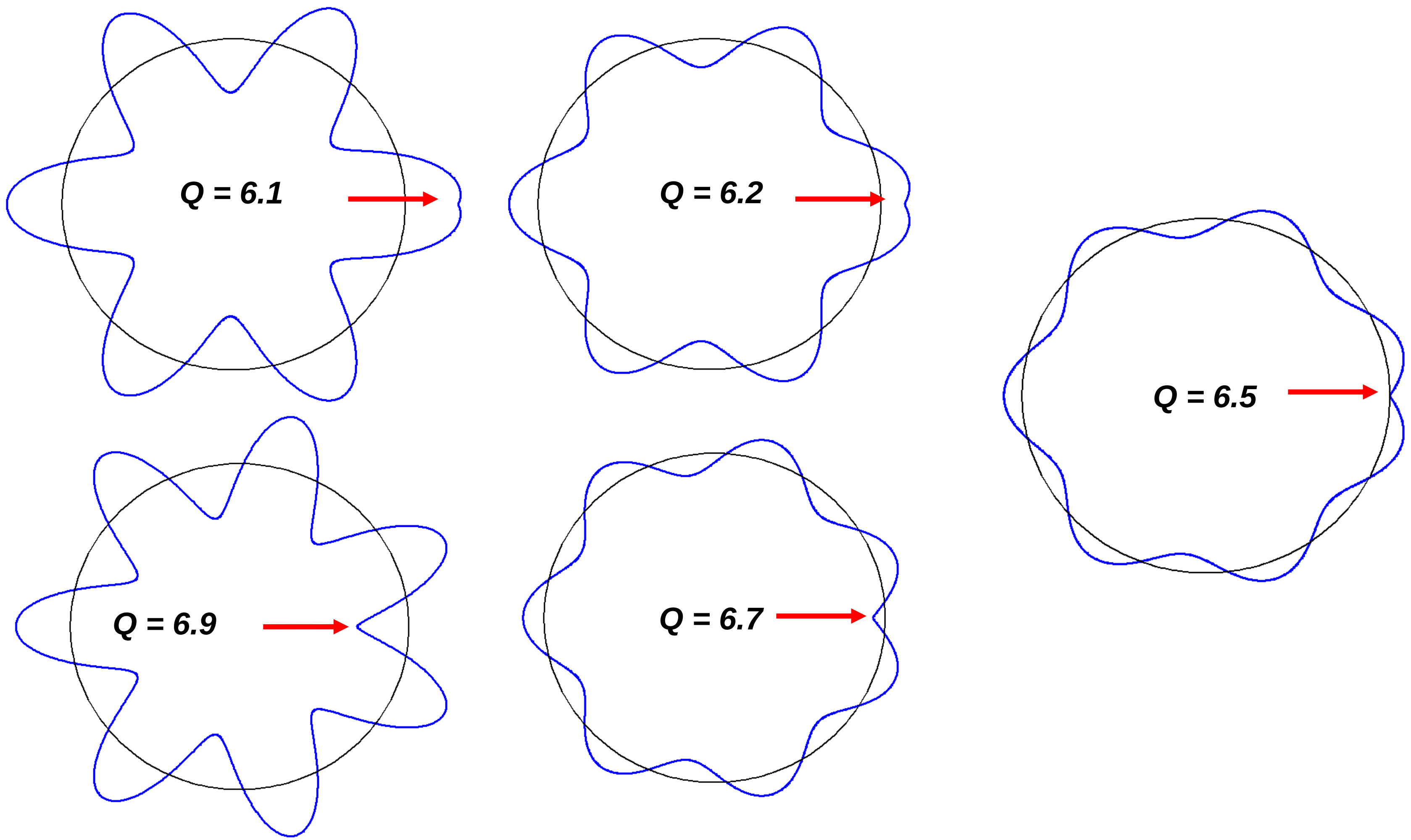
$$u(s) = \theta \frac{\sqrt{\beta(s)\beta_0}}{2\sin(\pi Q)} \cos(\pi Q - |\psi(s) - \psi_0|)$$





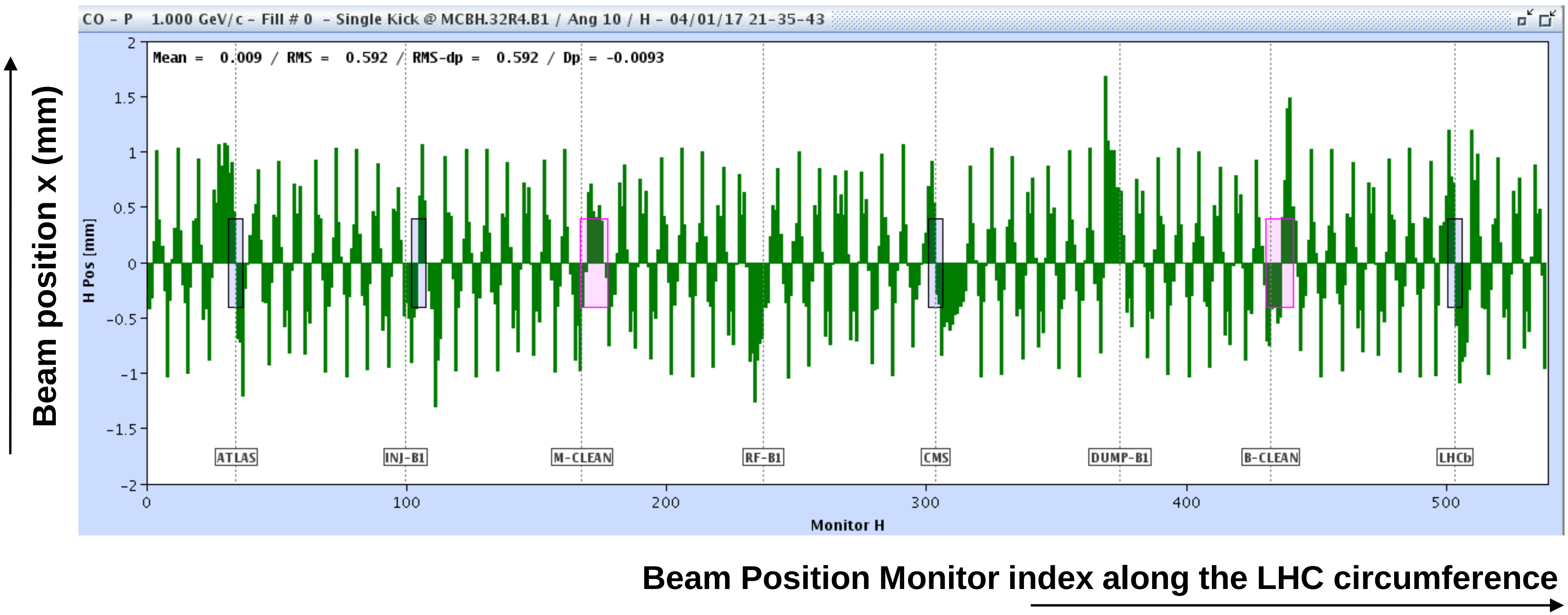
Closed orbit examples

- Example of horizontal closed orbit for a machine with tune Q = 6.x
- The **kink at the location of the deflection** (→) can be used to localize the deflection (if it is not known) → can be used for orbit correction.



A deflection at the LHC

- In the example below for the 26.7km long LHC, there is one undesired deflection, leading to a perturbed closed orbit.

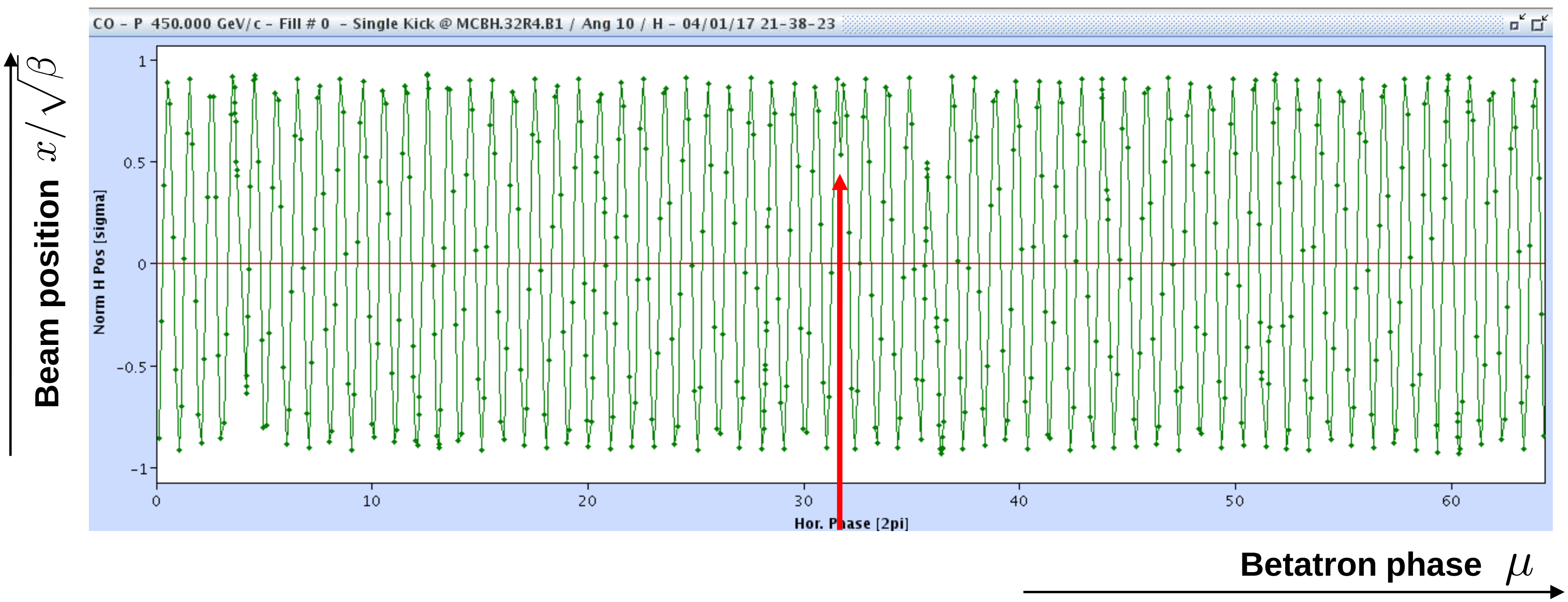


Where is the location of the deflection?

A deflection at the LHC

- To make our life easier we divide the position by $\sqrt{\beta(\sigma)}$ and replace the BPM index by its phase $\mu(\sigma)$→ transform into pure sinusoidal oscillation

$$\frac{u(s)}{\sqrt{\beta(s)}} = \theta \frac{\sqrt{\beta_0}}{2\sin(\pi Q)} \cos(\pi Q - |\psi(s) - \psi_0|)$$



Can you localize the deflection now?

Global orbit distortion

- **Orbit distortion due to many errors**

Courant and Snyder, 1957

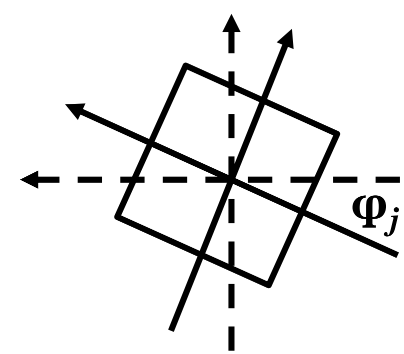
$$u(s) = \frac{\sqrt{\beta(s)}}{2\sin(\pi Q)} \int_s^{s+C} \theta(\tau)\sqrt{\beta(\tau)}\cos(\pi Q - |\psi(s) - \psi(\tau)|)d\tau$$

- **By approximating the errors as delta functions in *n* locations, the distortion at *i* observation points (Beam Position Monitors) is**

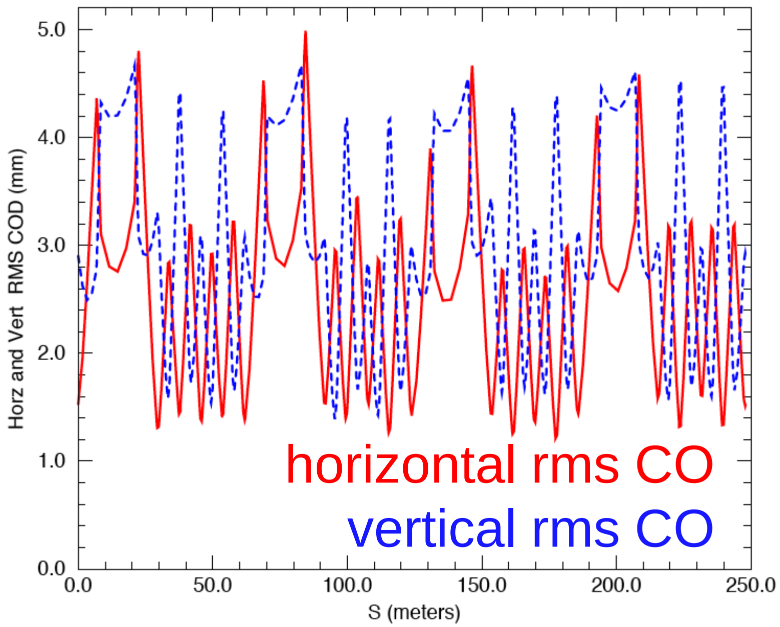
$$u_i = \frac{\sqrt{\beta_i}}{2\sin(\pi Q)} \sum_{j=i+1}^{i+n} \theta_j \sqrt{\beta_j}\cos(\pi Q - |\psi_i - \psi_j|)$$

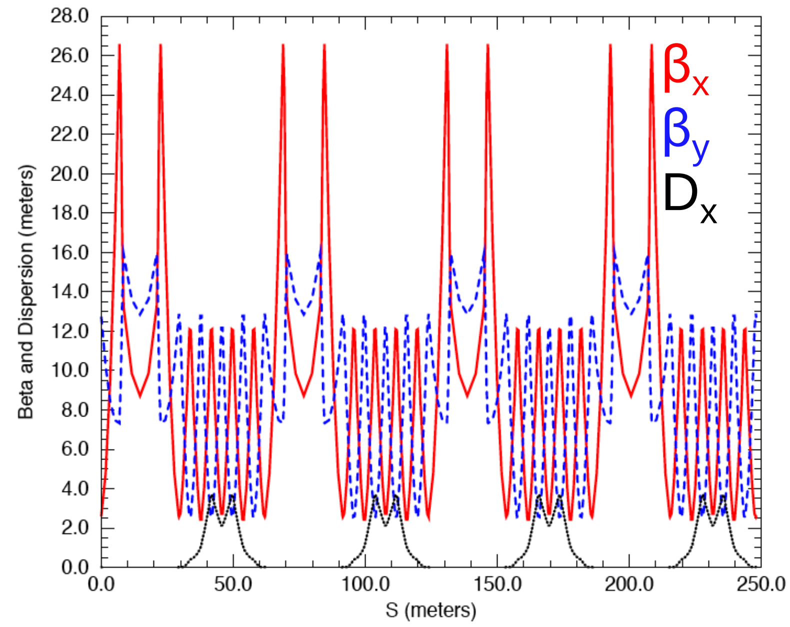
with the kick produced by the j^{th} error

- Integrated dipole field error $\theta_j = \frac{\delta(B_j l_j)}{B\rho}$
- Dipole roll $\theta_j = \frac{B_j l_j \sin\phi_j}{B\rho}$
- Quadrupole displacement $\theta_j = \frac{G_j l_j \delta u_j}{B\rho}$



Example: Orbit distortion in SNS





- In the SNS accumulator ring, the beta function is about **6 m** in the dipoles and about 30 m in the quadrupoles, the tune is **6.2**
- Consider dipole errors of **1 mrad**
- The maximum orbit distortion in dipoles is $u_0 = \frac{\sqrt{6 \cdot 6}}{2\sin(6.2\pi)} \cdot 10^{-3} \approx 5\text{mm}$
- For quadrupole displacement giving the same **1 mrad** kick (and betas of 30 m) the maximum orbit distortion is 25 mm, to be compared to magnet radius of 105 mm

Statistical estimation of orbit errors

- **Consider random distribution of errors in N magnets**
  - By squaring the orbit distortion expression and averaging over the angles (considering uncorrelated errors), the expectation (rms) value is given by

$$u_{\rm rms}(s) = \frac{\sqrt{\beta(s)}}{2\sqrt{2}|\sin(\pi Q)|}(\sum_i \sqrt{\beta_i}\theta_i)_{\rm rms} = \frac{\sqrt{N\beta(s)\beta_{\rm rms}}}{2\sqrt{2}|\sin(\pi Q)|}\theta_{\rm rms}$$

- **Example:**
  - In the SNS ring, there are **32** dipoles and **54** quadrupoles
  - The rms value of the orbit distortion in the dipoles

$$u_{\rm rms}^{\rm dip} = \frac{\sqrt{6\cdot 6}\sqrt{32}}{2\sqrt{2}\sin(6.2\pi)}\cdot 10^{-3} \approx 2{\rm cm}$$

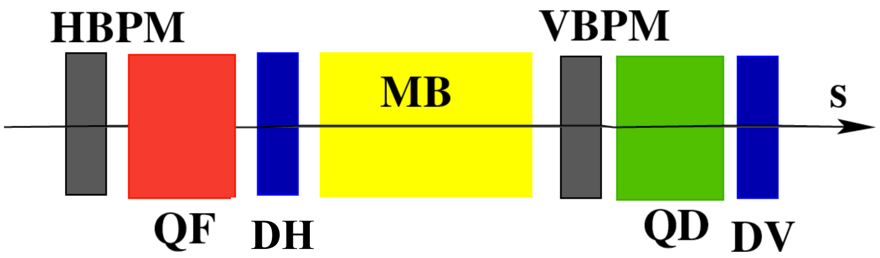
  - In the quadrupoles, for equivalent kick

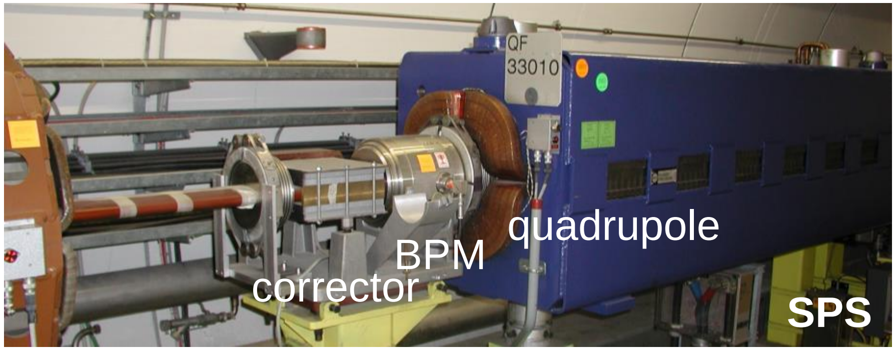
$$u_{\rm rms}^{\rm quad} = \frac{\sqrt{30\cdot 30}\sqrt{54}}{2\sqrt{2}\sin(6.2\pi)}\cdot 10^{-3} \approx 13{\rm cm}$$

Correcting closed orbit distortion

- **Horizontal dipole correctors and BPMs close to focusing quads + Vertical dipole correctors and BPMs next to defocusing quads**
  - Highest sensitivity / effect on closed orbit due to beta-function maxima

BPM: Beam Position Monitor
DH, DV: correctors

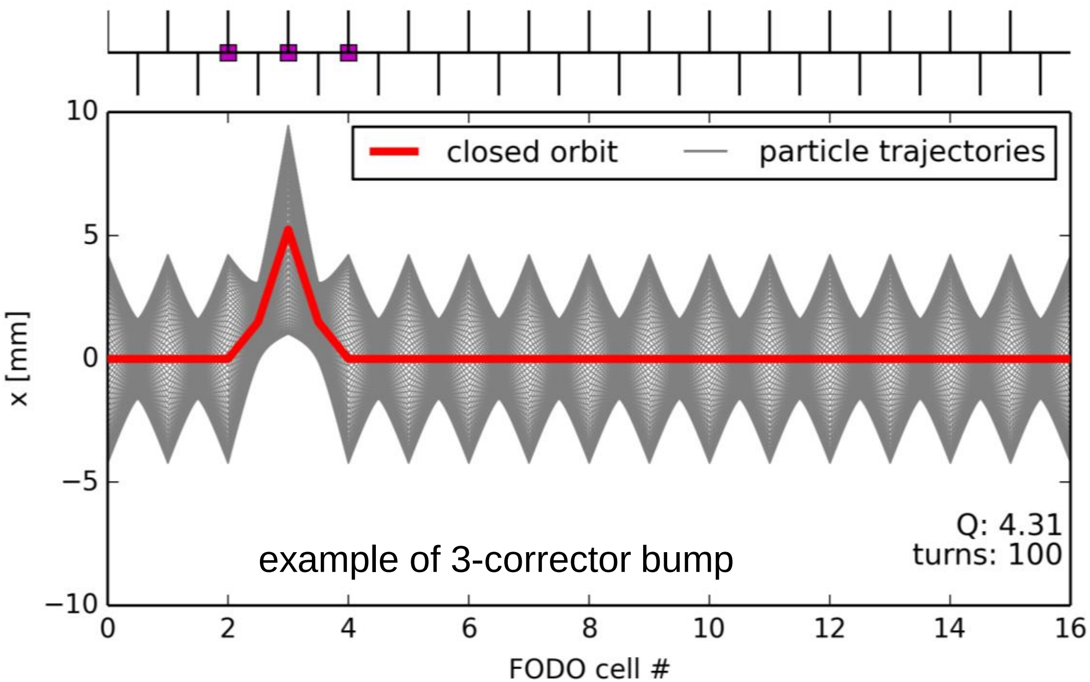




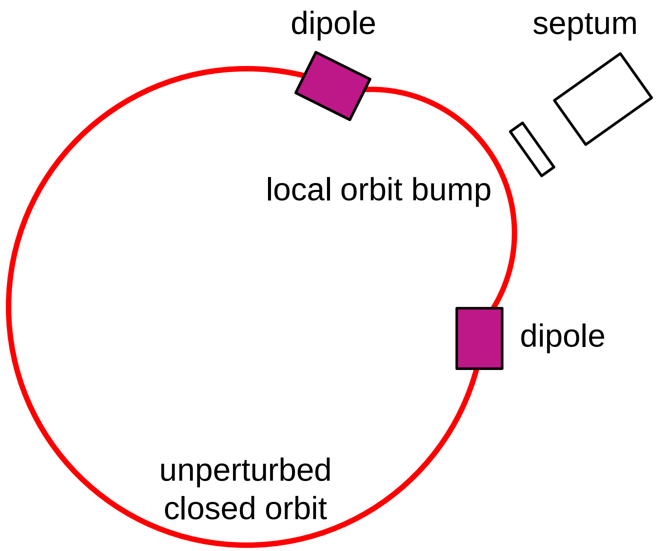
- **Measure orbit in BPMs and minimize orbit distortion**
  - Locally
    - Closed orbit bumps
    - Singular Value Decomposition (SVD)
  - Globally
    - Harmonic: minimizing components of orbit frequency response from Fourier analysis
    - MICADO: finding the most efficient corrector for minimizing the rms orbit
    - Least square minimization using orbit response matrix of correctors

Closed orbit bumps

- **Often it is needed to steer the closed-orbit away from the nominal trajectory in a localized part of a synchrotron**
  - Injection / extraction
  - Local orbit correction (or steering around local aperture restrictions)
- **Standard bump configurations exist**
  - π-bump (with 2 correctors)
  - 3 and 4-corrector bumps

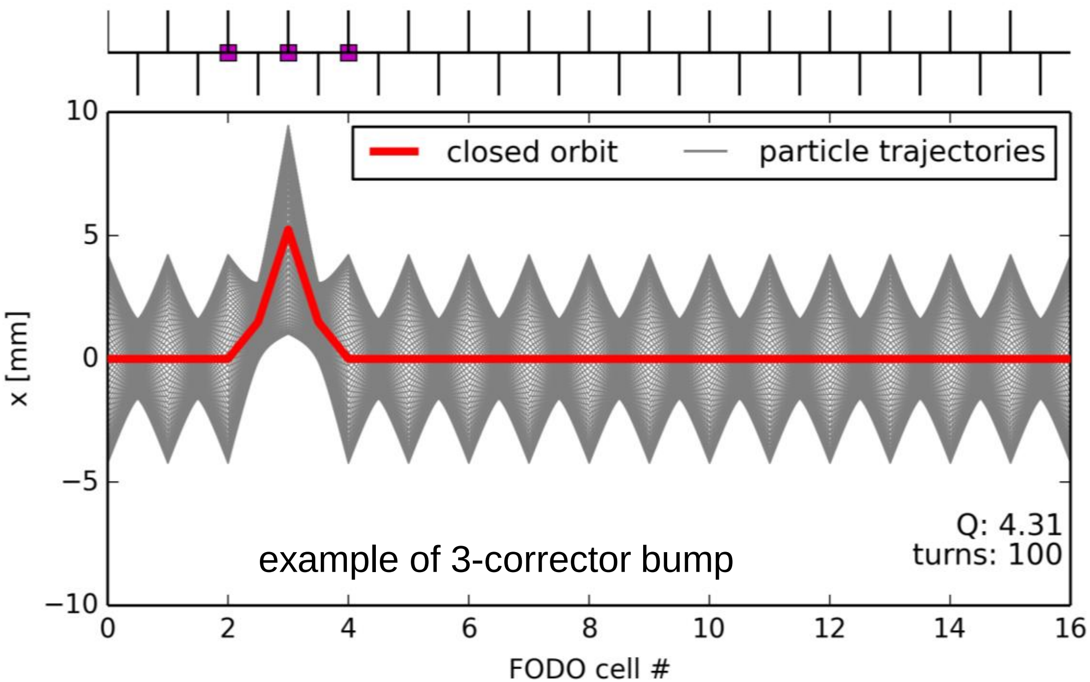


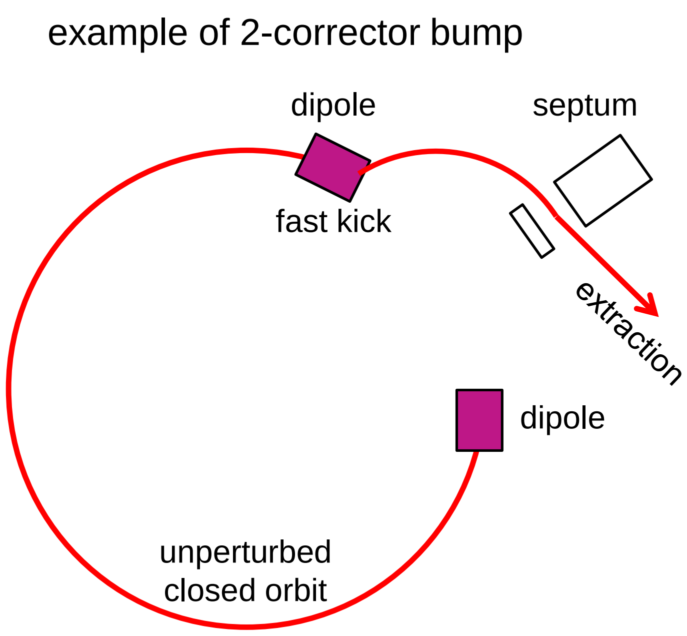
example of 2-corrector bump



Closed orbit bumps

- **Often it is needed to steer the closed-orbit away from the nominal trajectory in a localized part of a synchrotron**
  - Injection / extraction
  - Local orbit correction (or steering around local aperture restrictions)
- **Standard bump configurations exist**
  - π-bump (with 2 correctors)
  - 3 and 4-corrector bumps





Transport of closed orbit distortion

- Consider a transport matrix between positions 1 and 2

$$\mathcal{M}_{1\to 2} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

- The transport of transverse coordinates is written as

$$u_2 = m_{11}u_1 + m_{12}u'_1$$
$$u'_2 = m_{21}u_1 + m_{22}u'_1$$

- Consider a single dipole kick at position 1

$$\theta_1 = \frac{\delta(Bl)}{B\rho}$$

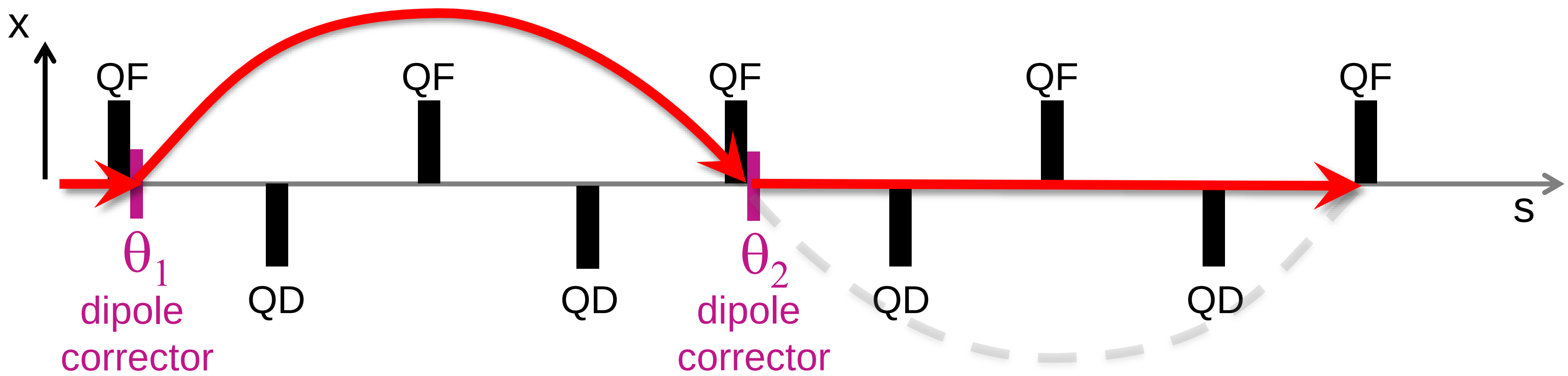
- Then, the first equation may be rewritten

$$u_2 + \delta u_2 = m_{11}u_1 + m_{12}(u'_1 + \theta_1) \to \delta u_2 = m_{12}\theta_1$$

- Replacing the coefficient from the general betatron matrix

$$\delta u_2 = \sqrt{\beta_1\beta_2}\sin(\psi_{12})\theta_1$$
$$\delta u'_2 = \sqrt{\frac{\beta_1}{\beta_2}}[\cos(\psi_{12}) - \alpha_2\sin(\psi_{12})]\theta_1$$

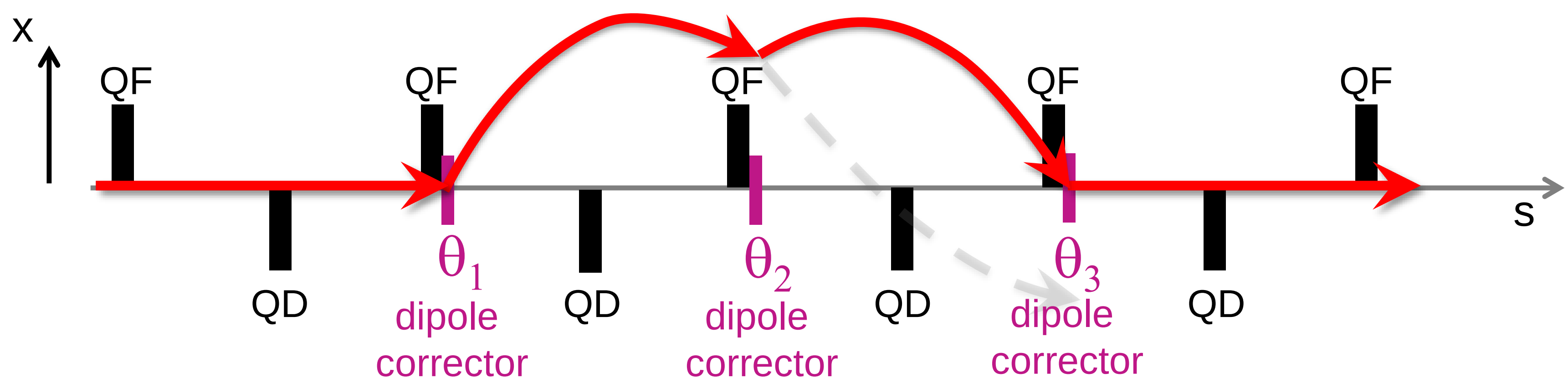
Orbit bumps: 2-corrector bump



- Consider a cell in which correctors are placed close to the focusing quads
- The orbit shift at the 2nd corrector is $\delta u_2 = \sqrt{\beta_1 \beta_2} \sin\left(\psi_{12}\right) \theta_1$
- This orbit bump can be closed by choosing a phase advance equal to π between correctors (this is called a "π-bump")
- The angle should satisfy the following equation

$$\theta_2 = \delta u_2' = -\sqrt{\frac{\beta_1}{\beta_2}} \left[\cos(\psi_{12})\theta_1 - \alpha_2 \sin(\psi_{12})\right] = \sqrt{\frac{\beta_1}{\beta_2}} \theta_1$$

Orbit bumps: 3-corrector bump

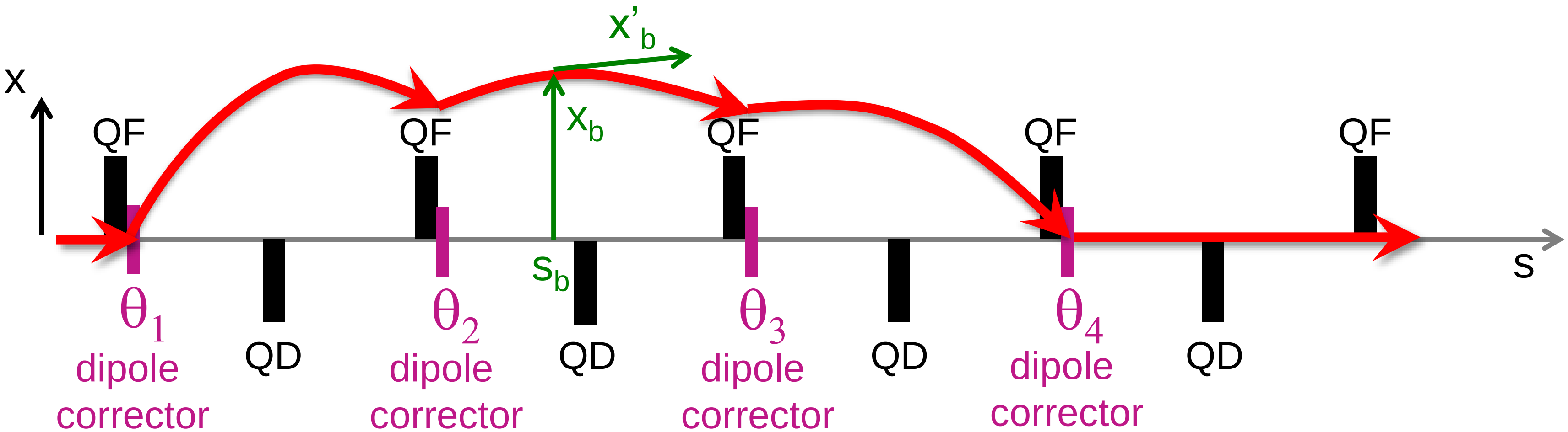


- Works for any phase advance if the three correctors satisfy

$$\frac{\sqrt{\beta_1}}{\sin\psi_{23}}\theta_1 = \frac{\sqrt{\beta_2}}{\sin\psi_{31}}\theta_2 = \frac{\sqrt{\beta_3}}{\sin\psi_{12}}\theta_3$$

- Angle of the closed orbit in the center of the bump is defined by above condition (cannot be adjusted independently of bump amplitude)

Orbit bumps: 4-corrector bump



$$\theta_1 = +\frac{1}{\sqrt{\beta_1\beta_b}}\frac{\cos\psi_{2b} - \alpha_b \sin\psi_{2b}}{\sin\psi_{12}}x_b - \sqrt{\frac{\beta_b}{\beta_1}}\frac{\sin\psi_{2b}}{\sin\psi_{12}}x'_b$$

$$\theta_2 = -\frac{1}{\sqrt{\beta_2\beta_b}}\frac{\cos\psi_{1b} - \alpha_b \sin\psi_{1b}}{\sin\psi_{12}}x_b + \sqrt{\frac{\beta_b}{\beta_2}}\frac{\sin\psi_{1b}}{\sin\psi_{12}}x'_b$$

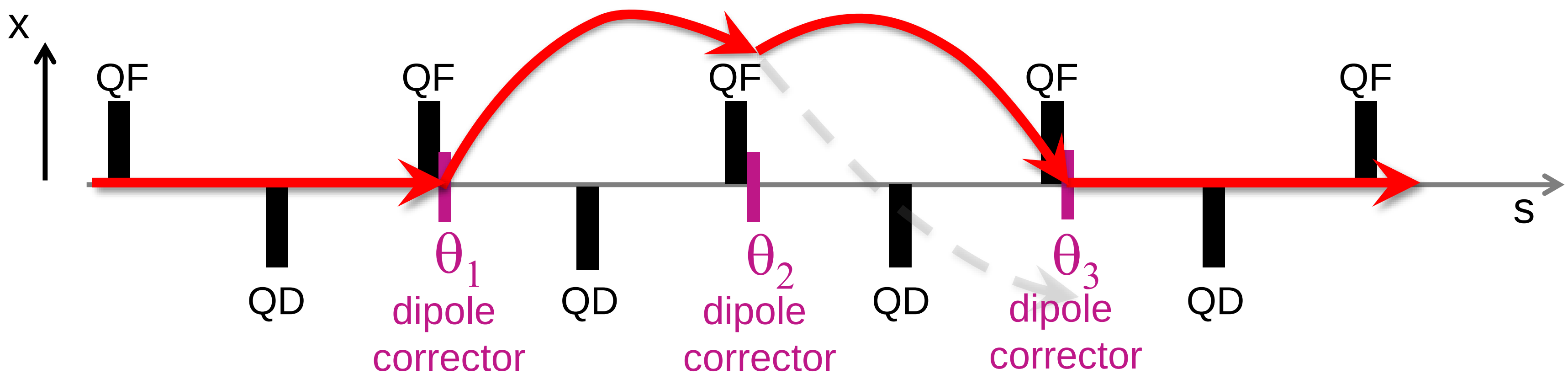
$$\theta_3 = -\frac{1}{\sqrt{\beta_3\beta_b}}\frac{\cos\psi_{b4} + \alpha_b \sin\psi_{b4}}{\sin\psi_{34}}x_b - \sqrt{\frac{\beta_b}{\beta_4}}\frac{\sin\psi_{b4}}{\sin\psi_{34}}x'_b$$

$$\theta_4 = +\frac{1}{\sqrt{\beta_4\beta_b}}\frac{\cos\psi_{b3} + \alpha_b \sin\psi_{b3}}{\sin\psi_{34}}x_b + \sqrt{\frac{\beta_b}{\beta_4}}\frac{\sin\psi_{b3}}{\sin\psi_{34}}x'_b$$

- Works for any phase advance
- Position x_b and angle x'_b of the bump at location s_b can be adjusted independently
- Can be used for aperture scanning, extraction bumps, …

Problem 2

Three correctors are placed at locations with phase advance of $\pi/4$ between them and beta functions of **12, 2** and **12 m**. How are the corrector kicks related to each other in order to achieve a closed **3-corrector bump** (i.e. what is the relative strength between the three kicks)?

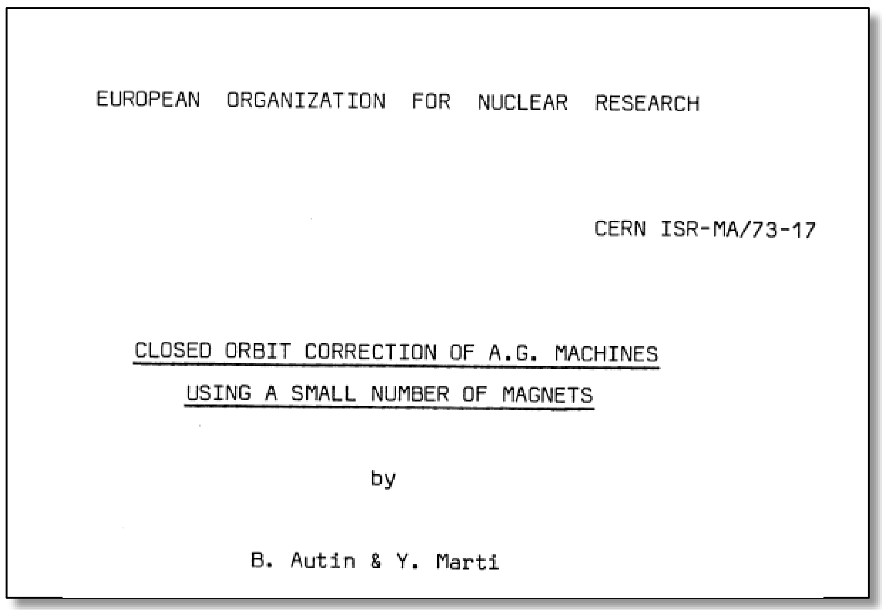


Closed orbit correction: MICADO

- The problem of correcting the orbit deterministically came up a long time ago in the first CERN machines.
- B. Autin and Y. Marti published a note in 1973 describing an algorithm that is still in use today (but in JAVA/C/C++ instead of FORTRAN) at ALL CERN machines: **MICADO***

* MInimisation des CArrés des Distortions d'Orbite.

(Minimization of the quadratic orbit distortions)



EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN ISR-MA/73-17

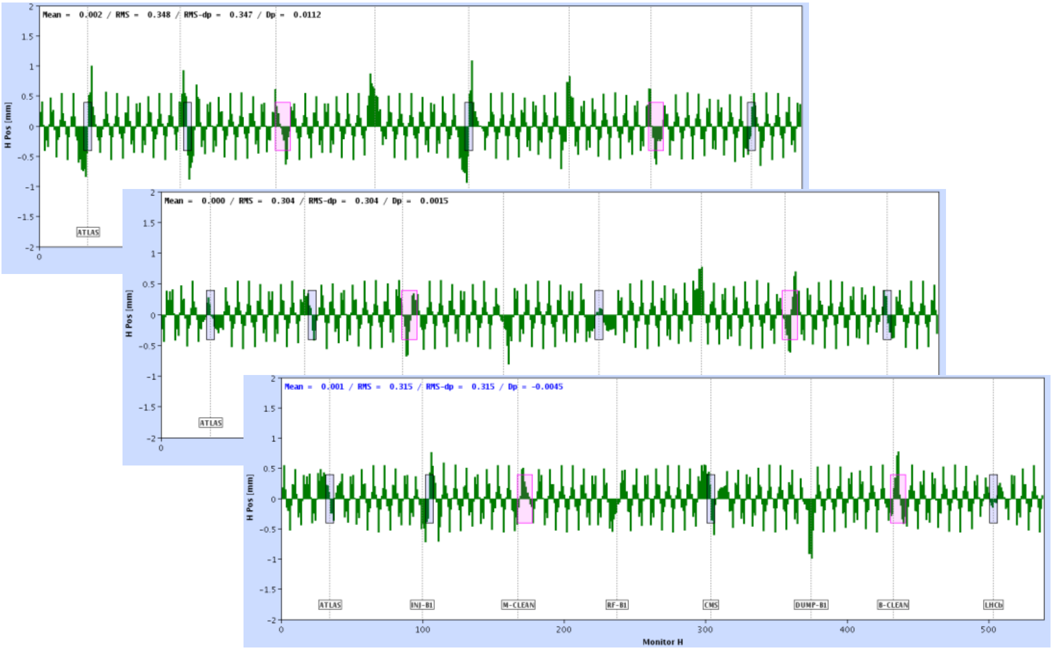
CLOSED ORBIT CORRECTION OF A.G. MACHINES

USING A SMALL NUMBER OF MAGNETS

by

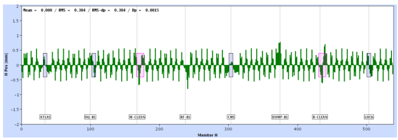
B. Autin & Y. Marti

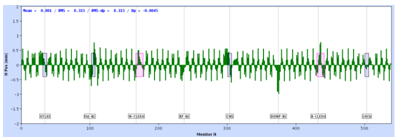
- The intuitive principle of MICADO is rather simple
- Need a model of the machine
- Compute for each orbit corrector what the effect (response) is expected to be on the orbit

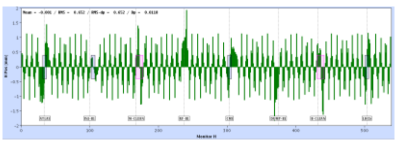


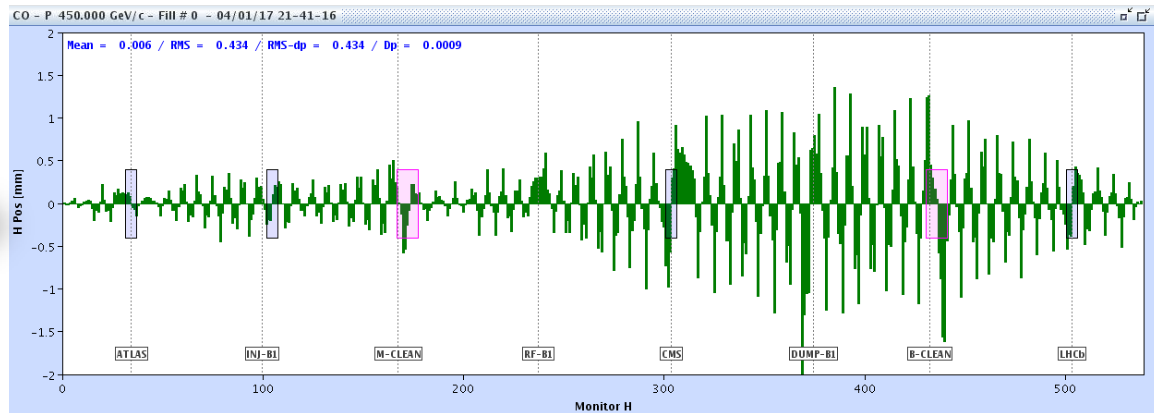
MICADO – how does it work?

- MICADO compares the response of every corrector with raw orbit







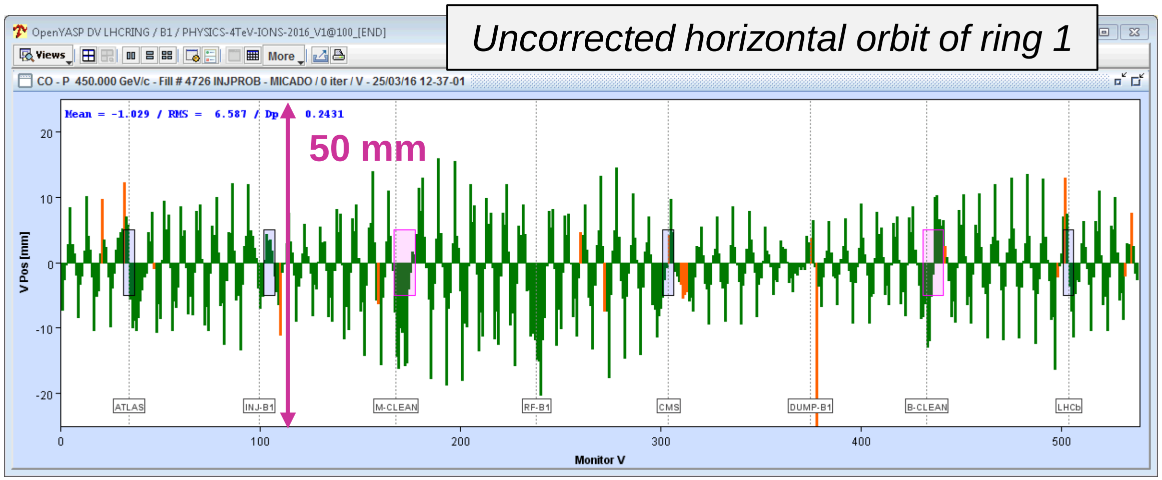


…

- MICADO picks out the corrector that has the best match with the orbit, and that will give the largest improvement to the orbit deviation rms
- The procedure can be iterated until the orbit is good enough (or as good as it can be)

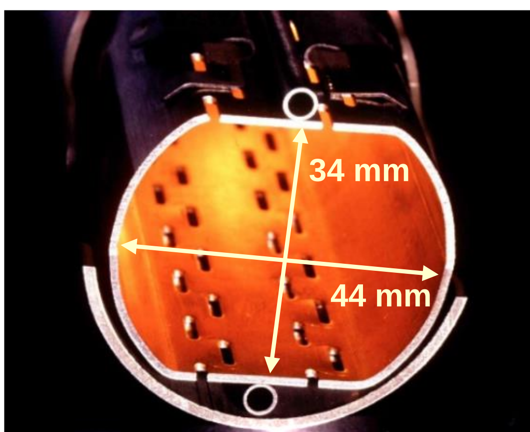
MICADO – LHC Orbit example

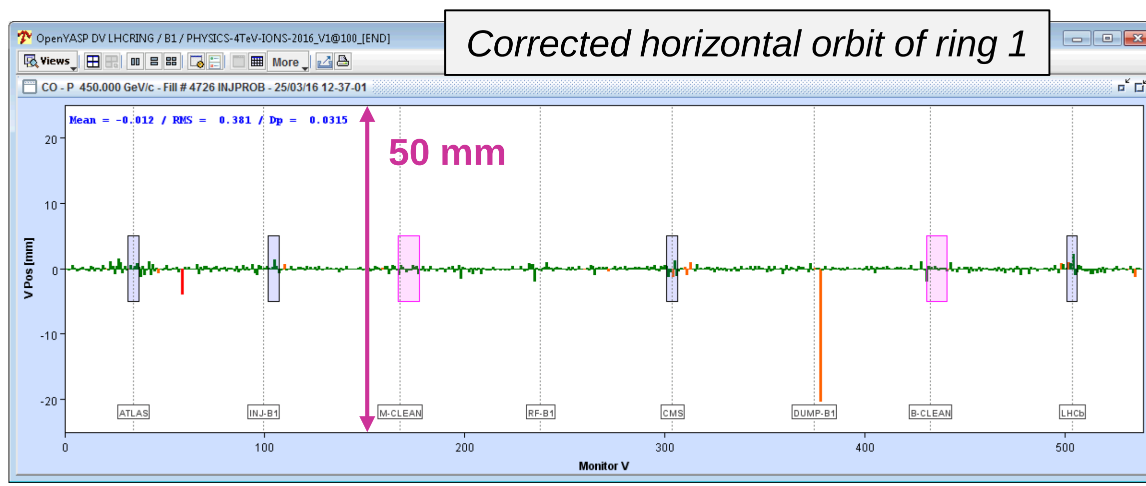
- The raw orbit at the LHC can have huge errors, but the correction (based partly on MICADO) brings the deviations down by a factor 20



MICADO & Co

LHC vacuum chamber





At the LHC a good orbit correction is vital !

Response matrix approach

- **This approach works for orbit correction when using the measured orbit distortion (but also for beta-beating when using Δβ/β, etc.)**
  - Available set of correctors: $\vec{c}$
  - Available observables (here the Beam Position Monitors): $\vec{m}$
  - Assume the linear approximation is good (small corrections): $\mathbf{A}\vec{c} = \vec{m}$
  - Use optics model to compute response matrix $\mathbf{A}$ (i.e. the orbit change in the ith monitor due to a unit kick from the jth corrector):

$$A_{i,j} = \frac{\sqrt{\beta_i \beta_j} \cos(\pi Q - |\psi_i - \psi_j|)}{2 \sin(\pi Q)}$$

… or use, e.g. MADX

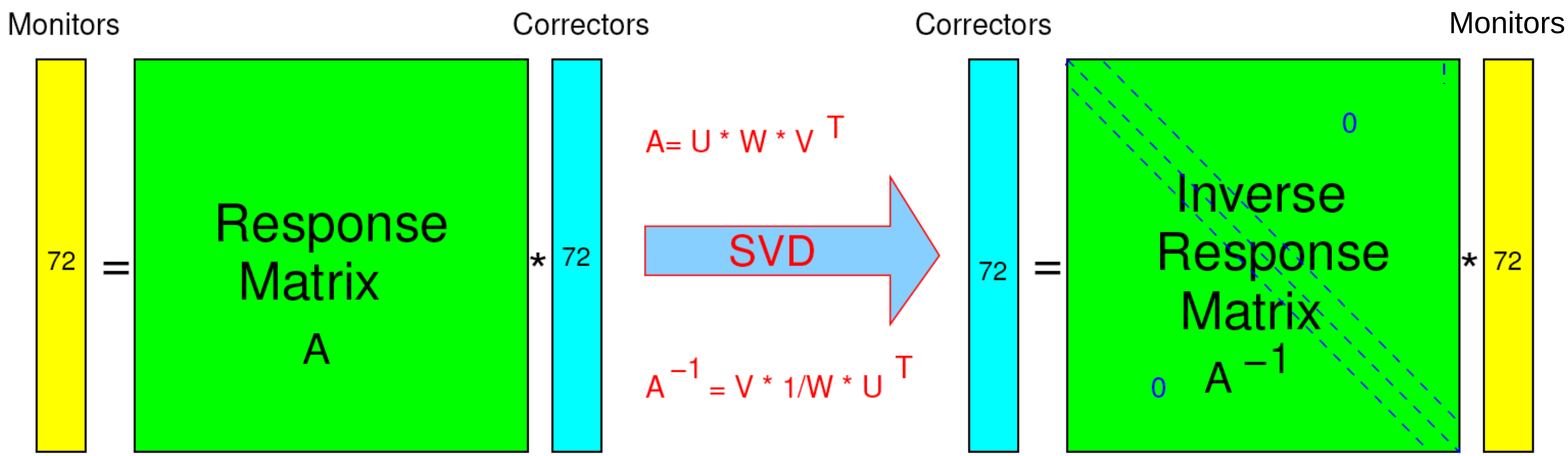
  - Invert or pseudo-invert the response matrix $\mathbf{A}$ to compute an effective global correction based on the measured $\Delta\vec{m}$:

$$\Delta\vec{c} = \mathbf{A}^{-1} \Delta\vec{m}$$

  - In case the number of correctors is not the same as the number of Beam Position Monitors one has to perform a pseudo matrix inversion, for example using the “Singular Value Decomposition (SVD)” algorithm

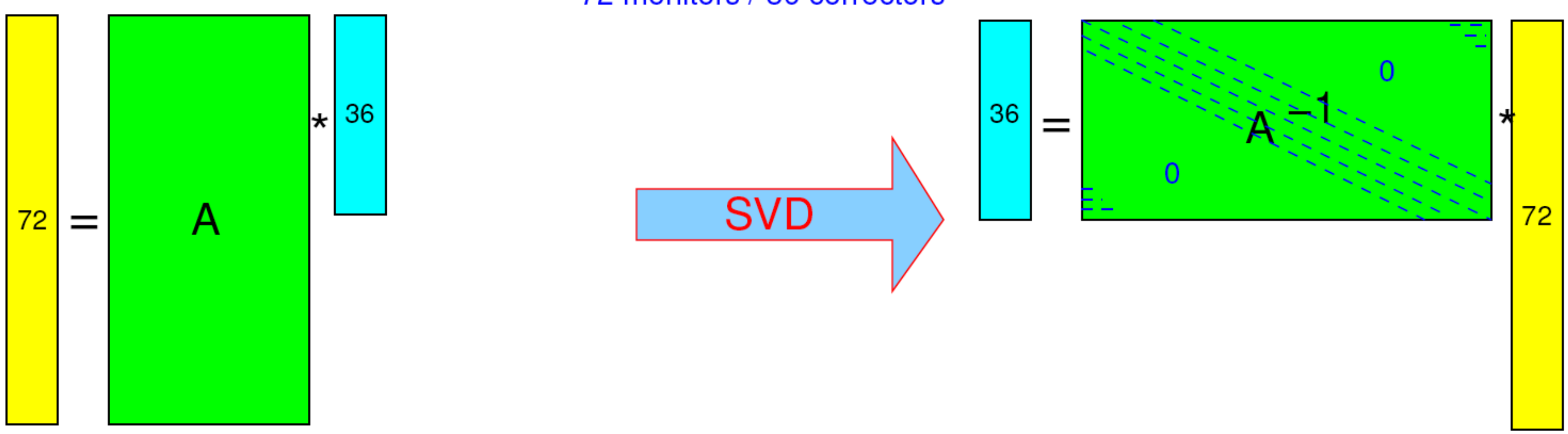
Singular Value Decomposition

72 monitors / 72 correctors



=> Minimization of the RMS orbit (=0 in case of "Matrix Inversion" using all Eigenvalues)

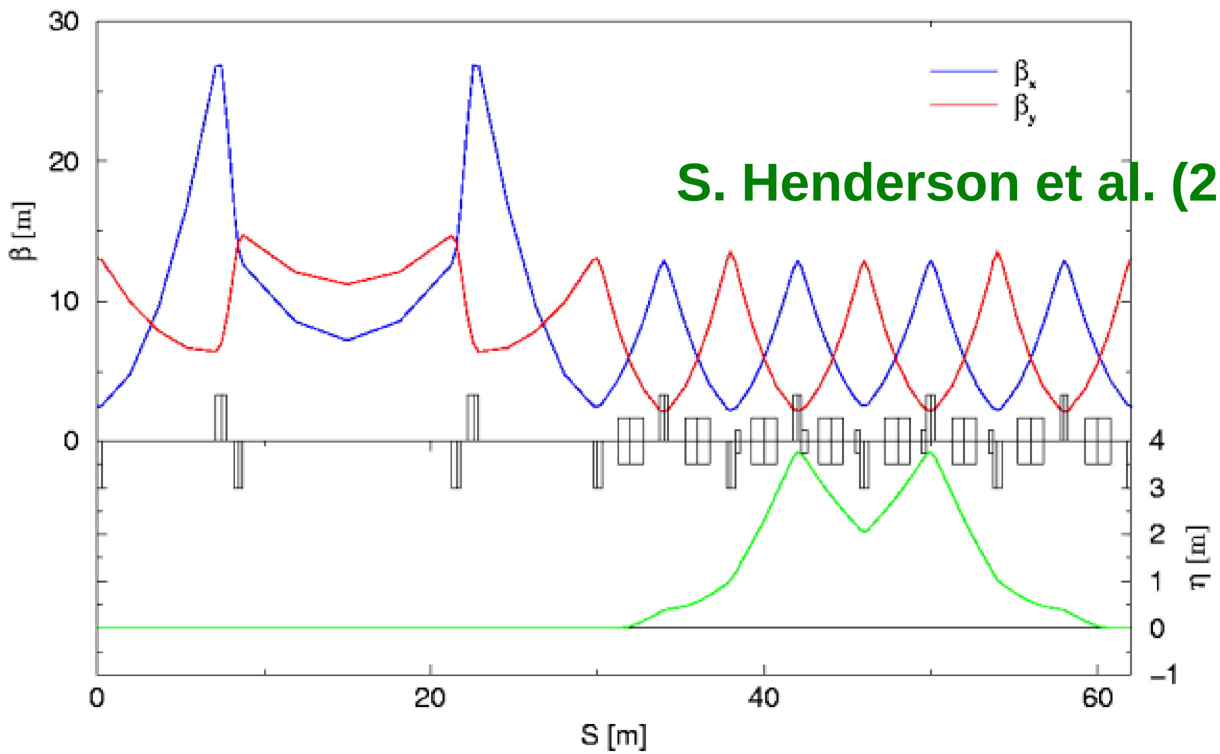
72 monitors / 36 correctors



=> Minimization of the RMS orbit (monitor averaging)

Problem 3

SNS: A **proton** ring with kinetic energy of 1 GeV and a **circumference of 248 m** has **18, 1 m-long** focusing quads with **gradient of 5 T/m**. In one of the quads, the horizontal and vertical **beta function** are **12 m** and **2 m** respectively. The **rms beta** function in both planes on the focusing quads is **8 m**. With a horizontal tune of **6.23** and a vertical of **6.2**, compute the expected horizontal and vertical orbit distortions on a single focusing quad given by **horizontal** and **vertical misalignments of 1 mm in all the quads**. What happens to the horizontal and vertical orbit distortions if the horizontal tune drops to **6.1** and **6.01**?



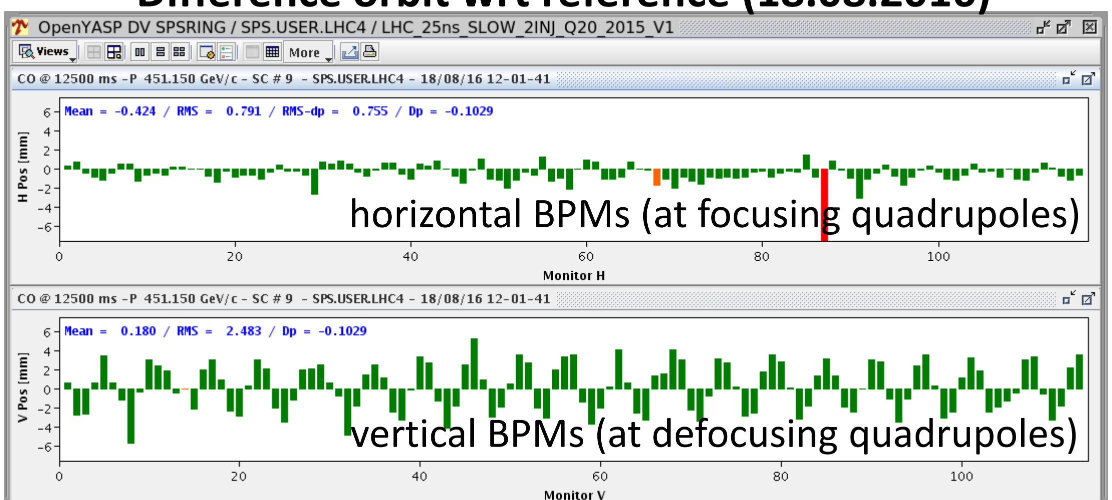
S. Henderson et al. (2005)

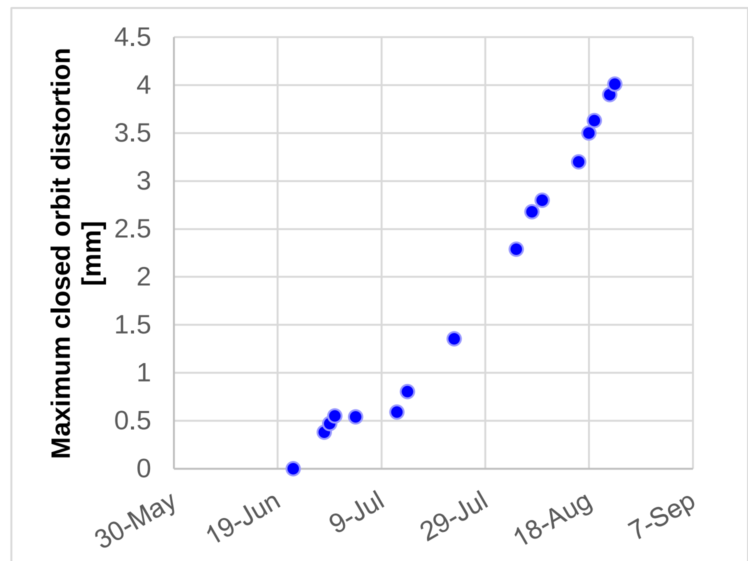
Problem 4

The SPS is a **400 GeV proton synchrotron** with a FODO lattice consisting of **108** focusing and **108** defocusing quadrupoles of length **3.22 m** and a gradient of **15 T/m**, with a **horizontal and vertical beta of 108 m and 30 m** in the focusing quads (**30 m and 108 m** for the defocusing ones). The tunes are **Q_x=20.13** and **Q_y=20.18**. Due to a mechanical problem, a focusing quadrupole was sinking down in 2016, resulting in an increasing closed orbit distortion compared to a reference taken earlier in the year.

- By how much the quadrupole had shifted down when the maximum vertical closed orbit distortion amplitude in defocusing quadrupoles reached 4 mm?
- Why was there no change of the horizontal orbit measured?
- How big would have been the maximum closed orbit distortion amplitude if it would have been a defocusing quadrupole?

Difference orbit wrt reference (18.08.2016)



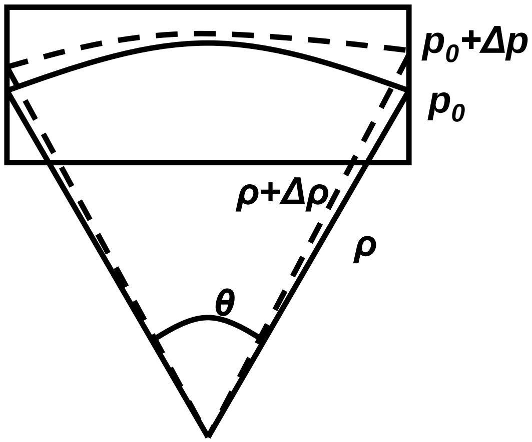


Beam orbit stability

- **Beam orbit stability is very critical**
  - Injection and extraction efficiency of synchrotrons
  - Stability of collision point in colliders
  - Stability of the synchrotron light spot in the beam lines of light sources
- **Consequences of orbit distortion**
  - Miss-steering of beams, modification of dispersion function, resonance excitation, aperture limitations, lifetime reduction, coupling, modulation of lattice functions, poor injection/extraction efficiency
- **Sources for closed orbit drifts**
  - **Long term (years - months)**: ground settling, season changes
  - **Medium term (days - hours)**: sun and moon, day-night variations (thermal), rivers, rain, wind, refills and start-up, sensor motion, drift of electronics, local machinery, filling patterns
  - **Short term (minutes - seconds)**: ground vibrations, power supplies, experimental magnets, air conditioning, refrigerators/compressors

Off-momentum particles in a dipole

- **Up to now all particles had the same momentum p_0**
- **What happens for off-momentum particles, i.e. particles with momentum $p_0+\Delta p$?**
  - Consider a dipole with field B and bending radius ρ
  - Recall that the magnetic rigidity is $B\rho = \frac{p_0}{q}$ and for off-momentum particles



$$B(\rho+\Delta\rho) = \frac{p_0+\Delta p}{q} \Rightarrow \frac{\Delta\rho}{\rho} = \frac{\Delta p}{p_0}$$

  - Considering the effective length of the dipole unchanged

$$\theta\rho = l = \text{const.} \Rightarrow \rho\Delta\theta + \theta\Delta\rho = 0 \Rightarrow \frac{\Delta\theta}{\theta} = -\frac{\Delta\rho}{\rho} = -\frac{\Delta p}{p_0}$$

  - Off-momentum particles get different deflection (different orbit)

$$\Delta\theta = -\theta\frac{\Delta p}{p_0}$$

Dispersion equation

- **Consider the equations of motion for off-momentum particles**

$$x'' + K_x(s)x = \frac{1}{\rho(s)}\frac{\Delta p}{p}$$

  - The solution is a sum of the **homogeneous** (on-momentum) and the **inhomogeneous** (off-momentum) equation solutions

$$x(s) = x_H(s) + x_I(s)$$

  - In that way, the equations of motion are split in two parts

$$\begin{aligned} x_H'' + K_x(s)x_H &= 0 \\ x_I'' + K_x(s)x_I &= \frac{1}{\rho(s)}\frac{\Delta p}{p} \end{aligned}$$

  - The **dispersion function** can be defined as $D_x(s) = \dfrac{x_I(s)}{\Delta p/p}$

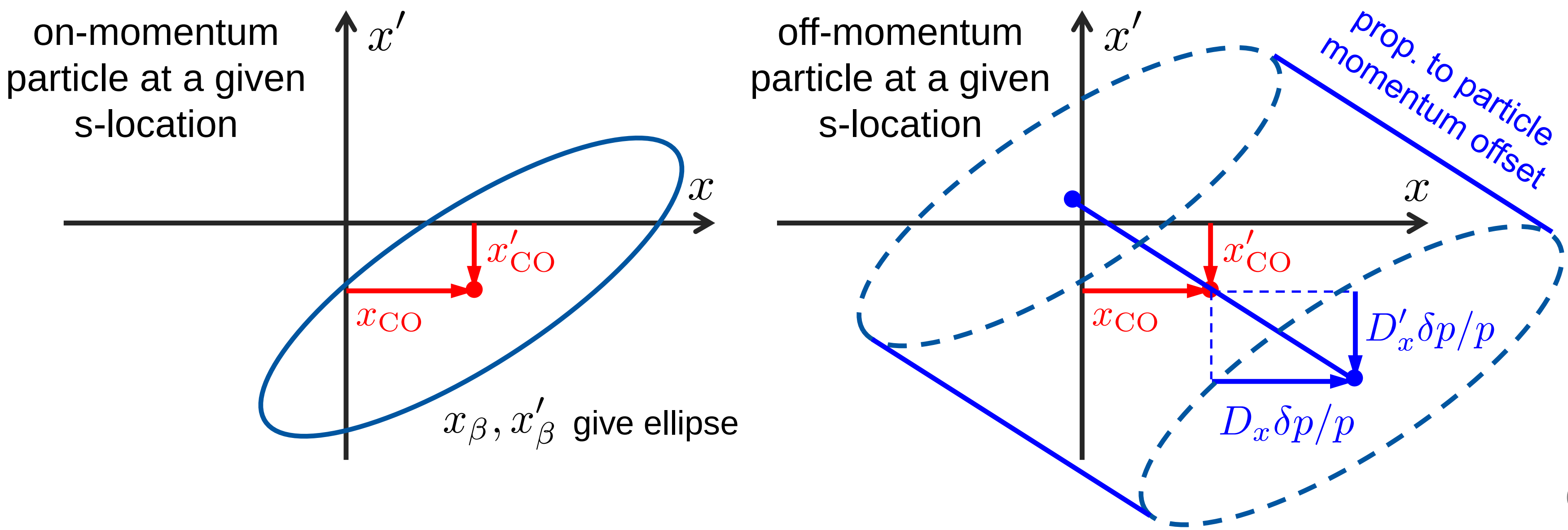
- **The dispersion equation is**

$$\boxed{D_x''(s) + K_x(s)D_x(s) = \frac{1}{\rho(s)}}$$

Closed orbit including dispersion

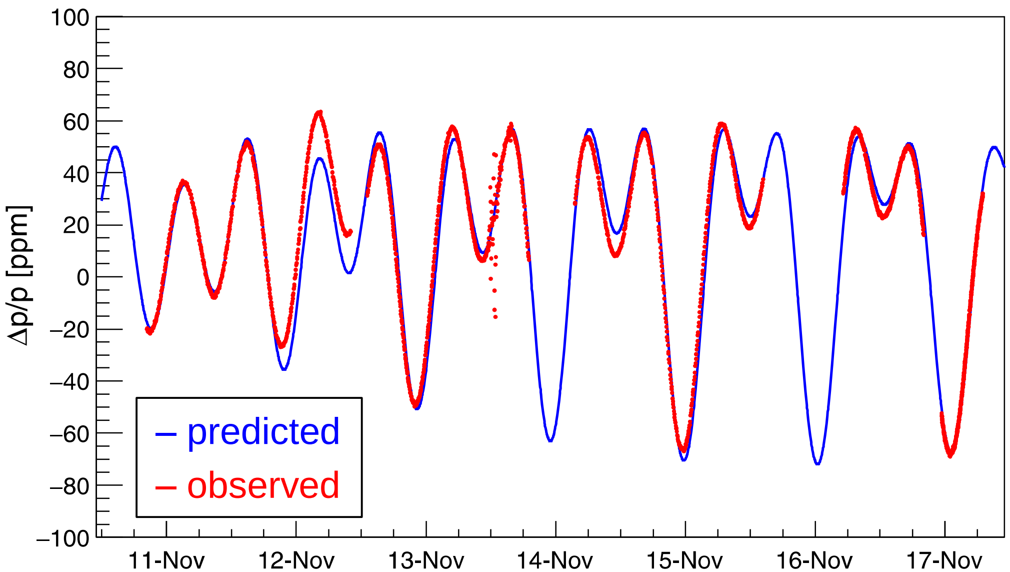
- Design orbit is defined by main dipole field
- On-momentum particles oscillate around the closed orbit (which is different compared to the design orbit in case of imperfections)
- Off-momentum particles oscillate around the chromatic closed orbit, defined by the dispersion function times the momentum offset added to the on-momentum closed orbit

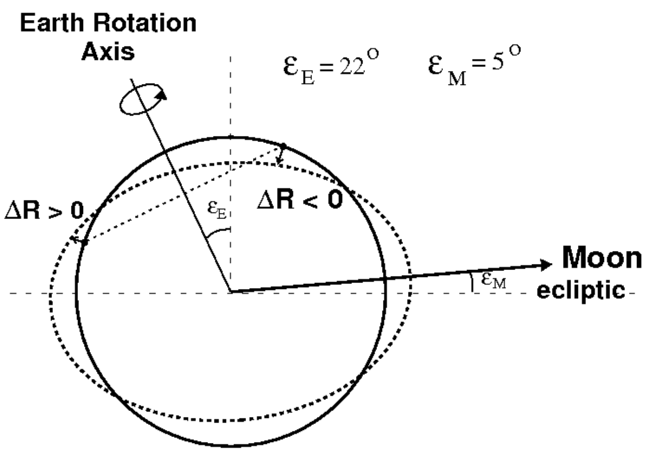
$$x = x_{\mathrm{CO}}(s) + x_\beta(s) + D_x(s)\delta p/p$$
$$x' = x'_{\mathrm{CO}}(s) + x'_\beta(s) + D'_x(s)\delta p/p$$



Impact of earth tides on LHC energy

- The LHC circumference is oscillating periodically due to Earth tides, caused by sun and moon, which move the Earth surface up and down (the Moon contributes ~2/3, the Sun contributes ~1/3)
- A change of beam energy of 0.014% is observed (through radial beam excursion for given RF frequency), corresponding to a change of circumference of 1.1 mm
- Very important for experiments for calibrating collision events!





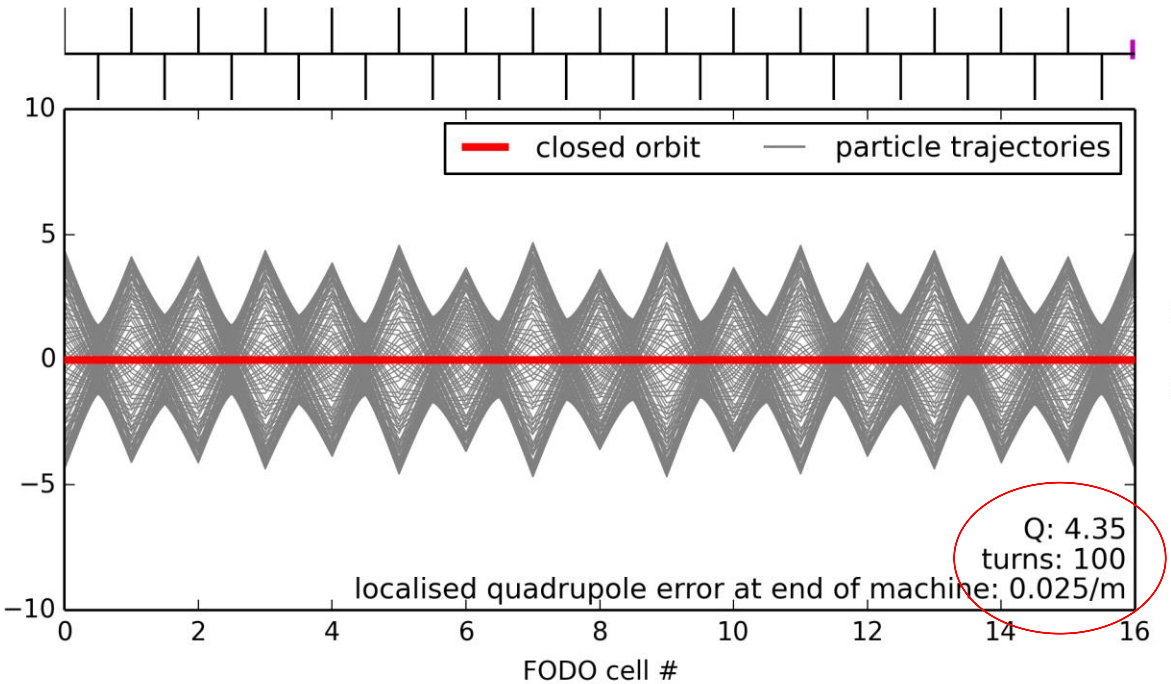
E. Todesco and J. Wenninger, PR-AB 20, 081003 (2017)

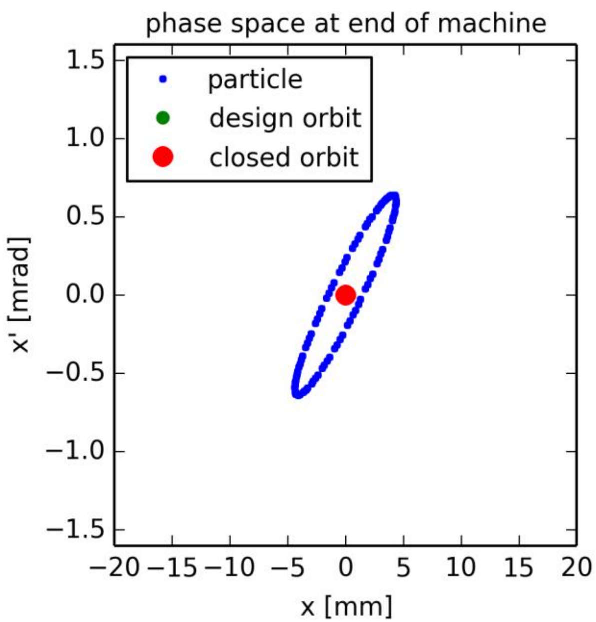
Outline

- **Introduction**
- **Closed orbit distortion (steering error)**
  - Beam orbit stability
  - Imperfections leading to closed orbit distortion
  - Effect of single and multiple dipole kicks
  - Closed orbit correction methods
  - Dispersion and chromatic orbit
- **Optics function distortion (gradient error)**
  - Imperfections leading to optics distortion
  - Tune-shift and beta distortion due to gradient errors
  - Gradient error correction
- **Coupling error**
  - Coupling errors and their effect
  - Coupling correction
- **Chromaticity**

Illustration of optics distortion

- **Ideal machine toy model with regular FODO lattice and quadrupole error at the end of circumference**
  - Particle injected with offset performs betatron oscillations but gets additional focusing from quadrupole error
  - There is a **tune-shift** (additional de-/focusing)
  - Beam envelope is distorted around the machine … “**beta-beating**”





Gradient error and optics distortion

- **Optics functions perturbation can induce aperture restrictions**
- **Tune perturbation can lead to reduced beam stability (dynamic aperture)**
- **Broken super-periodicity → excitation of all resonances**
  - In a ring made of *N* identical cells, only resonances with integer multiples of *N* can be excited
- **Sometimes control of optics is critical for machine performance**
  - Beta functions at collision points or at collimators (e.g. LHC)
- **Sources**
  - Errors in quadrupole strengths (random and systematic)
  - Injection elements
  - Higher-order multi-pole magnets and errors
- **Observables**
  - Tune-shift
  - Beta-beating
  - Excitation of integer and half integer resonances

Gradient error

- Consider the transfer matrix for 1-turn

$$\mathcal{M}_0 = \begin{pmatrix} \cos(2\pi Q) + \alpha_0 \sin(2\pi Q) & \beta_0 \sin(2\pi Q) \\ -\gamma_0 \sin(2\pi Q) & \cos(2\pi Q) - \alpha_0 \sin(2\pi Q) \end{pmatrix}$$

- Consider a gradient error in a quad. In thin element approximation the quadrupole matrix without and with error are

$$m_0 = \begin{pmatrix} 1 & 0 \\ -K_0(s)ds & 1 \end{pmatrix} \text{ and } m = \begin{pmatrix} 1 & 0 \\ -(K_0(s) + \delta K)ds & 1 \end{pmatrix}$$

- The new 1-turn matrix is $\mathcal{M} = m m_0^{-1} \mathcal{M}_0 = \begin{pmatrix} 1 & 0 \\ -\delta K ds & 1 \end{pmatrix} \mathcal{M}_0$

  which yields

$$\mathcal{M} = \begin{pmatrix} \cos(2\pi Q) + \alpha_0 \sin(2\pi Q) & \beta_0 \sin(2\pi Q) \\ -\delta K ds(\cos(2\pi Q) + \alpha_0 \sin(2\pi Q)) - \gamma_0 \sin(2\pi Q) & \cos(2\pi Q) - (\delta K ds \beta_0 + \alpha_0) \sin(2\pi Q) \end{pmatrix}$$

Gradient error and tune-shift

- Can also be written as a new matrix with a new tune $\chi = 2\pi(Q + \delta Q)$

$$\mathcal{M}^\star = \begin{pmatrix} \cos(\chi) + \alpha_0 \sin(\chi) & \beta_0 \sin(\chi) \\ -\gamma_0 \sin(\chi) & \cos(\chi) - \alpha_0 \sin(\chi) \end{pmatrix}$$

- The traces of the two matrices describing the 1-turn should be equal

$$\text{trace}\left(\mathcal{M}^*\right) = \text{trace}\left(\mathcal{M}\right)$$

which gives $2\cos(2\pi Q) - \delta K ds \beta_0 \sin(2\pi Q) = 2\cos(2\pi(Q + \delta Q))$

- Developing the right hand side

$$\cos(2\pi(Q+\delta Q)) = \cos(2\pi Q)\underbrace{\cos(2\pi\delta Q)}_{\approx 1} - \sin(2\pi Q)\underbrace{\sin(2\pi\delta Q)}_{\approx 2\pi\delta Q}$$

and finally $4\pi\delta Q = \delta K ds \beta_0$

- For a quadrupole of length *l* the tune shift is $\delta Q = \frac{1}{4\pi}\int_{s_0}^{s_0+l} \delta K \beta_0 ds$
- For distributed quadrupole errors

$$\delta Q = \frac{1}{4\pi}\oint \delta K(s)\beta(s)ds$$

Gradient error and beta distortion

- Consider the unperturbed transfer matrix for one turn

$$M_0 = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = B \cdot A \quad \text{with} \quad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

- Introduce a gradient perturbation between the two matrices

$$\mathcal{M}_0^\star = \begin{pmatrix} m_{11}^\star & m_{12}^\star \\ m_{21}^\star & m_{22}^\star \end{pmatrix} = B \begin{pmatrix} 1 & 0 \\ -\delta K ds & 1 \end{pmatrix} A$$

- Recall that $m_{12} = \beta_0 \sin(2\pi Q)$ and write the perturbed term as

a) $$m_{12}^\star = (\beta_0 + \delta\beta)\sin(2\pi(Q + \delta Q)) = m_{12} + \delta\beta \sin(2\pi Q) + 2\pi\delta Q \beta_0 \cos(2\pi Q)$$

where we used $\sin(2\pi\delta Q) \approx 2\pi\delta Q$ and $\cos(2\pi\delta Q) \approx 1$

- On the other hand

$$a_{12} = \sqrt{\beta_0 \beta(s_1)} \sin\psi,\ b_{12} = \sqrt{\beta_0 \beta(s_1)} \sin(2\pi Q - \psi)$$

b) $$m^{\star}_{12} = \underbrace{b_{11}a_{12} + b_{12}a_{22}}_{m_{12}} - a_{12}b_{12}\delta K ds = m_{12} - a_{12}b_{12}\delta K ds$$

- Equating the two terms

a) = b)

$$\delta\beta \sin(2\pi Q) + 2\pi\delta Q \beta_0 \cos(2\pi Q) = -\beta_0\beta(s_1)\sin\psi\sin(2\pi Q - \psi)\delta K ds$$

$$\delta\beta \sin(2\pi Q) + \frac{1}{2}\delta K ds \beta_0 \beta(s_1) \cos(2\pi Q) = -\beta_0\beta(s_1)\sin\psi\sin(2\pi Q - \psi)\delta K ds$$

- using $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$ and integrating yields

$$\frac{\delta\beta}{\beta_0} = -\frac{1}{2\sin(2\pi Q)}\int_{s_1}^{s_1+l}\beta(s)\delta K(s)\cos(2\psi - 2\pi Q)ds$$

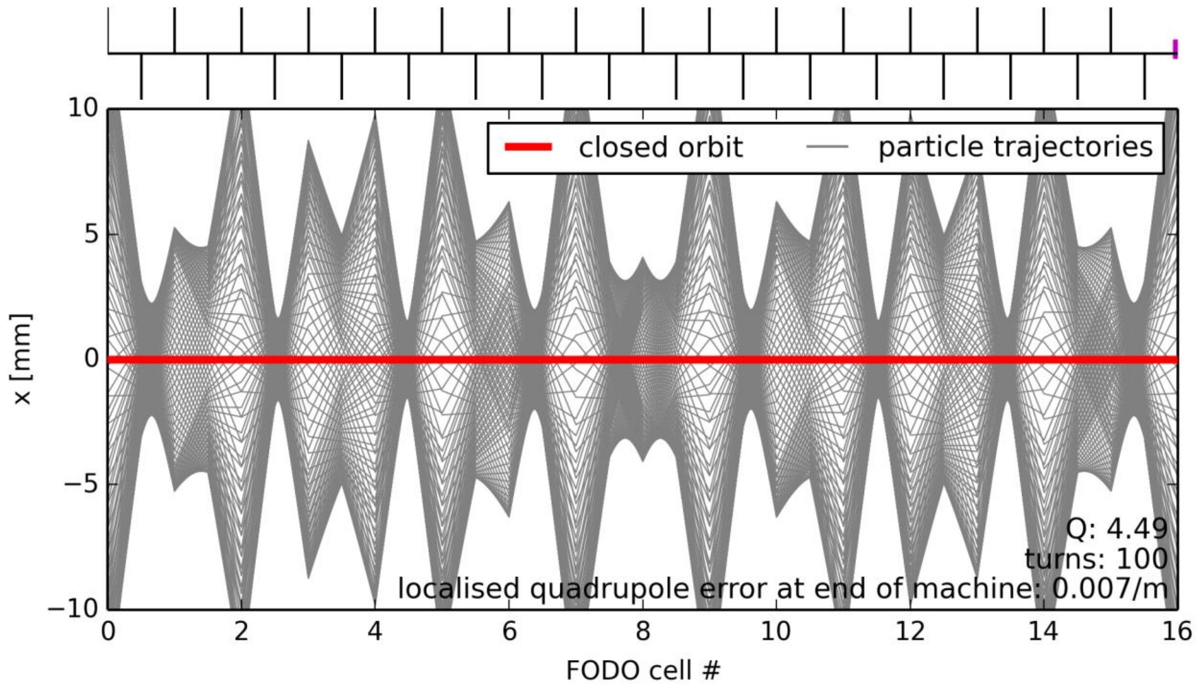
- for distributed errors around the machine

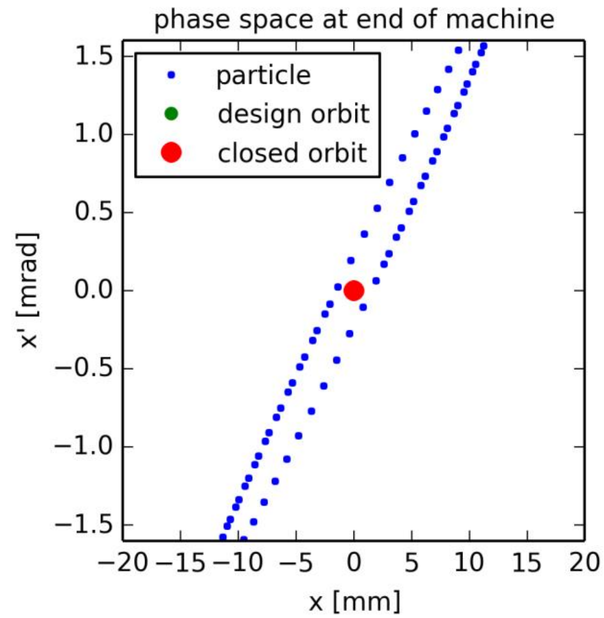
$$\frac{\delta\beta(s)}{\beta(s)} = -\frac{1}{2\sin(2\pi Q)}\int_{s}^{s+C}\beta(s_1)\delta K(s_1)\cos(|2\psi(s_1)) - 2\psi(s)| - 2\pi Q)ds_1$$

Optics distortion vs. tune

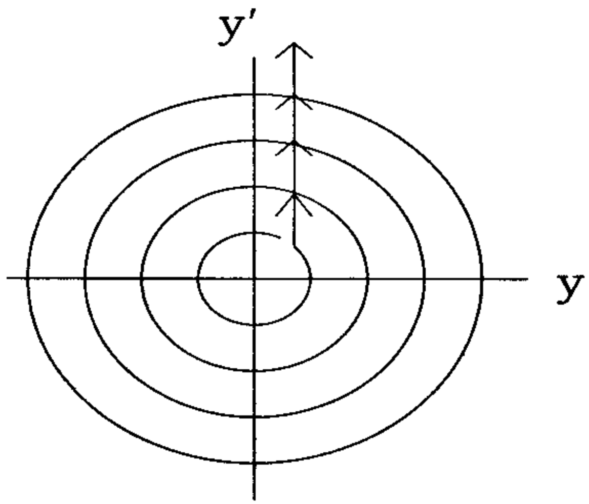
- Quadrupole errors have biggest impact **close to integer and half integer tunes → envelope (or beam size) becomes unstable**
- Optics distortion propagates with twice the tune (check the plot)

$$\frac{\delta\beta}{\beta_0} = -\frac{1}{2\sin(2\pi Q)}\int_{s_1}^{s_1+l}\beta(s)\delta K(s)\cos(2\psi - 2\pi Q)ds$$

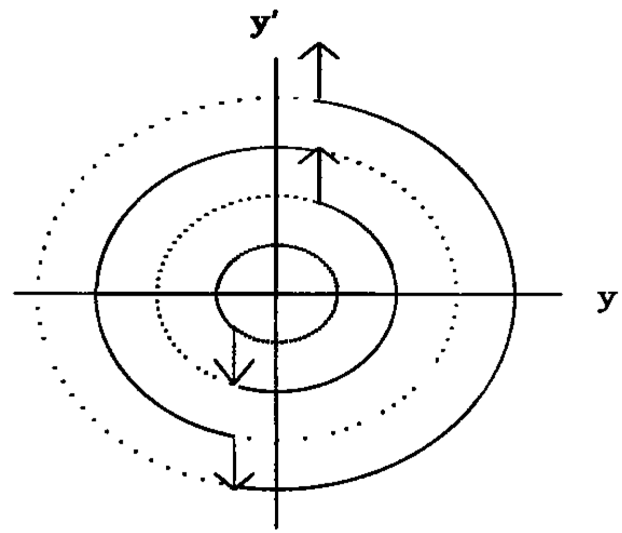




Quadrupole error in phase space



Q = n (integer)
→ kicks from quadrupoles add up (same as for kicks from dipoles)

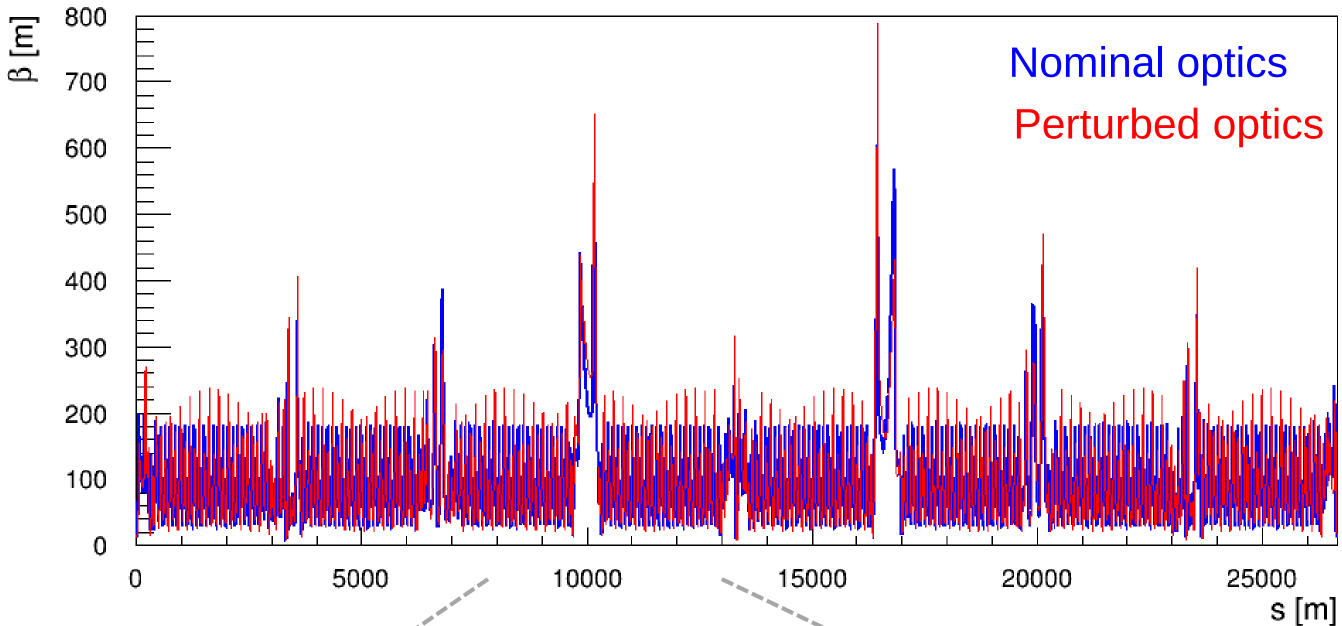


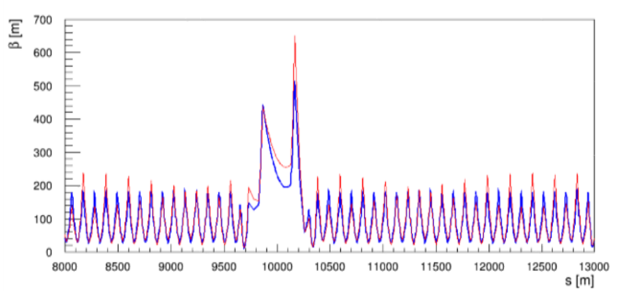
Q = n/2 (half integer)
→ kicks from quadrupoles add up (while kicks from dipoles cancel)

- Therefore **integer tunes and half integer tunes need to be avoided for machine operation** to avoid beam envelope becoming unstable due to quadrupole errors
- Recall: for integer tunes dipole errors drive the closed orbit unstable, but for half integer tunes they have minimum effect

- Let's take a look at the LHC …

Example: one quadrupole gradient is incorrect

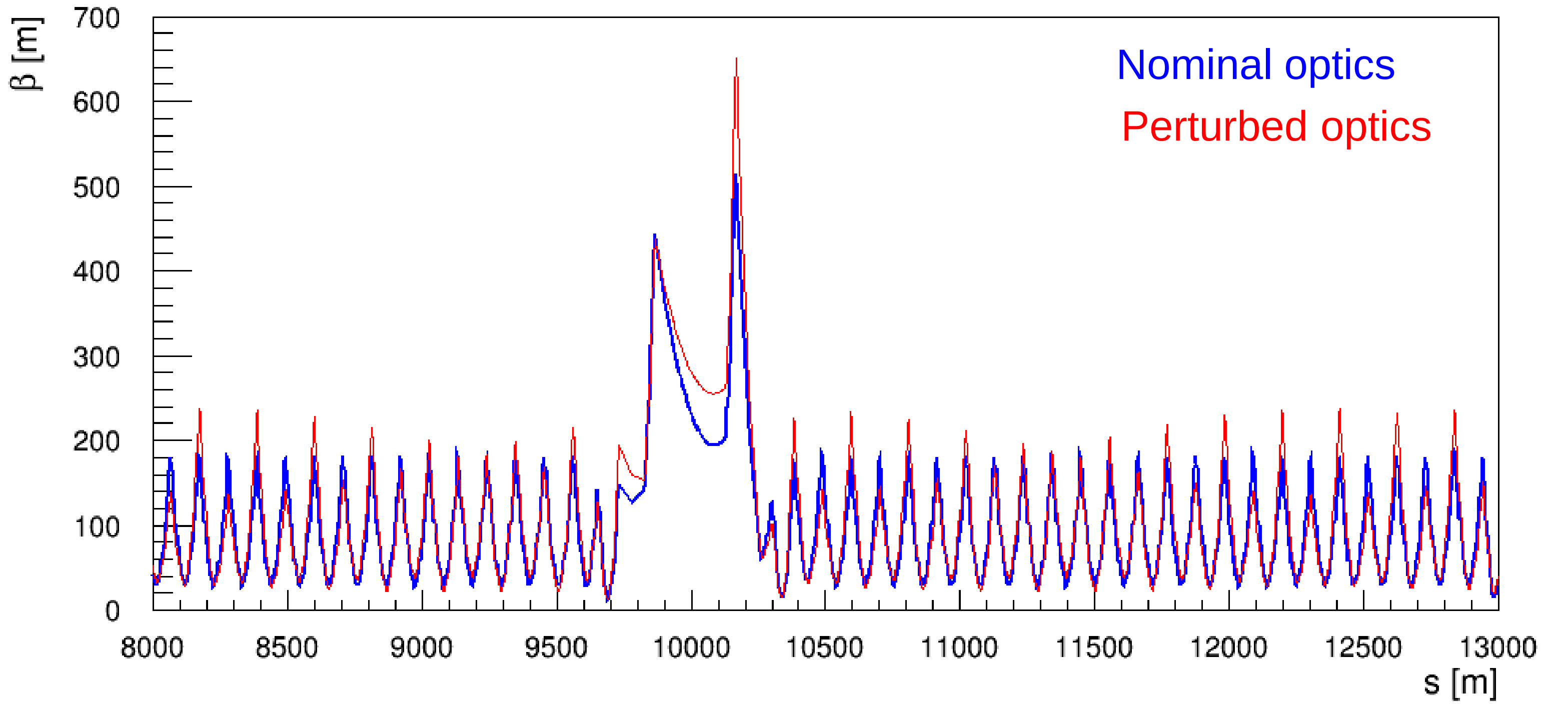




Zoom into a subsection

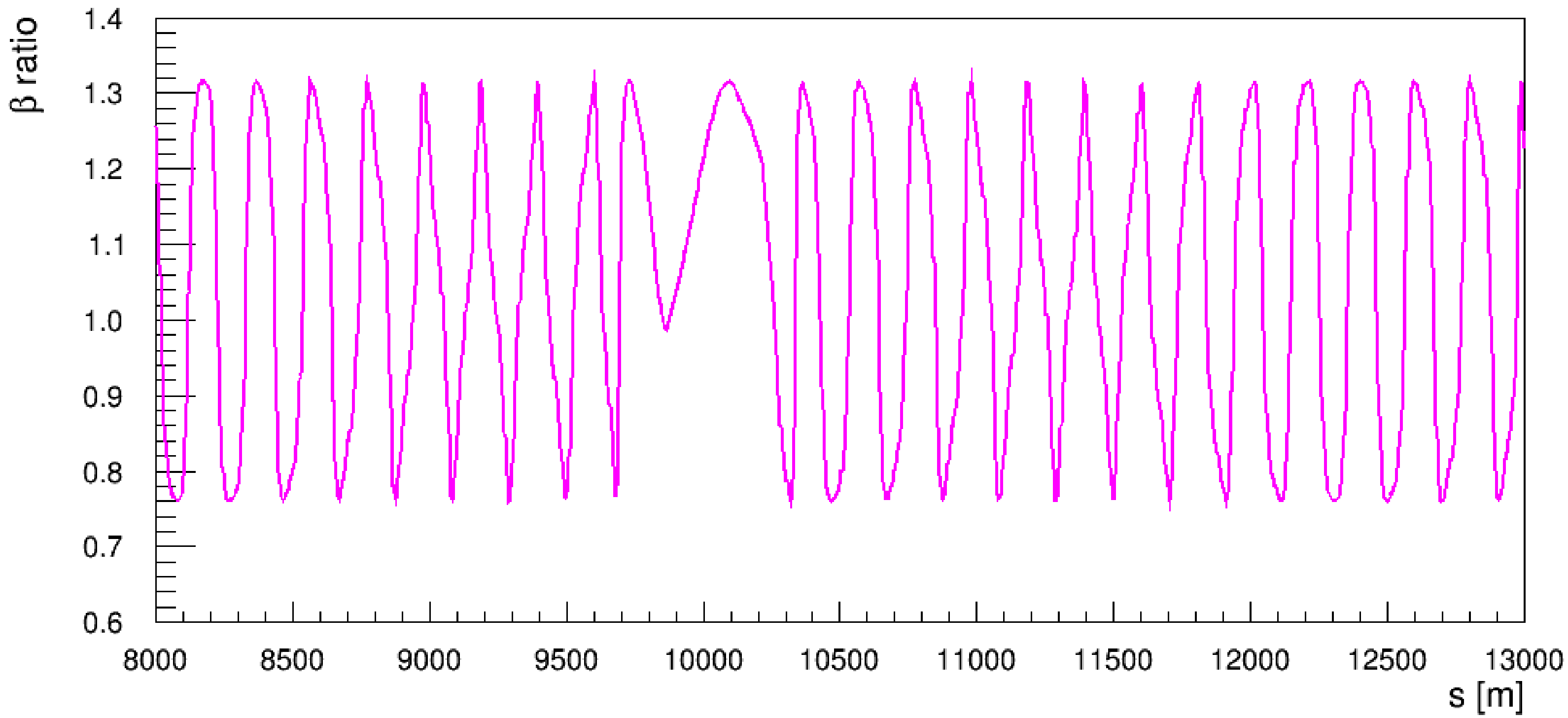
Optics distortion characteristics

- The local beam optics perturbation
- … note the oscillating pattern



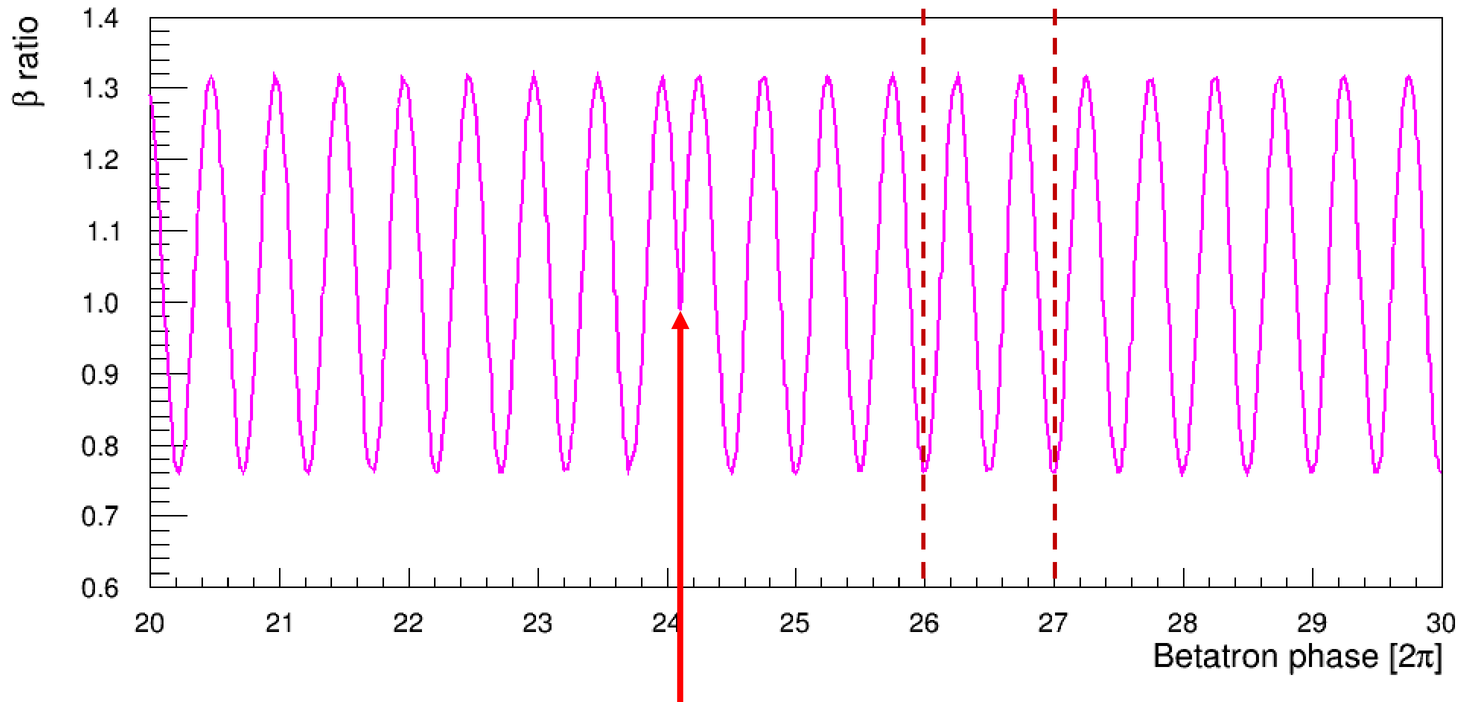
Optics distortion characteristics

- The error is easier to analyse and diagnose if one considers the ratio of the betatron function perturbed/nominal.
- The ratio reveals an oscillating pattern called the **betatron function beating** ('beta-beating'). The amplitude of the perturbation is the same all over the ring!

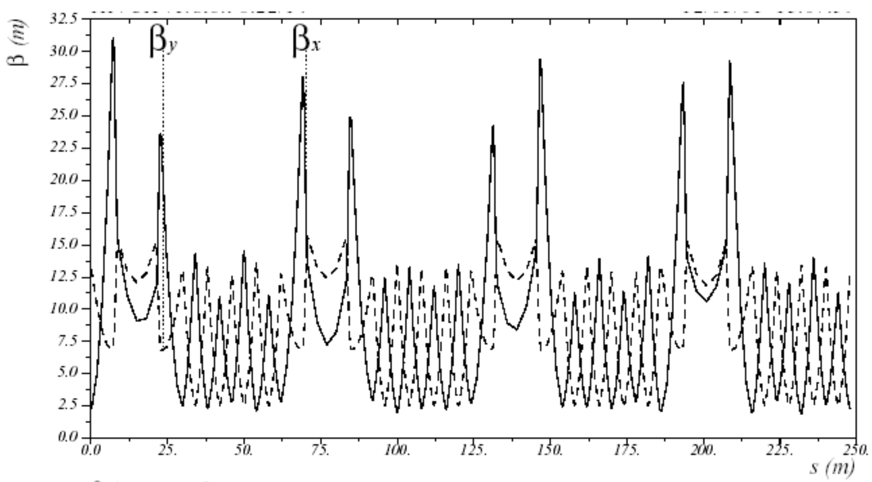


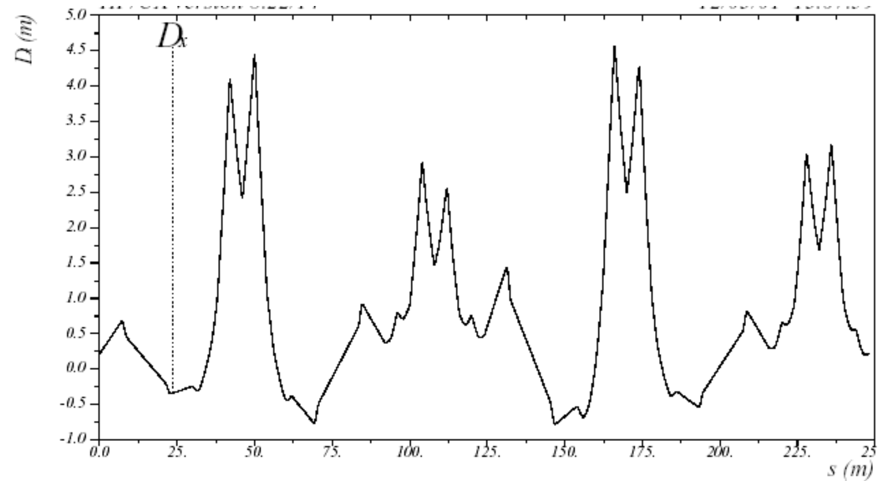
Optics distortion characteristics

- The beta-beating pattern comes out even more clearly if we replace the longitudinal coordinate with the betatron phase advance
- The result is very similar to the case of the closed orbit kick, the error reveals itself by a kink!
- If you watch closely you will observe that there are two oscillation periods per 2π (360 deg) phase. The beta-beating frequency is twice the frequency of the orbit!



Example: Gradient error in SNS



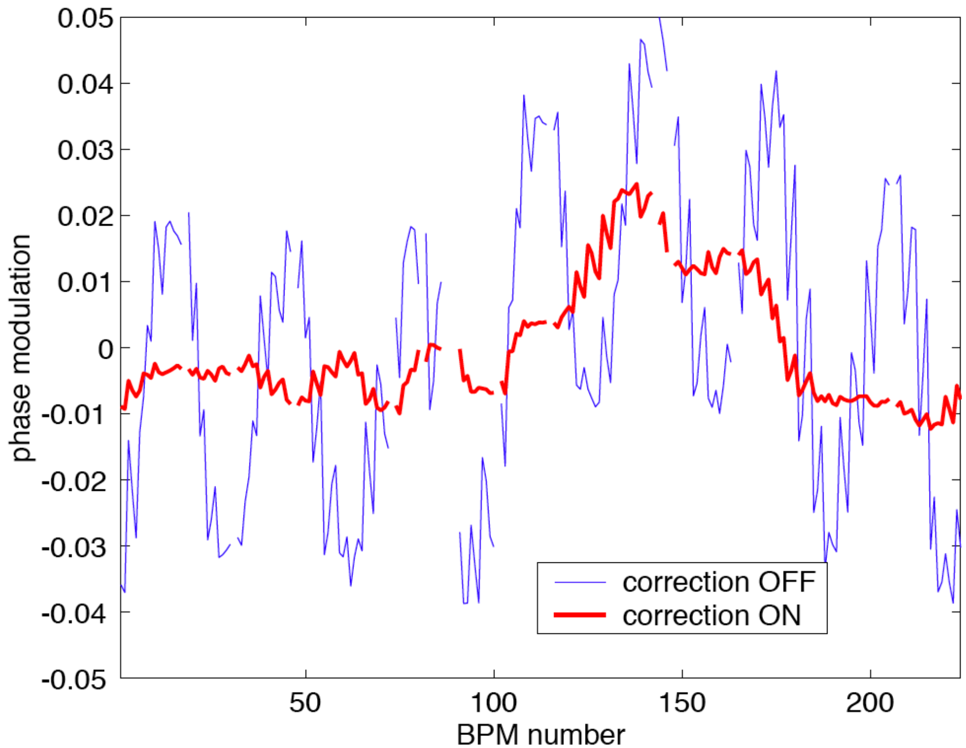


- Consider **18 focusing quads** in the SNS ring with **0.01 T/m** gradient error. In this location ***β* = 12 m**. The length of the quads is **0.5 m** and the magnetic rigidity is **5.6567 Tm**
  - The tune-shift is $\delta Q = \frac{1}{4\pi} 18 \cdot 12 \frac{0.01}{5.6567} 0.5 = 0.015$
- For a random distribution of errors the beta beating is

$$\frac{\delta\beta}{\beta_0}_{\text{rms}} = -\frac{1}{2\sqrt{2}|\sin(2\pi Q)|}\left(\sum_i \delta k_i^2 \beta_i^2\right)^{1/2}$$

  - Optics functions beating **> 20%** by random errors (1% of gradient) in high dispersion quads of the SNS ring … defines correctors strengths

Example: Gradient error in ESRF



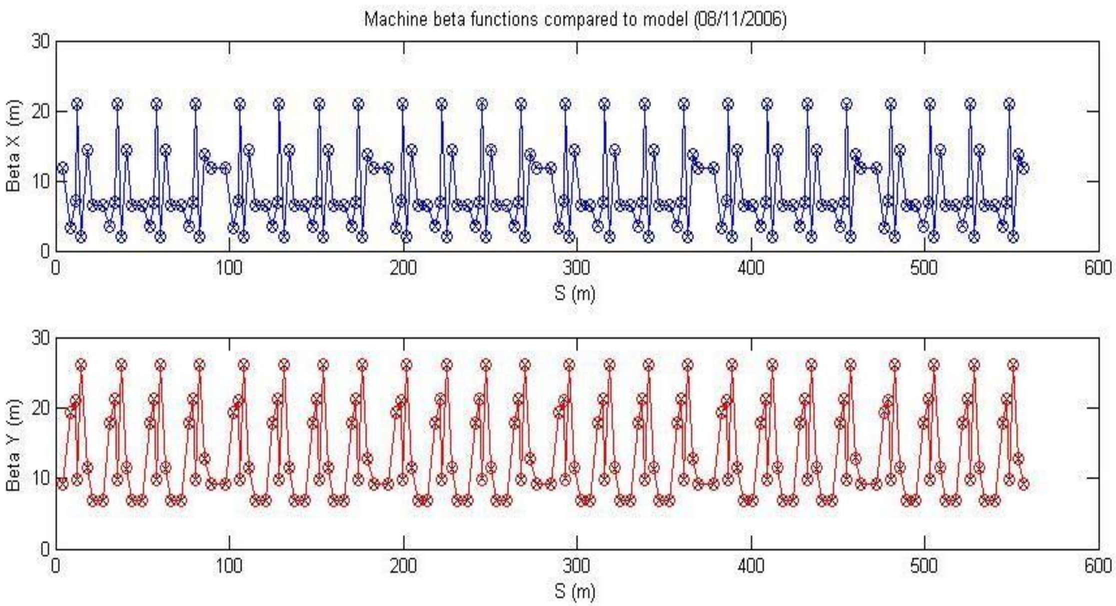
- Consider **128 focusing arc quads** in the ESRF storage ring with **0.001 T/m** gradient error. In this location β**= 30 m**. The length of the quads is around **1 m**. The magnetic rigidity of the ESRF is **20 Tm**.
- The tune-shift is $\delta Q = \frac{1}{4\pi} 128 \cdot 30 \frac{0.001}{20} 1 = 0.014$

Gradient error correction

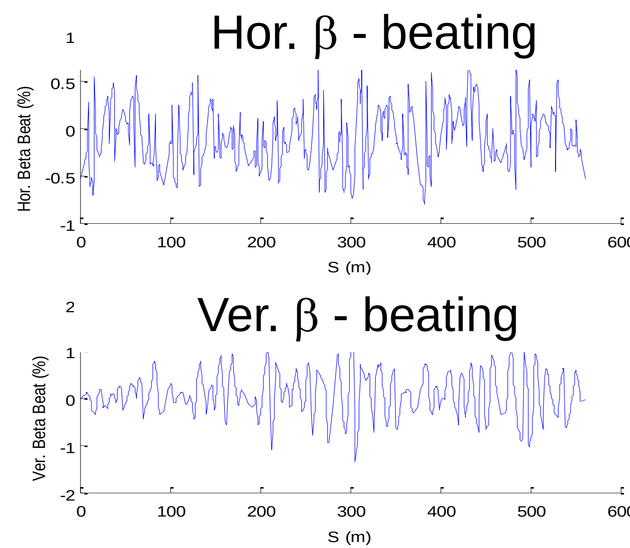
- **Quadrupole correctors**
  - Individual correction magnets
  - Windings on the core of the quadrupoles (trim windings)
  - Pairs of correctors at well-chosen locations for minimizing resonance
- **Methods & approaches**
  - Compute tune-shift and optics function beta distortion
  - Move working point close to integer and half integer resonance to increase sensitivity
  - Minimize beta wave or quadrupole resonance width with trim windings
  - Individual powering of trim windings can provide flexibility and beam based alignment of BPM
- **Modern methods of response matrix analysis (LOCO) can fit optics model to real machine and correct optics distortion**

Linear Optics from Closed Orbit

R. Bartolini, LER2010



J. Safranek et al.

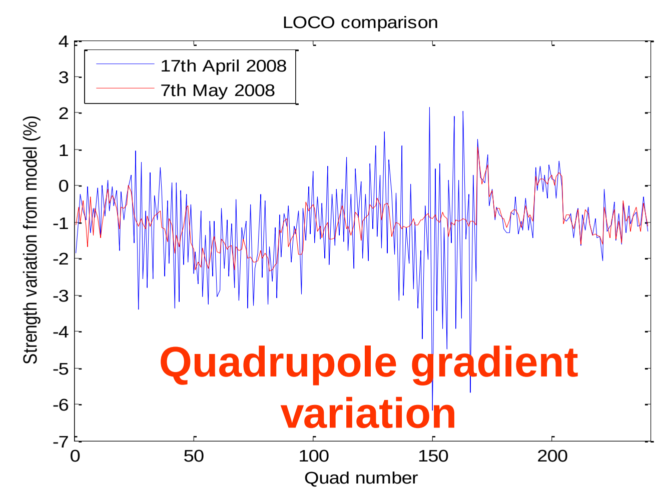


Modified version of LOCO with constraints on gradient variations **(see ICFA Newsletter, Dec' 07)**

β - beating reduced to 0.4% rms

Quadrupole variation reduced to 2%

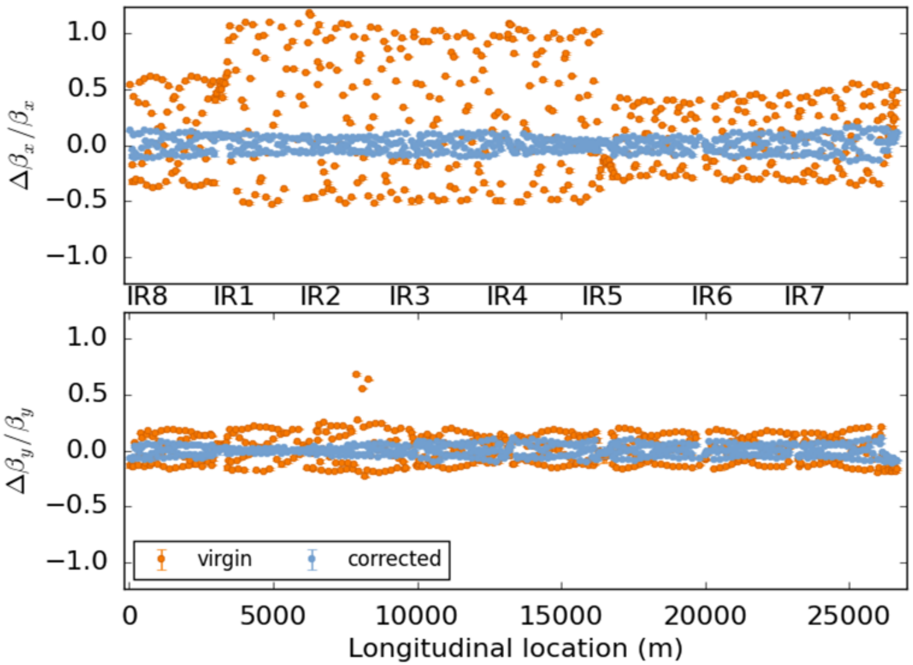
Results compatible with mag. meas. and calibrations



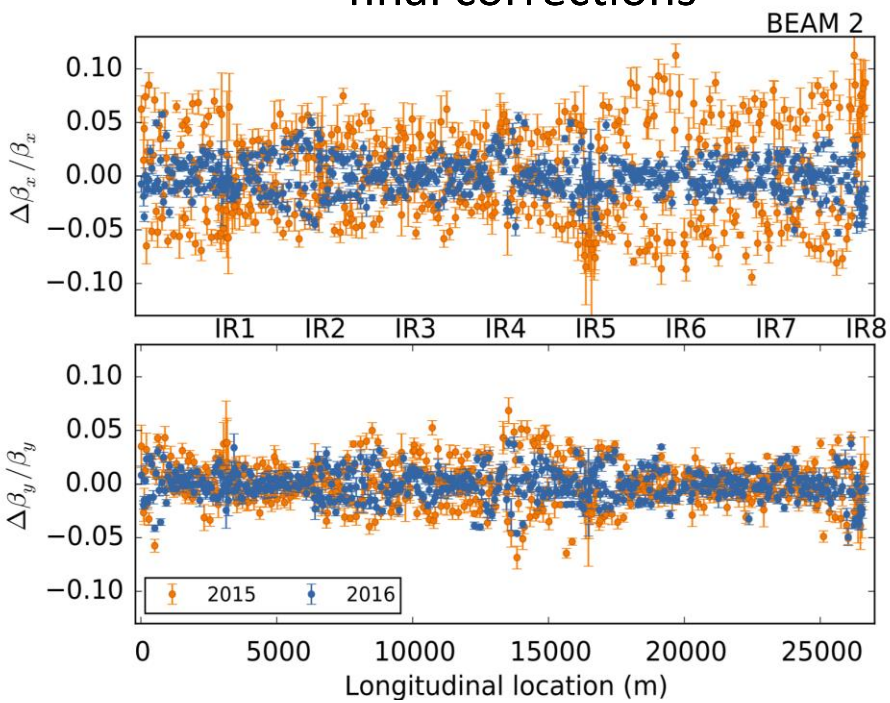
LOCO allowed remarkable progress with the correct implementation of the linear optics

- At β*=40cm, the bare machine has a beta-beat of more than 100%
- After global and local corrections, β-beating was reduced to few %

before and after local correction

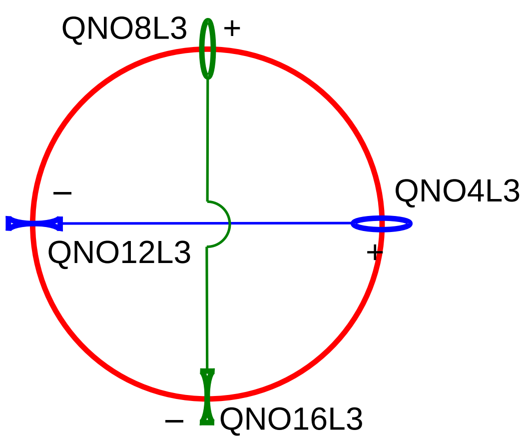


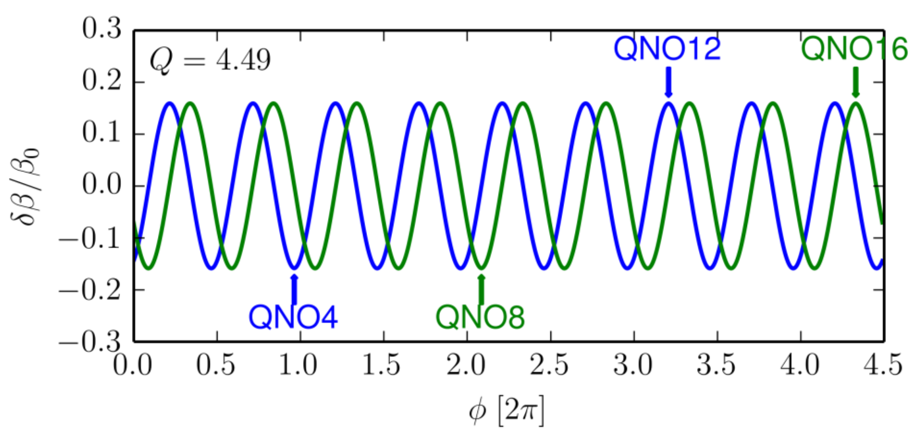
final corrections



R. Tomas et al. 2016

- **Compensation of quadrupole errors at half integer Q_y=4.5**
  - PSB has 16-fold symmetry
  - 2 families of normal quadrupole correctors
    - **+QNO4 and –QNO12** with $\Delta\mu_x = 2.25 * 2\pi$
    - **+QNO8 and –QNO16** with $\Delta\mu_x = 2.25 * 2\pi$
  - **Due to opposite polarity within each family, their contribution on beta-beating adds up** (beta-beat frequency is twice the tune!) **while there is no change of tune** (same change of focusing & defocusing)
  - The two families are orthogonal with respect to the half integer resonance driving term (every phase achievable)

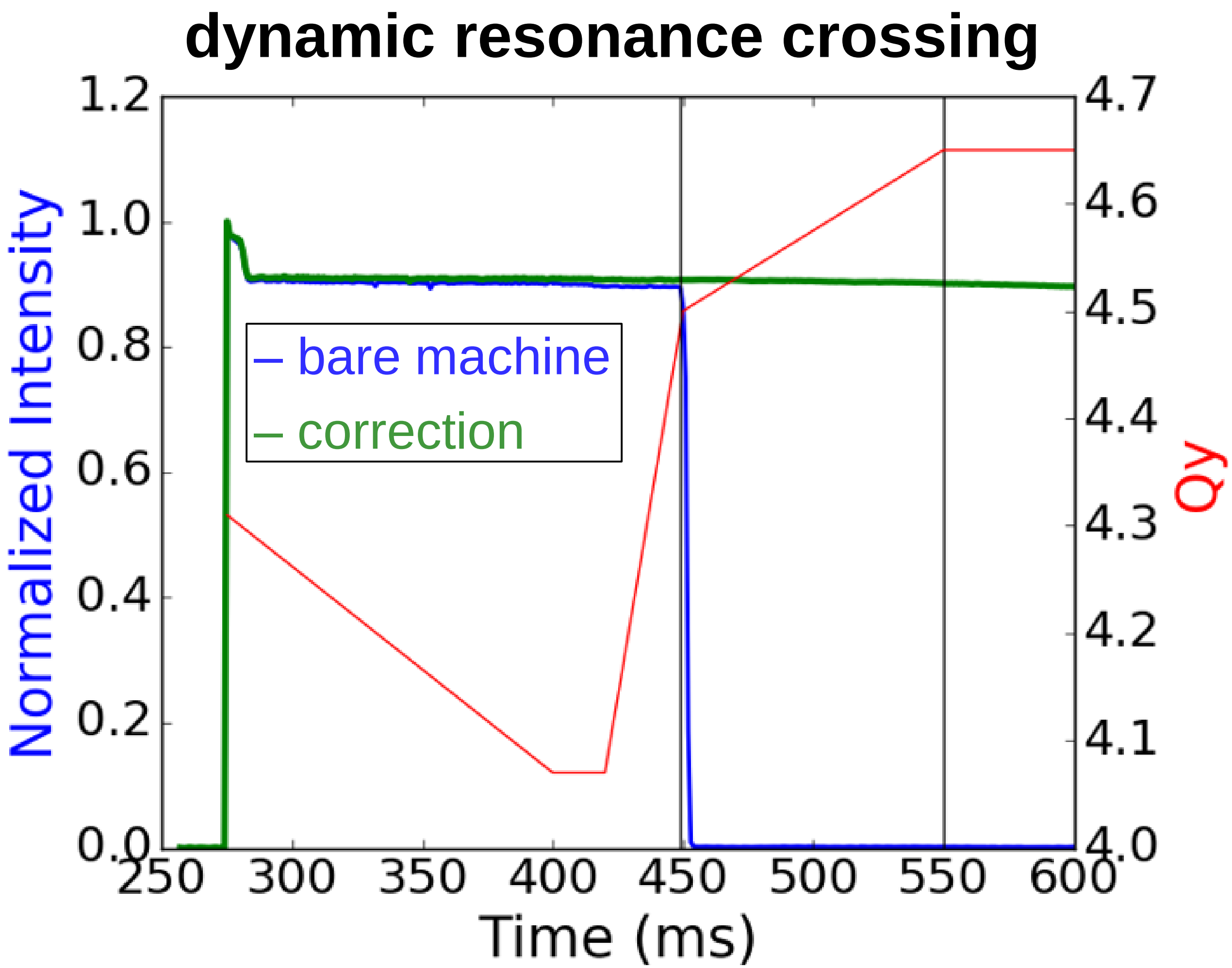




PSB half integer resonance correction

- **Experimental data!**

F. Asvesta, CERN



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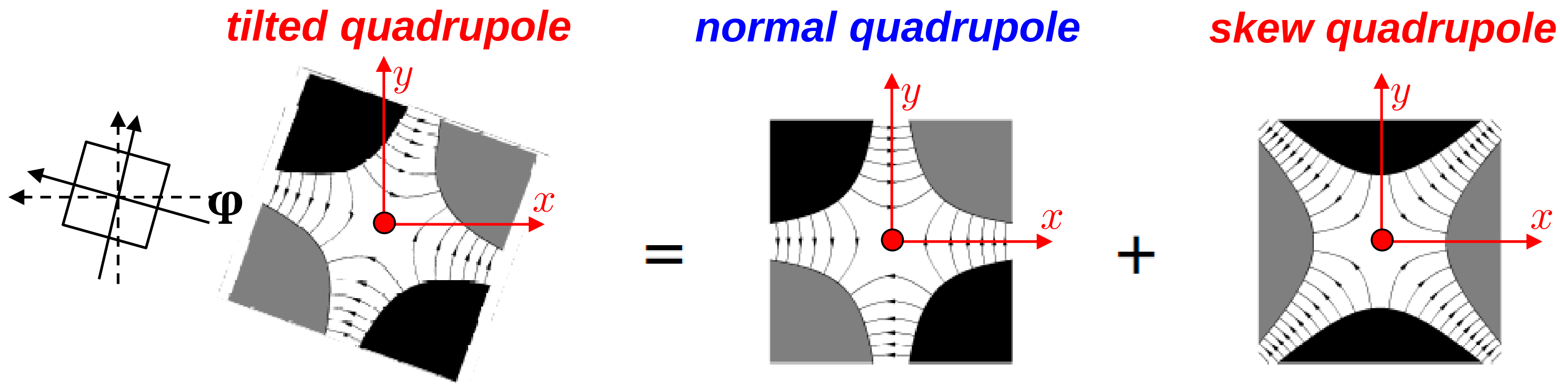
Coupling

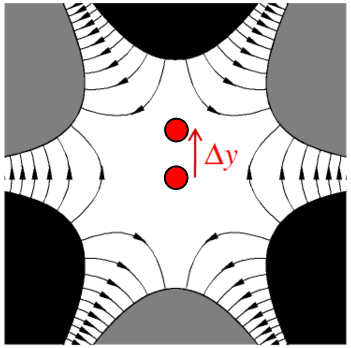
- Coupling may result from rotation of a quadrupole, so that the field contains a skew component



- A systematic vertical offset in a sextupole has the same effect as a skew quadrupole. For a displacement of δy the field becomes

$$B_x = 2B_2 x\bar{y} = 2B_2 xy + \underbrace{2B_2 x\delta y}_{\text{skew quadrupole}}$$

$$B_y = B_2(x^2 - \bar{y}^2) = \overbrace{-2B_2 y\delta y}^{\text{skew quadrupole}} + B_2(x^2 - y^2) - B_2(\delta y)^2$$



4x4 Matrices

- Combine the matrices for each plane

$$\begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = \begin{pmatrix} C_x(s) & S_x(s) \\ C'_x(s) & S'_x(s) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

$$\begin{pmatrix} y(s) \\ y'(s) \end{pmatrix} = \begin{pmatrix} C_y(s) & S_y(s) \\ C'_y(s) & S'_y(s) \end{pmatrix} \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}$$

to get a total 4x4 matrix

$$\begin{pmatrix} x(s) \\ x'(s) \\ y(s) \\ y'(s) \end{pmatrix} = \begin{pmatrix} C_x(s) & S_x(s) & 0 & 0 \\ C'_x(s) & S'_x(s) & 0 & 0 \\ 0 & 0 & C_y(s) & S_y(s) \\ 0 & 0 & C'_y(s) & S'_y(s) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \end{pmatrix}$$

Uncoupled motion

with skew quadrupoles these matrix elements are non-zero

Effect of coupling

- **Betatron motion is coupled in the presence of skew quadrupole components in the machine**
  - The field is $(B_x, B_y) = k_s(x, y)$ and Hill's equations are coupled
  - The motion is still linear with two new eigen-mode tunes, which are always split. For a thin skew quadrupole with k_s the induced tune split is

$$\delta Q \propto |k_s|\sqrt{\beta_x\beta_y}$$

  - Coupling coefficients represent the strength of coupling

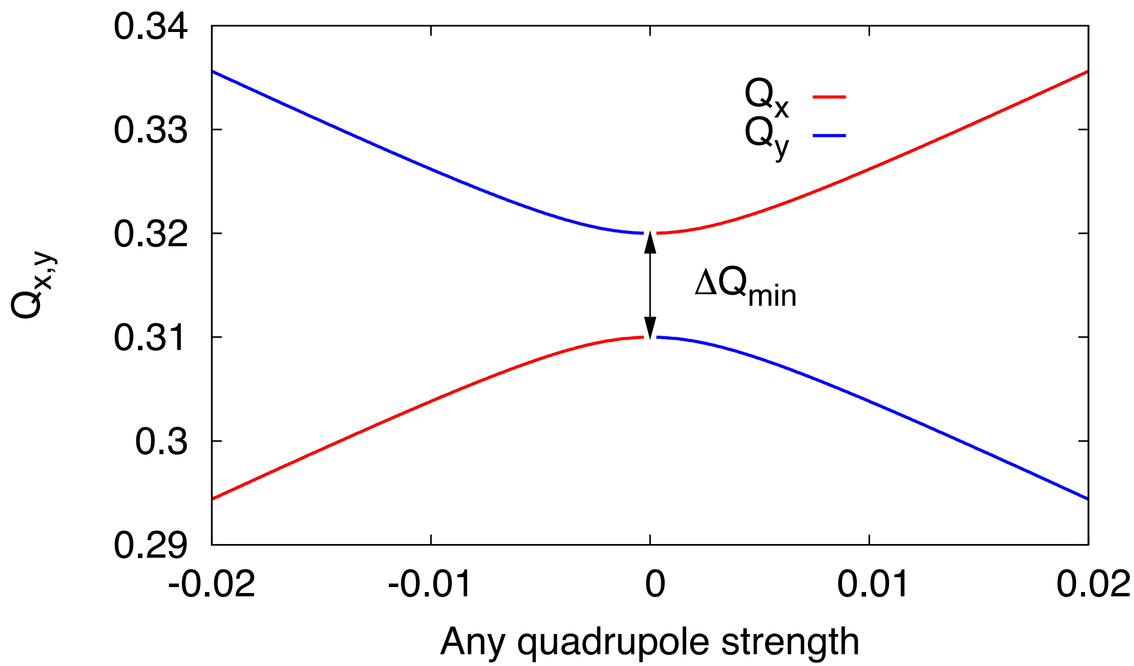
$$|C^{\pm}| = \left|\frac{1}{2\pi}\oint ds k_s(s)\sqrt{\beta_x(s)\beta_y(s)}e^{i(\psi_x\pm\psi_y-(Q_x\pm Q_y-q_{\pm})2\pi s/C)}\right|$$

  … complex number characterizing the difference resonance

$$Q_x - Q_y = N$$

- **As the motion is coupled, vertical dispersion and optics function distortion appears**

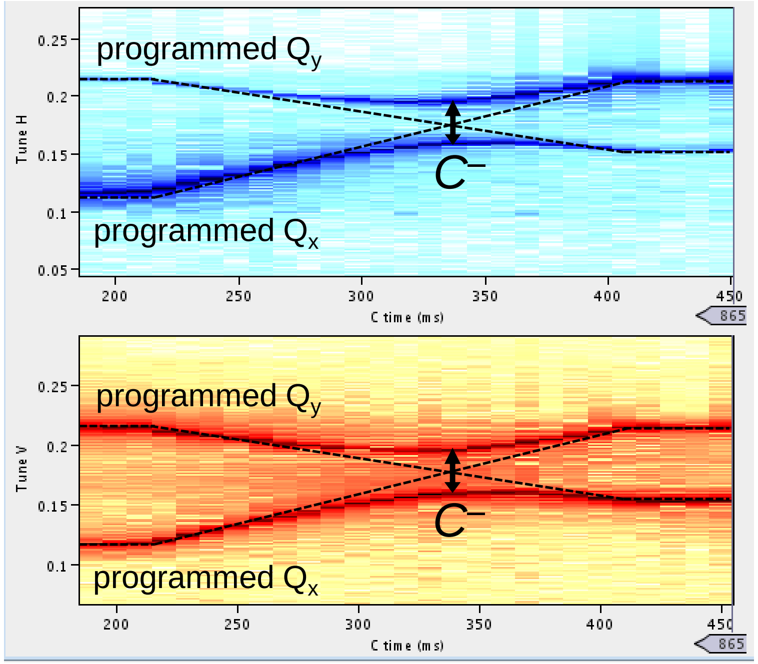
Closest tune approach



- Coupling makes it impossible to approach the tunes below $\Delta Q_{min} = |C^-|$, where C^- is again the coupling coefficient

Tune measurement in the CERN PS

quadrupole setting changed dynamically during storage time



tune peaks from both planes visible in Fourier spectra of horizontal and vertical motion

- Coupling makes it impossible to approach the tunes below $\Delta Q_{min} = |C^-|$, where C^- is again the coupling coefficient
- The coupling coefficient C^- can be measured very easily by trying to approach the tunes and measure the minimum distance

Linear coupling correction

- **Coupling correctors**
  - Introduce skew quadrupoles into the lattice
  - If skew quadrupoles are not available, one can make vertical closed orbit bumps in sextuple magnets (used in JPARC main ring until installation of skew quadrupole correctors)
- **Methods & approaches**
  - Correct globally/locally coupling coefficient (or resonance driving term)
  - Correct optics distortion (especially vertical dispersion)
  - Move working point close to coupling resonances and repeat
- **Remarks**
  - Correction especially important for beams with unequal emittances “flat beams” (coupling leads to emittance exchange)
  - The (vertical) orbit correction may be critical for reducing coupling (e.g. due to feed-down sextupoles)

Outline

- **Introduction**
- **Closed orbit distortion (steering error)**
  - Beam orbit stability
  - Imperfections leading to closed orbit distortion
  - Effect of single and multiple dipole kicks
  - Closed orbit correction methods
  - Dispersion and chromatic orbit
- **Optics function distortion (gradient error)**
  - Imperfections leading to optics distortion
  - Tune-shift and beta distortion due to gradient errors
  - Gradient error correction
- **Coupling error**
  - Coupling errors and their effect
  - Coupling correction
- **Chromaticity**

Chromaticity from quadrupoles

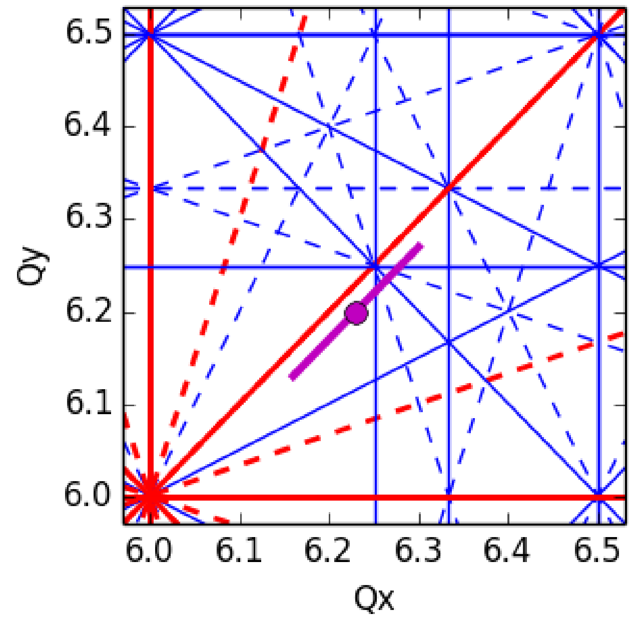
- Linear equations of motion depend on the energy (term proportional to dispersion)
- Chromaticity is defined as: $\xi_{x,y} = \frac{\delta Q_{x,y}}{\delta p/p}$
- Recall that the gradient is $k = \frac{G}{B\rho} = \frac{eG}{p} \rightarrow \frac{\delta k}{k} = \mp\frac{\delta p}{p}$
- This leads to dependence of tunes and optics function on the particle's momentum due to the **momentum dependent focusing of quadrupoles**
- For a linear lattice the tune shift is:

$$\delta Q_{x,y} = \frac{1}{4\pi}\oint \beta_{x,y}\delta K_{x,y}(s)ds = \pm\frac{1}{4\pi}\oint \beta_{x,y}\delta k(s)ds = \mp\frac{1}{4\pi}\frac{\delta p}{p}\oint \beta_{x,y}k(s)ds$$

- So the natural chromaticity is: $\xi_{x,y} = \mp\frac{1}{4\pi}\oint \beta_{x,y}k(s)ds$
- **Natural chromaticity is always negative** (since quadrupoles have to provide overall focusing)
- Sometimes the normalized chromaticity is quoted $\overline{\xi_{x,y}} = \frac{\xi_{x,y}}{Q_{x,y}}$

Chromaticity induced tune spread

- A beam consists of particles with momentum spread and through chromaticity this leads to a tune spread
- Example for SNS ring
  - In the SNS ring, the **natural chromaticity is –7** (in both planes)
  - Consider that **momentum spread $\delta p/p$ = ±1%**
  - Tune-shift for off-momentum particles



$$\delta Q_{x,y} = \xi_{x,y} \delta p/p = \pm 0.07$$

- To **correct chromaticity** need focusing for off-momentum particles

→ **Sextupoles**

Chromaticity from sextupoles

- The sextupole field component in the *x*-plane is: $B_y = B_2 x^2$
- In a location with non-zero dispersion the closed orbit is $x = x_o + D_x \frac{\delta p}{p}$
- Then the field is $B_y = B_2 x_0^2 + \underbrace{2B_2 D_x \frac{\delta p}{p} x_0}_{\text{quadrupole}} + \underbrace{B_2 D_x^2 \left(\frac{\delta p}{p}\right)^2}_{\text{dipole}}$
- With $k_2 = \frac{2B_2}{B\rho}$
  sextupoles introduce an equivalent focusing correction $\delta k = k_2 D_x \frac{\delta p}{p}$
- The sextupole induced chromaticity is

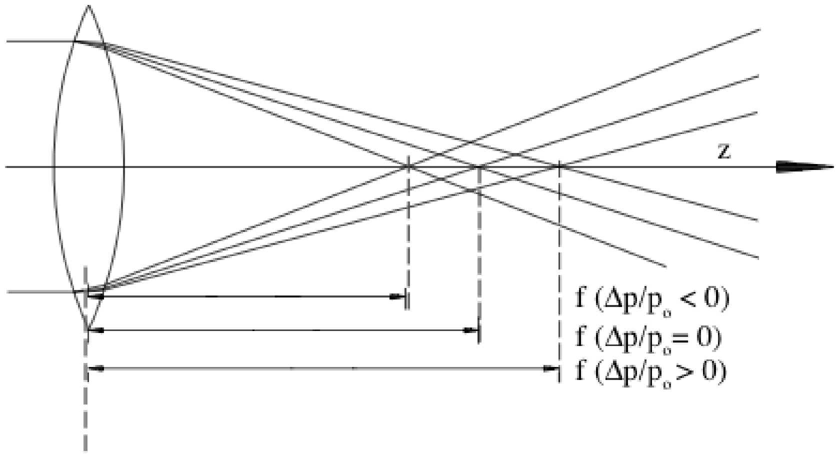
$$\xi^{\mathrm{S}}_{x,y} = \pm \frac{1}{4\pi} \oint \beta_{x,y} k_2(s) D_x(s) ds$$

- The total chromaticity is the sum of the natural and sextupole induced chromaticity

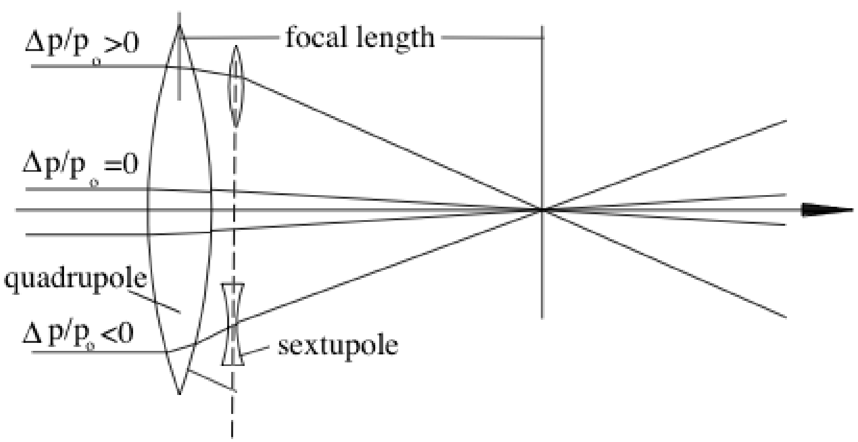
$$\xi^{\mathrm{tot}}_{x,y} = \mp \frac{1}{4\pi} \oint \beta_{x,y} (k(s) - k_2(s) D_x(s)) ds$$

Schematic of chromaticity correction

- Quadrupole focusing depends on particle momentum



- Sextupole in location with dispersion: closed orbit offset for off-momentum particles from dispersion results in feed-down in sextupole giving quadrupole effect (focusing or defocusing depending on sign of momentum)



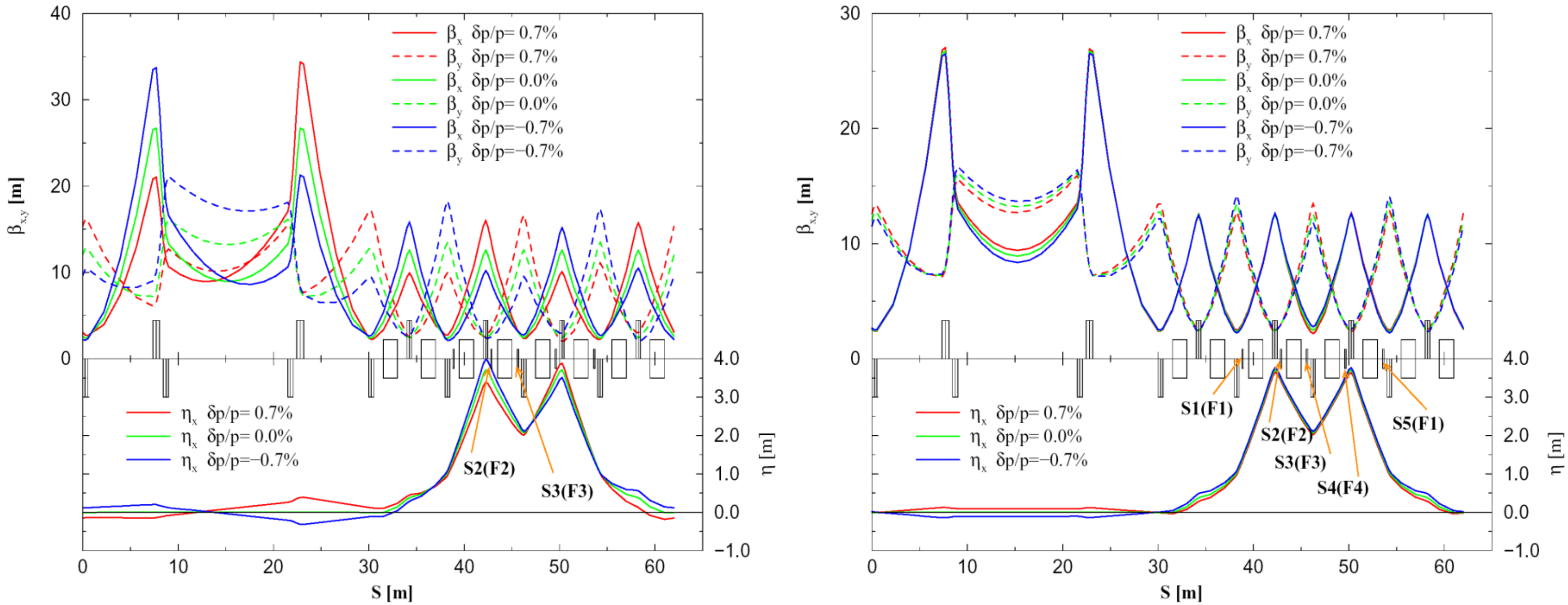
Chromaticity correction

- Install sextupoles in areas with high-dispersion and high beta
- Two families are able to control horizontal and vertical chromaticity (if installed in locations with different beta-functions in the two planes)
- Tune them to achieve desired chromaticity values
- Sextupoles introduce non-linear fields (can lead to chaotic motion)
- Sextupoles introduce tune-shift with amplitude
- Example:
  - The SNS ring has **natural chromaticity of –7**
  - Placing **two sextupoles of length 0.3 m** in locations where **β=12 m**, and the dispersion **D_x=4 m**
  - For getting 0 chromaticity, their strength should be

$$k_2 = \frac{7 \cdot 4\pi}{12 \cdot 4 \cdot 2 \cdot 0.3} \approx 3 \text{ m}^{-3}$$

or a gradient of **B_2=17.3 T/m^2**

Two vs. four sextupole families

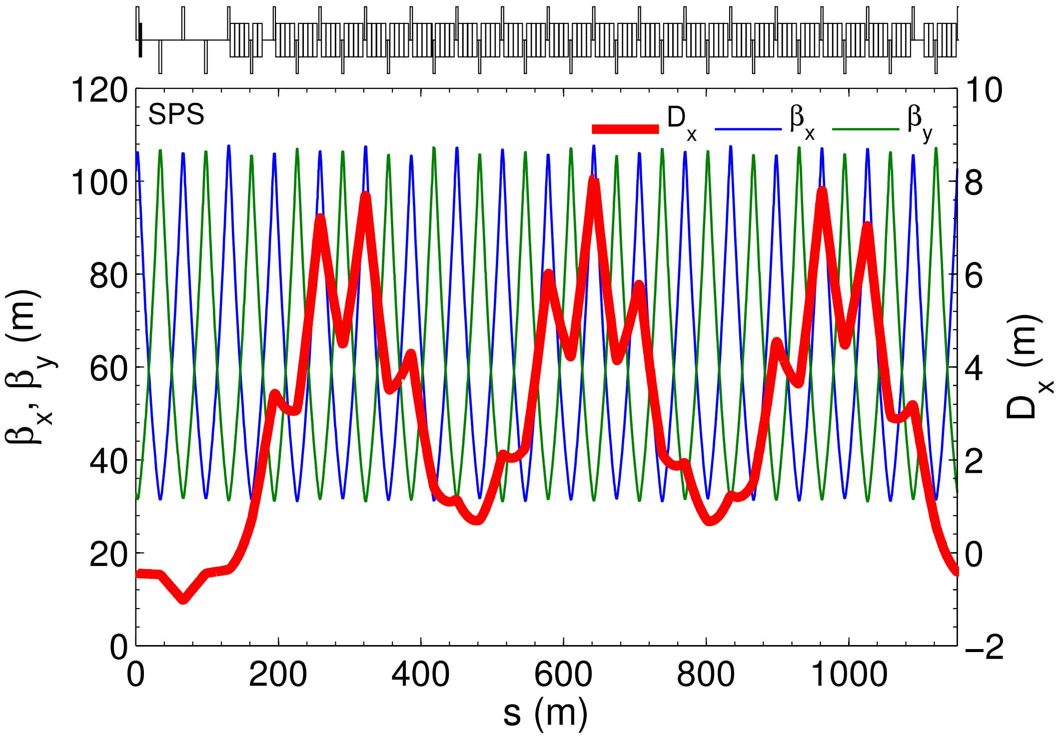


- Two families of sextupoles not enough for correcting off-momentum optics functions’ distortion and second order chromaticity
- Possible solutions:
  - Place sextupoles accordingly to eliminate second order effects (difficult)
  - Large optics function distortion for momentum spreads of ±0.7%,when using only two families of sextupoles → **more families** (4 in the case of the SNS ring) and optimize their settings to minimize off-momentum optics beating

Problem 5

The SPS is a **400 GeV proton synchrotron** with a FODO lattice consisting of **108** focusing and **108** defocusing quadrupoles of length **3.22 m** and a gradient of **15 T/m**, with a **horizontal and vertical beta of 108 m and 30 m** in the focusing quads (**30 m and 108 m** for the defocusing ones). The tunes are **Q_x=20.13** and **Q_y=20.18**.

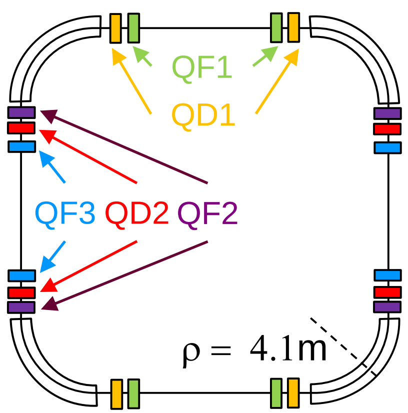
- Find the tune change for systematic gradient errors of 1% in the focusing and 0.5% in the defocusing quads.
- What is the natural chromaticity of the machine (without gradient errors)?

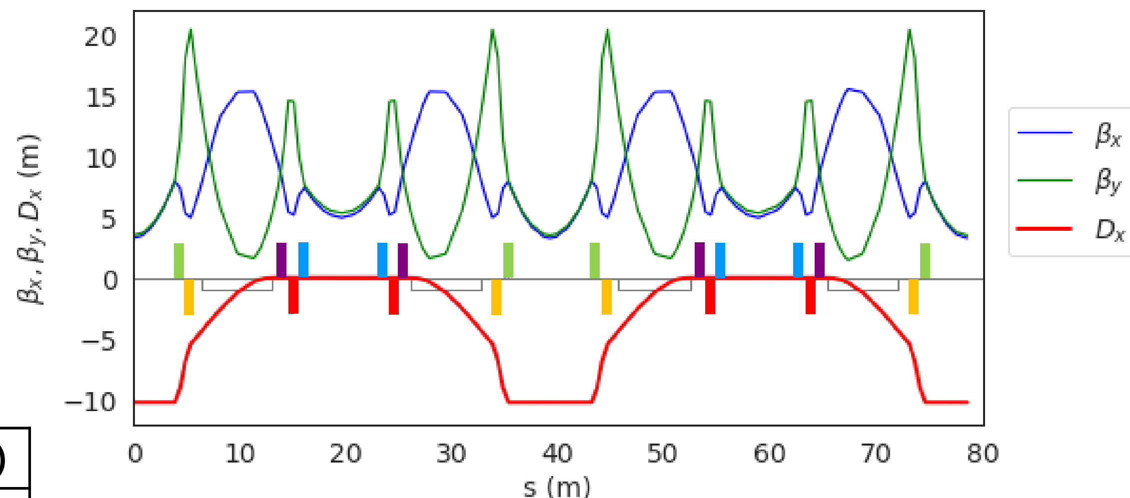


Problem 6

LEIR: Consider a heavy-ion **synchrotron** with **5 families of quadrupoles** (no FODO structure) and the optical functions from the plot and table below.

- What is the natural chromaticity of the machine? (quad length l_q=0.5m for all families, dipole length l_d=6.44m)
- What is the optimum placement for the sextupole magnets to correct the chromaticity? Give their estimated k_2 value to get $\xi_{x,y}^{tot} = 0$? (assume $l_s = l_q$)





	k_1 (m^{-2})	β_x (m)	β_y (m)	D_x (m)
QF1	0.9041	7.9	7.9	-10.9
QD1	-1.1303	5.3	18.0	-7.3
QF2	0.3088	7.2	10.8	0
QD2	-1.3181	5.4	14.5	0
QF3	0.7167	7.4	7.6	0

Summary

- Linear imperfections such as magnet misalignments and field errors are unavoidable in a real accelerator, but they can be corrected to some extent as summarized in this table:

Error	**Effect**	**Cure**
fabrication imperfections	unwanted multipolar components	better fabrication / multipolar correctors coils
transverse misalignments	feed-down effect	better alignment / correctors
dipole kicks	orbit distortion / residual dispersion	corrector dipoles
quad field errors	tune shift, beta-beating	trim special quadrupoles
quadrupole tilts	coupling x − y	better alignment / skew quads
chromaticity	tune spread	sextupoles
power supplies	closed orbit distortion / tune shift / modulation	improve power supplies and their calibration