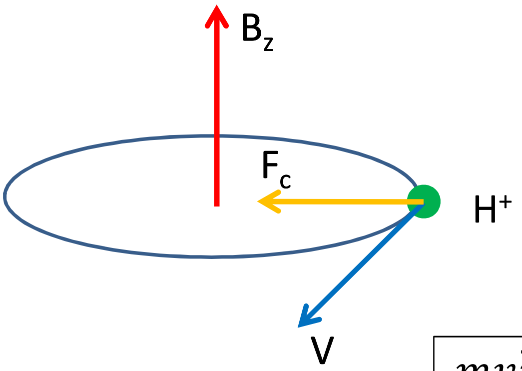
Applied cyclotrons: beam dynamics and magnetic design

- Introduction
- ….. about focusing and isochronism
- ….. about injection
- ….. about extraction

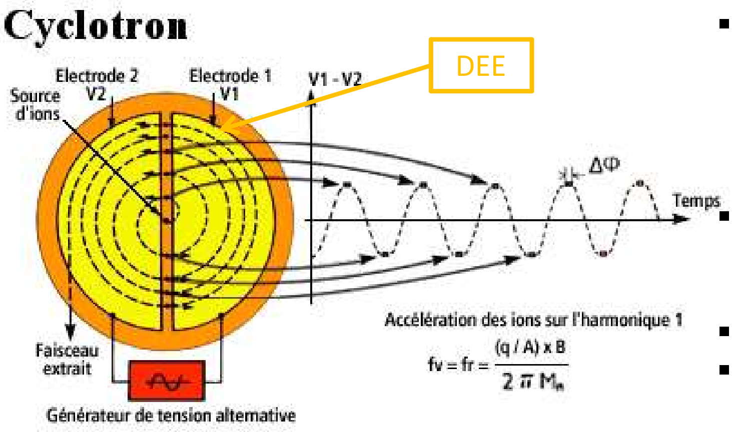
Introduction:
The most basic equation of the cyclotron



- A charged particle in a uniform magnetic field moves on a circle
- The centripetal force is equal to the Lorentz force acting on the particle
- **Thus the rotation frequency of the particle is constant => independent on radius ,velocity , energy or time** (in the non-relativistic limit)

$$\frac{mv^2}{r} = qvB \quad \Rightarrow \quad \omega = \frac{v}{r} = \frac{qB}{m}$$

Consequences of constant cyclotron frequency



Classical cyclotron:
Lawrence and Livingston, Phys. Rev. 40 (1932) 9

- Particles can be accelerated with an RF-system operating at constant frequency:

F_{RF}(MHz) = 15.2 h(Z/A) B (Tesla)

- The orbit starts in the center (injection) and spirals outward towards the pole radius (extraction)
- The magnet field is constant in time
- RF and magnetic stucture are completely integrated => **Same RF structure accelerates many times** =>compact and cost-effective
- CW-operation (continuous wave)

Classical cyclotron: where is the problem?

i. In a uniform magnetic field there is no vertical focusing (**metastable**)

ii. During acceleration, due to the relativistic mass increase, the revolution frequency decreases in a uniform magnetic field => loss of resonance between RF and the beam => loss of isochronism

iii. just increasing the magnetic field with radius is not possible => **vertically unstable**

$$\omega = \frac{qB}{m_0}\sqrt{1-\left(\frac{v}{c}\right)^2}$$

Another solution: **the synchro-cyclotron**

- Let the magnetic field gradually decrease with radius in order to obtain **weak vertical focusing** =>

$$\nu_z = \sqrt{k} \Rightarrow k = -\frac{r}{B}\frac{dB}{dr}$$

- Let the RF frequency gradually decrease with time in order to compensate for the drop of the magnetic field and for the increase of the mass

$$\omega = \frac{qB}{m}$$

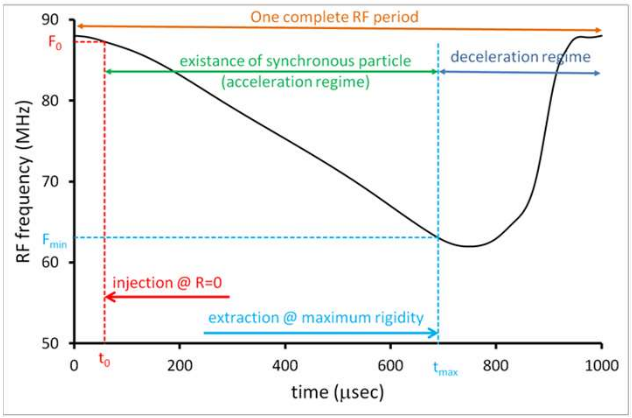
Veksler, J. Phys. USSR **9** (1945)153
McMillan, Phys. Rev. **68** (1945)143L

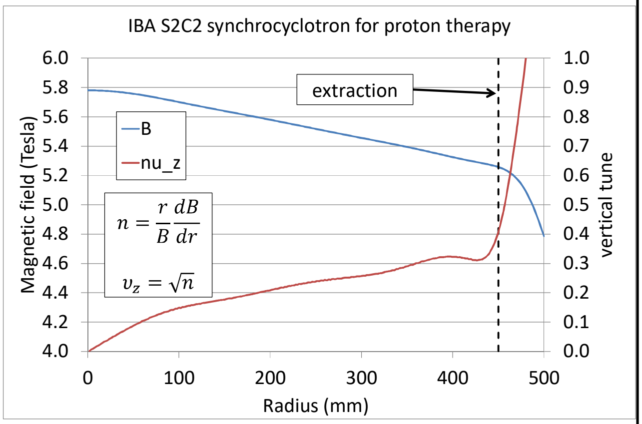
Note: synchrotron was invented at the same time

Some consequences

1. The RF (and beam) is pulsed but the magnetic field B=const ≠ B(t)
2. The mean beam intensity is much lower => OK for proton therapy
3. There is longitudinal beam dynamics similar to that of the synchrotron
4. Only during a short time-window, beam can be captured in the center
5. A more complicated RF because of the required frequency variation
6. The RF frequency can not be varied very fast (rotating capacitor) and therefore the acceleration must be slow => low energy gain per turn => many turns up to extraction => little RF power needed
7. There is only a very small turn-separation $\Rightarrow$ a special extraction method is needed to get the beam out of the machine (regenerative extraction)

Example: the IBA S2C2 for proton therapy





Repition rate = 1 kHz
Duty cycle about 100

Superconducting synchro-cyclotron
Extraction energy 230 MeV

Longitudinal dynamics in a synchro-cyclotron

- There is a definition of a synchronous particle: everywhere in the synchro-cyclotron, at any moment in time, the revolution frequency of the synchronous particle is equal to RF frequency
- There are oscillations (in energy and phase) of real particles around the synchronous particle
- There is a stability zone for these oscillations defined by a separatrix in the longitudinal phase space
- This separatrix is filled during the beam capture in the cyclotron center

Yet another solution: **the isochronous cyclotron**

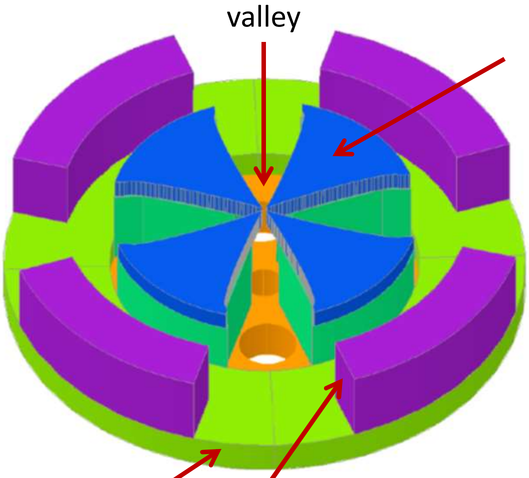
- Two contributions to vertical focusing:

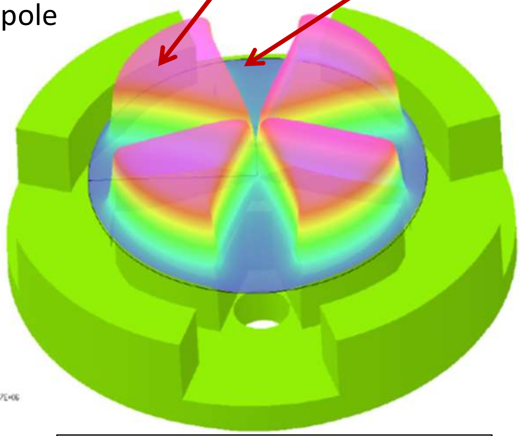
$$F_z = q(\vec{v} \times \vec{B})_z = -q(v_\theta B_r - v_r B_\theta)$$

  - $v_\theta B_r$ => obtained in the radially decreasing rotationally symmetric magnetic fields as in the classical cyclotron and the synchro-cyclotron
  - $v_r B_\theta$ => requires an azimuthal modulation of the magnetic field => introduce sectors (hills) with high field and valleys with low field => azimuthally varying field cyclotron=> the field variation creates the non-circular orbit

Some aspects of vertical focusing and isochronism

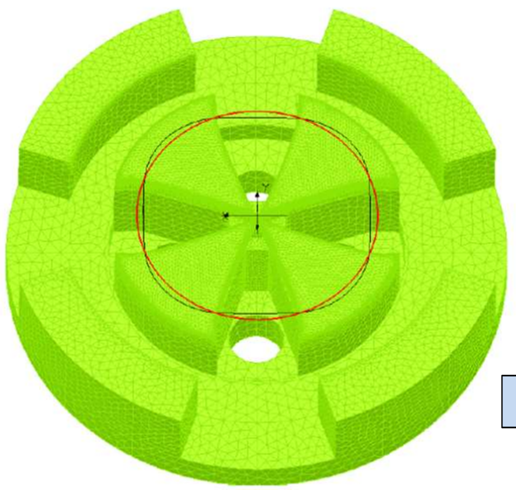
The Azimuthally Varying Field (AVF) cyclotron

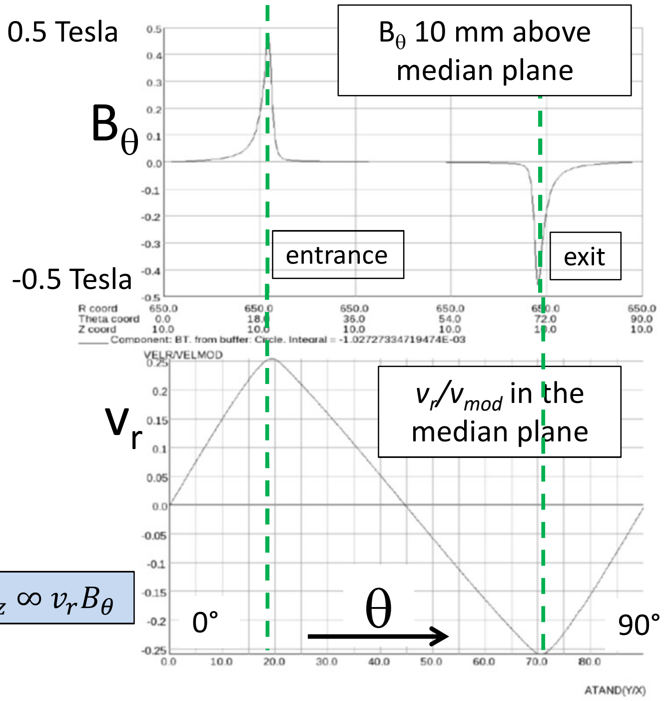




Magnetic field on hills and valleys

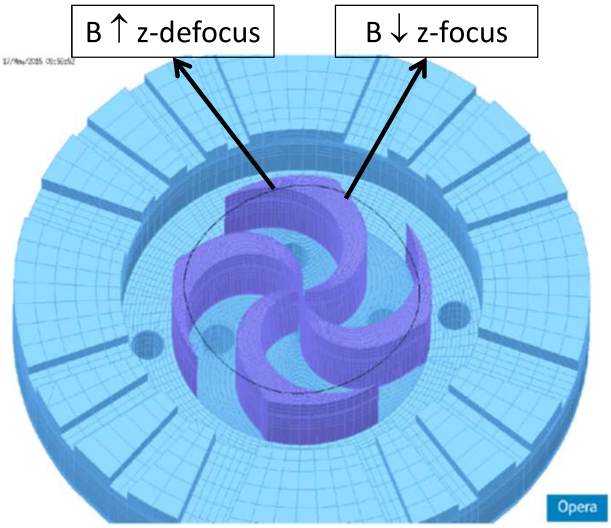
scaloping of the orbit => Vertical focusing





$F_z \propto v_r B_\theta$

More vertical focusing => pole spiraling



For straight sectors: equal vertical focusing at entrance and exit of sector

Spiraling of the pole changes the focusing strength at the entrance and exit of the sector:

Entrance: strong B-decrease => strong z-focusing

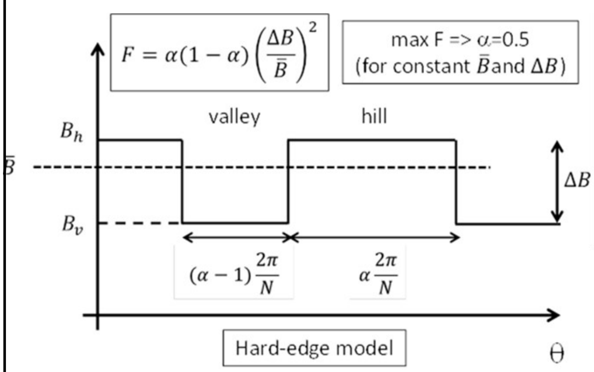
Exit: strong B-increase => strong z-defocus

ALTERNATING FOCUSING
This may give a very large contribution

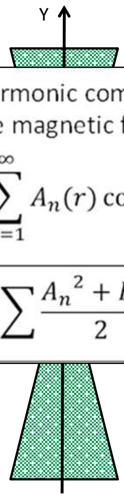
Flutter: a measure for the azimuthal field variation

Average of the field modulation

$$F(r) = \frac{\overline{B^2} - \bar{B}^2}{\bar{B}^2}$$



N=number of sectors
α='filling factor'



Fourier harmonic composition of the magnetic field

$$B(r,\theta) = \bar{B}(r)\left\{1 + \sum_{n=1}^{\infty} A_n(r)\cos n\theta + B_n(r)\sin n\theta\right\}$$

$$F = \sum \frac{{A_n}^2 + {B_n}^2}{2}$$

Formulas for focusing in an AVF cyclotron

$$\nu_z{}^2 = k + \frac{N^2}{N^2-1}F(1+2\tan^2\xi)$$

$$\nu_r{}^2 = (1-k) + \frac{3N^2}{(N^2-1)(N^2-4)}F(1+\tan^2\xi)$$

k = field index = $-\frac{r}{\bar{B}}\frac{d\bar{B}}{dr}$

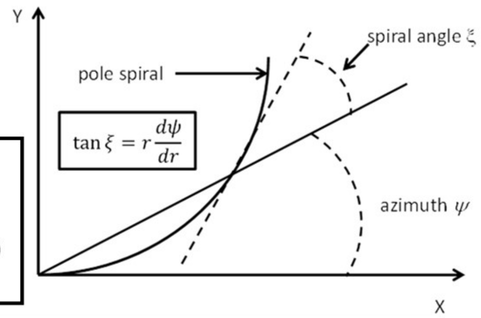
F = flutter

N = number of sectors

ξ = spiral angle

This is an approximation: There is also some dependency on radial gradients of the flutter. See: Hagedoorn and Verster, NIM **18,19** (1962) 201-228

Definition of spiral angle



NOTE: for an isochronous cyclotron:

$$k = 1 - \gamma_{\mathrm{rel}}{}^2$$

$$\nu_r \cong \gamma_{\mathrm{rel}}$$

Isochronism => the revolution frequency of the particle is constant everywhere in the cyclotron independent of the energy of the particle

Isochronous cyclotrons have to be isochronized by correct shaping of the average magnetic field as a function of radius

All cyclotron magnetic fields are mapped in the median plane

Precise mapping and iron pole shimming is needed in order to isochronize the magnetic field

⇒

It is not possible to obtain isochronism just from the design => required precission of <B> =>10^{-4} to 10^{-5}

- Move Hall-probe or a search coil (S2C2) on a 2D polar grid to obtain a full field-map => automized and computer controlled system

- Analyse the magnetic field on equilibrium orbits in order to evaluate isochronism
- Shim the hill sectors of the iron in order to improve the isochronism (reduce the RF phase slip)

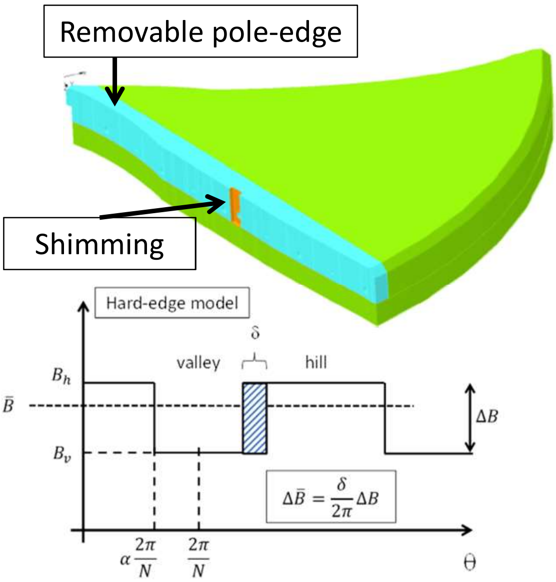
Essential information of a cyclotron field map

1. The level of isochronism => integrated RF phase slip
2. The transverse optical stability => tune functions
3. Crossing of dangerous resonances => operating diagram
4. Magnetic field errors
   - First and second harmonic errors => resonance drivers
   - Median plane errors => very difficult to measure
5. …

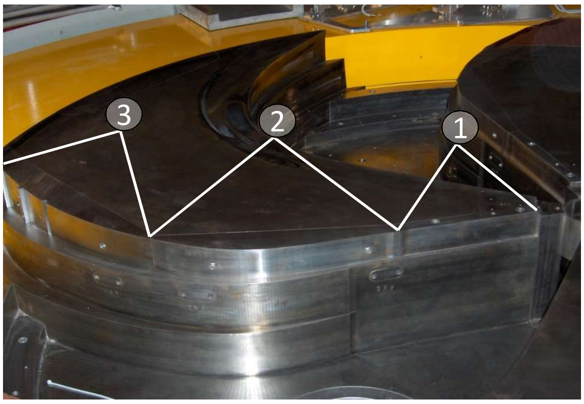
Analysis of a cyclotron field map

1. Static analysis => Acceleration is turned off
   - Computation of a family of closed orbits and their properties
2. Accelerated orbits => for special problems
   - Central region studies
   - Extraction studies
   - Study of resonance crossings
   - …

Isochronization by pole shimming



Removable pole edges in the C235

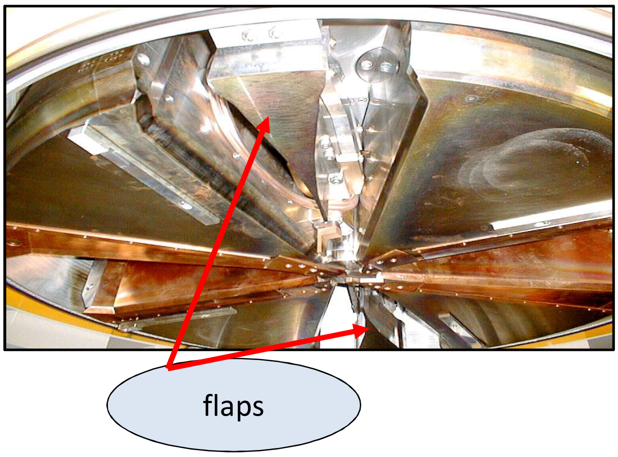


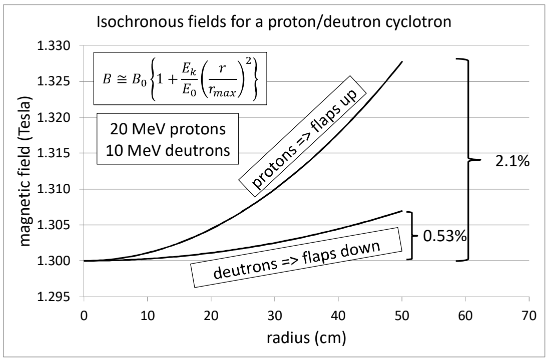
In this isochronous PT cyclotron, there are 3 removable pole edges (per pole) for shimming the average field as needed for isochronism

Isochronization for two different particles

Example: a proton/deuteron isotope production cyclotron

By placing pieces (flaps) in the valleys which can be moved vertically close to the median plane (protons) or further away from the median plane (deuterons)





INJECTION INTO A CYCLOTRON

Transfer of the beam from the ion source onto the equilibrium orbit in the center of the cyclotron, two appoaches:

1. Internal Ion Source:
   – Ion source placed in the center of the cyclotron
   – Source is 'integrated part' of the accelerating stucture
   – Is used in PT cyclotrons as well as isotope production cyclotrons
2. External Ion Source:
   – Ion source placed oustside of the machine
   – An injection line with magnets and electrostatic inflector is needed
   – Used in high intensity isotope production cyclotrons (and in IBA C400)

Injection: some important design goals

1. Place the beam on the correct equilibrium orbit given by the injection energy (horizontal beam centering)
2. Vertical centering with respect to the median plane
3. Longitudinal matching =>bunching => compressing the DC beam from the ion source into shorter packages at the frequency of the RF
4. Matching of the beam phase space into the cyclotron acceptance or eigenellipse (if possible)
5. Preserve as well as possible the beam quality with minimum losses between the ion source and the cyclotron center

Injection: internal ion source

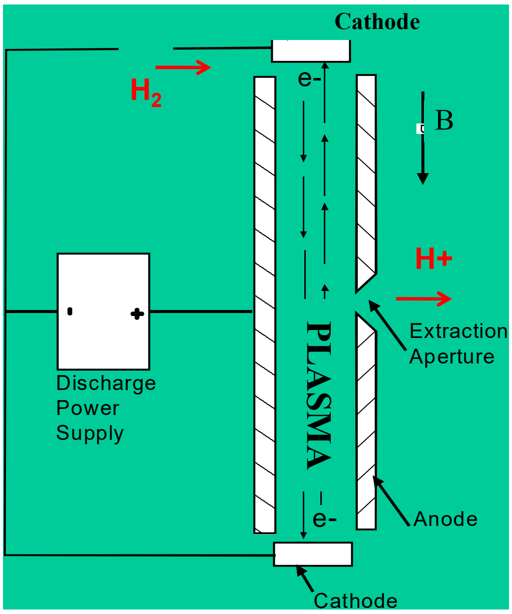
Some advantages

- Simple and cost-effective: simple ion source; no injection line needed
- Compact:
  - two ion sources can be placed simultaneously
  - Can be used in the high-field (6 to 9 Tesla) superconducting cyclotrons

Some disavantages/limitation

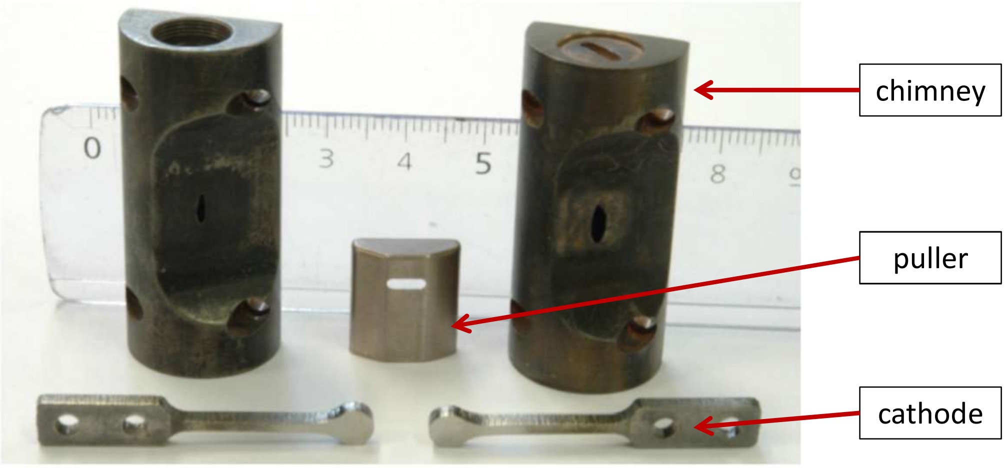
- Low to moderate beam intensities
- Simple ion species (H^+,H^-,deuterons,He-3, He-4)
- Beam matching/bunching/manipulation not possible
- Gas-leak directly into the cyclotron (bad for negative ions)
- Machine has to be vented for ion source maintenance

Injection: cold cathode PIG ion source



- Electron emission due to electrical potential on the cathodes
- Electron confinement due to the magnetic field along the anode axis
- Electrons produced by thermionic emission and ionic bombardment
  - Start-up: 3 kV to strike an arc
  - At the operating point : 100 V
- cathodes heated by the plasma (100 V is enough to pull an outer e- off the gas atoms)
- Hot cathode PIG => heated with filament

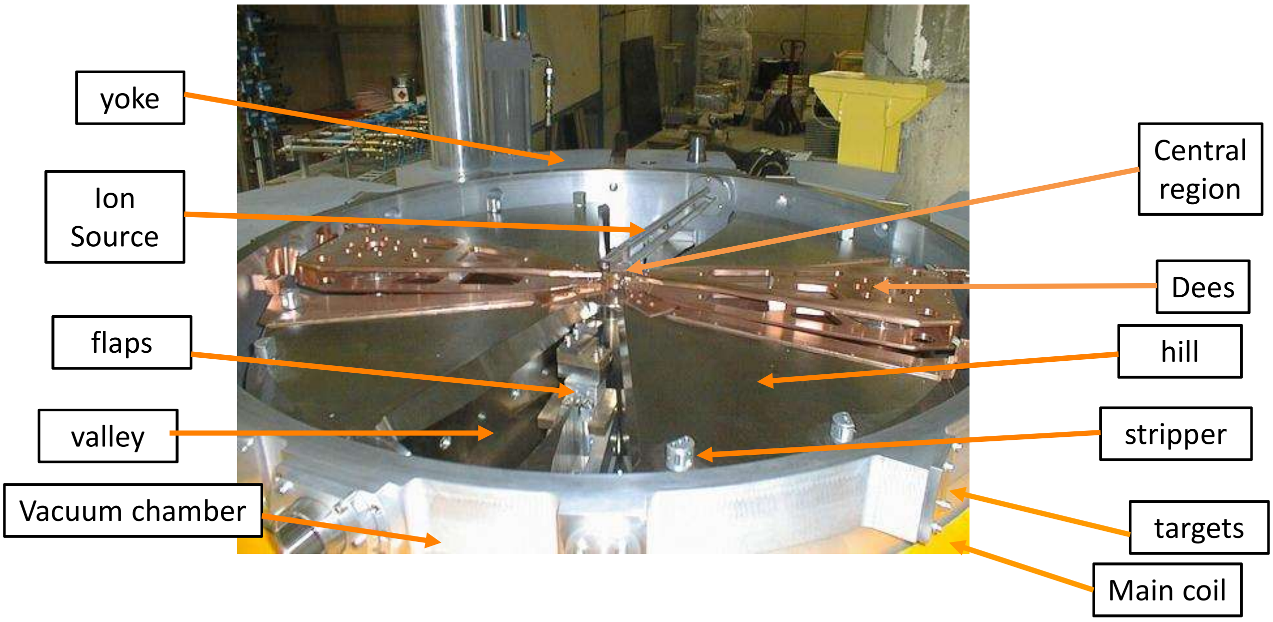
Chimney, cathodes and puller



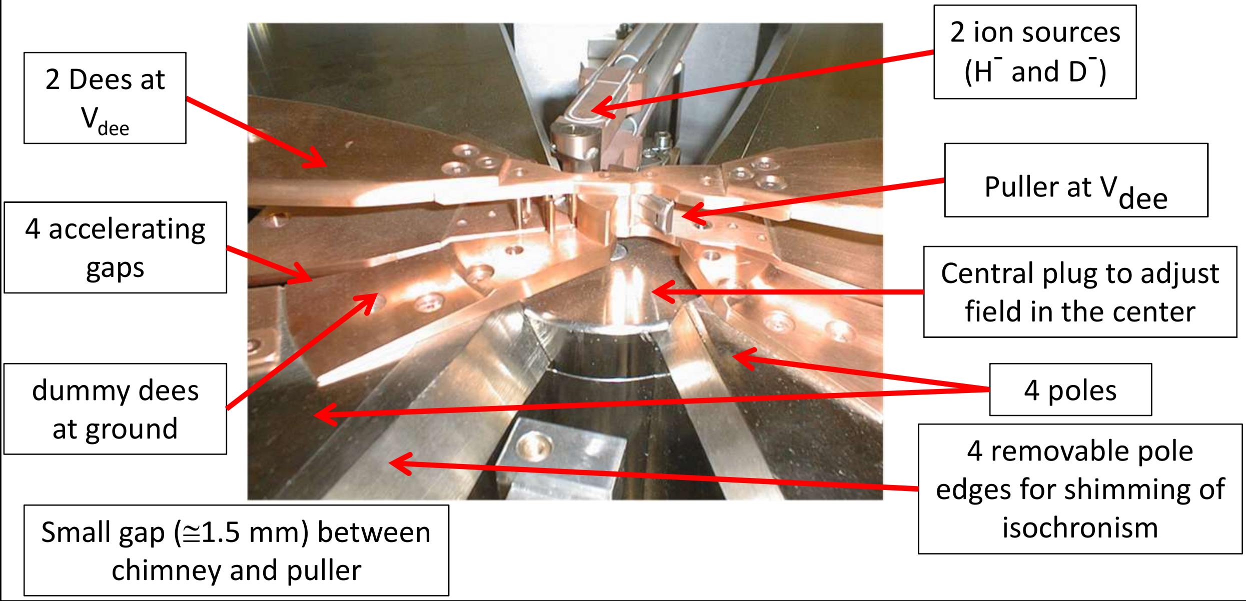
Chimney: copper-tungsten ⇒ good heat properies; machinable

Cathodes: tantallum ⇒ high electron emission; shaped to reduce heat conduction

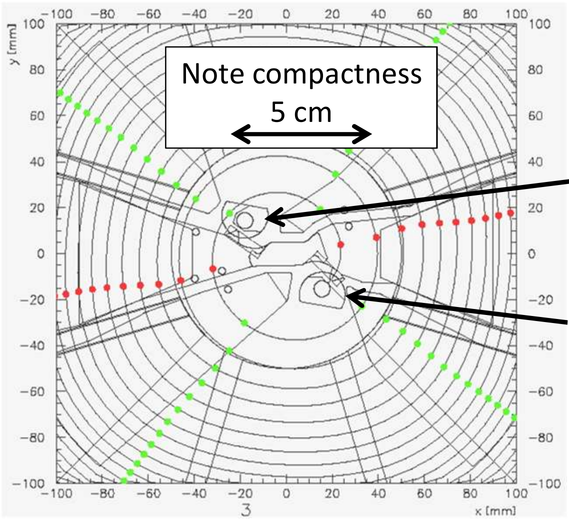
Compact Deep-valley Cyclotron Design



Example: central region of a compact cyclotron



Orbit tracking (C18/9 isotope production cyclotron)



E-fields => from Opera
B-fields => measured or Opera

D^- source; h=4

H^- source h=2

D^- source is placed further out because of larger orbit

Cut D^- chimney for H^- passage

Red dots: position of particle when V_{dee} =0

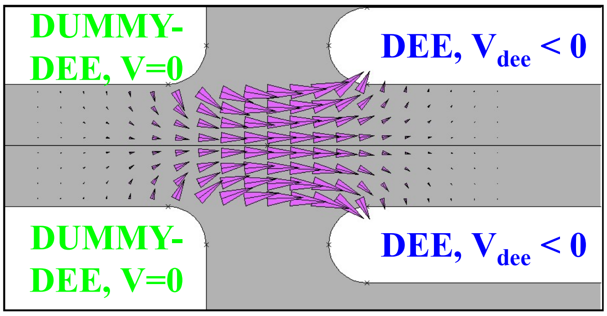
Green dots: position of particle when V_{dee} =V_{max}

Vertical focusing in the center

- Azimuthal Field Variation (AVF) goes to zero in the cyclotron center ⇒ magnetic vertical focusing disappears
- Two remedies
  - Add a magnetic field bump in the center ⇒ negative field gradient creates vertical focusing: field bump of a few hundred Gauss ⇒ central plug
  - The first few accelerating gaps provide electrical focusing ⇒ proper positioning of accelerating gaps during the design to get some phase focusing

Vertical Electrical Focusing in accelerating gap: two contributions

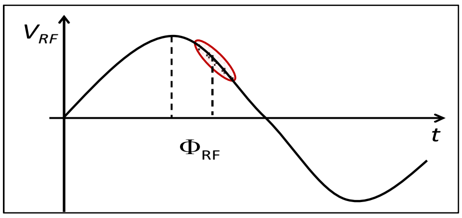
Vertical cross section



1st half => focusing   2nd half => defocusing

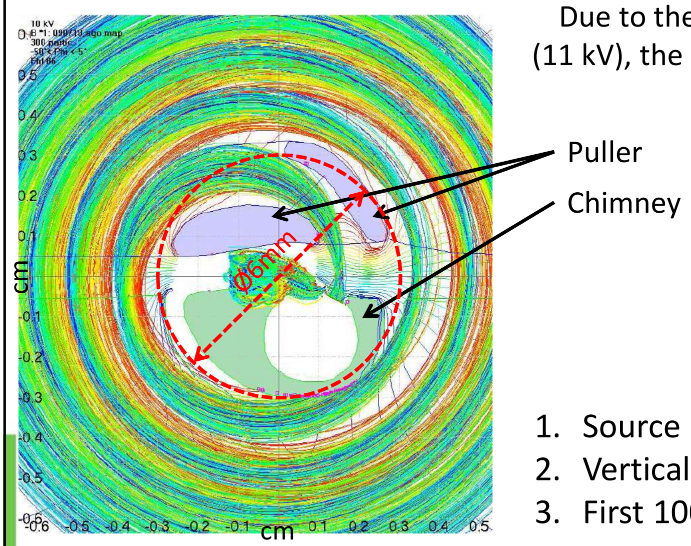
Falling slope of RF ⇒ net focusing (phase focusing)

i. Due to the shape of electric field lines in the gap: first half is focusing and second half is defocusing => total effect is focusing => comparable to **Einzel lens**
ii. Due to RF effect: If E-field is decreasing in time at moment of acceleration => falling slope of RF sine wave => second defocusing half is less important => net focusing (phase focusing)

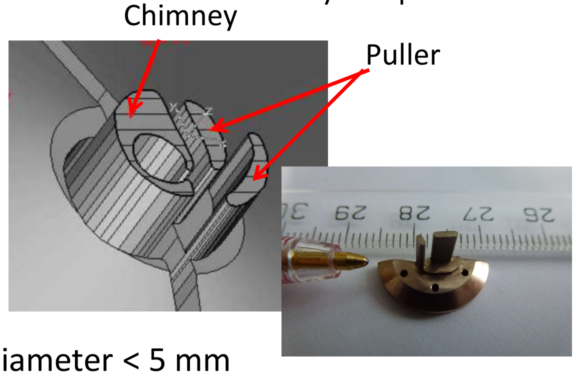


Example: Ion source and central region of the S2C2

Central region size with a very compact cold cathode PIG source



Due to the high field (5,74T) and the low dee voltage (11 kV), the source has to be extremely compact:



1. Source diameter < 5 mm
2. Vertical gap in the center 6 mm
3. First 100 turns within a radius of 3 cm

By the way: why a SC synchrocyclotron for PT

- An isochronous cyclotron needs flutter
- Flutter can only be created by the iron (not by the coil)
- Maximum achievable field modulation about 2 Tesla
- If average field is pushed too far up (using a SC coil) then no longer enough flutter => not enough vertical focusing
- In a synchro-cyclotron this problem does not occur

In a synchrocyclotron you can fully exploit the potential offered by superconductivity

Axial Injection

<u>Axial injection</u> $\Rightarrow$ most relevant for compact cyclotrons

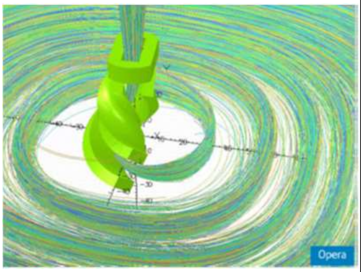
- Along the vertical symmetry axis of the cyclotron
- In the center, the beam is bent by 90° into the median plane
- For this an electrostatic inflector device is used

Spiral inflector for Axial Injection

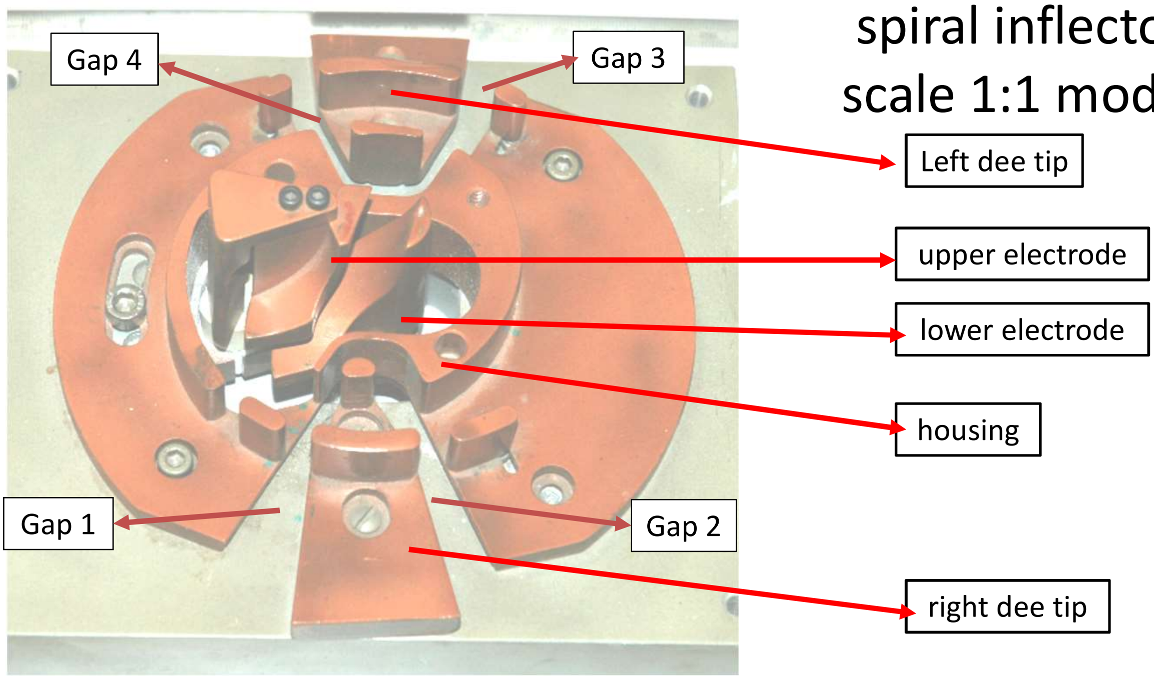
- The E-field between 2 electrodes bends the beam 90° from vertical to horizontal. The presence the cyclotron B-field creates a 3D orbit
- The spiral inflector basicly a cylindrical capacitor which is gradually twisted in order to take into account the spiraling of the trajectory induced by the vertical magnetic field
- E-field always perpendicular to velocity ⇒ orbit on equipotential ⇒ this allows for low electrode voltage

$$\frac{qV}{E} = \frac{2d}{A}$$

- Two free design parameters available to obtain orbit centering
  1. Electric radius A (equivalent to height of inflector)
  2. Tilt parameter k' (equivalent to a change of magnetic field)
- Very compact geometry
- Complicated electrode structure needs a 5 axis milling machine

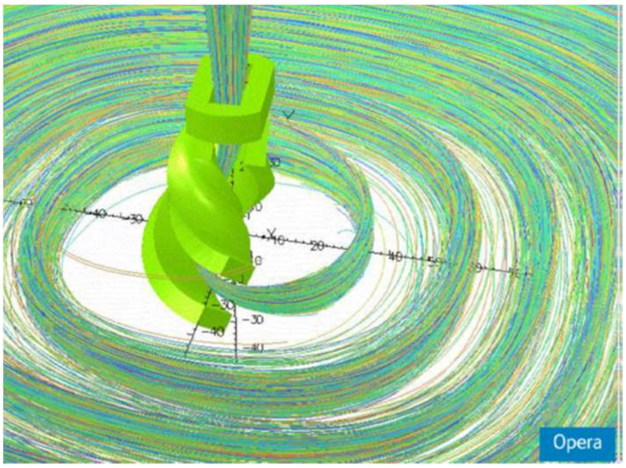


spiral inflector scale 1:1 model



Inflector simulations

Calculated orbits imported in Opera3D

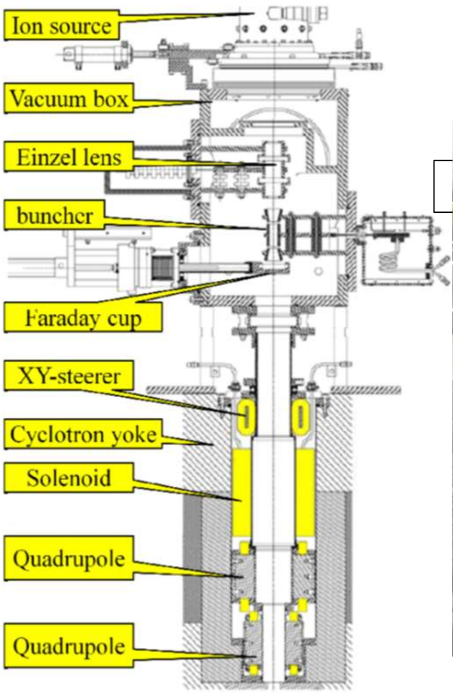


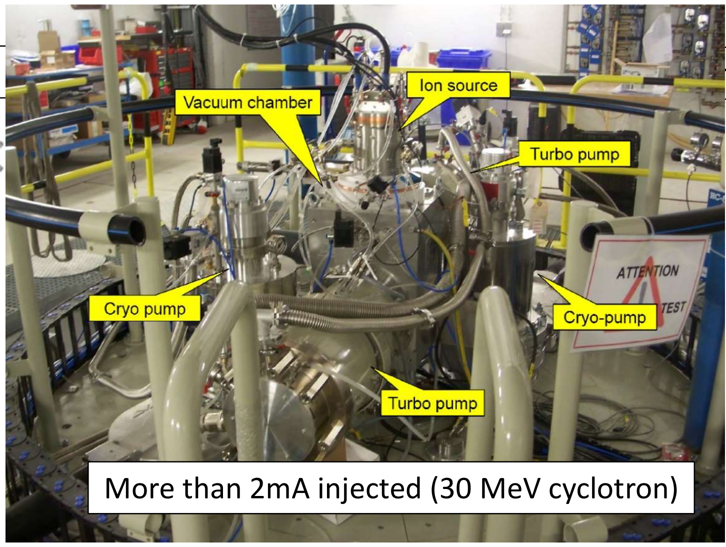
- Spiral inflector is a complex 3D problem
- 3D fields (B,E) are needed => Opera3d
- In house developed tracking code
- Calculated orbits are imported in Opera3d post-processor
- Tilt is seen as the electrode-rotation at the exit

C70-example

An additional horizontal deflector is needed for multi-particle cyclotron

Injection line





Some aspects of extraction

Extraction from a cyclotron

- Extraction: transfer of the beam from an internal orbit to the application outside of the magnetic field
- Often a difficult process. Why?
  1. The magnetic field acts like a trap: When the particle enters into the radial fringe field of the pole, it runs out of RF phase and will be decelerated $\Rightarrow$particle is « reflected » inwards (if nothing is done to prevent this)
  2. The orbits pile up at high radii $\Rightarrow$ smaller and smaller turn-separation

     $$R \infty \sqrt{E}$$

  3. The beam quality is quickly destroyed in the non-linear fringe field

Different ways of extraction

1. No extraction at all => place an internal target
   - Can be done for isotope production (a little bit dirty)
2. Stripping extraction (H^- cyclotrons; or H_2^+)
   - Isotope production cyclotrons
3. Extraction with an electrostatic deflector (ESD)
   - Proton therapy cyclotrons (Varian, IBA, SHI)
4. Regenerative extraction => synchrocyclotron
   - Proton therapy cyclotrons (Mevion, IBA)
5. Self-extraction => suitable shaping of the magnetic field
   - One IBA prototype cyclotron but needs further improvement

Cases 3 and 4 require some way to increase the turn separation before extraction

Stripping Extraction (1)

Beam passes through a thin foil to remove electrons and suddenly change of the orbit curvature

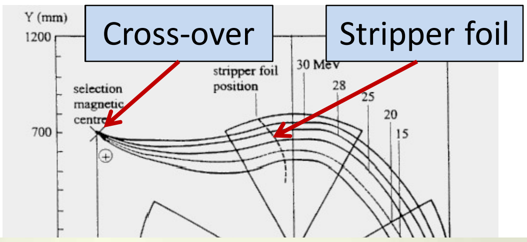
$$\rho_f = \frac{Z_i}{Z_f}\frac{M_f}{M_i}\rho_i$$

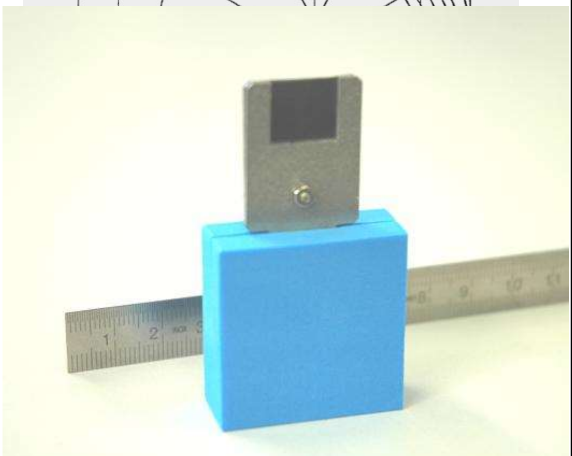
- Example H-minus, $H^- \Rightarrow H^+ + 2\,e^-$ (IBA C18/9, C30, ACS TR30, GE)
  - => Radius of curvature changes sign $\rho_f = -\rho_i$
- Example $H_2^+ \Rightarrow 2\,H^+ + e^-$ (IBA C400)
  - Requires a much larger machine, because the extracted energy reduces with a factor 4 compared to protons
  - Only works when there is enough flutter

$$\rho_f = \frac{\rho_i}{2}$$

H^- stripping extraction (2)

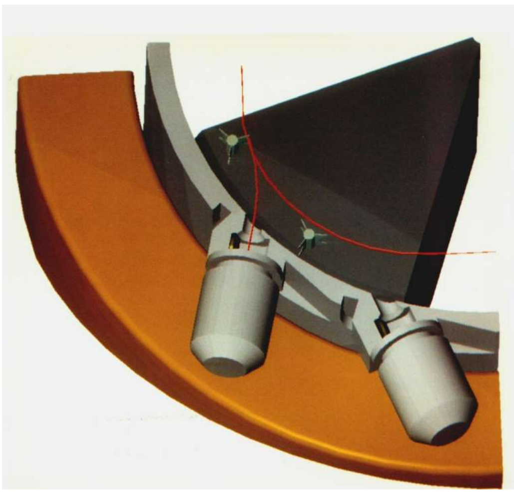
- Stripper foil removes the two electrons of the H^- ion and orbit curvature changes sign
- Energy variation by moving stripper position
- All energies go to one crossover point by proper foil azimuthal position
- Place combination magnet at crossover
- Ideal solution for industrial cyclotrons





Stripping Extraction (3)

- Other advantages
  - Simple and 100 % extraction efficiency
  - Multiple targets around the machine
  - Dual beam extraction
  - Good extracted beam optics
- Limitations due to stripping losses
  - Low B-field ⇒large magnet (Triumf 500 MeV/3 kG)
  - Good vacuum required (expensive)
  - OK for isotope production but not for proton therapy



A side step: why cyclotrons for isotope production?

- Cost-effective machines for achieving:
  - required energies (<100 MeV) and
  - high currents (upto 1 to 2 mA)
- Efficient use of RF power => same accelerating structure used multiple times
- Compact =>
  - magnet and RF integrated into one system
  - Single stage => no injector accelerator needed
- Moderate magnetic fields: 1 to 2 Tesla
- Simple RF system:
  - Constant RF-frequency (10-100 MHz) => CW operation
  - Moderate voltages (10-100 kVolt)
- Relative easy injection (internal ion source or axial injection)
- Simple extraction (stripping for H^- ions with carbon foil => 100% efficiency)

IBA was founded in 1986.
Since then **almost 500 isotope production cyclotrons** have been sold by IBA
Many more by competitors

Some commercial cyclotron vendors/manufacturers

SIEMENS	Germany (RP)	VARIAN medical systems	USA (PT)
GE	GE, USA (RP)	MEVION medical systems	USA (PT)
Best	Canada (RP)	SUMITOMO	Japan (RP+PT)
		C235	
ADVANCED CYCLOTRON SYSTEMS, INC.	Canada (RP)	iba Particle Therapy	Belgium (RP+PT)

Extraction continued: turn-separation in a cyclotron

A Coherent beam oscillation is an oscillation around the equilibrium orbit ⇒

$$r(\theta) = \overbrace{r_0(\theta)}^{\text{EO}} + \overbrace{\underbrace{x(\theta)}_{\text{amplitude}} \sin(\underbrace{v_r\theta + \theta_0}_{\text{phase}})}^{\text{betatron oscillation}}$$

⇓

There are three different mechanisms to create turn separation ⇒

$$\Delta r(\theta_i) = \overbrace{\Delta r_0(\theta_i)}^{\text{acceleration}} + \overbrace{\Delta x \sin(2\pi n(v_r - 1) + \theta_0)}^{\text{resonance}}$$

$$\overbrace{+2\pi(v_r - 1)x \cos(2\pi n(v_r - 1) + \theta_0)}^{\text{precession}}$$

How to create turn-separation for extraction

I. By acceleration ⇒ high dee-voltage — ⇒ IBA/SHI C235

II. By resonances (coherent beam oscillations)

- Precessional extraction (more subtle) — ⇒ Varian SC cyclotron
  - Create oscillation amplitude with 1st harmonic or beam off-centering
  - Accelerate into fringe field where ν_r~0.7
  - Turn separation obtained from betatron phase advance
- Regenerative extraction (even more subtle) — (IBA S2C2, Mevion Monarch)
  - Second harmonic gradient bump: $2\nu_r$=2; ν_r is locked to 1 in the stopband
  - Exponential growth of betatron amplitude

Deflecting and guiding the beam out

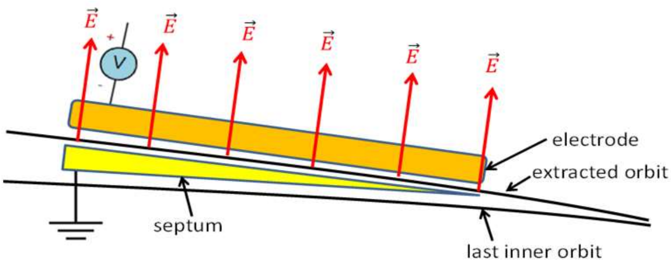
A generic method of precessional extraction in a few steps

i. Create an oscillation amplitude ⇒ by harmonic coils, trim rods or initial beam off-centering (at the ion source)
   - Obtain turn-separation by precession
ii. Provide an initial radial kick
   ⇒ Electostatic deflector ESD (peel off last turn)
iii. Reduce B-field and minimize optical damage when passing the fringe field ⇒ Gradient corrector channels
iv. Re-focus the beam as quickly as possible to handle beam divergencies created in the fringe field
   ⇒ First quadrupole doublet (in return yoke)

Non-adiabatic effect needed =>

Example of a harmonic coil

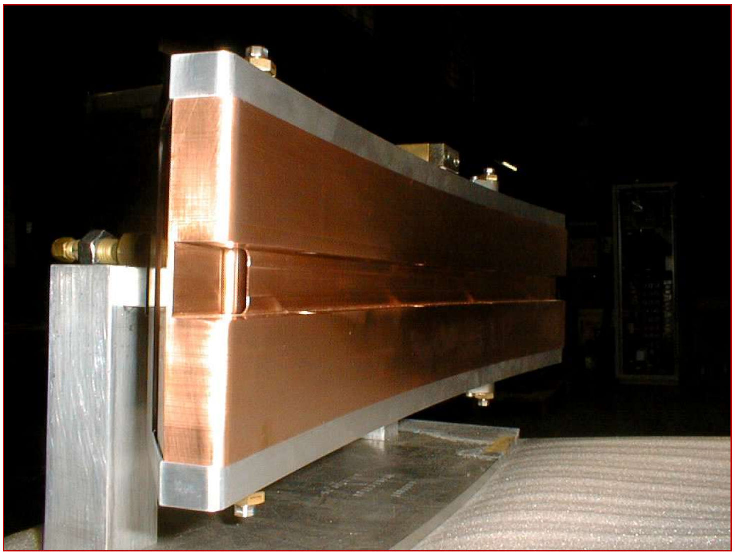
- DC radial E-field creates initial angular kick to deflect beam
- Inner electrode (septum) on ground potential
  - No disturbance on inner orbits
  - Knife thin (0.1 mm) and
  - V-shape at entrance (distribute heat)
  - Water cooled ⇒ limitation for maximum beam intensity
- Outer electrode on negative potential
- Electrode shape = orbit shape

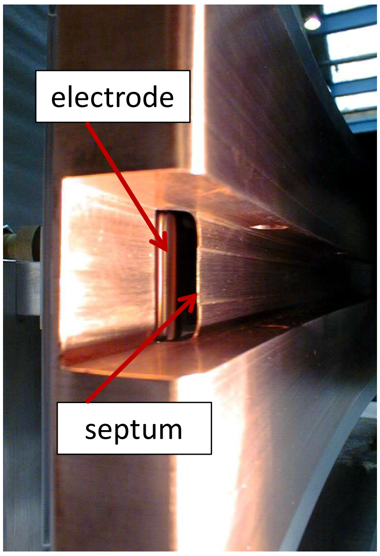


Electrostatic Deflector

ESD for IBA C235

C235 Electrostatic deflector

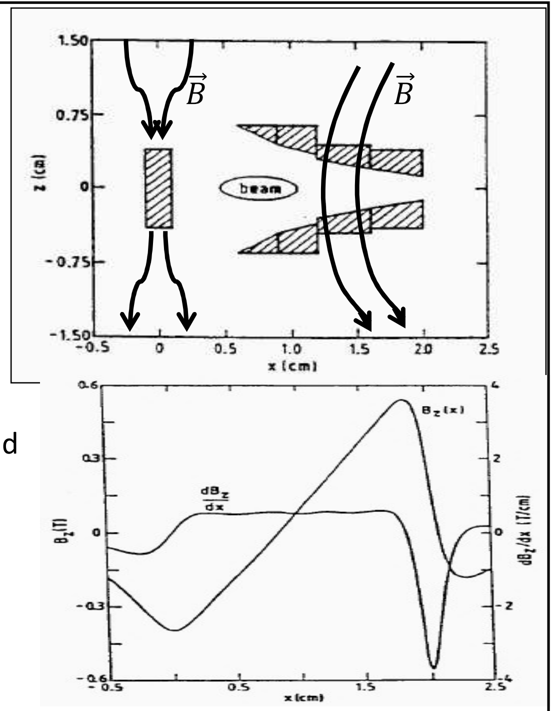




51

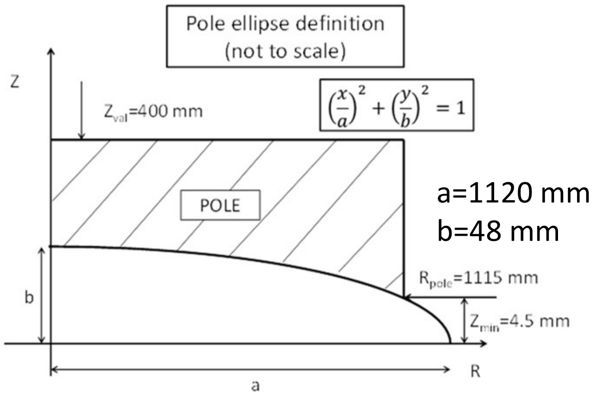
Gradient Corrector focusing Channel

- Goal:
  - Guide the beam through the fringe field
  - Lower magnetic field on extraction path
  - reduce vertical/increase radial focusing through fringe field
- Different types
  - Passive: soft iron magnetized by the main field
  - Active:
    - Using permanent magnets
    - Using coils
- Designed in such a way as to minimize adverse effects on internal orbits

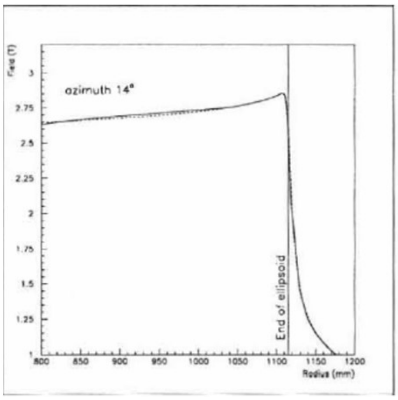


Extraction in the IBA C235

A very sharp transition from stable to unstable

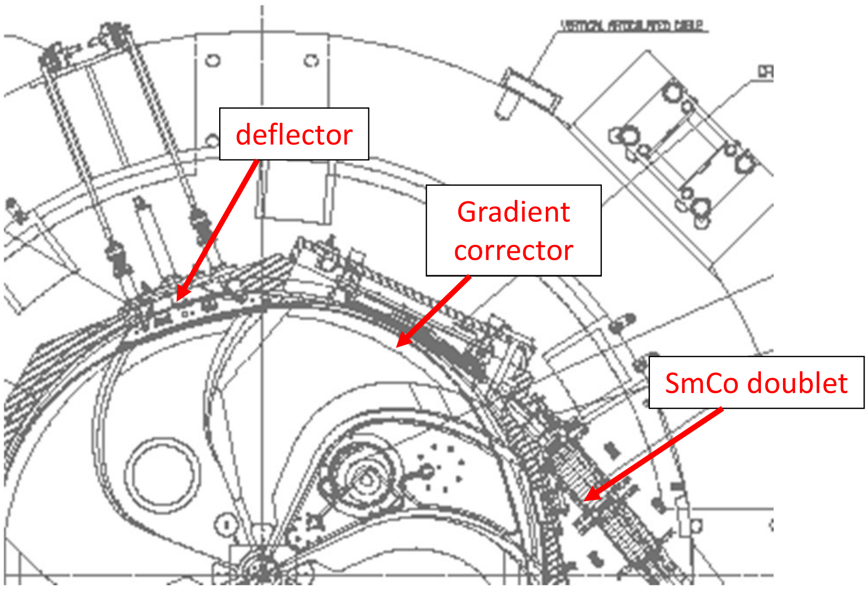


a=1120 mm
b=48 mm



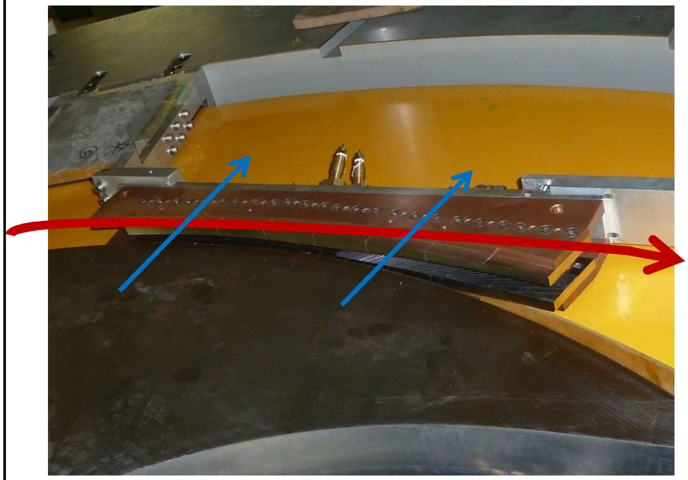
The pole gap in the C235 has an elliptical form.
This allows to obtain a good field region very close to the radius of the pole
Therefore particles can be accelerated very close to the radius of the pole
Only a small kick is needed to extract the beam => orbit is extracted in ¼ of a turn

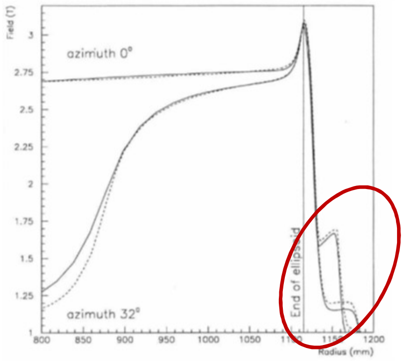
C235 Extraction Scheme



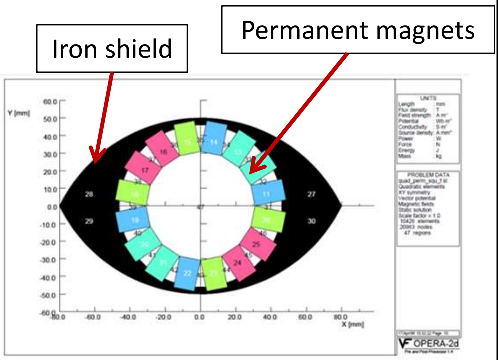
C235 Gradient Corrector

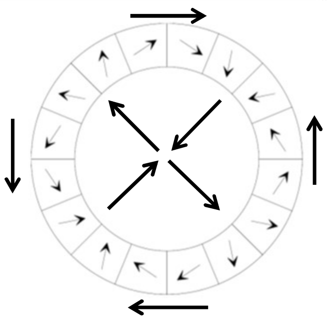
- A passif channel, magnetized by the cyclotron magnetic field
- Placed between the main coils, against (almost touching) the hill sector.
- A descending 'slider' of gradually decreasing magnetic field that guides the beam gently through the fringe field





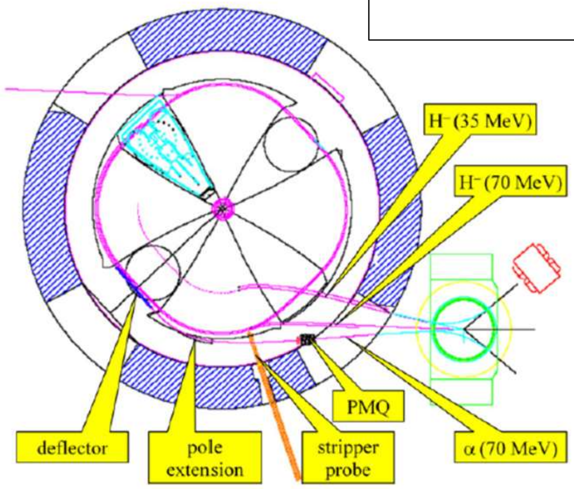
C235 Permanent Magnet Doublet Placed in the return yoke





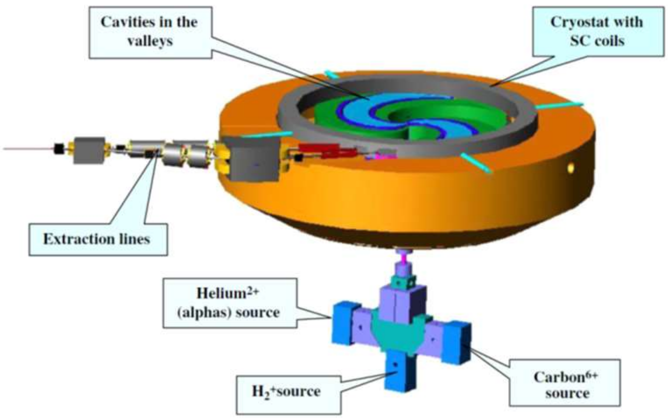
Two extraction systems in one cyclotron

C70 multiple particle cyclotron for Arronax in Nantes
H^-, D^-, α, H_2^+



- Stripping extraction for negative particles
- ESD for α-particle and H_2^+
- Two opposite exit ports
- Simultaneous dual beam capability for H- and D-
- Variable energy for H- and D-
- External switching magnet to direct different energies and particle into the beam lines

The IBA C400 cyclotron



particles	$^{12}C^{6+}$; H_2^+ ; $^4He^{2+}$
Final energy	
ions	400 MeV/A
protons	265 MeV
Bending limit	K=1600
Weight	700 t
Diameter	6.6 m
Hill field	4.5 Tesla
Valley field	2.45 Tesla
Number of cavities	2
RF frquency	75 MHz; h=4
Vdee	80-160 kV
Number of turns	2000
SC coil	NbTi; Helium cooled
Ischronism of H_2^+	Coil in 2 parts

Y. Jongen et.al., NIM A624 (2010) 47-53

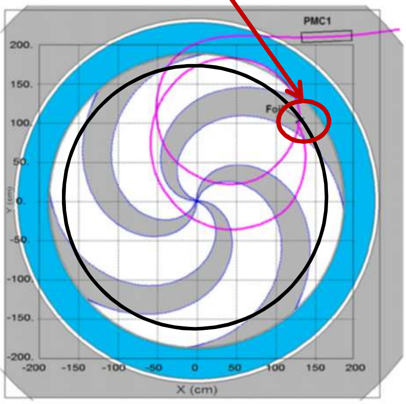
- Full detailed design study was done in collaboration with JINR
- Possibly/hopefully to be industrialized by the French company Normandy Hadrontherapy in which IBA is minority shareholder

Extraction from the C400

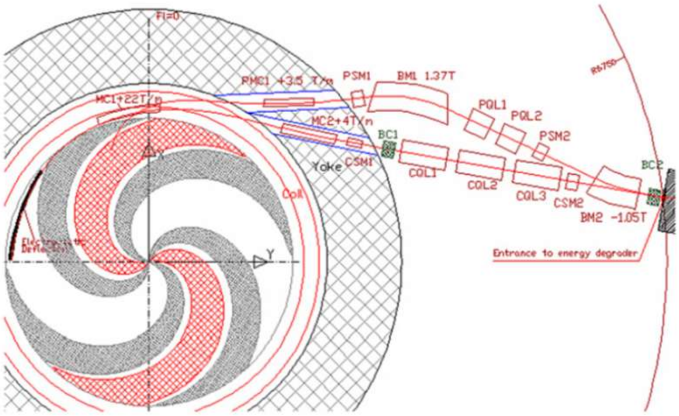
Protons => stripping of H_2^+

$^{12}C^{6+}$ => Electrostatic deflector

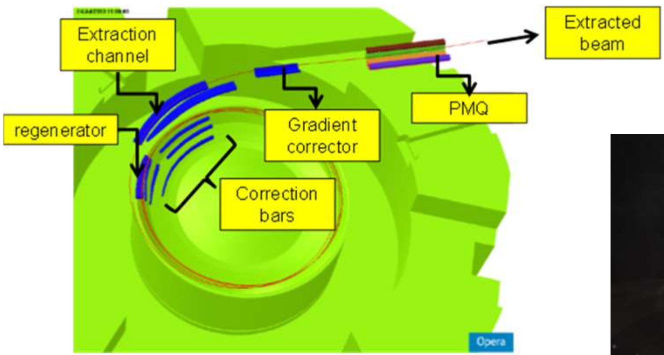
H_2^+ two-turn extraction after stripping



Combining both beams into one beam line



The IBA S2C2 extraction system



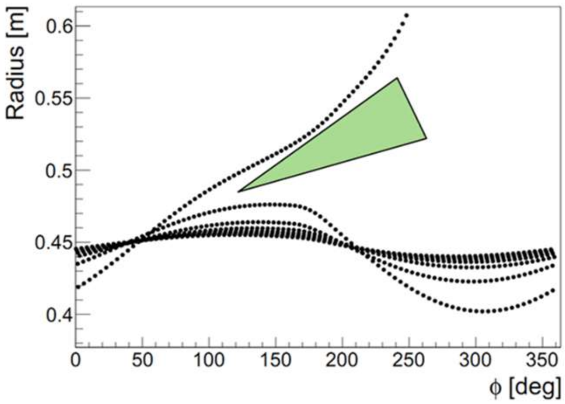
- Horizontal focusing by gradient corrector and permanent magnet quadrupole (PMQ) in strongly

r

- Fully passive system => only soft iron
- Use resonant extraction based on $2Q_h$
- Strong local field bump produced by r frequency and locks it to unity.
- Unstable orbit is pushed towards the

Regenerative extraction based on $2\nu_r=2$ resonance

- A strong regenerator bump increases ν_r and locks it to 1
- A steady shift of the beam towards the extraction channel builts up



- Avoid Walkinshaw resonance ($\nu_r=2\nu_z$)