Magnets, Superconductors and Cryostats
TE-MSC

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Practical Definitions & Formulae for Normal Conducting Magnets

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Summary
This synthetic document represents a practical reference with hints and formulae for the design of normal conducting magnets.
Certain formulae have been re-elaborated with practical units (for example the ones relevant to water cooling) for ease of use, in other cases analytical approximate formulae (for example the inductance of quadrupole and sextupole magnets) have been specifically drawn for this write-up.

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<u>Distribution</u>:
TE-MSC scientific staff, F. Bordry

1 Basic principles

1.1 Mass, velocity, momentum, energy

The energy E associated to a given particle with mass m follows the Einstein equation:

$$E = m\gamma c^2$$

where c= 299 792 458 m/s is the speed of light and γ is the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

where v is the speed of the particle, and $\beta = v/c$.

The energy of a particle increases then with velocity. The total energy can then be expressed as the sum of a rest energy plus a kinetic energy:

$$E = E_0 + E_K$$

In particle physics the energy is expressed in eV, where 1eV=1.6022x10^{-19} J is the energy acquired by an electron when accelerated by a difference of potential of 1 Volt.

For an electron: m = 9.11x10^{-31} kg; E_0=512 keV

For a proton m = 1.67x10^{-27} kg; E_0=938 MeV

We also define the momentum p of a particle as:

$$p = m\gamma v = \beta\frac{E}{c} = \frac{\sqrt{E^2 - E_0^2}}{c}$$

For practical purposes we will express the momentum in *GeV/c*. We observe that when the speed of the particle is close to that of light $\beta \approx 1$ energy and momentum have similar values.

We remark that 1GeV/c=10^9x1.6022·10^{-19}/(299 792 458) J/(m/s) = 5.34436x10^{-19} J/(m/s)

Table I summarizes the beam energy & momentum at the exit of the CERN proton injectors.

Table I: Beam energy & momentum at the exit of the CERN proton injectors					
Accelerator	**E_K [GeV]**	**E [GeV]**	**p [GeV/c]**	**γ=E/E_0**	**β=v/c**
LINAC 2	0.05	0.988	0.31	1.05	0.31
LINAC 4	0.16	1.098	0.57	1.17	0.52
PSB	1.40	2.34	2.14	2.49	0.92
PSB+	2.00	2.94	2.78	3.13	0.95
PS	25	26	26	27.7	1.00
SPS	449	450	450	480	1.00

1.2 Forces on particles, magnetic rigidity

We introduce the vector *magnetic field induction* $\boldsymbol{B}$ as a physical entity capable of producing a force $\boldsymbol{F}$ on an electrical charge q having a speed $\boldsymbol{v}$:

$$\vec{\boldsymbol{F}} = q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$$

Let's now consider a moving charge submitted to a dipolar field as shown in Fig.1.

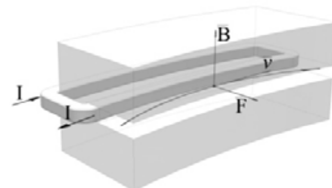


Fig.1 : force on a charged particle travelling in a perpendicular magnetic field

The velocity of the particle is perpendicular to the magnetic field, so we can just write F=qvB which, once confronted with the centripetal force and q is expressed in multiples z of the electron charge e, gives:

$$B\rho = \frac{p}{ze}$$

Expressing the momentum p in GeV/c and the energy in GeV, we obtain:

$$B\rho[Tm] = \frac{10^9}{c}\frac{p}{z} = 3.3356\frac{p}{z} = 3.3356\frac{\sqrt{E^2 - E_0^2}}{z}$$

The product $B\rho$ is known as *magnetic rigidity*, and is the fundamental starting point to define the requirements of a magnet from the particle momentum and charge.
Once, for a given beam type and energy, we have computed the magnetic rigidity, we can compute the field strength (or field integral) required to produce a given bending angle.
If we express the angle in radians, its measure α is the ratio between the arc length l and the radius ρ, so that the relation between required field integral and magnetic rigidity is:

$$Bl[Tm] = \alpha B\rho$$

For example for the 3.6° main magnets of the PS ($B\rho = 3.3356x26 = 86.7256$ Tm) we have:

$$Bl = \frac{\pi}{50}86.7256 = 5.449 \text{ Tm}$$

which are achieved with B=1.238 T and l=4.403 m.

In case of ions the kinetic energy is typically expressed in MeV/nucleon or MeV/u.
The rest energy of the ion is approximately equal to that of a nucleon (E_0=0.9315 GeV, note that it is smaller than that of a proton: the mass of an atom is smaller than the sum of the masses of its individual components because when these come together they release energy, typically in form of γ-ray, thus loosing mass. This is why one nucleon is lighter than a proton) times the mass number *A,* which is the number of nucleons in the atom, equal to the sum of the number of protons + neutrons.
We then have:

$$E_{ion} = AE_{nucleon}$$

The magnetic rigidity, computed from the energy per nucleon, becomes then:

$$B\rho[Tm] = 3.3356\frac{A\sqrt{E^2 - E_0^2}}{z}$$

1.3 Magnetic field components

If we consider a planar geometry like the 2D cross section of an infinitely long magnet, we can expand the horizontal and vertical magnetic field components B_x and B_y in a complex Fourier series:

$$B_y + iB_x = \sum_{n=1}^{\infty}(B_n + iA_n) \cdot (x + iy)^{n-1}$$

The coefficients B_n and A_n are the *multipoles* of the field, and determine the shape of the field lines.
As an example, a magnet in which only the term B_1 is non-zero, corresponds to a magnetic field:

$$B_x = 0 \ ; \ B_y = B_1$$

i.e. a perfect dipole field (constant in amplitude and direction) oriented in y direction. If the y direction is taken perpendicular to the plane of the accelerator (e.g. vertical), this is usually called a *normal* dipole, which provides bending in the plane of the accelerator (e.g. horizontal). A magnet in which only A_1 is non-zero results in a perfect and *skew* dipole field:

$$B_x = A_1 \ ; \ B_y = 0$$

which is in the plane of the accelerator (e.g. horizontal) and provides bending perpendicular to it (e.g. vertical). Multipoles B_2 and A_2 correspond to magnets generating a pure normal and skew quadrupole field. Higher order gradients (sextupole, octupole, etc.) are obtained by simple analogy in the continuation of the series.
In case of pure *normal* multipoles, we can write their amplitude as a function of the radius as:

$$|B| = B_n R^{n-1}$$

We can distinguish the:

- *dipole* the magnetic field is uniform, with amplitude $|B| = B_1$. Used to bend beams of particles.
- *quadrupole* the magnetic field amplitude increases linearly with the distance from the centre: $|B| = B_2 r$. The coefficient B_2 is called *gradient* (G) so that we can write $|B| = Gr$. We can introduce the *quadrupole strength k*, defined as the ratio between the gradient and the beam rigidity: $k=G/(B\rho)$. The meaning of the quadrupole strength is evident when we compute the angular deflection θ (in radians) of a particle passing at a distance x from the centre of a quadrupole of length l: $\theta=klx$. Quadrupole magnets are used to focus beams of particles.
- *sextupole* the magnetic field amplitude increases with the distance from the centre as: $|B| = B_3 r^2$. Sextupole magnets are used to correct the different amount of focusing provided by the quadrupoles to particles having different energies, in other words to correct the so called *chromaticity* (particles in a beam are not monochromatic as they have a distribution of energy). A sextupole is often described by the second derivative of the magnetic field, $B''=2B_3$.
- *octupole* the magnetic field amplitude increases with the distance from the centre as: $|B| = B_4 r^3$.
- *decapole* the magnetic field amplitude increases with the distance from the centre as: $|B| = B_5 r^4$.
- *dodecapole* the magnetic field amplitude increases with the distance from the centre as: $|B| = B_6 r^5$.

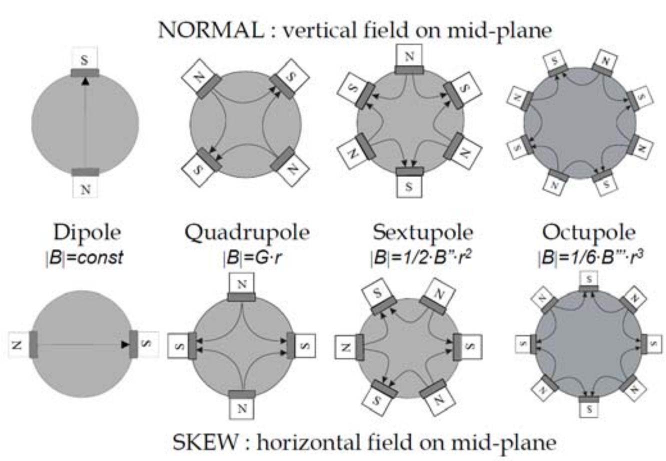


Fig.2 : graphical representation of normal and skew magnetic field components

1.4 Power

The power dissipated in a conductor of electrical resistivity ρ, cross section S and length l is:

$$P = \rho V J_{RMS}^2 = R I_{RMS}^2 \text{ with } R = \rho \frac{l}{S}$$

where V is the conductor volume, $J=I/S$ the current density and I the current flowing in the conductor. We recall that ρ depends on temperature: if T is the average temperature of the conductor in °C we have, for copper and aluminium respectively:

$$\rho_{Cu}[\Omega m] = 1.72 \cdot (1 + 0.0039(T - 20)) \cdot 10^{-8}$$

$$\rho_{Al}[\Omega m] = 2.65 \cdot (1 + 0.0040(T - 20)) \cdot 10^{-8}$$

The index *RMS* denotes an *effective* value, i.e. the equivalent steady DC (constant) value producing the same ohmic power dissipation as the relevant waveform, which is in general time varying.

The effective current I_{RMS} of a time varying current $i(t)$ over a time interval between t_1 and t_2 is:

$$I_{RMS} = \sqrt{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} i^2 dt}$$

Most of practical signals can be split into sinusoids, ramps and constants, for which we have:

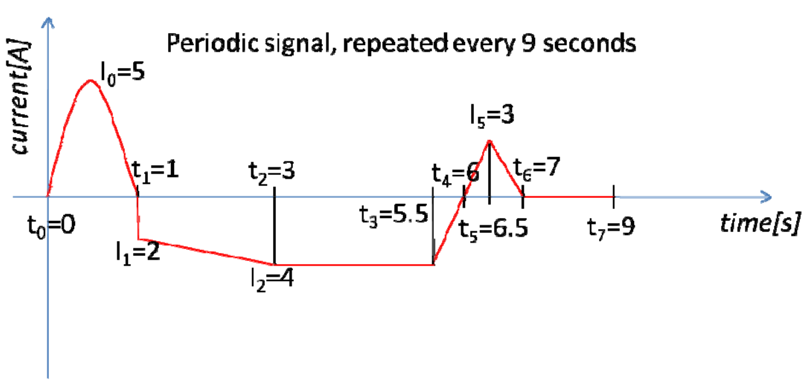
$$Constant: \; I_{RMS} = I_{MAX}$$

$$Sinusoid: \; I_{RMS} = I_{MAX}/\sqrt{2}$$

$$Ramp: \; I_{RMS} = I_{MAX}/\sqrt{3}$$

The effective current can then be computed as a quadratic average.

Example



$$I_{RMS} = \sqrt[2]{\frac{1}{t_7 - t_0} \cdot [\frac{I_0^2}{2} \cdot (t_1 - t_0) + \left(I_1 + \frac{(I_2 - I_1)}{\sqrt{3}}\right)^2 \cdot (t_2 - t_1) + I_2{}^2 \cdot (t_3 - t_2) + \frac{I_2^2}{3} \cdot (t_4 - t_3) + \frac{I_5^2}{3} \cdot (t_5 - t_4) + \frac{I_5^2}{3} \cdot (t_6 - t_5)]}$$

$$I_{RMS} = \sqrt[2]{\frac{1}{9} \cdot [\frac{25}{2} \cdot 1 + \left(2 + \frac{2}{\sqrt{3}}\right)^2 \cdot 2 + 16 \cdot 2.5 + \frac{16}{3} \cdot 0.5 + \frac{9}{3} \cdot 0.5 + \frac{9}{3} \cdot 0.5]} = 2.95\, A$$

2 Time varying fields

2.1 Penetration of a time varying electro-magnetic field in matter

The amplitude B_0 of a time varying magnetic field, of frequency *f*, penetrating at a distance *x* in a conducting material decreases according to the equation:

$$B(x) = B_0 e^{-x/\delta}$$

where δ is the *penetration depth*:

$$\delta[m] = \frac{1}{\sqrt{\pi \mu_0 \mu_r f \sigma}}$$

with $\mu_0=4\pi 10^{-7} H/m$, μ_r the relative magnetic permeability and σ the electrical conductivity.

Let's as an example check if a magnet yoke made with 1 mm thick laminations may be appropriate for operation at *f=10 Hz*. We consider, to remain into reasonable magnetic characteristics, μ_r =500. Recalling that for pure iron $\sigma=10\text{x}10^6\ (\Omega\text{m})^{-1}$ and for 3% silicon steel $\sigma=2\text{x}10^6\ (\Omega\text{m})^{-1}$, we obtain:

$\delta_{iron}=2$ mm and $\delta_{silicon\ steel}=5$ mm

A magnet yoke can efficiently drive the magnetic field when the thickness of its laminations is smaller than the penetration depth at a given frequency.
It is worth to remind that the electrical conductivity and the thickness of the lamination have an impact also on eddy current losses: this is why from 50 Hz, even if not strictly needed for magnetic reasons, silicon steel and laminations not exceeding 1 mm thick are commonly used.

what happens if the penetration depth is smaller than the lamination thickness?

Let's consider for example a septum magnet made with 0.35-mm-thick silicon steel laminations. Can it operate at a magnetic field in the iron of 0.5 T with a pulse equivalent frequency *f=20* kHz ?
The penetration depth is $\delta_{silicon\ steel}(20\ \text{kHz})=0.1$ mm, smaller than the lamination thickness.
In this case the magnetic field distribution inside a lamination will not be constant. To provide the required average field the edges have to work at a higher field: it is important to check this does not achieve saturation.
Fig.3 shows that, thanks to the relatively low magnetic field required our septum magnet can still work at the frequency *f=20* kHz: for a required average field $B_a= 0.5$T we have $B_0=1.7$T.
It is worth to remark that such a septum would in reality even work at a higher frequency, with a smaller magnetic permeability (which would increase the penetration depth).

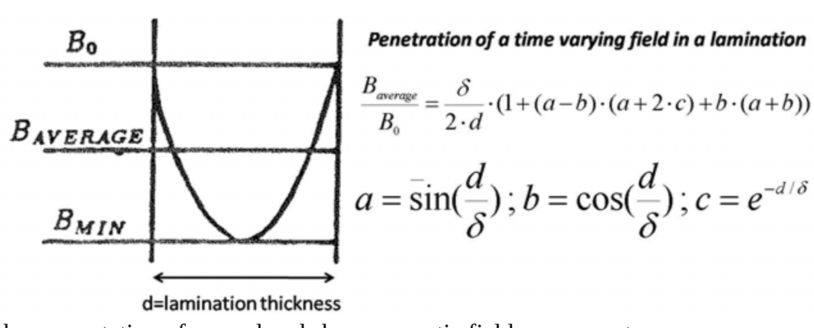


Fig.3 : graphical representation of normal and skew magnetic field components

2.2 Losses in the iron

Losses in the iron are due to eddy current losses and hysteresis losses.

Eddy current losses

The origin of eddy current losses is explained by the Faraday law stating that a variation of magnetic flux ϕ produces an induced voltage V, completed by Lenz with the sign "minus" because the induced voltage generates currents creating a counter-acting field to the main field, in practice eddy currents tend to decrease the magnetic field in the lamination (as already seen before):

$$V = -\frac{d\emptyset}{dt}$$

The currents produced by this induced voltage depend on the resistance of the circuit, in practice on the electrical resistance of the material and of the thickness of the lamination. To reduce these currents it is convenient to use thin laminations to reduce the section available for current flow, and add silicon to pure iron (silicon steel) to increase the resistivity of the material.

The estimate of eddy current losses in ferromagnetic laminations is very complicated, in particular when the magnetic field is not uniform within a lamination.

In a first approximation, for silicon steel, the eddy current losses P_e are:

$$P_e[\frac{W}{kg}] = 0.05(d\frac{f}{10}B_{ap})^2$$

where *d [mm]* is the lamination thickness in *mm, f* the frequency and B_{ap} *[T]* the average peak field in the lamination, corresponding to the peak field if $d<\delta$ (δ is the penetration depth)

Hysteresis losses

The energy used to magnetize and demagnetize a ferromagnetic material is the area of the hysteresis curve. The energy depends then on the amplitude of the magnetic field.

The resulting power depends on how many times-per-second the hysteresis loop is executed.

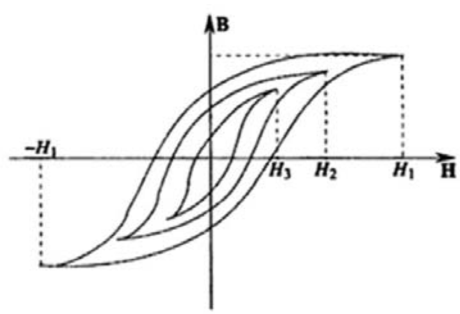


Fig.4 : hysteresis loops of diffeent amplitude

Using experimental results from 0.2 to 1.5 T, Steinmetz came up with the empirical law:

$$P_h[\frac{W}{kg}] = \eta f B_{ap}^{1.6}, \text{ with } \eta=0.01\div0.1$$

where B_{ap} is expressed in Tesla, and f in Hz. For silicon steel η~0.02

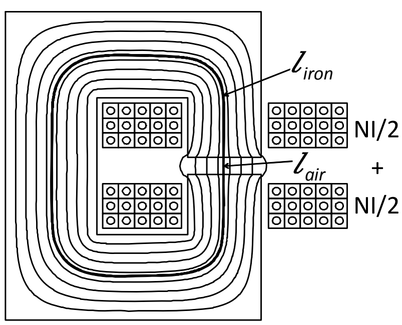
3 Design of Normal Conducting Magnets

3.1 Introduction

“Normal conducting”, and alternatively “resistive”, “warm” or “conventional”, are electro-magnets in which the magnetic field is generated by conductors like copper or aluminium, which oppose an electrical resistance to the flow of current. The magnetic field induction provided in the physical aperture of these magnets rarely exceeds 1.7 T ÷ 2.0 T, such that the working point of the ferromagnetic yoke remains below saturation. In these conditions the yoke provides a closure of the magnetic path with small use of magnetomotive force, and its pole profile determines the magnetic field quality.

3.2 Generation of the magnetic field in accelerator magnets

The Ampere’s law allows to easily obtain an analytical expression of the relationship between magnetic field and magnetomotive force in most of magnet configurations used in particle accelerators.
As an example, we illustrate in Fig.5 a non-saturated C-type dipole magnet, with physical vertical aperture l_{air}, made with two coils of *N/2* turns each, connected in series and supplied by a current I (so that the total magnetomotive force is *NI*).



$$NI = \oint H dl = H_{iron} l_{iron} + H_{air} l_{air} == \frac{B}{\mu_0 \mu_r} l_{iron} + \frac{B}{\mu_0} l_{air}$$

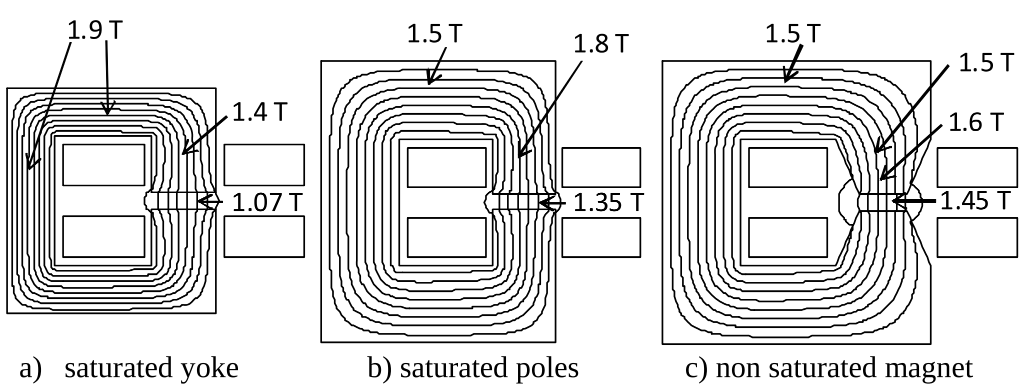
If $\mu_r \gg l_{iron}/l_{air}$, we can neglect the magnetomotive force “used” in the iron and obtain:

$$B \sim \mu_0 NI / l_{air}$$

In case part of the iron is saturated, its permeability will be lower and part of the *ampereturns NI* will be used to magnetize the iron as discussed later.

Fig.5 : a non-saturated C-type dipole with physical vertical aperture l_{air}, supplied by a total current *NI*

In case the magnetic field exceeds a value of typically about 1.5 T along the path corresponding to l_{iron} the magnetomotive force used in the iron may become no longer negligible with respect to that used in the air. As the iron yoke gathers also the stray field, the field induction in the iron poles is always higher than the one between poles. To reduce the iron portion working at fields above 1.5 T, the iron pole can be tapered (Fig.6). This allows designing iron-dominated magnets capable of producing magnetic fields intensities in their physical aperture rather close to the saturation limit of the iron, i.e. up to about 1.7 T-2.0 T.



a) saturated yoke b) saturated poles c) non saturated magnet

C-dipole magnetic circuit, of physical vertical aperture l_{air}=10 mm, supplied with $N_{tot}I$=120 000A.

Fig.6 : decreasing saturation in a ferromagnetic circuit

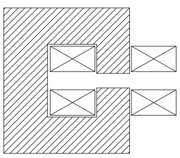
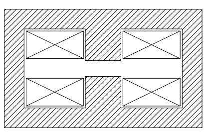
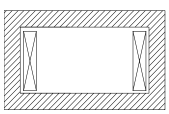
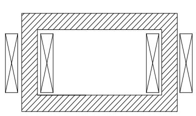
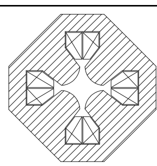
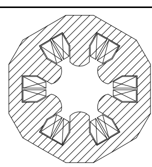
3.3 Transfer function and inductance

Transfer function and inductance can be computed starting from the Ampere’s law and considering the relationship between magnet inductance (L), current (I) and energy (E) as $E=½LI^2$.

In practical cases the theoretical transfer function of an ideal magnetic circuit is reduced by an “efficiency” η, typically of the order of η=0.95÷0.98, which depends on the length, stacking factor and working conditions of the magnetic yoke . The formulas in Table II provide an analytical formulation of the inductance values of different magnet configurations.

The inductance depends on how the pole geometry is trimmed (shims, tapered poles, chamfers) and on saturation. In particular for quadrupole and sextupole magnets different tapering of the poles can strongly modify the inductance. For such magnets these simplified formulas can cover only standard designs.

TABLE II: approximate pole shape, transfer function and inductance of basic magnets.

Magnet type	*Descriptions*	*Pole shape*	*Transfer function*	*Inductance*
	C-type dipole *w*: pole width *g*: distance between poles *NI*: total ampereturns *l*: yoke longitudinal length	$y=\pm g/2$	$B=\eta\mu_0 NI/g$	$L=\eta\mu_0 N^2 A/g$ $A\sim(w+1.2g)\cdot(l+g)$
	H-type dipole *w*: pole width *g*: distance between poles *NI*: total amperturns *l*: yoke longitudinal length	$y=\pm g/2$	$B=\eta\mu_0 NI/g$	$L=\eta\mu_0 N^2 A/g$ $A\sim(w+1.2g)\cdot(l+g)$
	Window-frame *d*: aperture between coils *t*: coil width *g*: physical vertical aperture *NI*: total ampereturns *l*: yoke longitudinal length	$y=\pm g/2$	$B=\eta\mu_0 NI/g$	$L=\eta\mu_0 N^2 A/g$ $A\sim(d+2/3t)\cdot(l+g)$
	Window-frame *d*: aperture between coils *t*: coil width *g*: physical vertical aperture *NI*: ampereturns on one leg *l*: yoke longitudinal length	$y=\pm g/2$	$B=\eta\mu_0 NI/g$	$L=\eta 2\mu_0 N^2 A/g$ $A\sim(d+2/3t)\cdot(l+g)$
	Quadrupole *R*: radius at pole tip d: distance centre-inner coil *NI*: ampereturns per pole *l*: yoke longitudinal length	$2xy=R^2$	$\lvert B\rvert(r)=G\cdot r$ $G=\eta 2\mu_0 NI/R^2$	$L=8\pi\mu_0 N^2 l_m\sqrt{d/R}$ $l_m=(l+2/3R)$
	Sextupole *R*: radius at pole tip d: distance centre-inner coil *NI*: ampereturns per pole *l*: yoke longitudinal length	$3x^2y-y^3=R^2$	$\lvert B\rvert(r)=S\cdot r^2$ $S=B''/2$ $S=\eta 3\mu_0 NI/R^3$	$L=12\pi\mu_0 N^2 l_m\sqrt{d/R}$ $l_m=(l+1/2R)$

3.4 Field quality

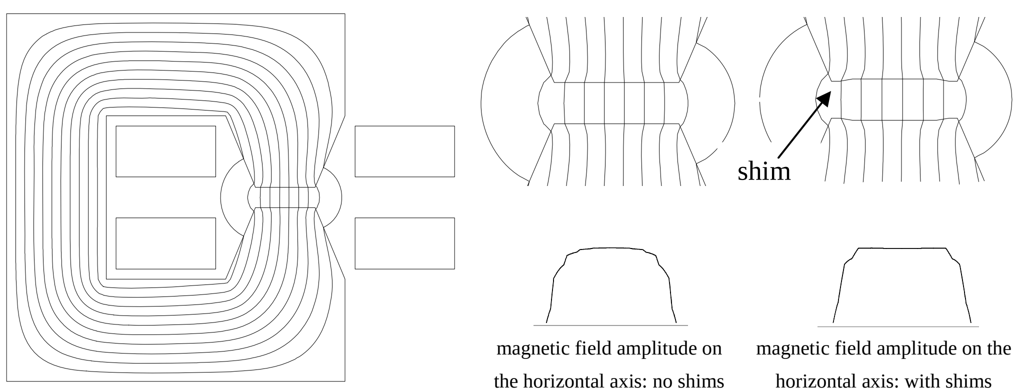
The field homogeneity typically required by an accelerator magnet within its *good field region* is of the order of few parts in 10^{-4}. Transfer line and corrector magnets may be specified with lower homogeneity.

Field quality in a given volume is determined by several factors:

- the size of the magnet aperture with respect to the good field region;
- the shape of the iron poles;
- manufacture and assembly tolerances;
- the proximity to active conductors (coils), in particular in window-frame magnets;
- the ferromagnetic properties at the working conditions of the steel used for the yoke;
- dynamic effects.

Trimming field quality is achieved considering all above aspects. In particular magnets operating below 2 T, as the ones treated in this chapter, are also described as iron dominated magnets because the shape of the magnetic field induction is determined by the shape of the ferromagnetic poles. At the interface between the magnet aperture and the poles, i.e. between steel and air, the component of the magnetic field induction $B_\perp$ perpendicular to the interface surface is the same on both media. The tangential component of the magnetic field H_t also remains the same in case no surface currents are present: this corresponds to a change of the tangential components B_t of the field induction by the ratio of the magnetic permeability between the two media. As a result, in case of infinite permeability the direction of the magnetic field induction in the air at the exit of a magnet pole is always perpendicular to the pole surface.

An example of trimming field quality with pole shims in a dipole magnet is shown in Fig.7.



C-dipole with pole shims. The shims extend the good field region

Fig.7 : improving field quality in a C-type dipole magnet

3.5 Coils

The coils generate the magnetomotive force necessary to produce the required magnetic field induction in the magnet physical aperture.
The effective current density J_{rms} determines coil size, power consumption and cooling: the designer shall consider a balance between requirements, technological limits, investment and operation cost. Typical values of J_{rms} are around 5 A/mm^2 for water cooled magnets, and around 1 A/mm^2 for air cooled magnets. Air cooled coils with favourable configurations (large perimeter-to-cross section ratio) can be operated at higher current densities.

Introducing l_{av} as the average coil turn length, the power dissipated in a magnet is:

$$P_{dipole} = \rho \frac{B_{rms} g}{\eta \mu_0} J_{rms} l_{av} \; ; \; P_{quadrupole} = 2\rho \frac{G_{rms} R^2}{\eta \mu_0} J_{rms} l_{av} \; ; \; P_{sextupole} = \rho \frac{B''_{rms} R^3}{\eta \mu_0} J_{rms} l_{av}$$

In case of water cooled magnets, heat is removed by water circulating in the coil (hollow) conductors.
To increase the temperature of 1 kg of water by 1°C we need 1 kcal=1/4.186 kJoules
⇒ one liter per second of water can draw 1/4.186 kW by increasing its temperature by 1°C.
In formulas, by expressing the water flow Q in liters/minute and the power P in kW, we have:

$$Q \left[\frac{l}{min}\right] = 14.3 \frac{P[kW]}{\Delta T}$$

In practice, accepting ΔT=30°C, every liter per minute can cool about 2 kW.
To be thermally effective, the water flow shall be moderately turbulent: this condition is fulfilled if the so called *Reynolds number* R_e, representing the ratio between the inertial forces and the viscous forces, is greater than 2000 and smaller than 10^5, where:

$$R_e = \frac{dv}{\nu}$$

where v is the water velocity, ν is the dynamic viscosity (about 0.7×10^{-6} m^2/s for water at 40°C), and d the equivalent hydraulic diameter of the cooling pipe.
For water at 40°C we have then:

$$R_e = \frac{dv}{\nu} \approx 1400 d[mm] v[\frac{m}{s}]$$

In practice, for efficient cooling, the water speed shall be greater than:

$$v[\frac{m}{s}] > \frac{1.4}{d[mm]}$$

To find the good combination of diameter and water velocity for a given flow this condition shall be iterated with:

$$Q \left[\frac{l}{min}\right] = 0.06\pi \frac{d[mm]^2}{4} v \left[\frac{m}{s}\right]$$

taking care of avoiding an excessive water velocity, causing erosion corrosion of the cooling pipe: in copper this starts taking place at $v>3m/s$.
The last parameter to compute is the pressure drop.
For hydraulically smooth pipes of length L and diameter d with inlet aand outlet at about the same height, as in magnet coils, with moderately turbulent flow, the pressure drop is given by the *Blasius* law which, in practical units, can be written as:

$$\Delta P = 60 L[m] \frac{Q[\frac{l}{min}]^{1.75}}{d[mm]^{4.75}}$$

- if the circuits are in parallel the pressure drop will be same across each of the circuits and the flow will be shared according to the hydraulic resistance of the circuits. If they all have the same length and diameter it is easy: the flow in each circuit will be the total flow divided by the number of circuits;
- if the circuits are in series the total pressure drop will be the sum of the pressure drops across each individual circuit.

In summary, the choice of cooling parameters and number of circuits is based on few main principles: set the water flow corresponding to the allowed temperature drop for a given power to be removed, have a moderate turbulent flow to provide an efficient cooling, keep the water velocity within reasonable limits to avoid erosion-corrosion and impingement of the cooling pipes and of the junctions, keep the pressure drop across the circuit within reasonable limits (typically within 10 bars).
Criteria and formulae for the determination of cooling circuits are summarized in Table III.

Table III: Criteria and formula for the determination of water cooling circuits.

Parameter	*Fundament*	*Formula (in practical units)*
Cooling flow	1 kcal=4186 J increases the temperature of 1 kg of water by 1°C. *Q* is the cooling flow, *P* the dissipated power, *ΔT* the allowed temperature drop	$Q[\frac{liter}{min}]{\sim}14.3\frac{P[kW]}{\Delta T[K]}$
Water velocity	$Q = v \cdot A$, where *Q* is the flow, *v* the water velocity and *A* the section of the pipe. In the formula *d* is the hydraulic diameter so that $A = \frac{\pi d^2}{4}$	$v\left[\frac{m}{s}\right] = \frac{1000}{15\pi d^2} Q[\frac{l}{min}]$
Turbolent flow	Reynolds number > 2000, where *Re=d·v/ν*, with *d* the hydraulic diameter of the pipe, *v* the fluid velocity and *ν* the kinematic viscosity	$R_e{\sim}1400d[mm]v\left[\frac{m}{s}\right] > 2000$ valid for water at ~ 40°C
Water velocity limit	Limited by erosion-corrosion and impingement, that might start already at $v > 1.5$ m/s in copper pipes, tee pieces and elbow fittings. Velocities up to 10 m/s can still be considered in particular cases, depending on water characteristics, temperature, sizing and layout of pipes and junctions.	$v < 3\ \frac{m}{s}$
Pressure drop	The pressure drop *ΔP* can be computed as a function of the cooling flow *Q* from the Blasius law. We consider a smooth pipe of length *L* and diameter *d*	$\Delta P[bar]{\sim}60L[m]\frac{Q[\frac{liter}{min}]^{1.75}}{d[mm]^{4.75}}$

Finally, coils are submitted to forces: their own weight and the electromagnetic forces produced by the interaction between magnetic field and current.
The interaction between a moving charge and the magnetic field is described by the so called Lorentz force, which for a wire carrying a current *I* is referred as *Laplace force:*

$$\vec{F} = I \cdot \vec{l} \times \vec{B}$$

where the length $\vec{l}$ is oriented towards the direction of the current flow. For example 1.5 meter of straight coil immerged in an average magnetic field component perpendicular to the coil of 0.5 T, carrying a total current of 60 000 ampereturns, is submitted to a force of:

$$F = 60000x1.5x0.5 = 45 \text{ kN}.$$

3.6 Yoke

The magnet yoke has the function of directing and shaping the magnetic field generated by the coils. While magnets operated in persistent mode can be built either with solid or with laminated steel, the yokes of cycled magnets are composed by laminations electrically insulated from each other to reduce the eddy currents generated by the change of magnetic field in time. This electrical insulation can be inorganic (oxidation, phosphating, Carlite) or organic (epoxy). Epoxy coating in the B-stage form can be used to glue laminations together, a technique widely used for small to medium magnets, possibly reinforced by welded bars on the yoke periphery. Larger magnets are typically made by stacked laminations bolted and welded (on the outer periphery only if cycled magnets) together.
The magnetic properties of steel depend on the chemical composition and on the temperature/mechanical history of the material.
Important parameters for accelerator magnets are the coercive field H_c and the saturation induction. The coercive field has an impact on the remanent field present in the magnet once the current is switched off and on the reproducibility of the magnetic field particularly at low currents. A typical requirement for the steel used in accelerator magnets is H_c < 80 A/m. Tighter constraints (H_c < 20 A/m) apply when the operation covers a large field factor starting from low field inductions (few hundred gauss). The saturation induction is highest with low carbon steel (carbon content in the final state < 0.006%). It is common to specify points along the normal magnetization curve, with the condition that the magnetic induction B shall exceed specification values at given field levels H.
To increase its electrical resistivity and at the same time narrow the hysteresis cycle, laminated steel used in cycled magnets usually contains 2÷3% of silicon.
With 3% of Si the electrical resistivity increases from $\rho = 2x10^{-7}\Omega$m to $\rho = 5x10^{-7}\Omega$m.

3.7 Costs

We distinguish

- *fixed costs*: design, coil tooling (winding, molding), yoke tooling (punching, stacking), quality assurance (including tools for specific measurements/checks, as magnetic measurements if requested)
- *unitary costs:* main materials (conductor, insulation, steel), manufacture of parts (coil, laminations, yoke), final assembly, ancillaries (connectors, interlocks, hoses), tests (mechanical, electrical, magnetic)
- *other systems:* cooling, power converters, controls and interlocks, electrical distribution. These parameters have to be taken into account at the magnet design phase: for example for cycled magnets a low inductance can minimize the voltage levels, however the corresponding higher current would require larger supply cables.
- *running costs:* electric power, maintenance over the life of the project.

A compromise between capital and operational cost is found with magnets operating with:

- current density~5 A/mm^2:higher current densities correspond to smaller coils and thereafter yoke, lower current densities to lower power consumption and smaller cooling plants
- field induction levels in the region between 1.2 T and 1.7 T: a given required integrated strength can be provided by short magnet with high field induction, long magnets with low field induction or a compromise. Since, below saturation, the pole width size depends on the good field region size and not on the field induction level, the highest possible field and the corresponding lowest magnetic length represent in most cases a cost-optimized yoke design.

4 Undulators, wigglers, permanent magnets.

Wigglers and *Undulators* produce a periodic field variation along the beam trajectory causing relativistic charged particles to wiggle emitting electromagnetic radiation with special properties, in particular with a small angle $\alpha = 1/\gamma$ where γ is the relativistic factor.
To a first approximation, these magnets produce a series of dipole fields with alternated directions, of period λ. This is typically obtained with conventional electromagnets when the period is relatively large allowing sufficient space for the coils, and with permanent magnets for shorter periods. Superconducting windings, in general cryocooled, are used in case the field exceeds 2 T and/or for small periods where a high current density is needed.
The difference between wigglers and undulators is in the nature of the radiation produced by the particle. When the amplitude of the beam excursion is small with respect to the angle of the synchrotron radiation emission, the device is called undulator: the emitted radiation is concentrated in a small opening angle and the radiation produced by the different periods interferes coherently producing sharp peaks at harmonics of a fundamental wavelength. Wigglers on the contrary produce particle displacements of larger amplitude: the emitted radiation is similar to the continuous spectrum generated by bending magnets, with in addition the effect coming from the superposition of radiation from individual poles.
It is useful to introduce the *deflection parameter* $K = \delta_0/\alpha$ as the ratio between the maximum trajectory deflection δ_0 and the emission angle α. For electrons

$$K = \frac{eB_{0\lambda}}{2\pi mc} = 93.4 \cdot B_0 \cdot \lambda$$

In case *K<1* the device is an undulator, in case *K>>1* the device is a wiggler.
As anticipated, these magnets are often built with use of permanent magnets.
Two types of high performance permanent magnet materials, both composed of rare earth elements, are available: Neodymium-Iron-Boron (NdFeB) and Samarium-Cobalt (in the form $SmCo_5$ or Sm_2Co_{17}, also referred as SmCo 1:5 and SmCo 2:17 types).
NdFeB materials show the highest remanent induction, up to $B_r \sim 1.4\ T$, they are ductile, but they require coating to avoid corrosion and have a relatively low stability versus temperature. Their relative change of remanent field induction with temperature (temperature coefficient) is $\frac{\Delta B_r}{B_r \cdot {}^\circ C} \sim -0.11\%$: field induction decreases when temperature increases.
SmCo magnets show a lower remanent induction, up to $B_r \sim 1.1T$, are brittle, but they are corrosion and radiation resistant. Furthermore, their temperature coefficient is about -0.03%, lower than that of NdFeB.

5 Solenoids.

Solenoids are made by electrical conductors wound in the form of a helix. The magnetic field induction inside a solenoid is parallel to the longitudinal axis and its intensity is $B = \mu_0 NI/l$ where *NI* is the total number of ampereturns and *l* the solenoid length. By introducing the coil thickness *t*, the formula can be written as $B = \mu_0 Jt$, where *J* is the current density.
Solenoids can be built as ironless magnets or can have an external ferromagnetic yoke, used for shielding and to increase the magnetic field uniformity particularly at the solenoid extremities. The design and construction of solenoids, thanks to their use in many electrical and electro-mechanical devices, is well assessed since more than a century. A comprehensive treatment of solenoid electromagnets was compiled by C.Underhill already in 1910. The treatment by Montgomery in 1969 is still a reference nowadays, in spite of new materials now available in particular for wire dielectric insulation and for the containment of stresses.
In solenoids electromagnetic forces can reach extremely high values capable of breaking the wires or even, especially for pulsed magnets, leading to explosion of the device. Their design shall consider conductor characteristics and reinforcements, winding tension during manufacture and containment structure.