

# ***CORRECTION SCHEMES***

***Lecture 4***  
***January 2020***

***P.J. Bryant***

# Introduction

- ❖ Consider the basic question of how to determine the orbit excursion  $\Delta z$  at a given point due to an upstream kick

$$\delta = \Delta z' = \frac{\ell \Delta B}{B \rho}. \quad (4.1)$$

- ❖ In ‘transfer line’ situations this is done by tracking the betatron oscillation (from Lecture 1),

$$z(s) = A \beta^{1/2}(s) \cos(\mu(s) + B) \quad (4.2)$$

$$z'(s) = -A \beta^{-1/2} [\alpha \cos(\mu + B) + \sin(\mu + B)] \quad (4.3)$$

- ❖ or using the transfer matrix (also from Lecture 1),

$$M(s_1 \rightarrow s_2) = \begin{pmatrix} \left( \frac{\beta_2}{\beta_1} \right)^{1/2} (\cos \Delta\mu + \alpha_1 \sin \Delta\mu) & (\beta_1 \beta_2)^{1/2} \sin \Delta\mu \\ -(\beta_1 \beta_2)^{-1/2} [(1 + \alpha_1 \alpha_2) \sin \Delta\mu + (\alpha_2 - \alpha_1) \cos \Delta\mu] & \left( \frac{\beta_1}{\beta_2} \right)^{1/2} (\cos \Delta\mu - \alpha_2 \sin \Delta\mu) \end{pmatrix} \quad (4.4)$$

**Note that ‘Transfer line’ situations include sections of rings used as a transfer line, e.g. for a closed bump.**

## *Introduction continued*

- ❖ However, in rings, there is a more holistic approach that expresses the closed orbit in the ring in the presence of kicks by using a perturbation equation. This equation is surprisingly accurate and widely used.

$$\Delta z(s) = \mp \frac{\beta_z^{1/2}(s)}{2 \sin \pi Q_z} \sum_n \beta_n^{1/2} \delta_n \cos[|\mu_n - \mu_z(s) - \pi Q_z|] \quad (4.5)$$

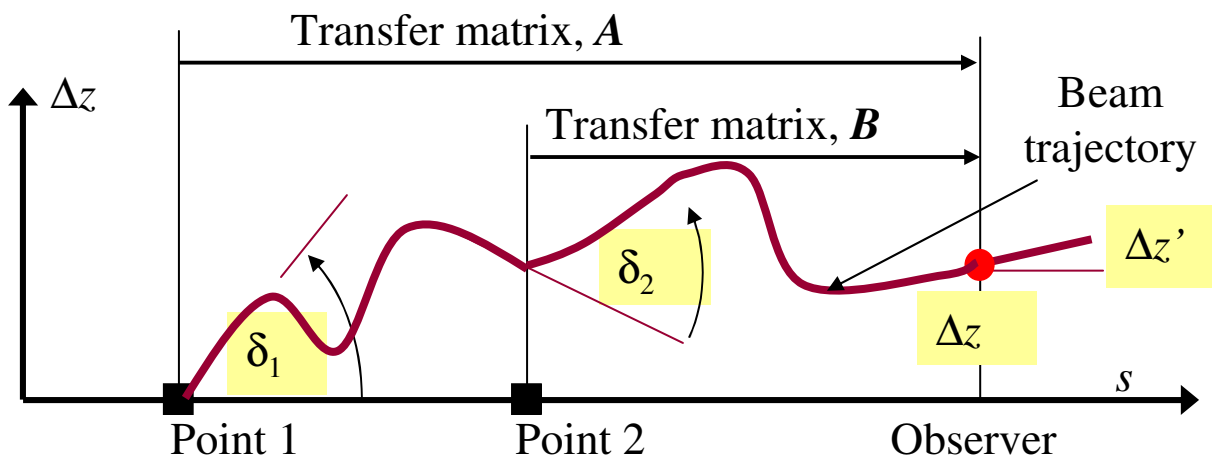
The upper sign is for the horizontal plane and the lower sign is for the vertical plane.  $\beta_z(s)$  and  $\mu_z(s)$  refer to the observer while  $\beta_n$  and  $\mu_n$  refer to the  $n$ th error.

**Note the modulus sign in the phase term !!!**

**Equation (4.5) should be covered in other parts of the course, but can be found in Ref. [1.2].**

# Transfer line steering

- ❖ How to change (steer) the position and angle ( $\Delta z, \Delta z'$ ) at a given point in a transfer line using two upstream dipole kicks ( $\delta_1, \delta_2$ ).



- ❖ The system is linear, so the effect of each kick at the Observer can be calculated and the effects added.

$$\begin{pmatrix} \Delta z \\ \Delta z' \end{pmatrix} = \underbrace{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} 0 \\ \delta_1 \end{pmatrix}}_{\text{Effect of } \delta_1 \text{ at Observer}} + \underbrace{\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} 0 \\ \delta_2 \end{pmatrix}}_{\text{Effect of } \delta_2 \text{ at Observer}} \quad (4.6)$$

# *Transfer line steering continued*

❖ which can be rewritten as,

$$\begin{pmatrix} \Delta z \\ \Delta z' \end{pmatrix} = \begin{pmatrix} a_{12} & b_{12} \\ a_{22} & b_{22} \end{pmatrix} \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} \quad (4.7)$$

where  $\Delta$  indicates the changes in the position and angle seen by the Observer.

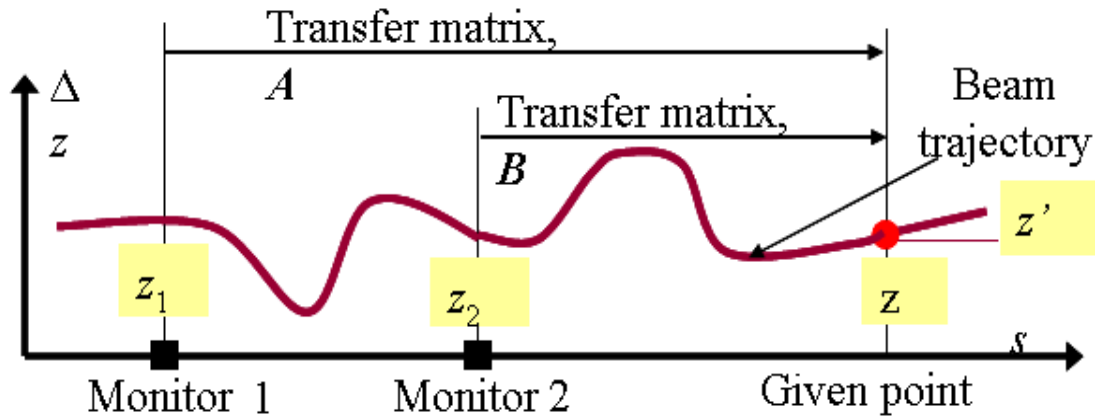
❖ Inverting (4.7) gives the kicks required for the position and angle changes,

$$\begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} = \frac{1}{(b_{22}a_{12} - b_{12}a_{22})} \begin{pmatrix} b_{22} & -b_{12} \\ -a_{22} & a_{12} \end{pmatrix} \begin{pmatrix} \Delta z \\ \Delta z' \end{pmatrix}_{\text{Observer}} \quad (4.8)$$

**Note that the modulus of the matrix is not unity and must be included.**

**A similar reasoning can be used to transform two independent orbit measurements into a position and angle measurement at a given point.**

# Position & angle measurement rings & transfer lines



- ❖ **Transfers to given point from each monitor,**

$$\begin{pmatrix} z \\ z' \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} z_1 \\ z_1' \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} z \\ z' \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} z_2 \\ z_2' \end{pmatrix}$$

- ❖ **Eliminate  $z_1'$  and  $z_2'$**

$$z_1 \left( \frac{a_{11}}{a_{12}} - \frac{a_{21}}{a_{22}} \right) = \frac{z}{a_{12}} - \frac{z'}{a_{22}} \quad \text{and} \quad z_2 \left( \frac{b_{11}}{b_{12}} - \frac{b_{21}}{b_{22}} \right) = \frac{z}{b_{12}} - \frac{z'}{b_{22}}$$

- ❖ **Solve for  $z$  and  $z'$**

$$\begin{aligned} z &= \frac{1}{(a_{22}b_{12} - a_{12}b_{22})} (z_1 b_{12} - z_2 a_{12}) \\ z' &= \frac{1}{(a_{22}b_{12} - a_{12}b_{22})} (z_1 b_{22} - z_2 a_{22}) \end{aligned} \quad (4.9)$$

**Note  $|A|=a_{11}a_{22}-a_{12}a_{21}=1$  and  $|B|=b_{11}b_{22}-b_{12}b_{21}=1$ , but (4.9) is not based on a unit matrix.**

# Half-wavelength bump rings & transfer lines

- ❖ Recalling (4.2) and (4.3), we have,

$$z(s) = A\beta(s)^{1/2} \cos[\mu(s) + B]$$

$$z'(s) = -A\beta^{-1/2} [\alpha \cos(\mu + B) + \sin(\mu + B)]$$

- ❖ If the first kick  $\delta_1 (= z'_1)$  is put at  $\mu_1 = -\pi/2$ , then  $B = 0$  and  $A = \delta_1\beta_1^{1/2}$ . At  $\mu = \pi/2$ , just half a wavelength later, the excursion will again be zero. At this point the oscillation can be killed by a second kick  $\delta_2$ , which is equal and opposite to the trajectory slope at this point, so that  $\delta_2 = -z'_2$ , which gives the conditions,

Imposed condition:  $\mu_2 - \mu_1 = \pi$

Derived condition:  $\delta_1\beta_1^{1/2} = \delta_2\beta_2^{1/2} \quad (4.10)$

- ❖ The bump height can be controlled at any point by scaling the kicks, but the angle of the trajectory is a feature of the lattice geometry and cannot be controlled.

**It would also be possible to use the perturbation equation (4.5) for this derivation in a ring.**

# 3-magnet bump rings & transfer lines

- ❖ It is rare that magnets can be placed with a phase separation of exactly  $\pi$ . Even when possible, this makes the lattice inflexible for future developments.
- ❖ It is therefore useful to know how to correct the residual error of an imperfect 2-magnet bump with a third dipole.
- ❖ Use (4.4) to track forwards from kick 1 to kick 2.

$$\begin{cases} z_{2,f} = (\beta_1 \beta_2)^{1/2} \sin(\Delta\mu_{2,1}) \delta_1 \\ z'_{2,f} = (\beta_1 / \beta_2)^{1/2} [\cos(\Delta\mu_{2,1}) - \alpha_2 \sin(\Delta\mu_{2,1})] \delta_1 \end{cases}$$

- ❖ Also use (4.4) to track backwards from kick 3 to kick 2.

$$\begin{cases} z_{2,b} = (\beta_3 \beta_2)^{1/2} \sin(-\Delta\mu_{2,3}) \delta_3 \\ z'_{2,b} = (\beta_3 / \beta_2)^{1/2} [\cos(-\Delta\mu_{2,3}) - (-\alpha_2) \sin(-\Delta\mu_{2,3})] \delta_3 \end{cases}$$



## *3-magnet bump rings & transfer lines continued*

- ❖ The forward- and back-track amplitudes at  $\delta_2$  must be identical and the difference in the derivatives must be matched by the dipole kick  $\delta_2$ , i.e.

$$z_{2,f} = z_{2,b} \quad \text{and} \quad \delta_2 = -z'_{2,b} - z'_{2,f}$$

- ❖ Some manipulation of the above equations yields (the ‘sine rule’),

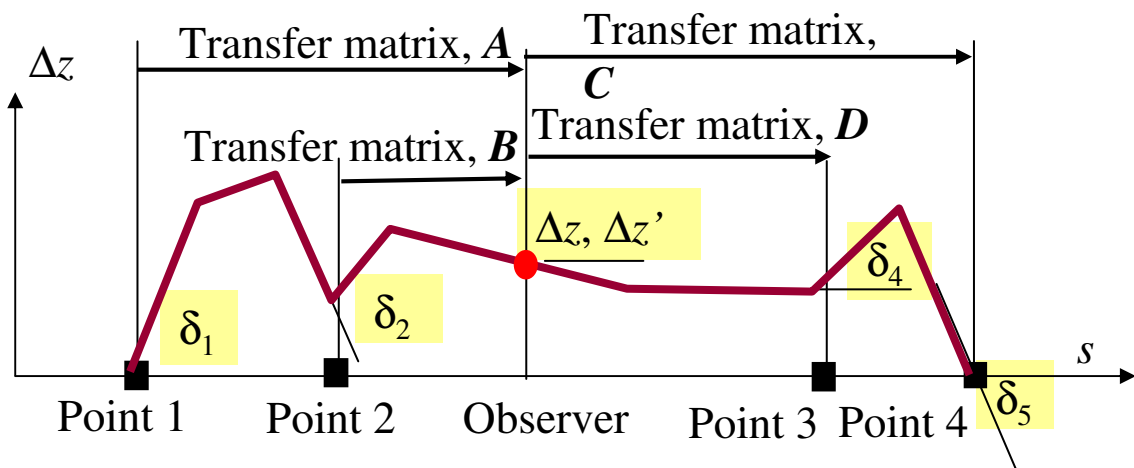
$$\frac{\delta_1 \beta_1^{1/2}}{\sin(\Delta\mu_{3,2})} = \frac{\delta_2 \beta_2^{1/2}}{\sin(\Delta\mu_{1,3})} = \frac{\delta_3 \beta_3^{1/2}}{\sin(\Delta\mu_{2,1})} \quad (4.11)$$

- ❖ As with the 2-magnet, half-wavelength bump, the excursion of the trajectory can be controlled at any point by scaling the kicks, but the angle of the trajectory is a feature of the lattice geometry and cannot be controlled.

It would also be possible to use the perturbation equation (4.5) for this derivation.

# 4-magnet bump rings & transfer lines

- ❖ Often a local bump is required that controls both the position and angle of the beam at some particular position. This requires four magnets with one pair upstream of the control point and one pair downstream.



- ❖ Calculate the kicks  $\delta_1$  and  $\delta_2$  to achieve the displacement  $\Delta z, \Delta z'$  at the Observer position by using the steering equation (4.8).

$$\begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} = \frac{1}{(b_{22}a_{12} - b_{12}a_{22})} \begin{pmatrix} b_{22} & -b_{12} \\ -a_{22} & a_{12} \end{pmatrix} \begin{pmatrix} \Delta z \\ \Delta z' \end{pmatrix}_{\text{Observer}} \quad (4.8)$$

# 4-magnet bump rings & transfer lines continued

❖ Eqn (4.8) can also be used to specify the downstream kicks, but because the transfer matrices and kicks are defined in the beam direction, the downstream kicks that close the bump are found by back-tracking.

❖ Remember,

$$C^{-1} = \frac{1}{|C|} \begin{pmatrix} c_{22} & -c_{12} \\ -c_{21} & c_{11} \end{pmatrix} = \begin{pmatrix} c_{22} & -c_{12} \\ -c_{21} & c_{11} \end{pmatrix} \text{ since } |C|=1$$

$$D^{-1} = \frac{1}{|D|} \begin{pmatrix} d_{22} & -d_{12} \\ -d_{21} & d_{11} \end{pmatrix} = \begin{pmatrix} d_{22} & -d_{12} \\ -d_{21} & d_{11} \end{pmatrix} \text{ since } |D|=1$$

❖ This gives,

$$\begin{pmatrix} \Delta z \\ -\Delta z' \end{pmatrix}_{\text{Observer}} = \underbrace{\begin{pmatrix} c_{22} & -c_{12} \\ -c_{21} & c_{11} \end{pmatrix} \begin{pmatrix} 0 \\ -\delta_4 \end{pmatrix}}_{\text{Effect of } \delta_4 \text{ at Observer}} + \underbrace{\begin{pmatrix} d_{22} & -d_{12} \\ -d_{21} & d_{11} \end{pmatrix} \begin{pmatrix} 0 \\ -\delta_3 \end{pmatrix}}_{\text{Effect of } \delta_3 \text{ at Observer}}$$

$$\begin{pmatrix} \Delta z \\ -\Delta z' \end{pmatrix}_{\text{Observer}} = \begin{pmatrix} -c_{12} & -d_{12} \\ c_{11} & d_{11} \end{pmatrix} \begin{pmatrix} -\delta_4 \\ -\delta_3 \end{pmatrix} \quad (4.12)$$

# Overlapping orbit bumps rings & transfer lines

- ❖ To correct a distorted orbit, a first reflex could be to apply correcting bumps to all peaks in the distortion.
- ❖ Consider a regular lattice with phase advance  $\mu_0$  per cell. Let correctors and monitors be co-incident with one set per cell with  $\beta_0$  and let consecutive groups of 3 make well-shaped, overlapping closed bumps.

For example, 3 FODO cells of  $\mu_0 = \pi/3$  with monitor and corrector pairs at each F-quad for the horizontal orbit.

- ❖ Clearly the distortion at each monitor can be made zero by the appropriate bump. Because monitors and correctors are co-incident, adjacent monitors are unaffected by a bump.
- ❖ Use (4.4) to track forwards in a bump from kick 1 to kick 2.

$$z_2 = \beta_0 \sin(\mu_0) \delta_1$$

- ❖ Now use (4.11), the ‘sine rule’,

$$\frac{\delta_1}{\sin(\mu_0)} = \frac{\delta_2}{\sin(-2\mu_0)} = \frac{\delta_3}{\sin(\mu_0)}$$

to express each kick in terms of  $z_2$

$$\delta_1 = \frac{1}{\beta_0 \sin(\mu_0)} z_2, \quad \delta_2 = -\frac{2 \cos(\mu_0)}{\beta_0 \sin(\mu_0)} z_2, \quad \delta_3 = \frac{1}{\beta_0 \sin(\mu_0)} z_2$$

(4.13)

# Overlapping orbit bumps rings & transfer lines continued

- ❖ If the orbit is measured and all bumps applied, then the net kick at the  $n$ th corrector will be,

$$\delta_n = \frac{1}{\beta_0 \sin(\mu_0)} z_{n-1} - \frac{2 \cos(\mu_0)}{\beta_0 \sin(\mu_0)} z_n + \frac{1}{\beta_0 \sin(\mu_0)} z_{n+1} \quad (4.14)$$

- ❖ Equation (4.14) defines the method, but there is a fear that a betatron oscillation in an *error-free region* will lead to the unnecessary excitation of correctors.
- ❖ In such a region, the oscillation will have the form

$$z_n = A \beta_0^{1/2} \cos(n\mu_0 + \phi) \quad (4.15)$$

where  $A$  and  $\phi$  will be constants.

- ❖ Substitution of (4.15) into (4.14) shows this excitation will be zero thus making the overlapping bumps method viable (attributed to T. Collins).

**In theory, the above applies to rings and transfer lines, but is better for transfer lines.**

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$$\begin{aligned} \delta_n &= \frac{A\sqrt{\beta_0} \cos([n-1]\mu_0 + \phi)}{\beta_0 \sin(\mu_0)} - \frac{2 \cos(\mu_0) A\sqrt{\beta_0} \cos(n\mu_0 + \phi)}{\beta_0 \sin(\mu_0)} + \frac{A\sqrt{\beta_0} \cos([n+1]\mu_0 + \phi)}{\beta_0 \sin(\mu_0)} \\ &= \cos([n\mu_0 + \phi] - \mu_0) - 2 \cos(\mu_0) \cos(n\mu_0 + \phi) + \cos([n\mu_0 + \phi] + \mu_0) \\ &= 2 \cos(n\mu_0 + \phi) \cos(\mu_0) - 2 \cos(\mu_0) \cos(n\mu_0 + \phi) = 0 \end{aligned}$$

# *The 'best' corrector for rings*

- ❖ In machines with customised lattices, it is more useful to search for a small group of the most effective correctors.
- ❖ The simplest approach is to scan through the correctors one by one applying a correlation test.
- ❖ Let  $\Delta z_n$  be the excursion calculated in the  $n$ th monitor due to a kick  $\delta_m$  from the  $m$ th corrector.

$$\Delta z_n = t_{n,m} \delta_m$$

where  $t_{n,m}$  is the matrix element that relates  $\delta_m$  to the orbit shift  $\Delta z_n$ .  $t_{n,m}$  can be found theoretically from (4.5).

- ❖ If  $z_n$  is the measured orbit error, then the corrected orbit using the  $m$ th corrector would be  $(z_n - \Delta z_n)$ .
- ❖ The quality of the correction could be quantified by the expression

$$S_m = \sum_n (z_n - t_{n,m} \delta_m)^2 \quad (4.16)$$

# The 'best' corrector for rings continued

- ❖ Differentiating (4.16) finds the turning point and hence the optimum kick for the  $m$ th corrector,

$$\frac{\partial S_m}{\partial \delta_m} = 0 = -2 \sum_n (z_n - t_{n,m} \delta_m) t_{n,m}$$

$$\delta_m = \frac{\sum_n z_n t_{n,m}}{\sum_n t_{n,m}^2} \quad (4.17)$$

- ❖ By performing this simple calculation (4.17) on the corrector with the minimum value of  $S_m$  an optimum correction can be applied.
- ❖ The newly corrected orbit can then be re-measured and re-corrected in the same way, so extending the method to the 'next best' correctors.
- ❖ **The advantages are speed, scalability and simplicity.**
- ❖ **The disadvantage is that a correction once made is 'frozen in'.**

# *Closed-orbit correction for rings*

- ❖ Closed-orbit correction is a compromise. The orbit is measured at only a finite number of positions and there is only a finite number of correctors. In general, the correction cannot be guaranteed either at the measured points or between them.
- ❖ In transfer lines, the *overlapping bump* method works well when there are 6 or more monitors and correctors per betatron wavelength.
- ❖ In rings, provided that the number of measurement points is sufficiently large and the points are evenly enough spaced to measure the Fourier harmonics in the distortion up to an order of a few units above the tune, the gamble usually pays off.
- ❖ This is because ‘naturally’ arising errors excite orbits dominated by harmonics around the tune value and the high-order harmonics that could excite large excursions between the measured points are suppressed by a factor  $Q^2/(Q^2-k^2)$  where  $k$  is the order of the harmonic.
- ❖ Finding a set of efficient correctors is basically a problem of matrix inversion, but the solution also needs to accommodate practical problems,
  - ❖ Unequal numbers of monitors and correctors.
  - ❖ The suppression of faulty monitors and correctors.
  - ❖ Noise in monitor readings.
  - ❖ Limitations on corrector strengths.



## MICADO Ref. [4.1]

- ❖ To avoid the ‘Freeze-in effect’, a group of  $M$  correctors have to be optimised together.
- ❖ Looking at (4.16), we can formulate a measure of the quality of a correction by  $M$  correctors to the  $N$  measured orbit errors ( $N \geq M$ ) as,

$$S = \sum_{n=1}^N \left( z_n - \sum_{m=1}^M t_{n,m} \delta_m \right)^2 \quad (4.18)$$

- ❖ As before we differentiate  $S$  to find the minimum with respect to all correctors  $\delta_m$ ,

$$0 = \sum_n \left( z_n - \sum_m t_{n,m} \delta_m \right) t_{n,m} \quad (4.19)$$

- ❖ After some manipulation, this can be written as,

$$[\delta] = -\left(A^T A\right)^{-1} A^T [z] \quad (4.20)$$

where  $A$  is known as the *Design Matrix* that holds the relations  $t_{n,m}$  between kicks and excursions.  $[\delta]$  and  $[z]$  are column matrices holding the correction kicks and the orbit errors respectively.

It is standard practice to compute a series of closed orbit corrections comprising 1 to a medium sized fraction of the total number of correctors in the machine. The best correction is then selected and set by the operator.

# *Tune shifts in rings*

- ❖ Let a gradient perturbation  $\Delta k\ell$  be represented by a ‘thin lens’. Since a ‘thin lens’ has zero length, it can be multiplied into the single-turn matrix equation (1.9) without affecting the geometry.

$$T_{(s+Cl_s)} = \begin{pmatrix} \cos 2\pi Q_0 + \alpha \sin 2\pi Q_0 & \beta \sin 2\pi Q_0 \\ -\gamma \sin 2\pi Q_0 & \cos 2\pi Q_0 - \alpha \sin 2\pi Q_0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \Delta k\ell & 1 \end{pmatrix}$$

- ❖ The perturbed machine is then

$$\begin{pmatrix} \cos 2\pi Q_0 + \alpha \sin 2\pi Q_0 + \beta \Delta k\ell \sin 2\pi Q_0 & \beta \sin 2\pi Q_0 \\ -\gamma \sin 2\pi Q_0 + \Delta k\ell (\cos 2\pi Q_0 - \alpha \sin 2\pi Q_0) & \cos 2\pi Q_0 - \alpha \sin 2\pi Q_0 \end{pmatrix} \quad (4.21)$$

- ❖ The perturbed machine can also be written as

$$\begin{pmatrix} \cos 2\pi(Q_0 + \Delta Q) + \alpha \sin 2\pi(Q_0 + \Delta Q) & \beta \sin 2\pi(Q_0 + \Delta Q) \\ -\gamma \sin 2\pi(Q_0 + \Delta Q) & \cos 2\pi(Q_0 + \Delta Q) - \alpha \sin 2\pi(Q_0 + \Delta Q) \end{pmatrix} \quad (4.22)$$

- ❖ Equating the Traces of (4.21) and (4.22) gives,

$$2 \cos 2\pi(Q_0 + \Delta Q) = 2 \cos 2\pi Q_0 + \beta \Delta k\ell \sin 2\pi Q_0$$

- ❖ and for  $\Delta Q$  small,

$$\cos 2\pi(Q_0 + \Delta Q) \approx \cos 2\pi Q_0 - 2\pi \Delta Q \sin 2\pi Q_0$$

# Tune shifts in rings continued

❖ So that

$$\Delta Q = -\frac{1}{4\pi} \beta \Delta k \ell \quad (4.23)$$

❖ For several small gradient errors

$$\Delta Q = \mp \frac{1}{4\pi} \int_0^C \beta(s) \Delta k(s) ds \quad \text{or}$$
$$\Delta Q = \mp \frac{1}{4\pi} \sum_n (\beta \Delta k \ell)_n \quad (4.24)$$

**The upper sign applies to the horizontal plane and the lower sign to the vertical plane.**

- ❖ Intuitively one would fear that adding gradients to a machine would change the betatron amplitude functions around the machine (as indeed it does) rendering equations (4.23) and (4.24) inaccurate.
- ❖ In practice, equations (4.23) and (4.24) are surprisingly accurate and linear over an adequate tuning range. A more likely problem is that the machine is moved into an unstable region in the *tune diagram*.

# Tune control

- ❖ Controlling the tunes in a ring can be done by:
  - ❖ Using the main quadrupoles, or
  - ❖ Installing two series families of smaller quads distributed around the ring (**recommended**),
  - ❖ Adding backleg windings to the main quadrupoles (**not recommended**).
- ❖ Choose the quadrupole families so that  $\beta_x > \beta_y$  in the F-series and  $\beta_y > \beta_x$  in the D-series. This gives a degree of orthogonality between the families.

$$\begin{aligned}
 \Delta Q_x &= -\Delta k_F \underbrace{\sum_{i=1}^{NF} \left( \frac{\ell \beta_x}{4\pi} \right)_i}_{A_{11}} - \Delta k_D \underbrace{\sum_{i=1}^{ND} \left( \frac{\ell \beta_x}{4\pi} \right)_i}_{A_{12}} \\
 \Delta Q_y &= \Delta k_F \underbrace{\sum_{i=1}^{NF} \left( \frac{\ell \beta_y}{4\pi} \right)_i}_{A_{21}} + \Delta k_D \underbrace{\sum_{i=1}^{ND} \left( \frac{\ell \beta_y}{4\pi} \right)_i}_{A_{22}} \quad (4.25)
 \end{aligned}$$

where  $NF$  and  $ND$  are the numbers in each family.

- ❖ Control of  $\Delta Q_x$  and  $\Delta Q_y$  is then achieved by inverting the matrix. If non-linearities cause a small error, then re-cycle the calculation.

$$\begin{bmatrix} \Delta Q_x \\ \Delta Q_y \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} -\Delta k_F \\ \Delta k_D \end{bmatrix} \quad (4.26)$$

# Natural chromaticity

- ❖ Although the gradient in a quadrupole is constant over the whole aperture, the focusing strength varies inversely with the momentum of the ions.
- ❖ This is a chromatic effect that gives rise to the so-called '*natural chromaticity*' of the lattice.
- ❖ Remember (4.24)

$$\Delta Q = \mp \frac{1}{4\pi} \sum_n (\beta \Delta k \ell)_n \quad (4.24)$$

- ❖ where  $\Delta k$  is defined as,

$$k = \frac{1}{B\rho} \frac{dB_y}{dx} = \frac{e}{p} \frac{dB_y}{dx}$$

$$k = \frac{e}{p_0} \left( 1 - \frac{\Delta p}{p_0} + \dots \right) \frac{dB_y}{dx}$$

$$k = k_0 \left( 1 - \frac{\Delta p}{p} + \dots \right)$$

$$\Delta k = -k_0 \frac{\Delta p}{p} \quad (4.27)$$

- ❖ From (4.24) and (4.27)

*Natural chromaticity*

$$Q' = \frac{\Delta Q}{\Delta p / p} = \pm \frac{1}{4\pi} \sum_n (\beta k_0 \ell)_n \quad (4.28)$$

# Achromatic quadrupole

- ❖ A sextupole can be regarded as a quadrupole whose gradient varies linearly across the aperture and passes through zero on central orbit.
- ❖ In a region of finite dispersion, the beam momenta are spread quasi-linearly across the aperture.
- ❖ Thus a sextupole can be used to correct the focusing error in a quadrupole due to  $\Delta p/p$ .
- ❖ The gradient seen by an off-momentum particle in a sextupole of strength  $k'$  and with dispersion  $D_x$  is

$$\Delta k = k' D_x \frac{\Delta p}{p}$$

- ❖ The corresponding gradient error in a quadrupole comes from (4.27)

$$\Delta k = -k_0 \frac{\Delta p}{p}$$

- ❖ Opposing these two effects gives the condition for an achromatic quadrupole

$$k_0 \ell_{\text{Quad}} = k' D_x \ell_{\text{Sext}} \quad (4.29)$$

In the past, machines have been designed with the sextupole component built into the pole profile of the quadrupole, but it is more flexible to place a small sextupole next to the quadrupole.

# Chromaticity control

- ❖ Controlling the horizontal and vertical chromaticity in a ring can be done by installing two series families of sextupoles distributed around the ring.
- ❖ Choose the sextupole families so that  $\beta_x > \beta_y$  in the F-series and  $\beta_y > \beta_x$  in the D-series. This gives a degree of orthogonality between the families.

$$\begin{aligned} \Delta Q'_x &= -\Delta k'_F \underbrace{\sum_{i=1}^{NF} \left( \frac{\ell \beta_x D_x}{4\pi} \right)_i}_{B_{11}} - \Delta k'_D \underbrace{\sum_{i=1}^{ND} \left( \frac{\ell \beta_x D_x}{4\pi} \right)_i}_{B_{12}} \\ \Delta Q'_y &= \Delta k'_F \underbrace{\sum_{i=1}^{NF} \left( \frac{\ell \beta_y D_x}{4\pi} \right)_i}_{B_{21}} + \Delta k'_D \underbrace{\sum_{i=1}^{ND} \left( \frac{\ell \beta_y D_x}{4\pi} \right)_i}_{B_{22}} \end{aligned} \quad (4.30)$$

where  $NF$  and  $ND$  are the numbers in each family.

- ❖ Control of  $\Delta Q'_x$  and  $\Delta Q'_y$  are then achieved by inverting the matrix. Unlike the case of tune control, there will be no nonlinearity because the sextupoles are added as a perturbation and do not affect the linear lattice.

$$\begin{bmatrix} \Delta Q'_x \\ \Delta Q'_y \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} -\Delta k'_F \\ \Delta k'_D \end{bmatrix} \quad (4.31)$$

# *Chromaticity control continued*

- ❖ When planning the F and D sextupole families,
  - ❖ Have an even number of sextupoles in each family,
  - ❖ Separate the sextupoles in betatron phase advance by  $\pi/3$ .
  - ❖ The F sextupoles  $\beta_x/\beta_y$  should be equal for all members and be as large as possible.
  - ❖ The D sextupoles  $\beta_y/\beta_x$  should be equal for all members and be as large as possible.
  - ❖ These precautions will make the resonance driving terms cancel, which may be important for the dynamic aperture.

**Note a F-sextupole strengthens a F-quadrupole for positive  $x$ . A D-sextupole strengthens a D-quadrupole for positive  $x$ .**

**Higher-order chromaticity corrections will be discussed in the last lecture.**



# Prognosis

- ❖ When designing a machine it is useful to know the expected amplitude of closed-orbit distortions.
- ❖ Consider first equation (4.5)

$$\Delta z(s) = \mp \frac{\beta_z^{1/2}(s)}{2 \sin \pi Q_z} \sum_n \beta_n^{1/2} \delta_n \cos[|\mu_n - \mu_z(s)| - \pi Q_z].$$

- ❖ The expected rms distortion around the machine and averaged over many sample machines would be

$$\langle \Delta z^2(s) \rangle^{1/2} = \frac{\bar{\beta}_z^{1/2}(s)}{2\sqrt{2} |\sin \pi Q|} \left[ \sum_n \beta_n \langle \delta_n^2 \rangle \right]^{1/2}.$$

- ❖ For errors from many positions, this can be simplified to

$$\langle \Delta z^2(s) \rangle^{1/2} = \frac{\bar{\beta}_z(s)}{2\sqrt{2} |\sin \pi Q|} \left[ \sum_n \langle \delta_n^2 \rangle \right]^{1/2}. \quad (4.32)$$

- ❖ It remains to estimate all the errors. These are likely to be worst at injection and can be from many sources, for example,
  - ❖ Steel quality, dimensions and geometry of the dipoles.
  - ❖ Longitudinal shifts that appear as missing field at the entry and additional field at the exit.
  - ❖ Off-centred quadrupoles and tilted dipoles.
  - ❖ Powering errors, ripple and transients.

## *Prognosis continued*

- ❖ Equation (4.32) can identify important error sources and set tolerances for manufacture, alignment and powering.
- ❖ But, it is equally important to know how well the closed orbit can be corrected, the likely effect of missing monitors and correctors, and how often a full machine re-alignment will be needed.
- ❖ For these problems, it is necessary to resort to a lattice program and to run a large number of machines through an orbit correction routine with randomly generated errors.
- ❖ Degradation of the machine alignment with time depends on how the machine's foundations settle. In the absence of expert help, try:

$$\Delta x_{\text{RMS}} = \Delta x_{\text{RMS}} + n\delta e; \quad \Delta z_{\text{RMS}} = \Delta z_{\text{RMS}} + n\delta e; \quad \text{etc.}$$

$$\Delta \theta_{\text{RMS}} = \Delta \theta_{\text{RMS}} + n\delta \varepsilon;$$

where  $n$  = number of years;  $\delta e = 0.1$  mm/yr and  $\delta \varepsilon = 0.1$  mrad/year. After 5 years, the rate of movement is likely to decrease.

## *Prognosis continued*

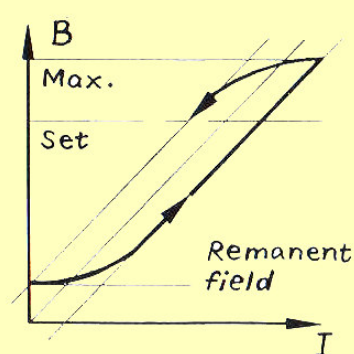
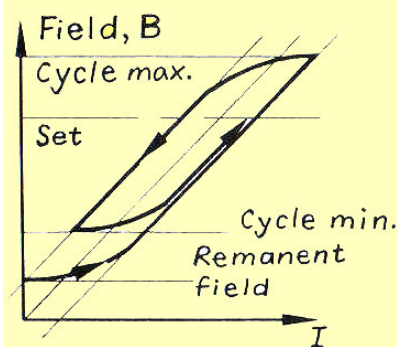
- ❖ The closed orbit is a prime target for prognosis, but it is also important to look at tune fluctuations.
- ❖ Re-arranging (4.24) for fractional errors gives

$$\langle (\Delta Q)^2 \rangle^{1/2} = \frac{1}{4\pi} \left[ \sum_n (\beta_{kl})_n^2 \right]^{1/2} \langle (\Delta k / k)^2 \rangle^{1/2} \quad (4.33)$$

- ❖ It is also standard practice to make a similar prognosis of the coupling to be expected and its correction. There is insufficient time to include this here, but a full suite of routines are available in the WinAGILE program.

# Reproducibility

- ❖ Sometimes the closed orbit, or the application of closed-orbit bumps, or the shape and position of the working line suffer from poor reproducibility. This is likely to be due to hysteresis, **especially if corrections are being applied by backleg windings.**
- ❖ The first precaution is to use a standard current cycle when measuring the magnets.
- ❖ Ideally, the same standard cycle should be used when setting the magnets and the final setting should always be on the same ramp direction (up or down).
- ❖ Cycling is not possible with a circulating beam. In this case, a hysteresis model can be constructed from the magnetic measurements and used for setting.



- ❖ When accelerating the beam, slow  $dI/dt$  to prevent overshoot (rst control loop).

