Outline:

- Time and frequency domain treatment & Fourier Transformation
- Signal generation → transfer impedance
- Capacitive *button* BPM for high frequencies
- Capacitive *shoe-box* BPM for low frequencies
- Electronics for position evaluation
- BPMs for measurement of closed orbit, tune and further lattice functions
- Summary

Usage of BPMs

A *B*eam *P*osition *M*onitor is an non-destructive device for bunched beams

It has a low cut-off frequency i.e. dc-beam behavior can not be monitored
The abbreviation BPM and pick-up PU are synonyms

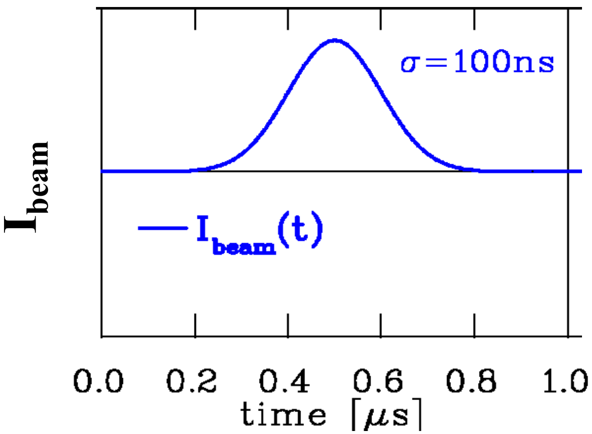
1. It delivers information about the transverse center of the beam

- ***Trajectory:*** Position of an individual bunch within a transfer line or synchrotron
- ***Closed orbit***: Central orbit averaged over a period much longer than a betatron oscillation
- ***Single bunch position***: Determination of parameters like tune, chromaticity, ***β***-function
- Bunch position on a large time scale: bunch-by-bunch → turn-by-turn → averaged position
- Time evolution of a single bunch can be compared to 'macro-particle tracking' calculations
- Feedback: fast bunch-by-bunch damping ***or*** precise (and slow) closed orbit correction

2. Information on longitudinal bunch behavior (see next chapter)

- **Bunch shape and evolution** during storage and acceleration
- For proton LINACs: the beam **velocity** can be determined by two BPMs
- For electron LINACs: **Phase** measurement by Bunch Arrival Monitor
- ***Relative*** low current measurement down to 10 nA.

Time domain: Recording of a voltage as a function of time:

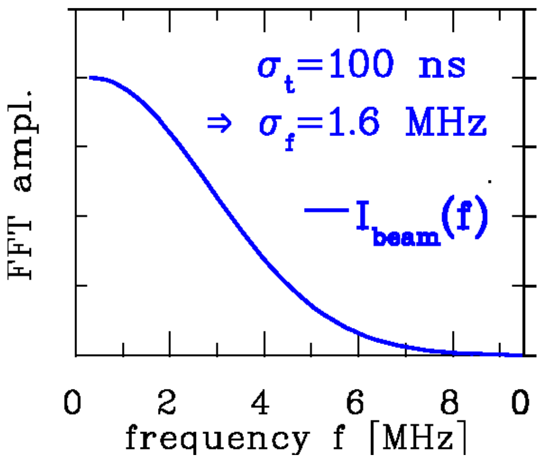


Instrument: Oscilloscope

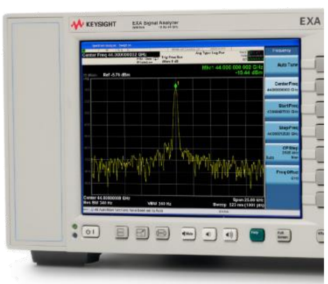
Fourier Transformation:

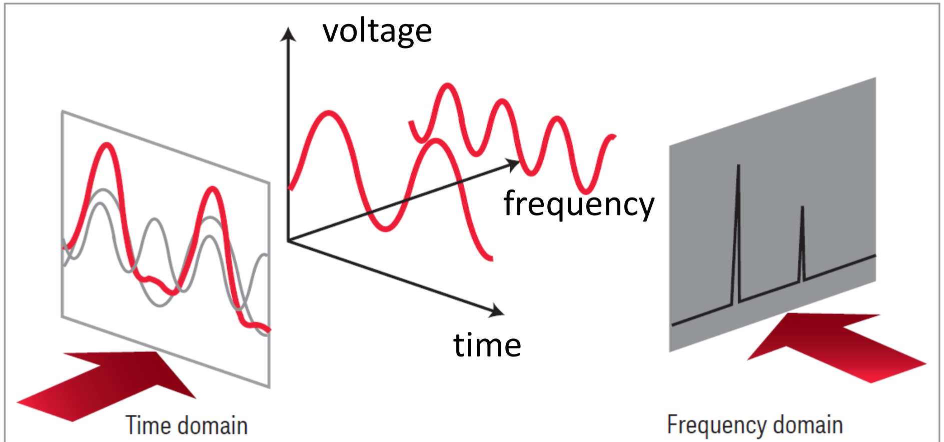
$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

Frequency domain: Displaying of a voltage as a function of frequency:



Instrument: Spectrum An





courtesy company Keysight

Care: Fourier Transformation of time domain data contains amplitude ***and*** phase

Excurse: Properties of Fourier Transformation

Fourier Transform.: $\tilde{f}(\omega) \equiv \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$ **Inv. F. T.:** $f(t) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(\omega) e^{i\omega t} d\omega$

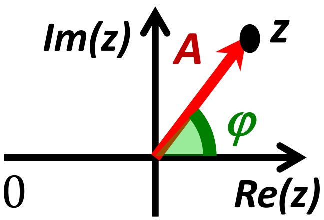
tech. *DFT(f)* tech. *IDFT(f)*

$\Rightarrow$ a process can be described either with $f(t)$ **time domain**' or $\tilde{f}(\omega)$ '**frequency domain**'

We assume $t, \omega, f(t) \in \mathbb{R}$ but $\tilde{f}(\omega) \in \mathbb{C}$; note: integral from $-\infty$ to ∞

FT: $\tilde{f}(\omega) \in \mathbb{C} \rightarrow$ amplitude $A(\omega) = \left|\tilde{f}(\omega)\right|$ or power $P(\omega) \propto \left|\tilde{f}(\omega)\right|^2$

& phase $\varphi(\omega) = \arctan \frac{Im(\tilde{f})}{Re(\tilde{f})}$ **tech:** displayed for $|\,\omega\,| \geq 0$



No loss of information: If $\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$ exists, than $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(\omega)\, e^{i\omega t} d\omega$

$\rightarrow$ the original function *f(t)* is 'recovered', or 'you neither gain nor lose information'

Linearity: $\int_{-\infty}^{\infty} [a f_1(t) + b f_2(t)] e^{-i\omega t} dt = a\tilde{f}_1(\omega) + b\tilde{f}_2(\omega)$ for $a, b \in \mathbb{R}$

Similarity Law: For $a \neq 0$ it is for $f(at)$: $|1/a| \cdot \tilde{f}(\omega/a) = \int_{-\infty}^{\infty} f(at) e^{-i\omega t} dt$

$\rightarrow$ the properties can be scaled to any frequency range; $\Leftrightarrow$ 'shorter time signal has wider FT' ,

e.g. Gaussian $f(t) = \exp\left(-\frac{t^2}{2\,{\sigma_t}^2}\right) \Rightarrow \tilde{f}(\omega) = \exp\left(-\frac{\omega^2}{2\,{\sigma_\omega}^2}\right)$ with $\sigma_\omega = \frac{1}{\sigma_t}$

Excurse: Properties of Fourier Transformation

Differentiation Law: For ***n***th derivative ***f***$^{(n)}$***(t)*** it is: $$(i\omega)^n \cdot \tilde{f}(\omega) = \int_{-\infty}^{\infty} f^{(n)}(t) e^{-i\omega t} dt$$

$\rightarrow$ differentiation in time domain corresponds to multiplication with $\boldsymbol{i\omega}$ in frequency domain

Frequency shift: Modulation by ω_0 it is $\int_{-\infty}^{\infty} f(t) e^{i\omega_0 t}\, e^{-i\omega t}\, dt = \tilde{f}(\omega - \omega_0)$

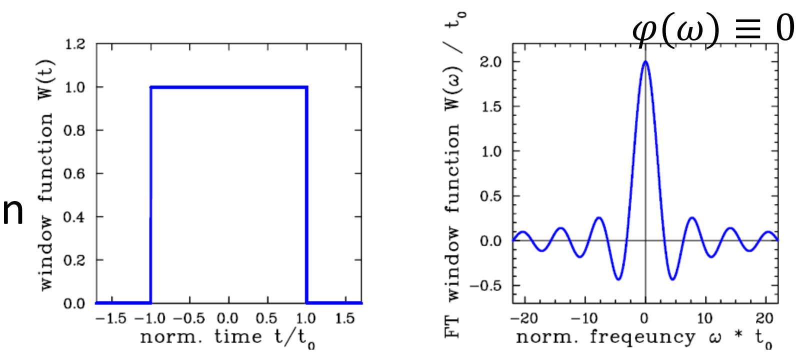
Condition for FT: If $\int_{-\infty}^{\infty} |f(t)|^2\, dt < \infty$ and convergent, than $f(t)$ is Fourier transformable

$\Rightarrow$ e.g. $f(t) = \sin(\omega_0 t)$, polynomials $f(t) = \Sigma\, a_i\, t^i$ and $f(t)=e^t$ are **not** Fourier transformable !

Windowing: Def. $\tilde{f}_W(\omega) \equiv \int_{-\infty}^{\infty} W(t) \cdot f(t) e^{-i\omega t}\, dt$

with a window function $W(t) \in \mathbb{R} \Rightarrow$ finite integration

$\Rightarrow \tilde{f}_W(\omega) = \widetilde{W}(\omega) * \tilde{f}(\omega)$ as convolution



Example: $W(t) = \begin{cases} 1 & \text{for } -t_0 \le t \le t_0 \\ 0 & \text{for } |t| > 1 \end{cases} \quad \Rightarrow \quad \widetilde{W}(\omega) = 2\frac{\sin(\omega t_0)}{\omega} \equiv 2t_0\, \mathrm{sinc}(\omega t_0)$

Zero-crossing at: $\sin(\omega t_0) = 0 \;\Leftrightarrow\; n\pi = \omega t_0 \;\Leftrightarrow\; \omega = n \cdot \frac{\pi}{t_0}$ with $n \in \mathbb{Z}\backslash\{0\}$

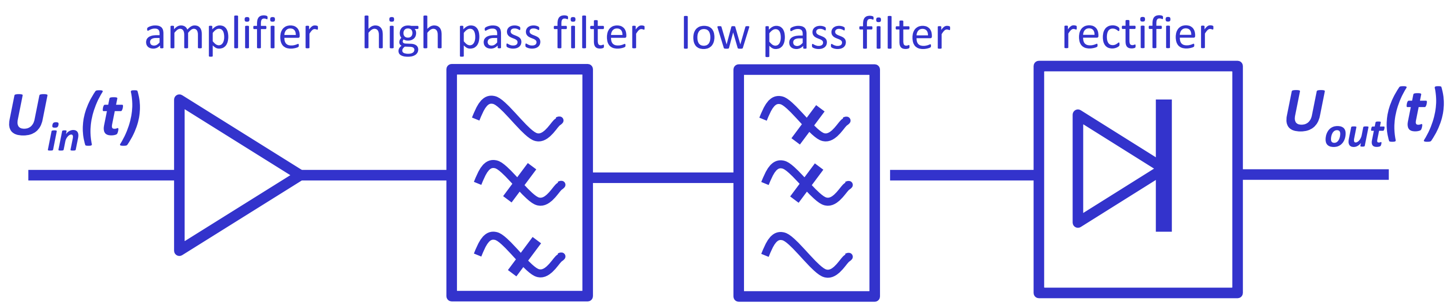
Scaling of maxima: $\max(\omega) = \frac{2}{\omega} \;\Leftrightarrow\; \sin(\omega t_0)=1 \;\Leftrightarrow \omega = \frac{2n+1}{2} \cdot \frac{\pi}{t_0}$ with $n \in \mathbb{Z}\backslash\{0\}$

Convolution Law: For the convolution $f(t) = f_1(t) * f_2(t) \equiv \int_{\infty}^{\infty} f_1(\tau) \cdot f_2(t-\tau) d\tau$

$\Rightarrow$ FT can be calculated as $\tilde{f}(\omega) = \tilde{f}_1(\omega) \cdot \tilde{f}_2(\omega)$

→ convolution in time domain can be expressed as multiplication of FT in frequency domain

Application: Chain of electrical elements calculated in frequency domain more easily parameters are more easy in frequency domain (bandwidth, ***f***-dependent amplification.....)



Engineering formulation for finite number of discrete samples:

Digital Fourier Transformation *DFT(f):* corresponds to math. FT for finite number time

Fast Fourier Transformation: *FFT(f)* dedicated algorithm for **fast** calc. with 2^n increments

Transfer function *H(ω)* and ***h(t)*** are used to describe electrical elements

Calculation with ***H(ω)*** in frequency domain or ***h(t)*** time domain

→ 'Finite Impulse Response' FIR filter or 'Infinite Impulse Response' IIR filter

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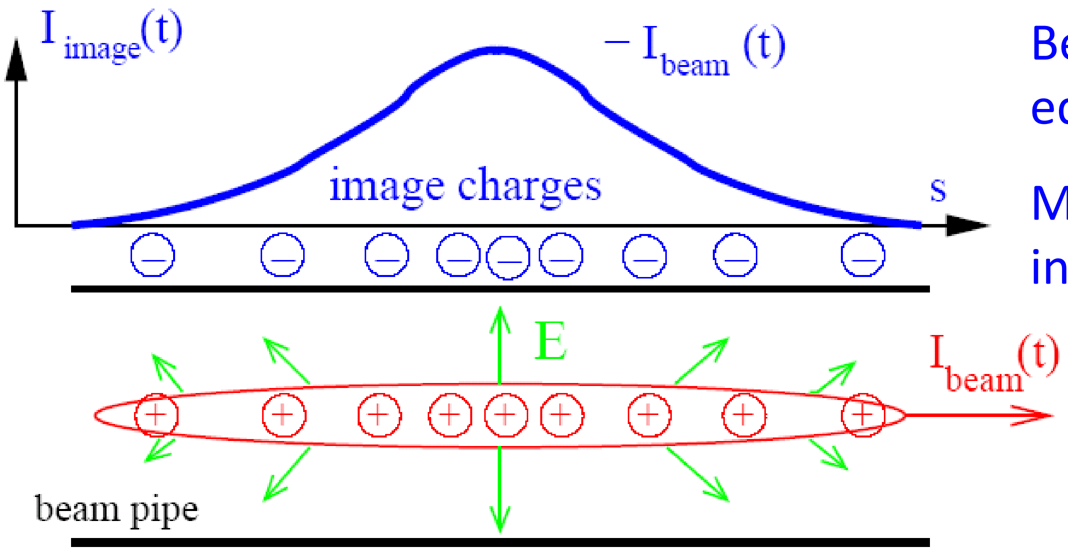
The image current at the beam pipe is monitored on a high frequency basis i.e. the ac-part given by the bunched beam.



Beam Position Monitor **BPM** equals Pick-Up **PU**

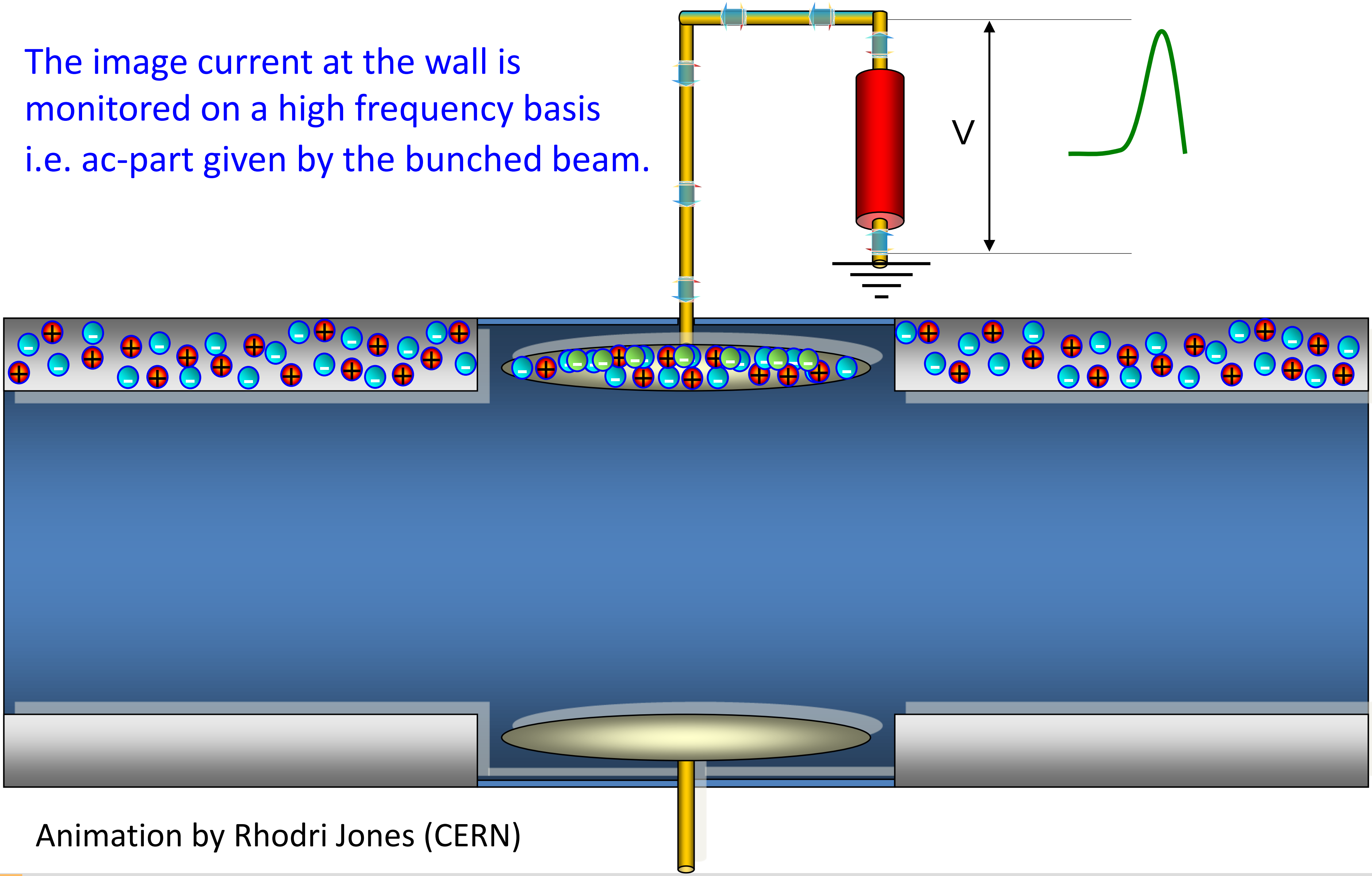
Most frequent used instrument!

For relativistic velocities, the electric field is transversal:

$$E_{\perp,lab}(t) = \gamma \cdot E_{\perp,rest}(t')$$

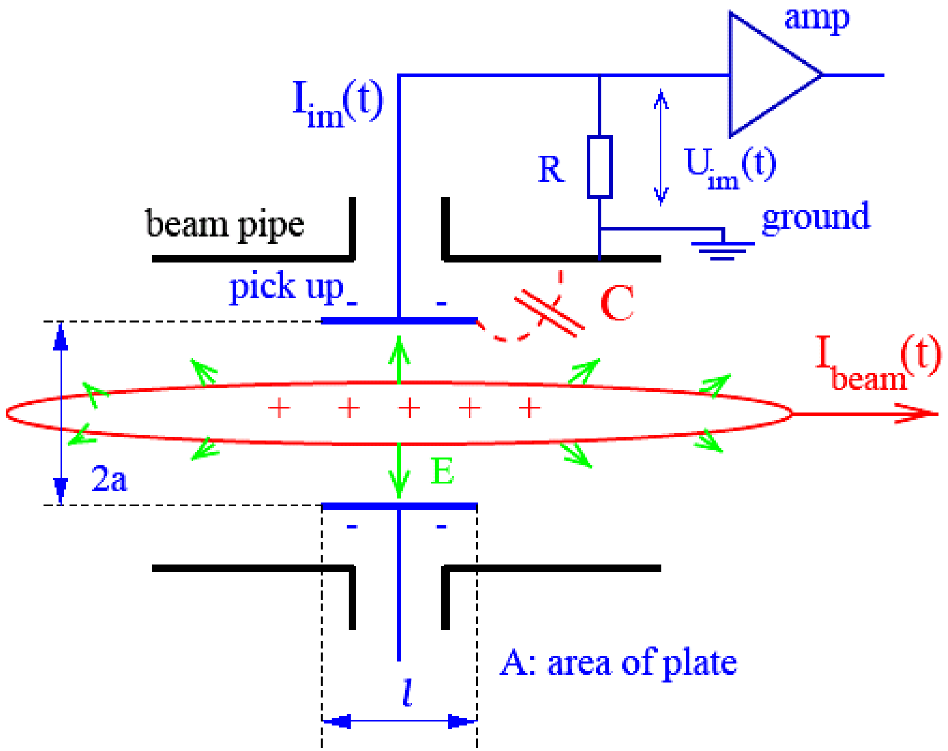
Principle of Signal Generation of capacitive BPMs

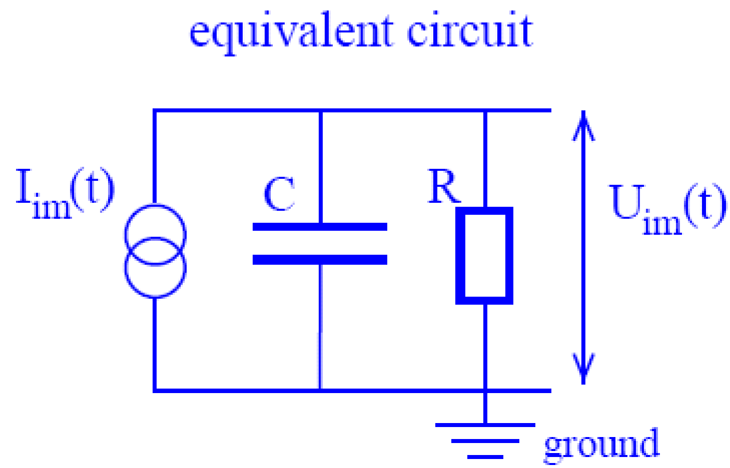
The image current at the wall is monitored on a high frequency basis i.e. ac-part given by the bunched beam.



Animation by Rhodri Jones (CERN)

The wall current is monitored by a plate or ring inserted in the beam pipe:





The image current I_{im} at the plate is given by the beam current and geometry:

$$I_{im}(t) = -\frac{dQ_{im}(t)}{dt} = \frac{-A}{2\pi al} \cdot \frac{dQ_{beam}(t)}{dt} = \frac{-A}{2\pi a} \cdot \frac{1}{\beta c} \cdot \frac{dI_{beam}(t)}{dt} = \frac{A}{2\pi a} \cdot \frac{1}{\beta c} \cdot i\omega I_{beam}(\omega)$$

Using a relation for Fourier transformation: $I_{beam} = I_0 e^{-i\omega t} \Rightarrow dI_{beam}/dt = -i\omega I_{beam}$.

Transfer Impedance for a capacitive BPM

At a resistor ***R*** the voltage U_{im} from the image current is measured.
The transfer impedance Z_t is the ratio between voltage U_{im} and beam current I_{beam}
in ***frequency domain***: $U_{im}(\omega) = R \cdot I_{im}(\omega) = Z_t(\omega, \beta) \cdot I_{beam}(\omega)$.

Capacitive BPM:

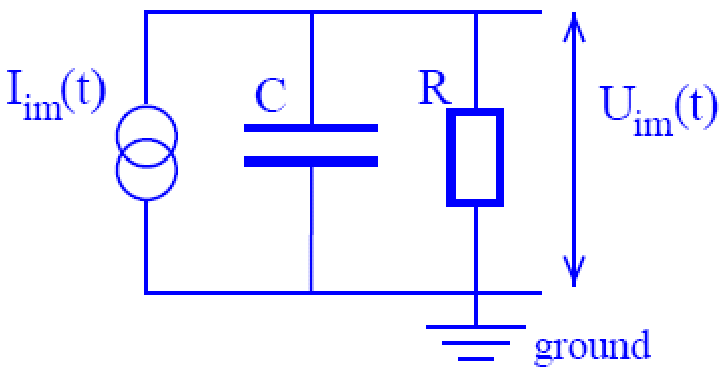
- The pick-up capacitance ***C***:
  plate ↔ vacuum-pipe and cable.
- The amplifier with input resistor ***R***.
- The beam is a high-impedance current source

$$U_{im} = \frac{R}{1+i\omega RC} \cdot I_{im}$$

$$= \frac{A}{2\pi a} \cdot \frac{1}{\beta c} \cdot \frac{1}{C} \cdot \frac{i\omega RC}{1+i\omega RC} \cdot I_{beam}$$

$$\equiv Z_t(\omega, \beta) \cdot I_{beam}$$

equivalent circuit



$$\frac{1}{Z} = \frac{1}{R} + i\omega C \Leftrightarrow Z = \frac{R}{1+i\omega RC}$$

This is a high-pass characteristic with $\omega_{cut} = 1/RC$:

Amplitude: $$|Z_t(\omega)| = \frac{A}{2\pi a} \cdot \frac{1}{\beta c} \cdot \frac{1}{C} \cdot \frac{\omega / \omega_{cut}}{\sqrt{1+\omega^2/\omega^2_{cut}}}$$

Phase: $$\varphi(\omega) = \arctan(\omega_{cut}/\omega)$$

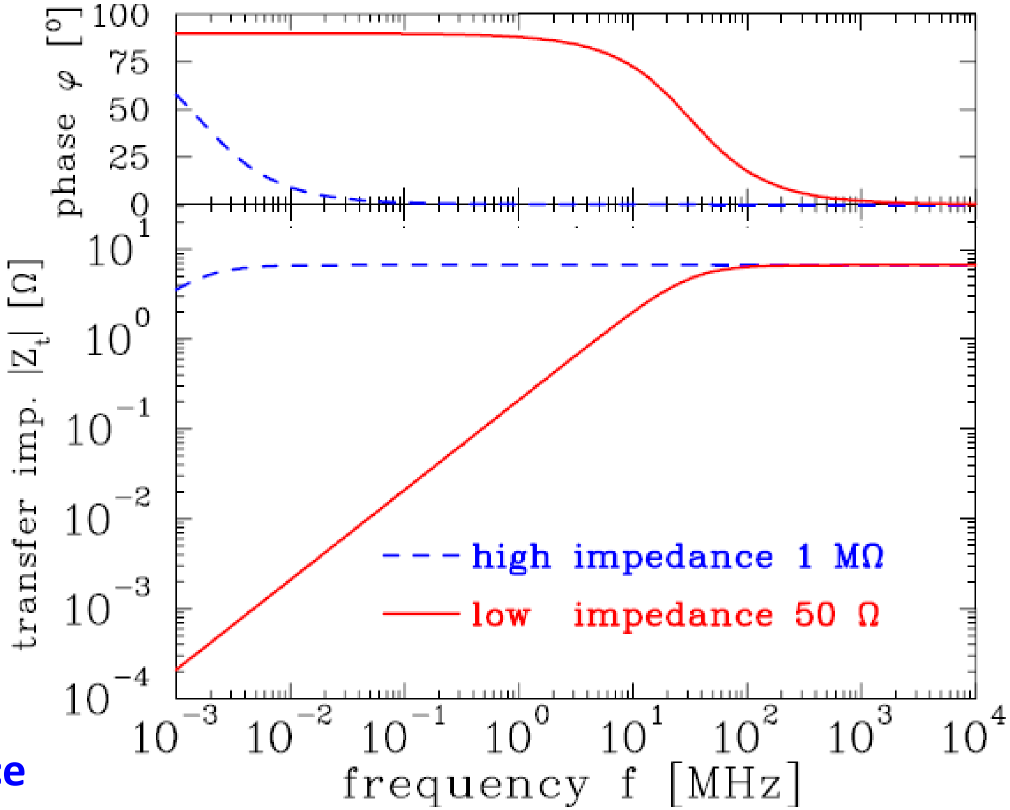
Example of Transfer Impedance for Proton Synchrotron

The high-pass characteristic for typical synchrotron BPM:

$$U_{im}(\omega)=Z_t(\omega)\cdot I_{beam}(\omega)$$

$$|Z_t| = \frac{A}{2\pi a}\cdot\frac{1}{\beta c}\cdot\frac{1}{C}\cdot\frac{\omega/\omega_{cut}}{\sqrt{1+\omega^2/\omega^2_{cut}}}$$

$$\varphi = \arctan(\omega_{cut}/\omega)$$



Parameter for shoe-box BPM:

C=100pF, ***l***=10cm, ***β***=50%

$f_{cut}= \omega/2\pi=(2\pi RC)^{-1}$

for ***R*=50 Ω** ⇒ f_{cut}= 32 MHz

for ***R*=1 MΩ** ⇒ f_{cut}= 1.6 kHz

Large signal strength → **high impedance**

Smooth signal transmission → **50 Ω**

Remark: For $\omega \rightarrow 0$ it is $Z_t \rightarrow 0$ i.e. **no** signal is transferred from dc-beams e.g.

- de-bunched beam inside a synchrotron
- for slow extraction through a transfer line

Depending on the frequency range ***and*** termination the signal looks different:

- ***High frequency range $\omega >> \omega_{cut}$*** :

$$Z_t \propto \frac{i\omega/\omega_{cut}}{1+i\omega/\omega_{cut}} \to 1 \Rightarrow U_{im}(t) = \frac{1}{C}\cdot\frac{1}{\beta c}\cdot\frac{A}{2\pi a}\cdot I_{beam}(t)$$

$\Rightarrow$ **direct image** of the bunch. Signal strength $Z_t \propto A/C$ i.e. nearly independent on length

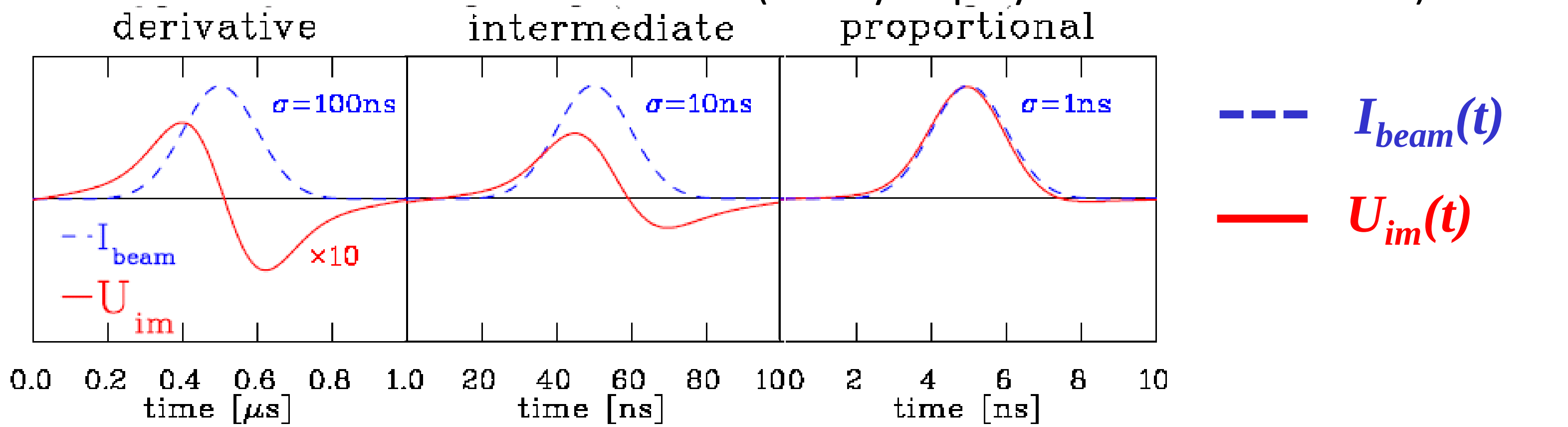
- ***Low frequency range $\omega << \omega_{cut}$*** :

$$Z_t \propto \frac{i\omega/\omega_{cut}}{1+i\omega/\omega_{cut}} \to i\frac{\omega}{\omega_{cut}} \quad \Rightarrow \quad U_{im}(t) = R\cdot\frac{A}{\beta c\cdot 2\pi a}\cdot i\omega I_{beam}(t) = R\cdot\frac{A}{\beta c\cdot 2\pi a}\cdot\frac{dI_{beam}}{dt}$$

$\Rightarrow$ **derivative** of bunch, single strength $Z_t \propto A$, i.e. (nearly) independent on C

- ***Intermediate frequency range $\omega \approx \omega_{cut}$***: Calculation using Fourier transformation

Example: Synchrotron BPM with 50 Ω termination (reality at p-synchrotron : σ>>1 ns):



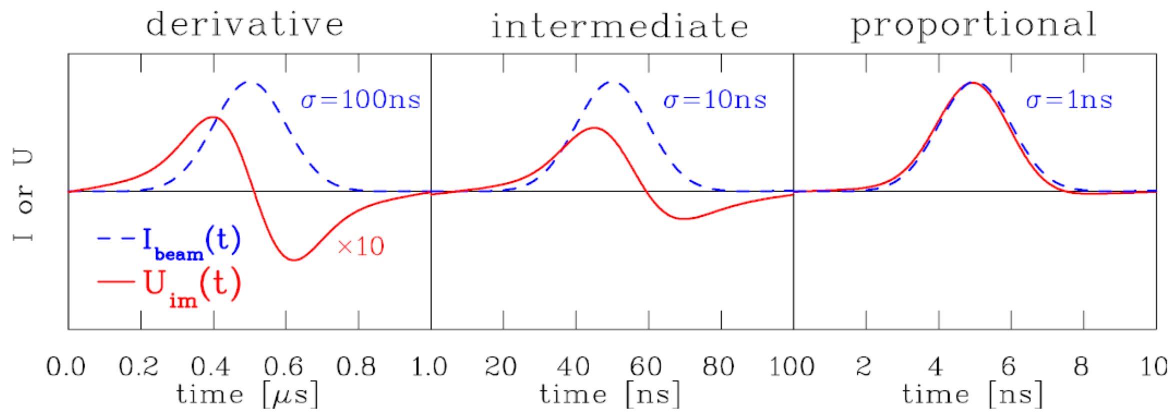
Calculation of Signal Shape (here single bunch)

The transfer impedance is used in frequency domain! The following is performed:

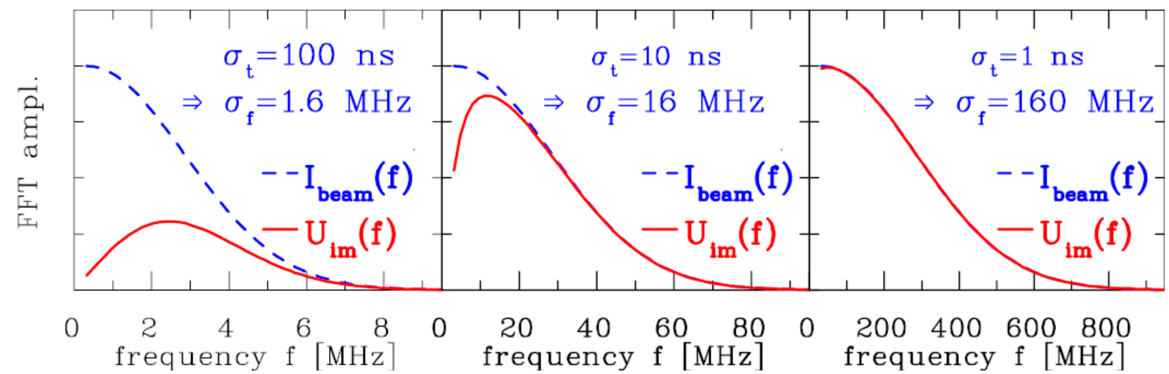
1. Start: Time domain Gaussian function $I_{beam}(t)$ having a width of σ_t

Fourier trans.

inverse Fourier trans.



2. FFT of $I_{beam}(t)$ leads to the frequency domain Gaussian $I_{beam}(f)$ with $\sigma_f = (2\pi\sigma_t)^{-1}$



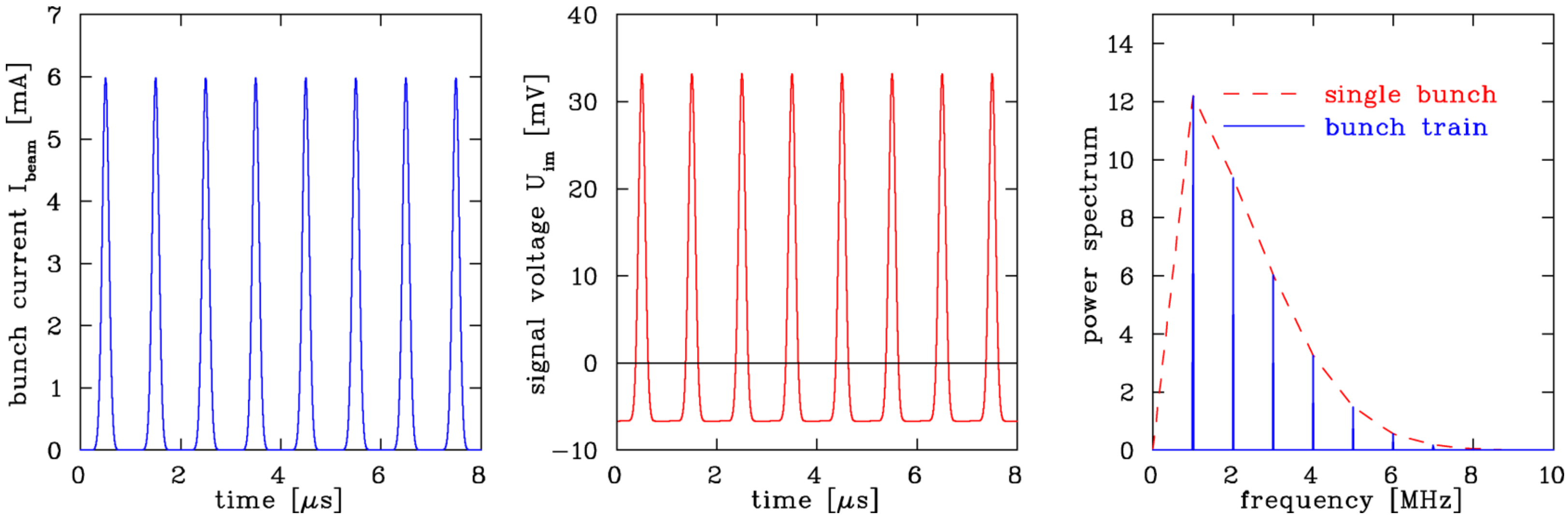
3. Multiplication with $Z_t(f)$ with f_{cut}=32 MHz leads to $U_{im}(f) = Z_t(f) \cdot I_{beam}(f)$

4. Inverse FFT leads to $U_{im}(t)$

Calculation of Signal Shape: repetitive Bunch in a Synchrotron

Synchrotron filled with 8 bunches accelerated with f_{acc}=1 MHz

BPM terminated with R= 1 MΩ $\Rightarrow f_{acc} >> f_{cut}$:



Parameter: R=1 MΩ $\Rightarrow f_{cut}$=2 kHz, Z_t=5 Ω , all buckets filled

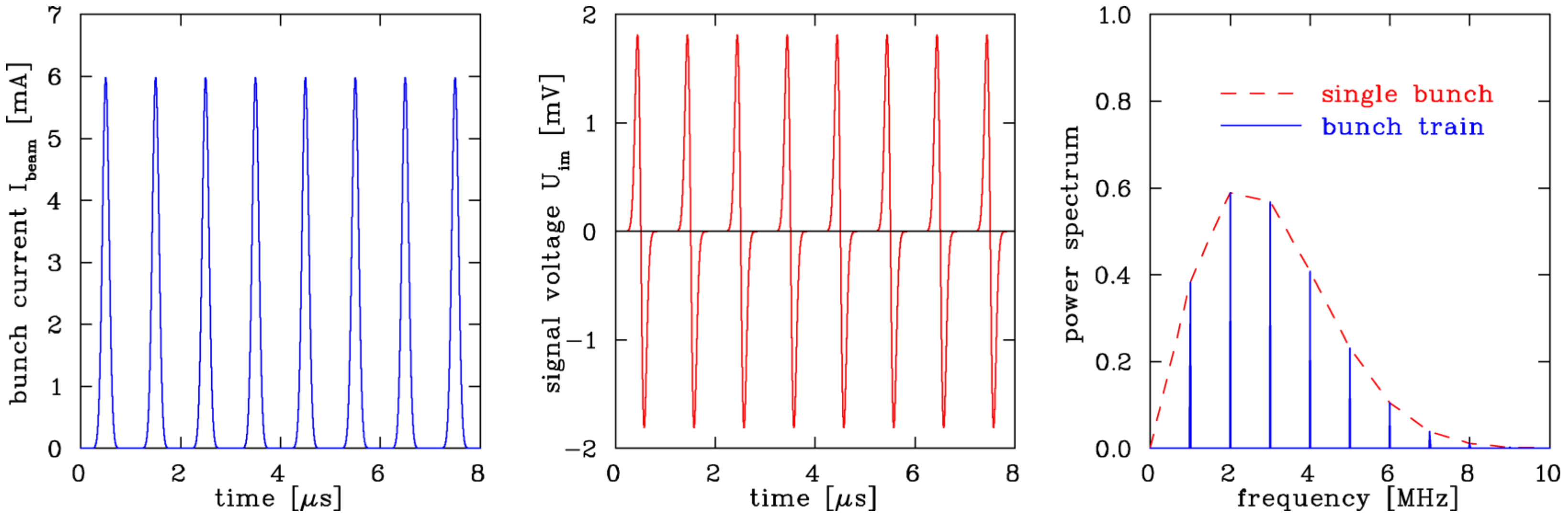
C = 100pF, l = 10cm, β = 50%, σ_t = 100 ns $\Rightarrow \sigma_l$ = 15m

- Fourier spectrum is composed of lines separated by acceleration f_{rf}
- Envelope given by single bunch Fourier transformation
- Baseline shift due to ac-coupling

Remark: 1 MHz< f_{rf} <10MHz $\Rightarrow$ Bandwidth ≈100MHz=10·f_{rf} for broadband observation

Synchrotron filled with 8 bunches accelerated with f_{acc}=1 MHz

BPM terminated with R=50 Ω $\Rightarrow f_{acc} << f_{cut}$:

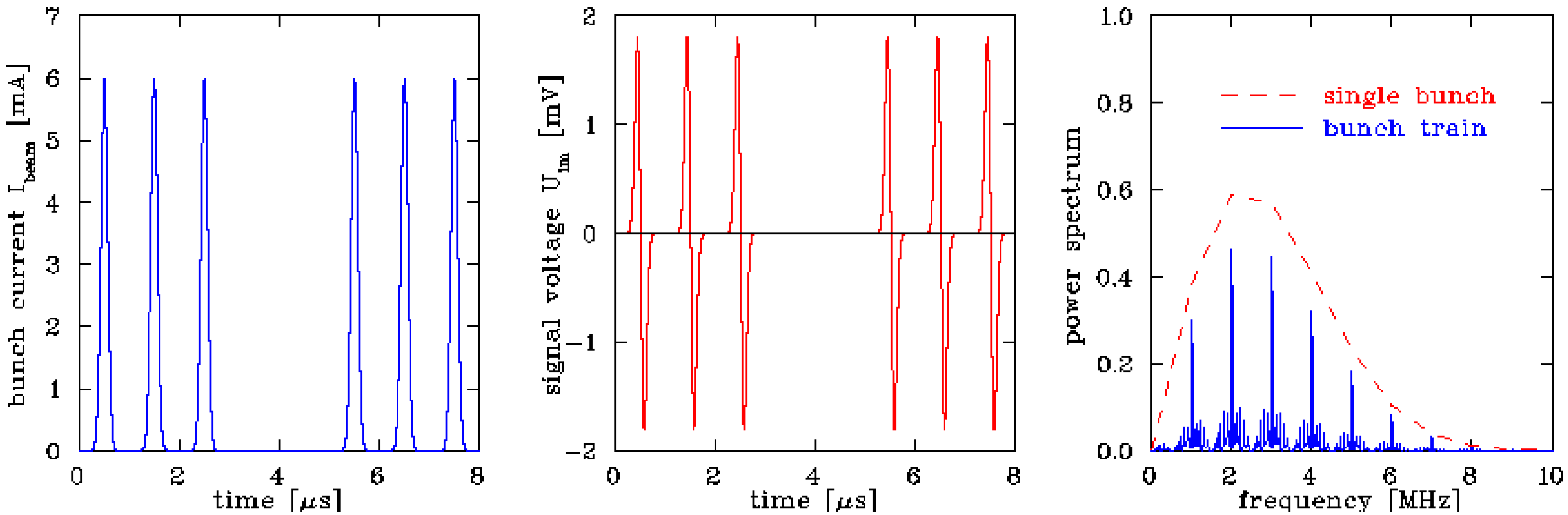


Parameter: R=50 Ω ⇒ f_{cut}=32 MHz, all buckets filled

C = 100pF, l = 10cm, β = 50%, σ_t = 100 ns ⇒ σ_l = 15m

- Fourier spectrum is concentrated at acceleration harmonics with single bunch spectrum as an envelope.
- Bandwidth up to typically 10*f_{acc}

Synchrotron during filling: Empty buckets, R=50 Ω:

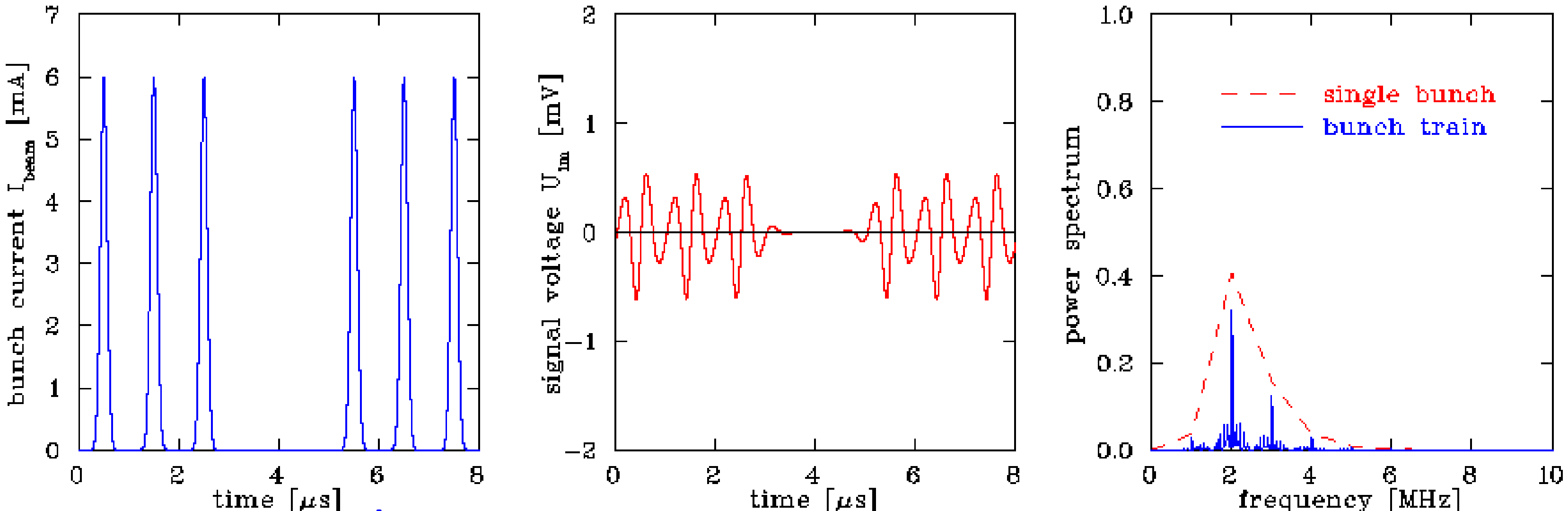


Parameter: R=50 Ω ⇒ f_{cut}=32 MHz, 2 empty buckets

C = 100pF, l = 10cm, β = 50%, σ_t = 100 ns ⇒ σ_l = 15m

- Fourier spectrum is more complex, harmonics are broader due to sidebands

Effect of filters, here bandpass:



Parameter: R=50 Ω, 4th order Butterworth filter f_{cut}=2 MHz

C = 100pF, l = 10cm, β = 50%, σ = 100 ns

- Ringing due to sharp cutoff
- Other filter types more appropriate

*n^{th} order Butterworth filter, math. simple, but **not** well suited:*

$$|H_{low}| = \frac{1}{\sqrt{1+(\omega/\omega_{cut})^{2n}}} \quad \text{and} \quad |H_{high}| = \frac{(\omega/\omega_{cut})^{n}}{\sqrt{1+(\omega/\omega_{cut})^{2n}}}$$

$$H_{filter} = H_{high} \cdot H_{low}$$

Generally: $Z_{tot}(\omega) = H_{cable}(\omega) \cdot H_{filter}(\omega) \cdot H_{amp}(\omega) \cdot \ldots \cdot Z_t(\omega)$

Remark: For numerical calculations, time domain filters (FIR and IIR) are more appropriate

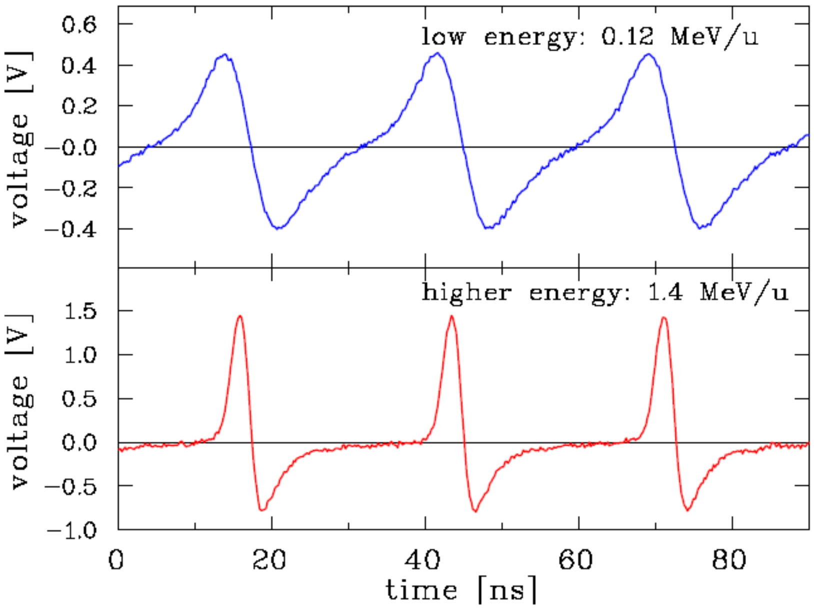
Proton LINAC, e⁻-LINAC&synchtrotron:

100 MHz < f_{rf} < 1 GHz typically

R=50 Ω processing to reach bandwidth

C≈5 pF ⇒ $f_{cut} = 1/(2\pi RC) \approx$ 700 MHz

Example: 36 MHz GSI ion LINAC



Proton synchtrotron:

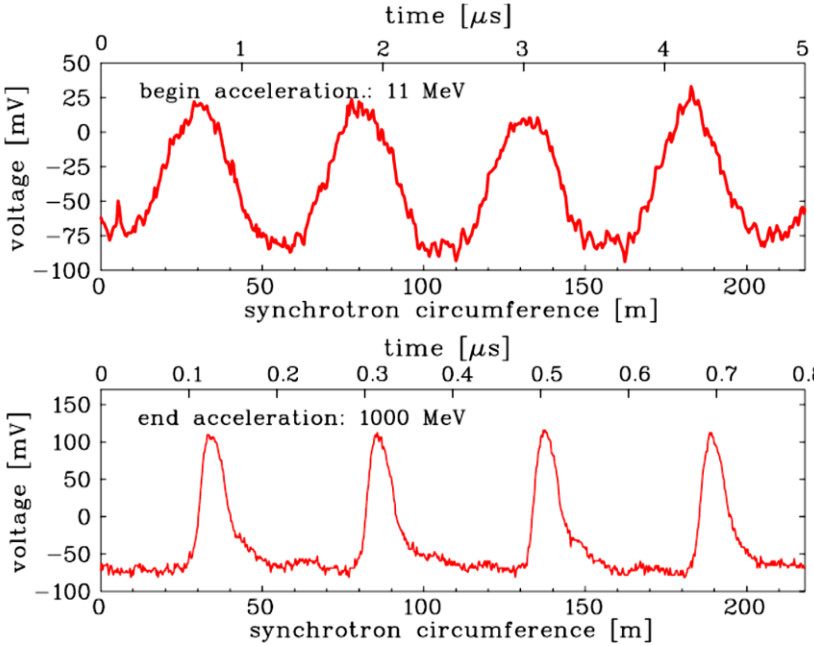
1 MHz < f_{rf} < 30 MHz typically

R=1 MΩ for large signal i.e. large Z_t

C≈100 pF ⇒ $f_{cut} = 1/(2\pi RC) \approx$10 kHz

Example: non-relativistic GSI synchrotron

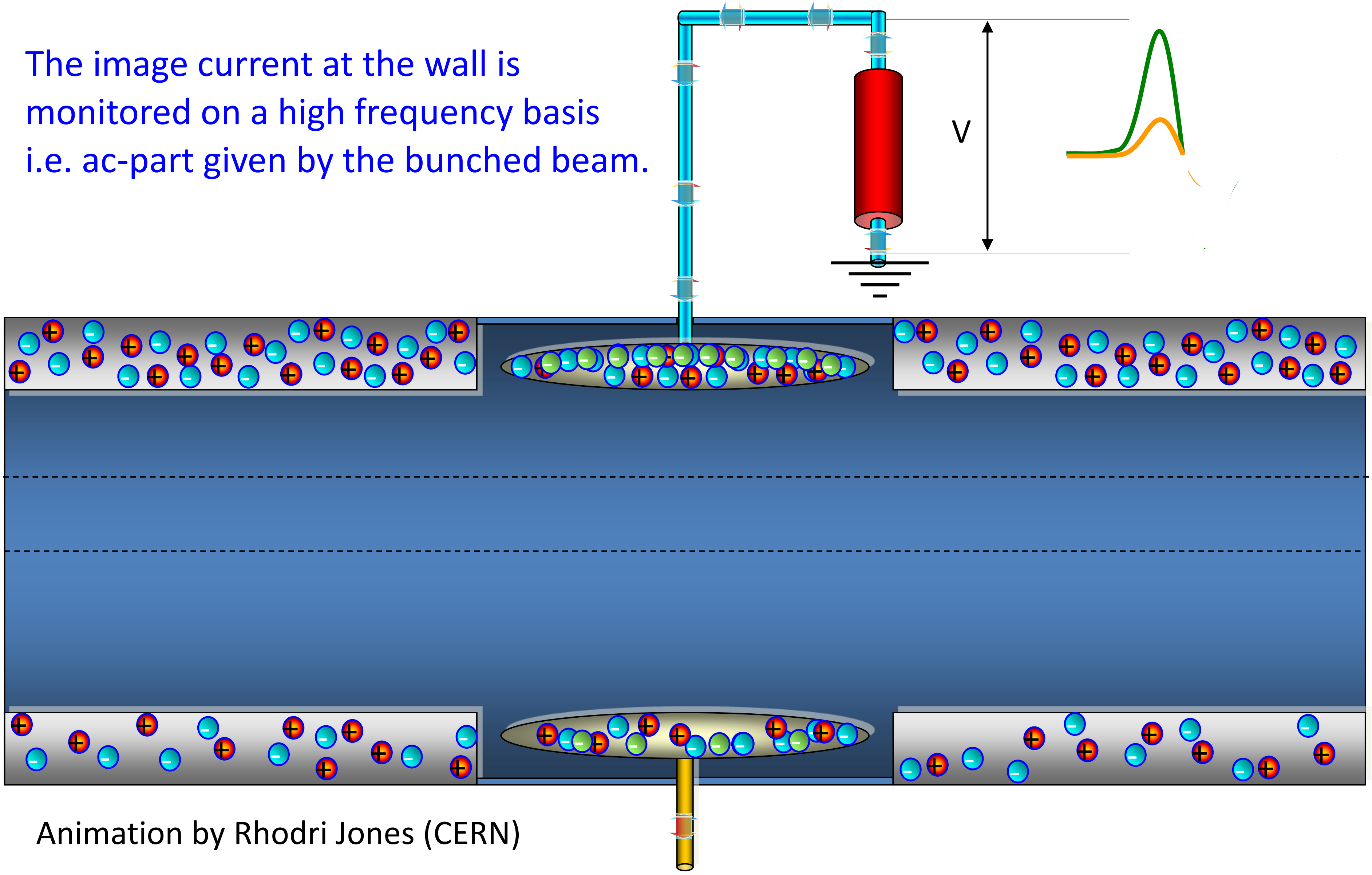
f_{rf} : 0.8 MHz → 5 MHz



Remark: During acceleration the bunching-factor is increased: 'adiabatic damping'.

Principle of Signal Generation of a BPMs: off-center Beam

The image current at the wall is monitored on a high frequency basis i.e. ac-part given by the bunched beam.



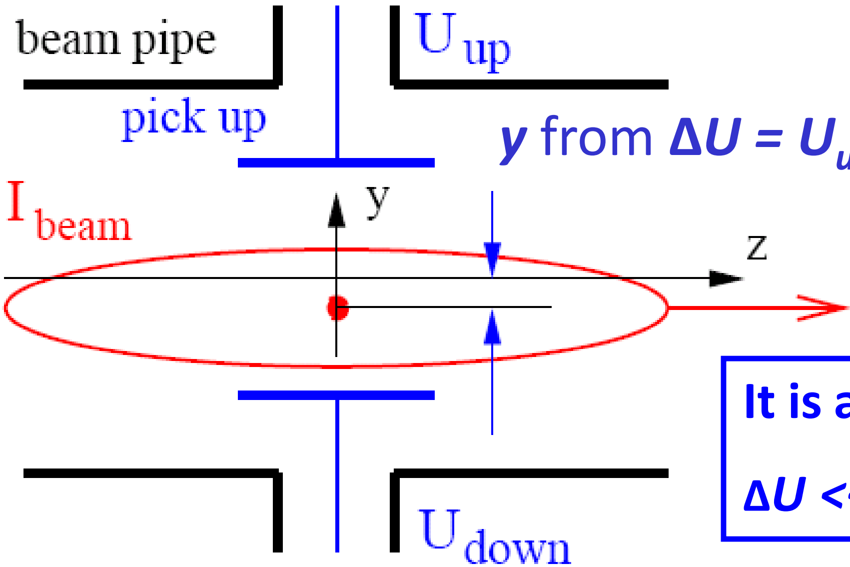
Animation by Rhodri Jones (CERN)

The difference voltage between plates gives the beam's center-of-mass
→**most frequent application**

'Proximity' effect leads to different voltages at the plates:

$$y = \frac{1}{S_y(\omega)} \cdot \frac{U_{up} - U_{down}}{U_{up} + U_{down}} + \delta_y(\omega)$$

$$\equiv \frac{1}{S_y} \cdot \frac{\Delta U_y}{\Sigma U_y} + \delta_y$$



y **from** $\boldsymbol{\Delta U = U_{up} - U_{down}}$

Correspondingly:

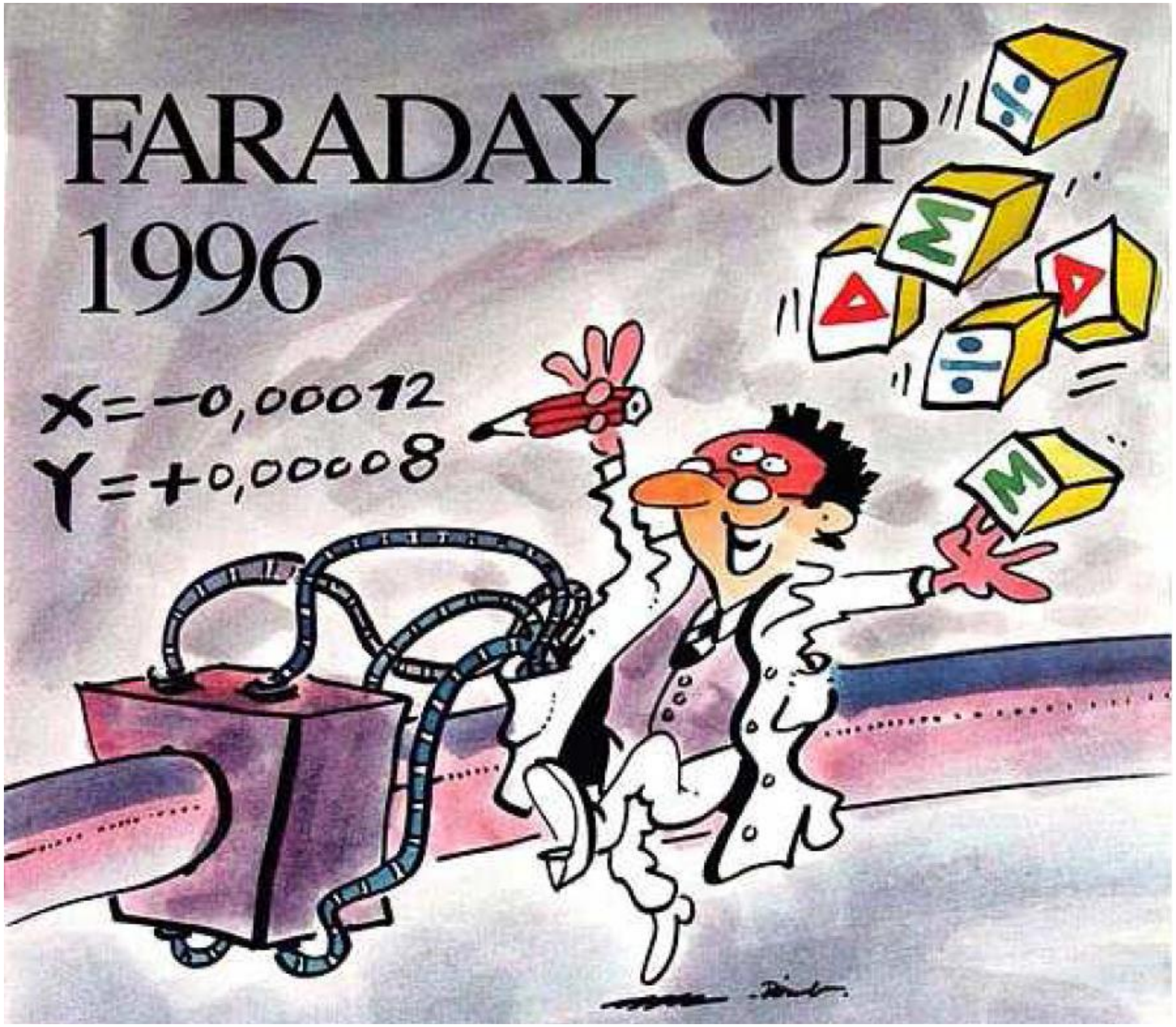
$$x = \frac{1}{S_x(\omega)} \cdot \frac{U_{right} - U_{left}}{U_{right} + U_{left}} + \delta_x(\omega)$$

It is at least:

$\boldsymbol{\Delta U \ll \Sigma U / 10}$

S(ω,x) is called **position sensitivity,** sometimes the inverse is used ***k(ω,x)=1/S(ω,x)***
S is a geometry dependent, non-linear function, which have to be optimized
Units: ***S***=[%/mm] and sometimes ***S***=[dB/mm] or ***k***=[mm].

The Artist's View of a BPM



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- Signal generation $\rightarrow$ transfer impedance
- Capacitive ***button*** BPM for high frequencies

  used at most proton LINACs and electron accelerators
- Capacitive *shoe-box* BPM for low frequencies
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‘Proximity effect’: larger signal for closer plate

Ideal 2-dim model: Cylindrical pipe → image current density via ‘image charge method’ for ‘pencil’ beam:

$$j_{im}(\phi) = \frac{I_{beam}}{2\pi a} \cdot \left(\frac{a^2 - r^2}{a^2 + r^2 - 2ar \cdot \cos(\phi - \theta)} \right)$$

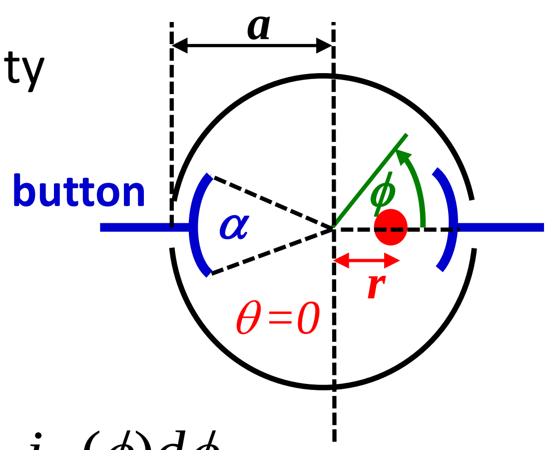
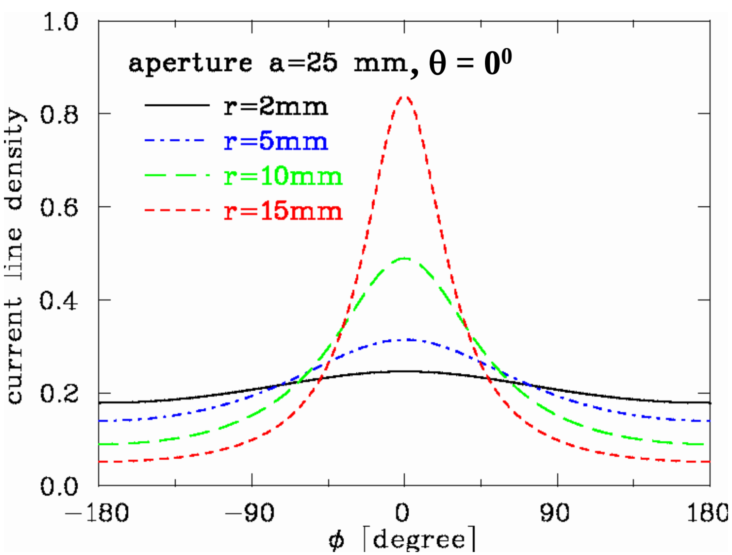
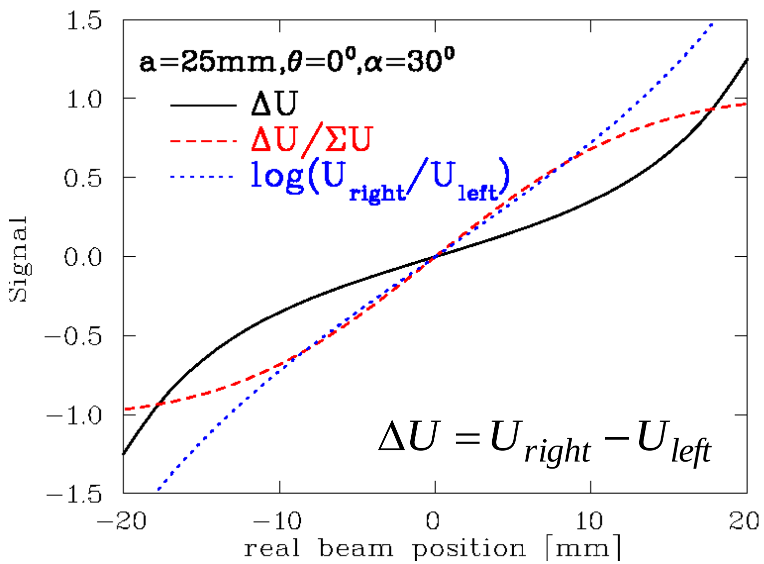


Image current: Integration of finite BPM size: $I_{im} = a \cdot \int_{-\alpha/2}^{\alpha/2} j_{im}(\phi) d\phi$





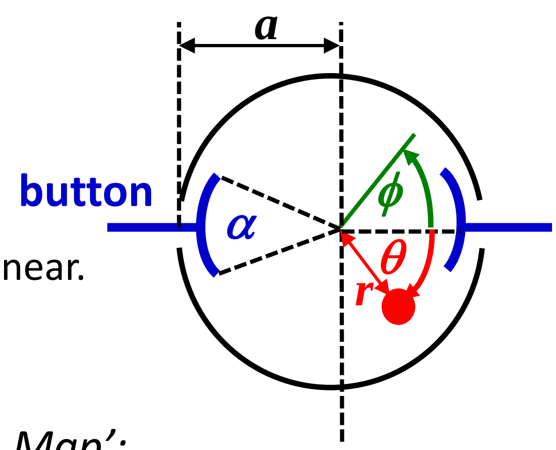
Ideal 2-dim model: Non-linear behavior and hor-vert coupling:

Sensitivity $\boldsymbol{S}$ is converts signal to position $x = \frac{1}{S} \cdot \frac{\Delta U}{\Sigma U}$

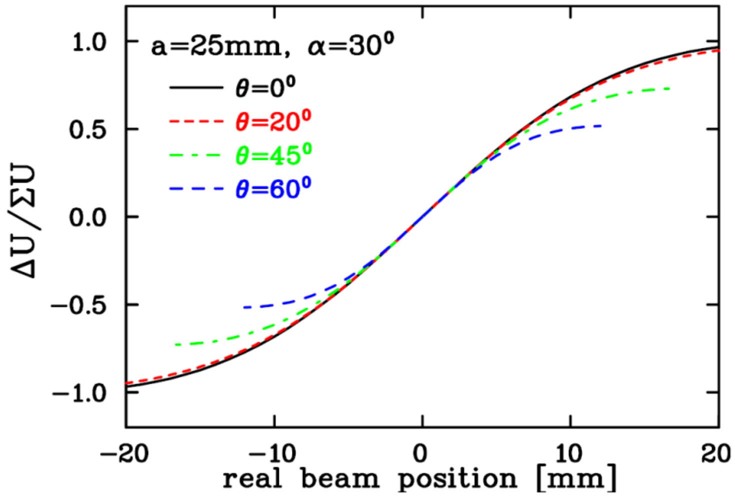
with S [%/mm] or [dB/mm]

i.e. $\boldsymbol{S}$ is the derivative of the curve $S_x = \frac{\partial(\frac{\Delta U}{\Sigma U})}{\partial x}$, here $S_x = S_x(x, y)$ i.e. non-linear.

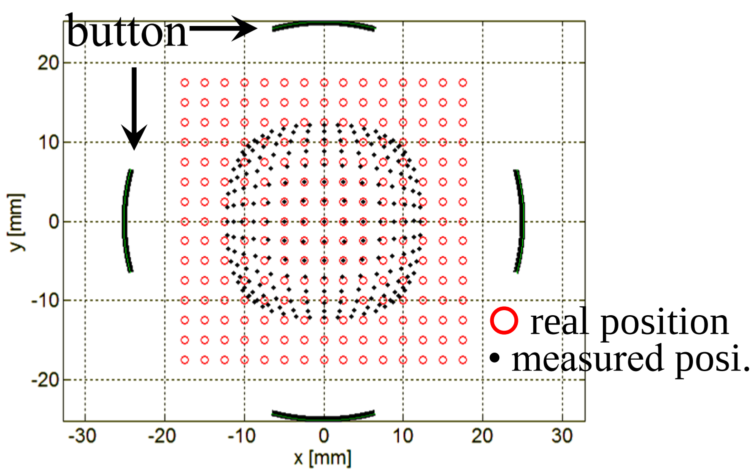
For this example: center part *S*=7.4%/mm ⇔ *k*=1/*S*=14mm



Horizontal plane:



'Position Map':



Button BPM Realization

LINACs, e⁻-synchrotrons: 100 MHz < f_{rf} < 3 GHz → bunch length ≈ BPM length

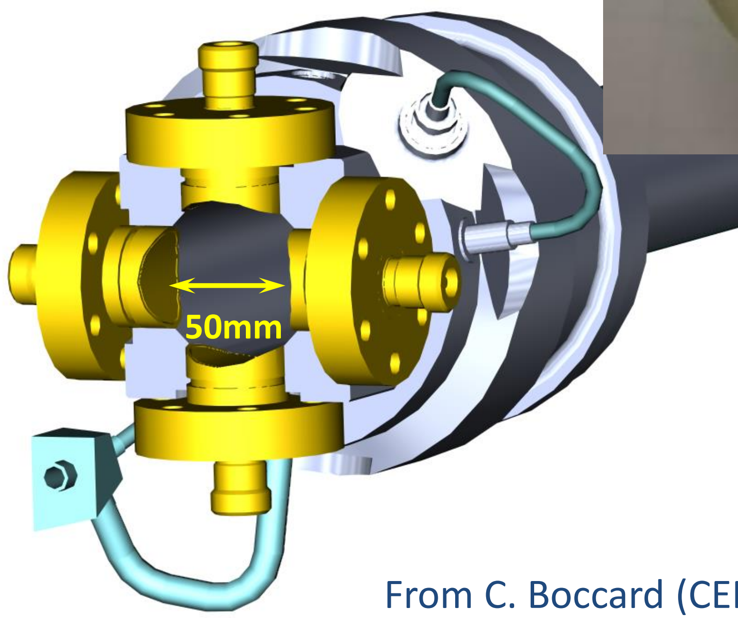
→ 50 Ω signal path to prevent reflections

Button BPM with 50 Ω ⇒ $U_{im}(t$

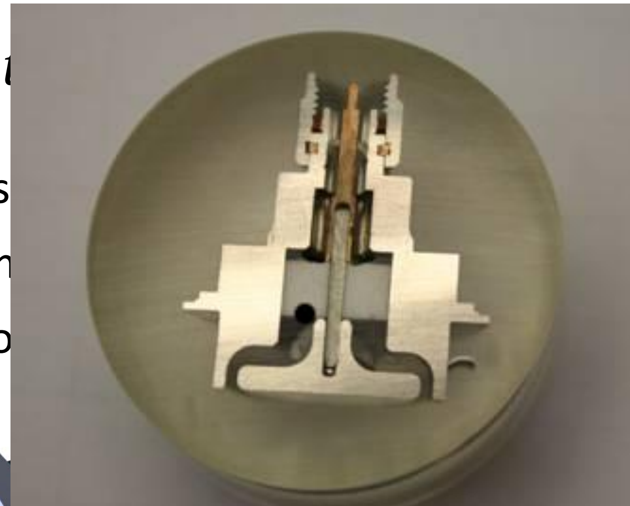
Example: LHC-type inside cryos

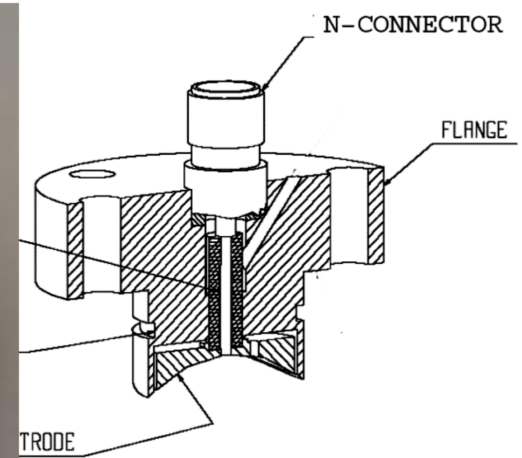
∅24 mm, half aperture a=25 m

⇒ f_{cut}=400 MHz, Z_t = 1.3 Ω abo



From C. Boccard (CE

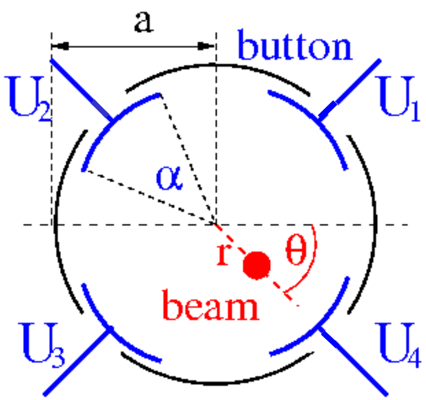




The button BPM can be rotated by 45^0

to avoid exposure by synchrotron light:

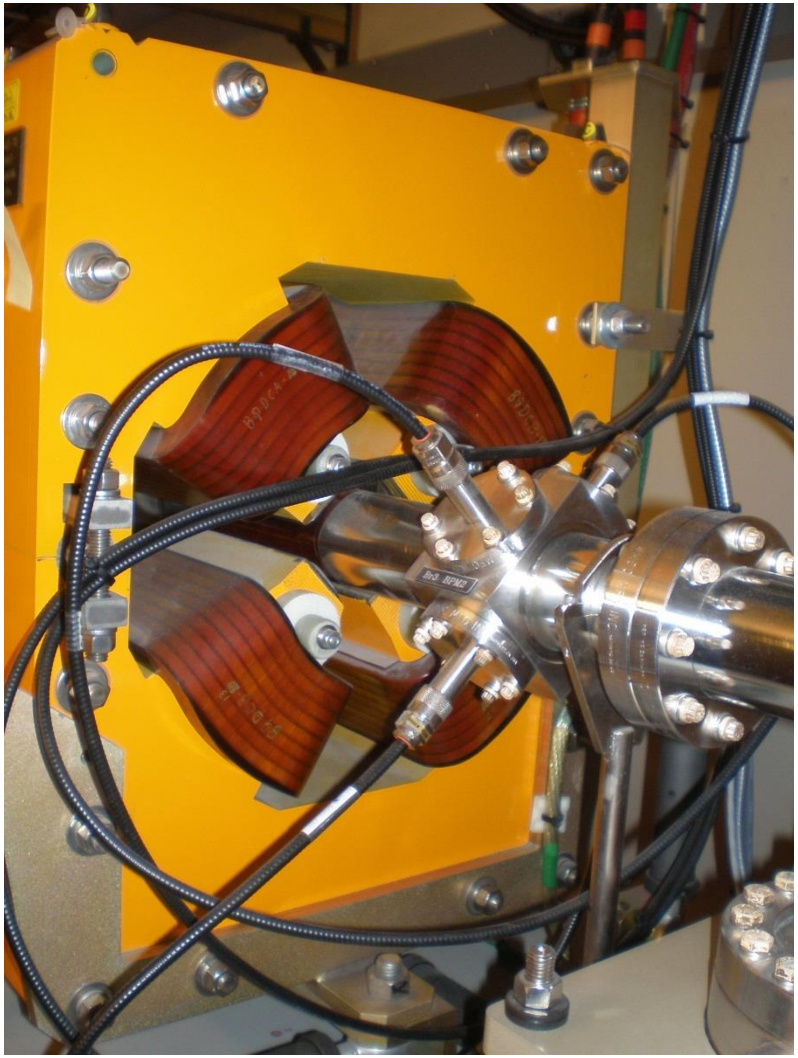
Frequently used at boosters for light sources



$$\text{horizontal} \ : x = \frac{1}{S} \cdot \frac{(U_1 + U_4) - (U_2 + U_3)}{U_1 + U_2 + U_3 + U_4}$$

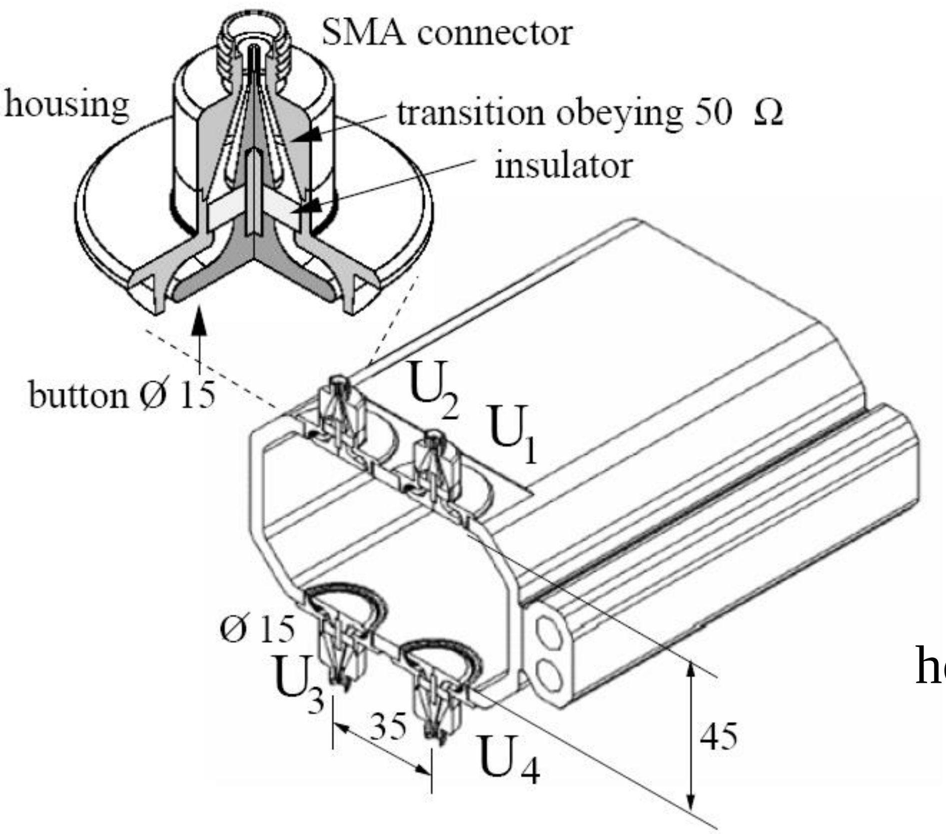
$$\text{vertical} \ : \quad y = \frac{1}{S} \cdot \frac{(U_1 + U_2) - (U_3 + U_4)}{U_1 + U_2 + U_3 + U_4}$$

Example: Booster of ALS, Berkeley

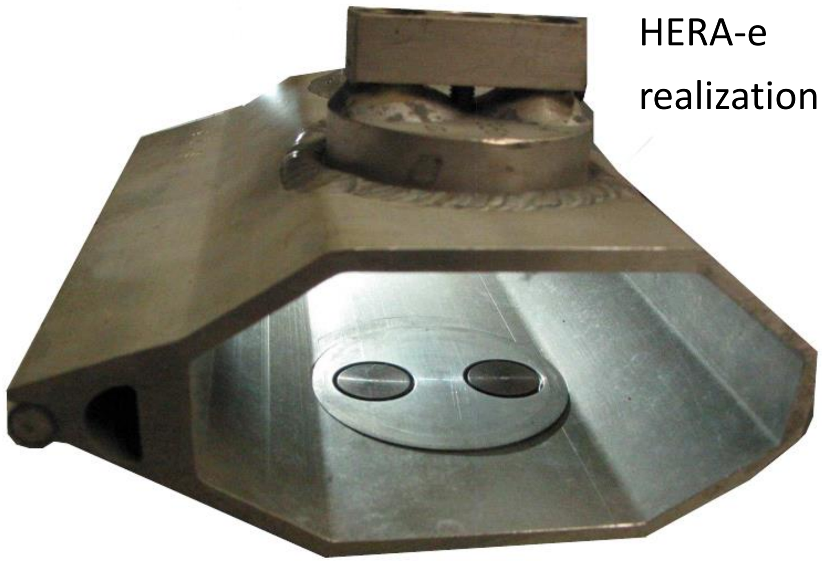


Button BPM at Synchrotron Light Sources

Due to synchrotron radiation, the button insulation might be destroyed
$\Rightarrow$buttons only in vertical plane possible $\Rightarrow$ increased non-linearity



PEP-realization



HERA-e realization

$$\text{horizontal}: x = \frac{1}{S_x} \cdot \frac{(U_1 + U_4) - (U_2 + U_3)}{U_1 + U_2 + U_3 + U_4}$$

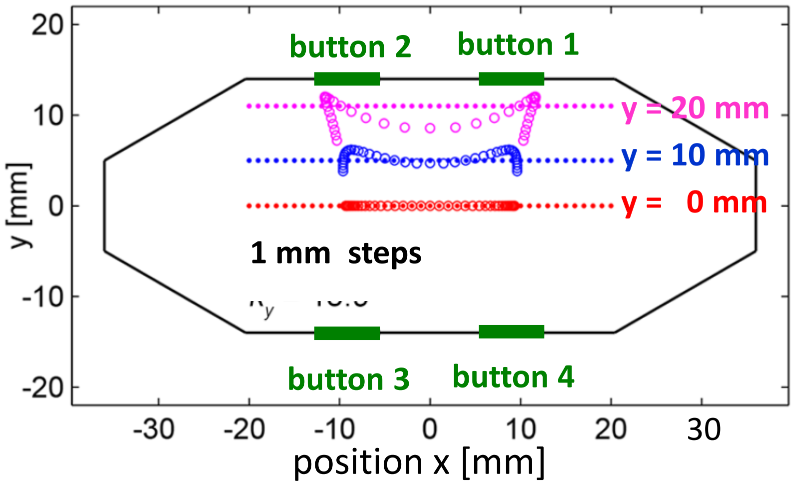
$$\text{vertical}: y = \frac{1}{S_y} \cdot \frac{(U_1 + U_2) - (U_3 + U_4)}{U_1 + U_2 + U_3 + U_4}$$

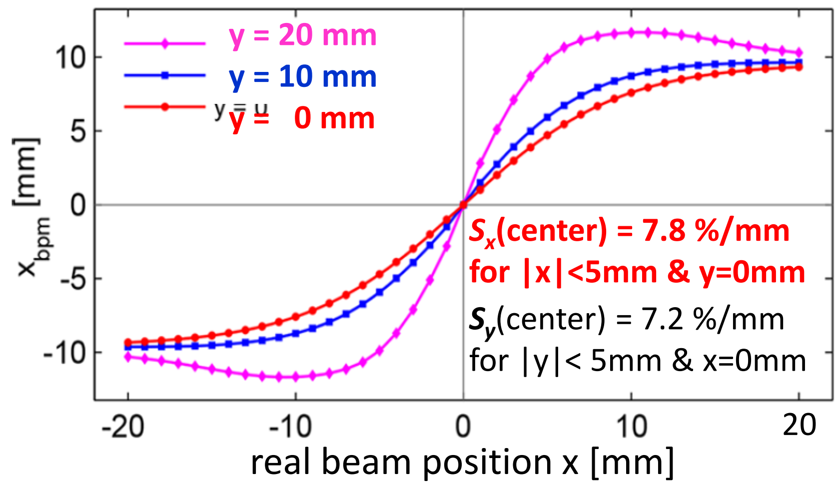
Simulations for Button BPM at Synchrotron Light Sources

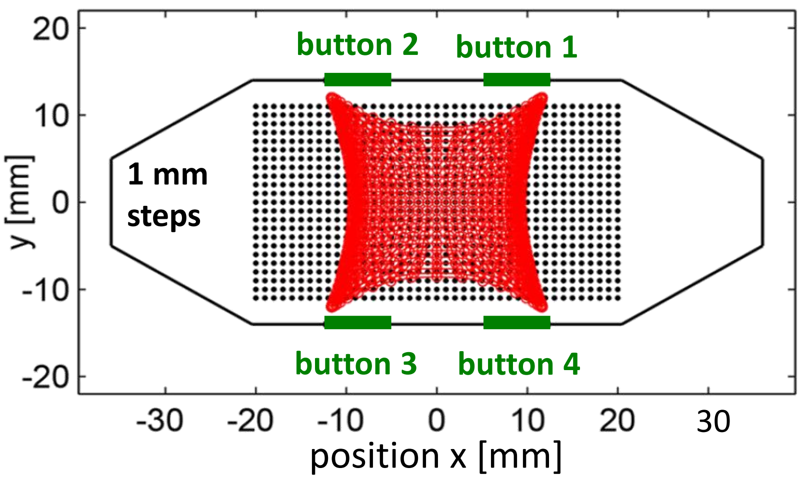
Example: Simulation for ALBA light source for 72 x 28 mm² chamber

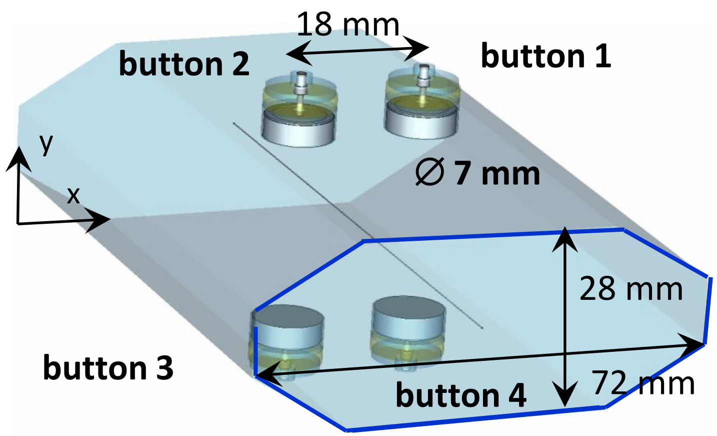
Optimization: horizontal distance and size of buttons

from A.A. Nosych et al., IBIC'14









Result: non-linearity and ***xy***-coupling occur in dependence of button size and position

Outline:

- Signal generation → transfer impedance
- Capacitive *button* BPM for high frequencies

  used at most proton LINACs and electron accelerators
- Capacitive ***shoe-box*** BPM for low frequencies

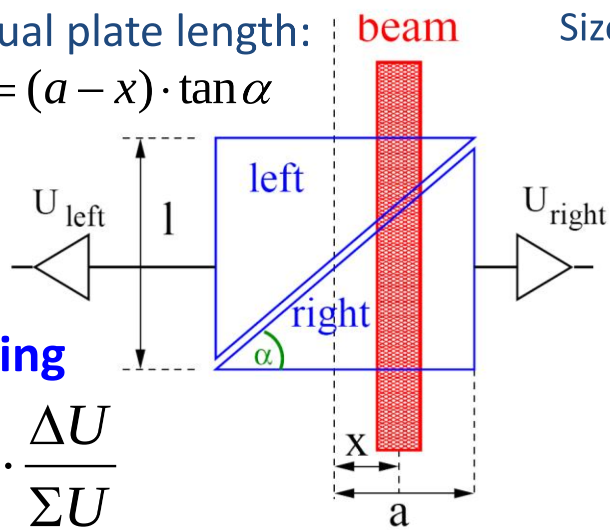
  used at most proton synchrotrons due to linear position reading
- Electronics for position evaluation
- BPMs for measurement of closed orbit, tune and further lattice functions
- Summary

Frequency range: 1 MHz < f_{rf} < 10 MHz ⇒ bunch-length >> BPM length.

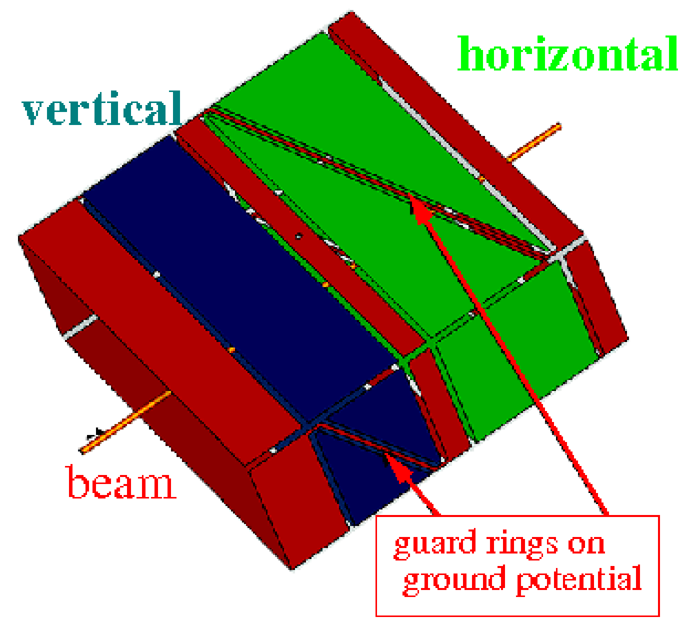
Signal is proportional to actual plate length:

$$l_{\text{right}} = (a + x) \cdot \tan\alpha, \quad l_{\text{left}} = (a - x) \cdot \tan\alpha$$

$$\Rightarrow \quad x = a \cdot \frac{l_{\text{right}} - l_{\text{left}}}{l_{\text{right}} + l_{\text{left}}}$$

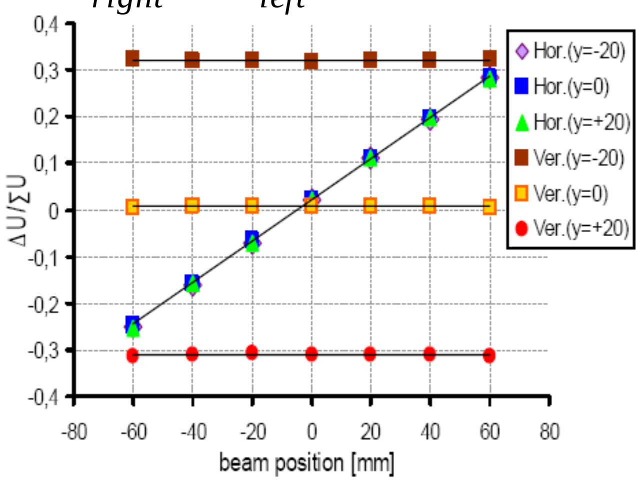


Size: 200x70 mm^2



In ideal case: linear reading

$$x = a \cdot \frac{U_{right} - U_{left}}{U_{right} + U_{left}} \equiv a \cdot \frac{\Delta U}{\Sigma U}$$



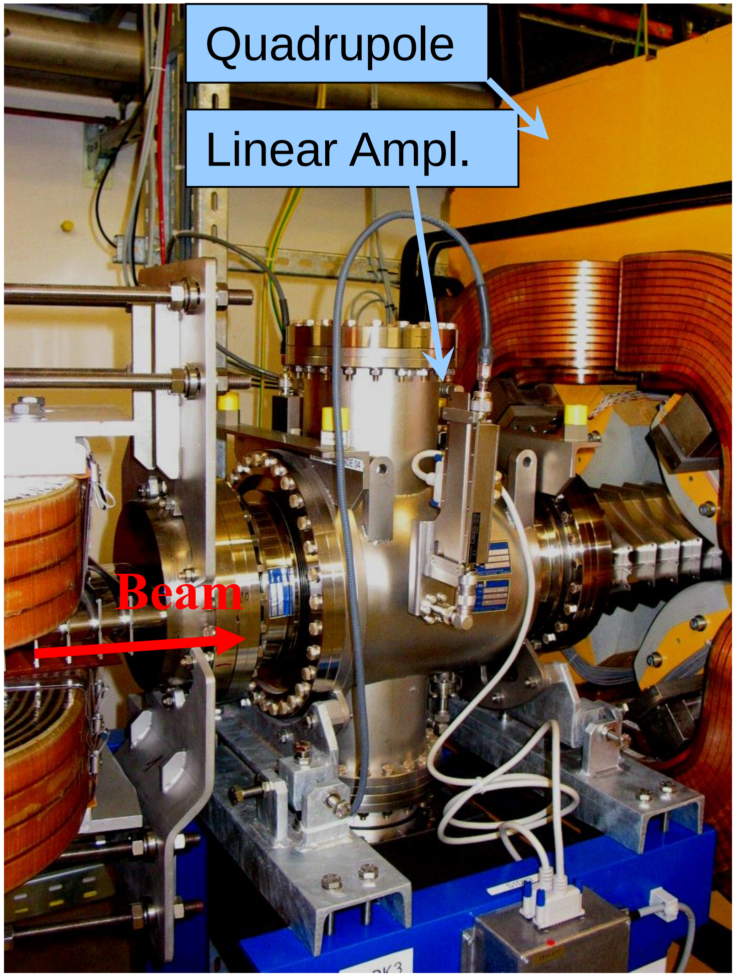
Shoe-box BPM:

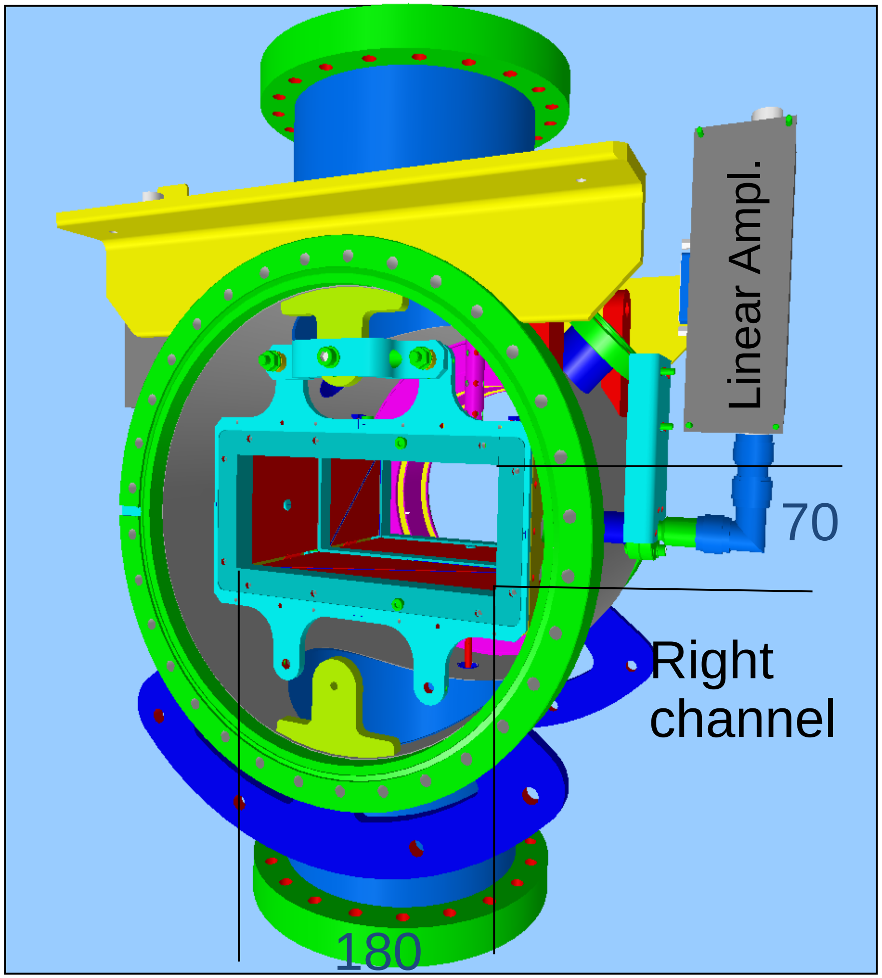
Advantage: Very linear, low frequency dependence
i.e. position sensitivity ***S*** is constant

Disadvantage: Large size, complex mechanics
high capacitance

Technical Realization of a Shoe-Box BPM

Technical realization at HIT synchrotron of 46 m length for 7 MeV/u→ 440 MeV/u
BPM clearance: 180x70 mm^2, standard beam pipe diameter: 200 mm.





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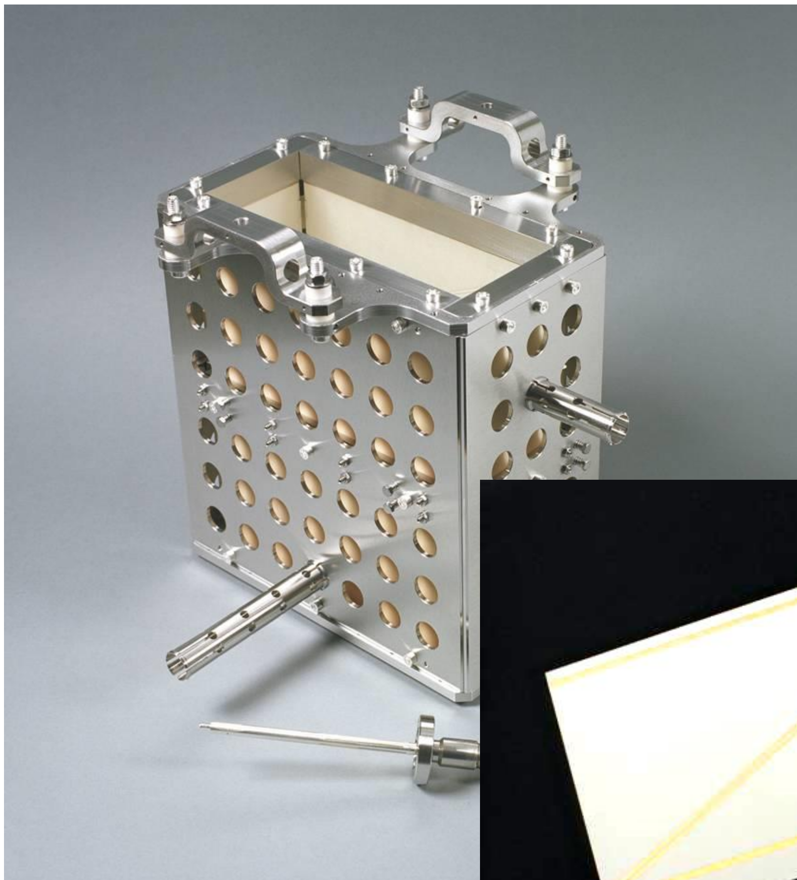
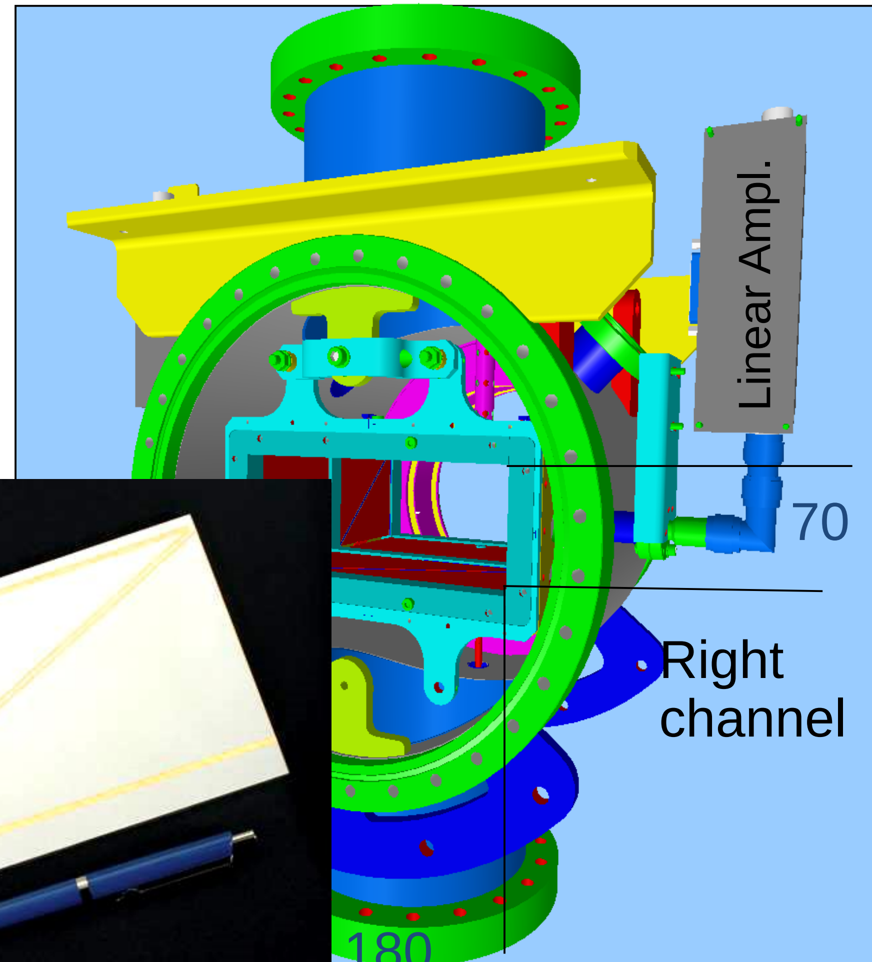
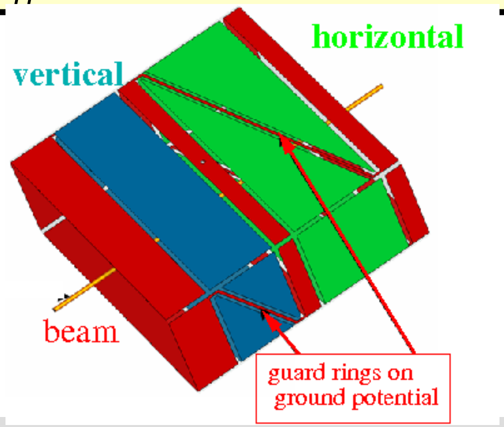
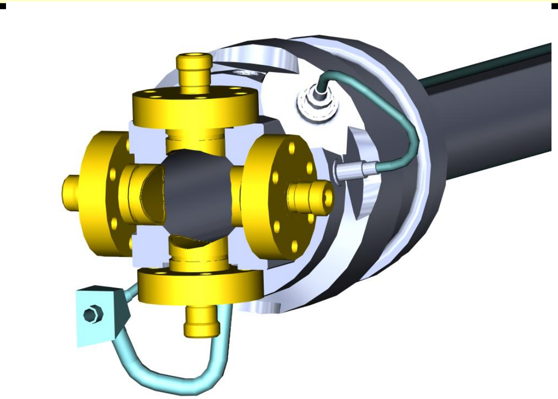




Comparison Shoe-Box and Button BPM

	Shoe-Box BPM	Button BPM
Precaution	Bunches longer than BPM	Bunch length comparable to BPM
BPM length (typical)	10 to 20 cm length per plane	∅1 to 5 cm per button
Shape	Rectangular or cut cylinder	Orthogonal or planar orientation
Bandwidth (typical)	0.1 to 100 MHz	100 MHz to 5 GHz
Coupling	1 MΩ or ≈1 kΩ (transformer)	50 Ω
Cutoff frequency (typical)	0.01… 10 MHz (C=30…100pF)	0.3… 1 GHz (C=2…10pF)
Linearity	Very good, no x-y coupling	Non-linear, x-y coupling
Sensitivity	Good, care: plate cross talk	Good, care: signal matching
Usage	At proton synchrotrons, f_{rf} < 10 MHz	All electron acc., proton Linacs, f_{rf} > 100 MHz





Outline:

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  used at most proton LINACs and electron accelerators
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  analog signal conditioning to achieve small signal processing
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- Summary

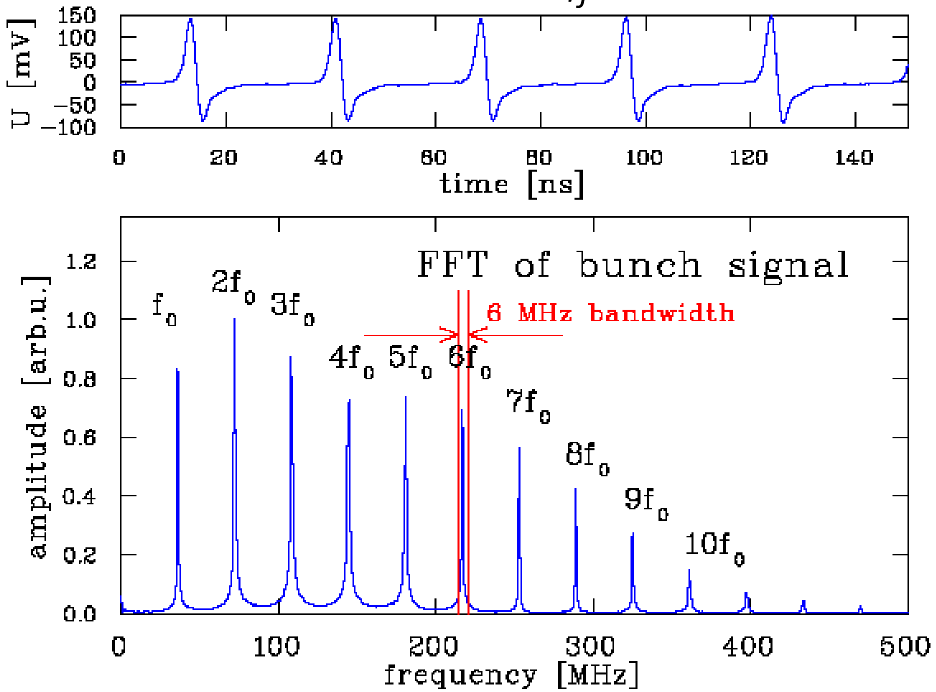
General: Noise Consideration

1. Signal voltage given by: $U_{im}(f) = Z_t(f) \cdot I_{beam}(f)$
2. Position information from voltage difference: $x = 1/S \cdot \Delta U / \Sigma U$
3. Thermal noise voltage given by: $U_{eff}(R, \Delta f) = \sqrt{4k_B \cdot T \cdot R \cdot \Delta f}$

⇒ Signal-to-noise $\Delta U_{im}/U_{eff}$ is influenced by:

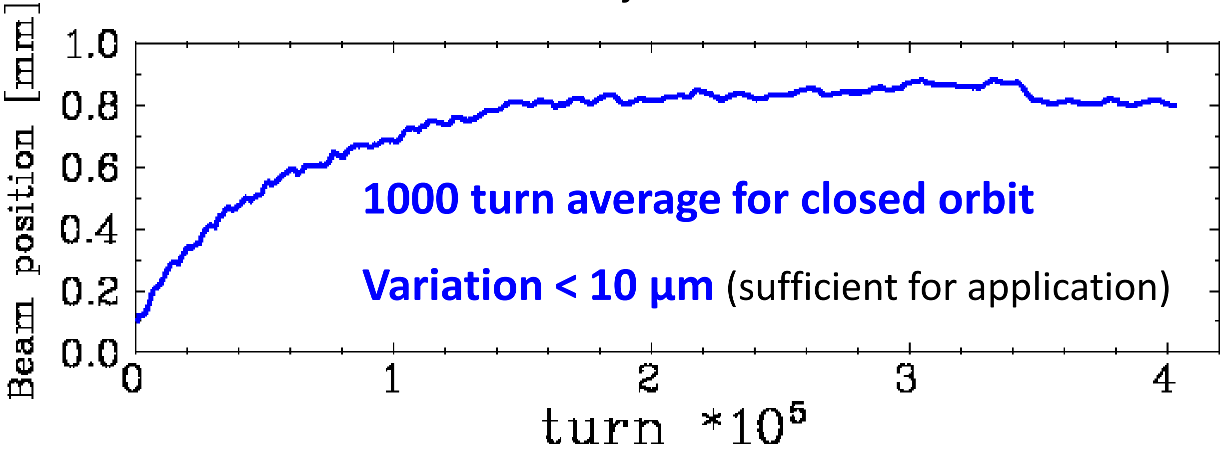
- Input signal amplitude
  → large or matched Z_t
- Thermal noise at ***R***=50 Ω for ***T***=300 K
  (for shoe box ***R*** = 1 kΩ ...1 MΩ)
- Bandwidth ***Δf***
  ⇒ Restriction of frequency width because the power is concentrated on the harmonics of f_{rf}

Example: GSI-LINAC with f_{rf}=36 MHz

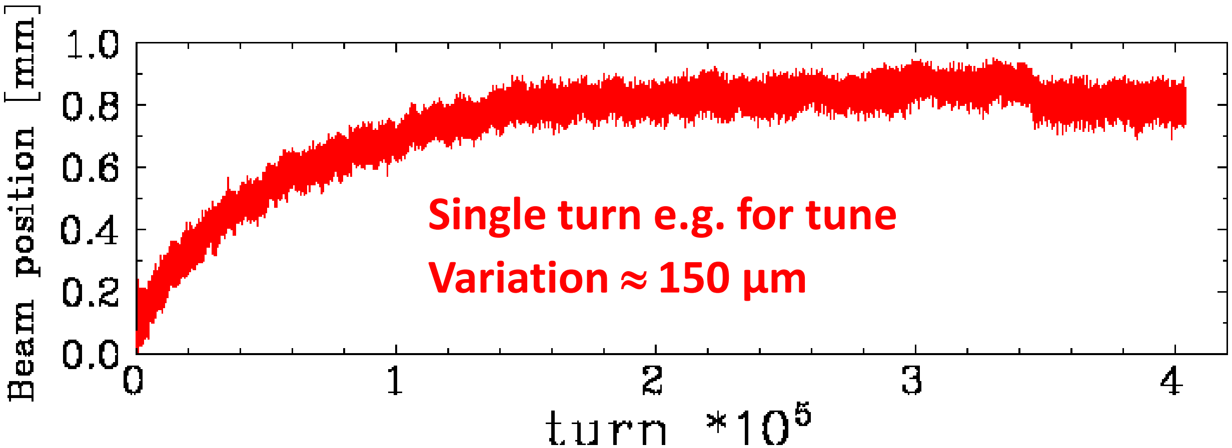


Remark: Additional contribution by non-perfect electronics typically a factor 2
Moreover, pick-up by electro-magnetic interference can contribute ⇒ good shielding required

Example: GSI Synchr.: U^{73+}, E_{inj}=11.5 MeV/u→ 250 MeV/u within 0.5 s, 10^9 ions

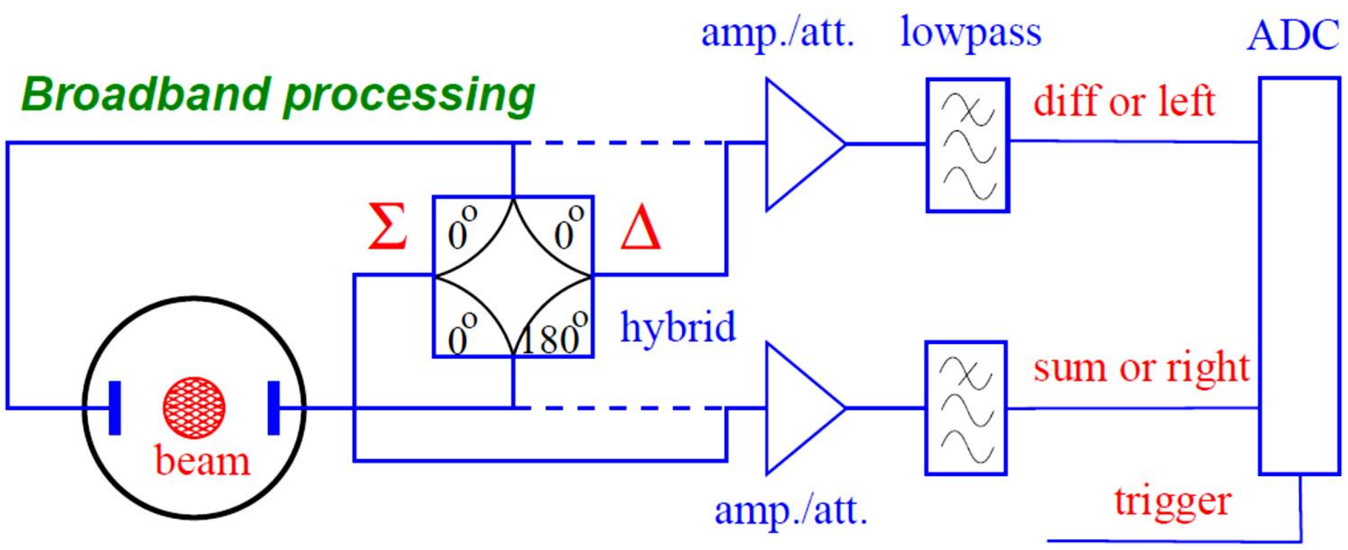


- Position resolution < 30 µm (BPM half aperture *a*=90 mm)
- average over 1000 turns corresponding to ≈1 ms or ≈1 kHz bandwidth



- Turn-by-turn data have much larger variation

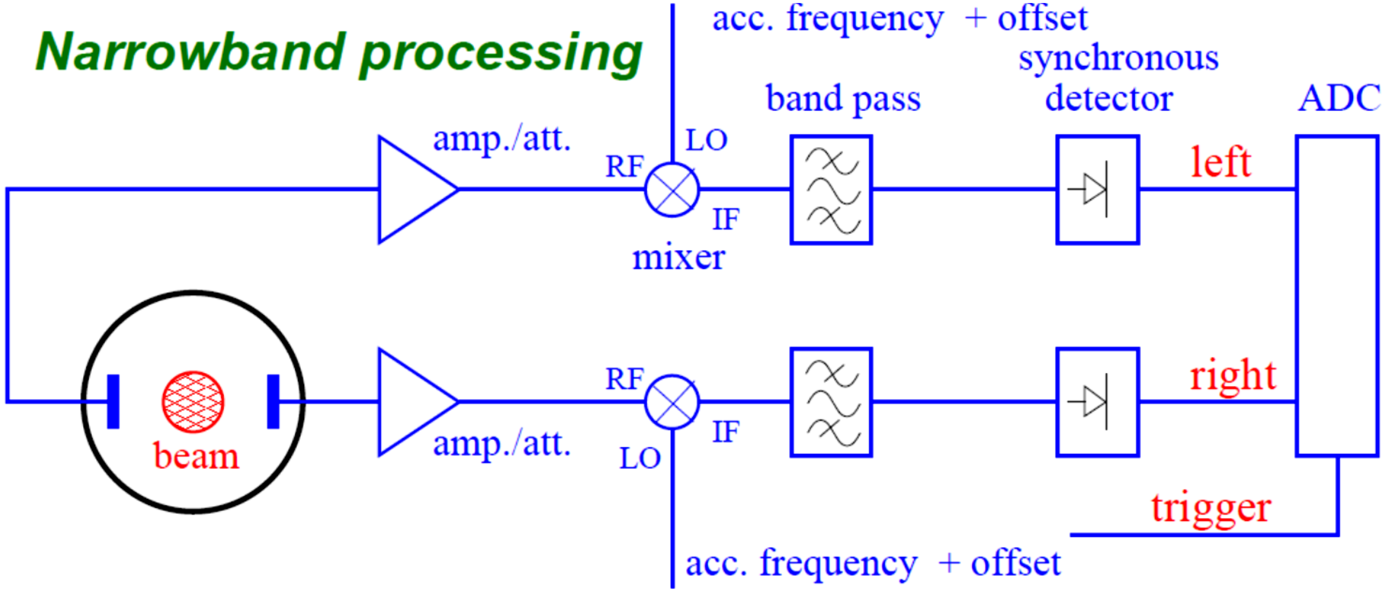
However: not only noise contributes but additionally **beam movement** by betatron oscillation
⇒ broadband processing i.e. turn-by-turn readout for tune determination.



- Hybrid or transformer close to beam pipe for analog ***ΔU*** & ***ΣU*** generation or U_{left} & U_{right}
- Attenuator/amplifier
- Filter to get the wanted harmonics and to suppress stray signals
- ADC: digitalization → followed by calculation of of ***ΔU /ΣU***

Advantage: Bunch-by-bunch possible, versatile post-processing possible

Disadvantage: Resolution down to ≈ 100 µm for shoe box type , i.e. ≈0.1% of aperture, resolution is worse than narrowband processing



Narrowband processing equals heterodyne receiver (e.g. AM-radio or spectrum analyzer)

- Attenuator/amplifier
- Mixing with accelerating frequency $f_{rf} \Rightarrow$ signal with sum and difference frequency
- Bandpass filter of the mixed signal (e.g at 10.7 MHz)
- Rectifier: synchronous detector
- ADC: digitalization → followed calculation of ***ΔU/ΣU***

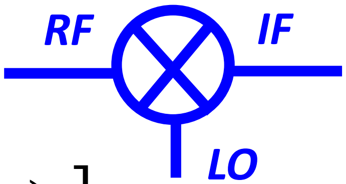
Advantage: Spatial resolution about 100 time better than broadband processing

Disadvantage: No turn-by-turn diagnosis, due to mixing = 'long averaging time'

For non-relativistic p-synchrotron: → variable f_{rf} leads via mixing to constant intermediate freq.

Mixer: A passive rf device with

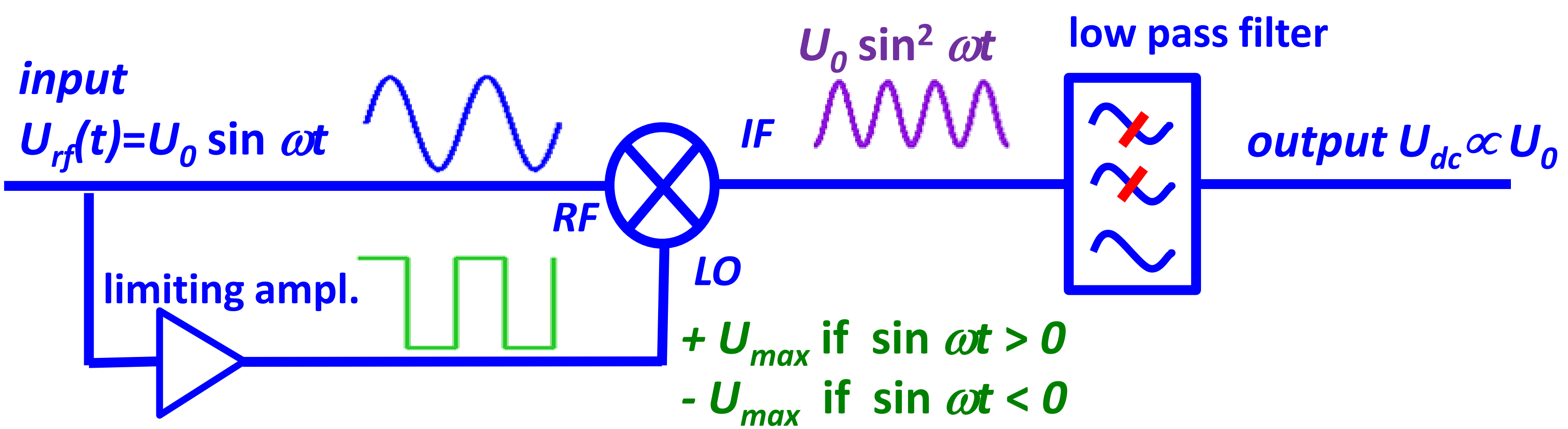
- Input RF (radio frequency): Signal of investigation $A_{RF}(t) = A_{RF} \cos \omega_{RF} t$
- Input LO (local oscillator): Fixed frequency $A_{LO}(t) = A_{LO} \cos \omega_{LO} t$
- Output IF (intermediate frequency)



$$A_{IF}(t) = A_{RF} \cdot A_{LO} \cos \omega_{RF} t \cdot \cos \omega_{LO} t$$
$$= A_{RF} \cdot A_{LO} \left[\cos(\omega_{RF} - \omega_{LO})t + \cos(\omega_{RF} + \omega_{LO})t \right]$$

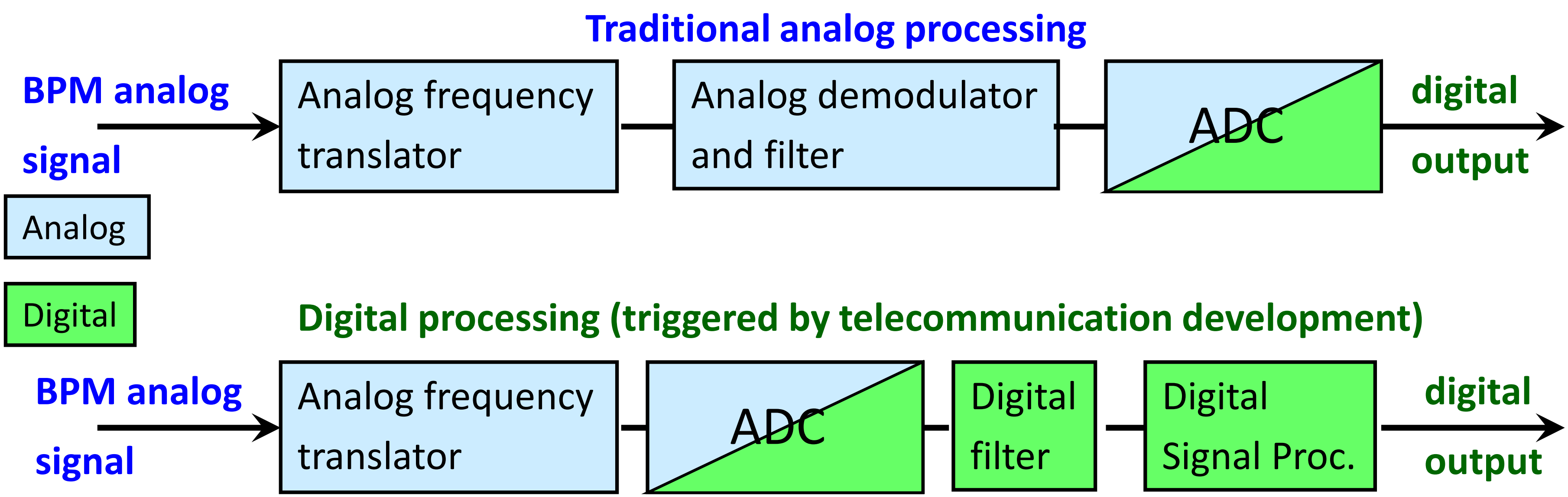
$\Rightarrow$ Multiplication of both input signals, containing the sum and difference frequency.

Synchronous detector: A phase sensitive rectifier



Analog versus Digital Signal Processing

Modern instrumentation uses **digital** techniques with extended functionality.



Digital receiver as modern successor of super heterodyne receiver

- Basic functionality is preserved but implementation is very different
- Digital transition just after the amplifier & filter or mixing unit
- Signal conditioning (filter, decimation, averaging) on digital electronics e.g. FPGA

Advantage of DSP: Versatile operation, flexible adoption without hardware modification

Disadvantage of DSP: **non**, good engineering skill requires for development, expensive

Comparison of BPM Readout Electronics (simplified)

Type	Usage	Precaution	Advantage	Disadvantage
Broadband	p-sychr.	Long bunches	Bunch structure signal Post-processing possible Required for fast feedback	Resolution limited by noise
Narrowband	all synchr.	Stable beams >100 rf-periods	High resolution	No turn-by-turn Complex electronics
Digital Signal Processing	all	Several bunches ADC 125 MS/s	Very flexible High resolution **Trendsetting technology for future demands**	Limited time resolution by ADC → under-sampling complex and expensive

Outline:

- Signal generation $\rightarrow$ transfer impedance
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  used at most proton LINACs and electron accelerators
- Capacitive *shoe-box* BPM for low frequencies

  used at most proton synchrotrons due to linear position reading
- Electronics for position evaluation

  analog signal conditioning to achieve small signal processing
- **BPMs for measurement of closed orbit, tune and further lattice functions**

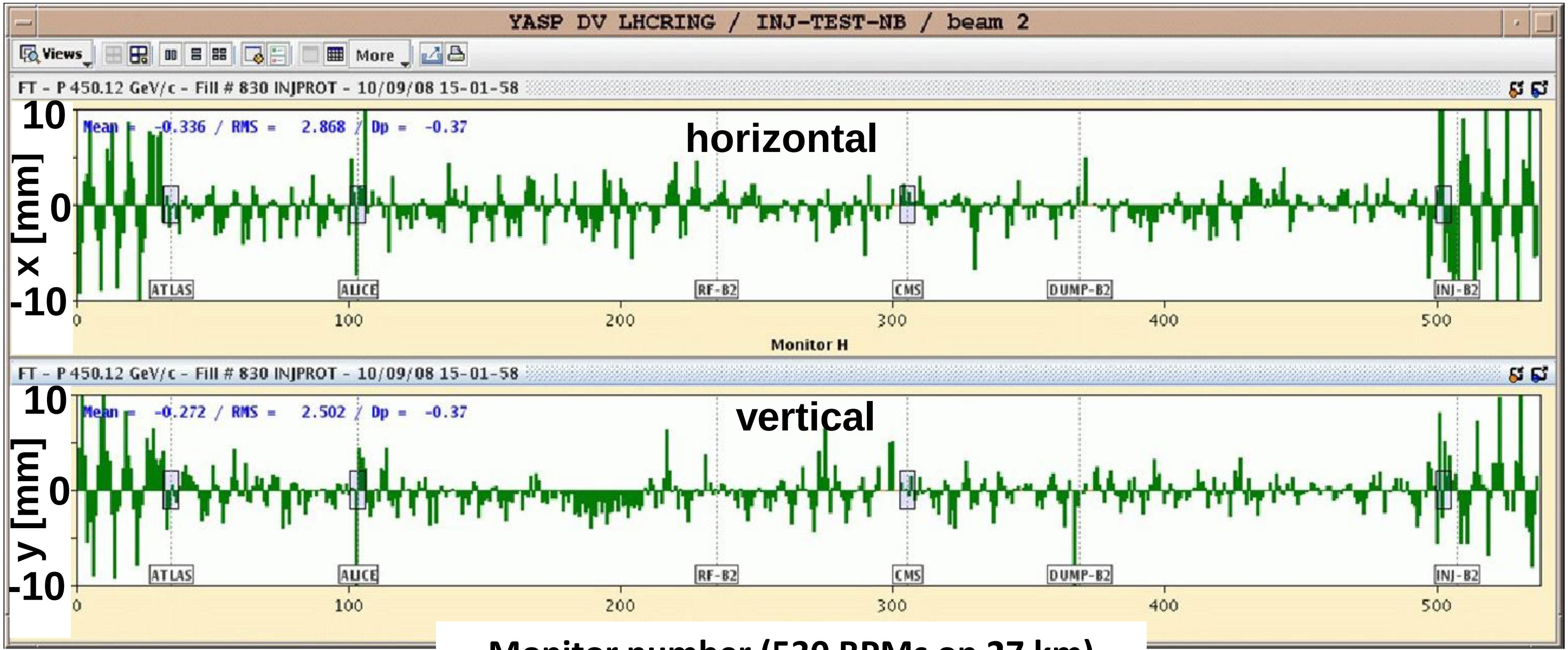
  frequent application of BPMs
- Summary

Trajectory:

The position delivered by an **individual bunch** within a transfer line or a synchrotron.

Main task: Control of matching (center and angle), first-turn diagnostics

Example: LHC injection 10/09/08 i.e. first day of operation !



Monitor number (530 BPMs on 27 km)

From R. Jones (CERN)

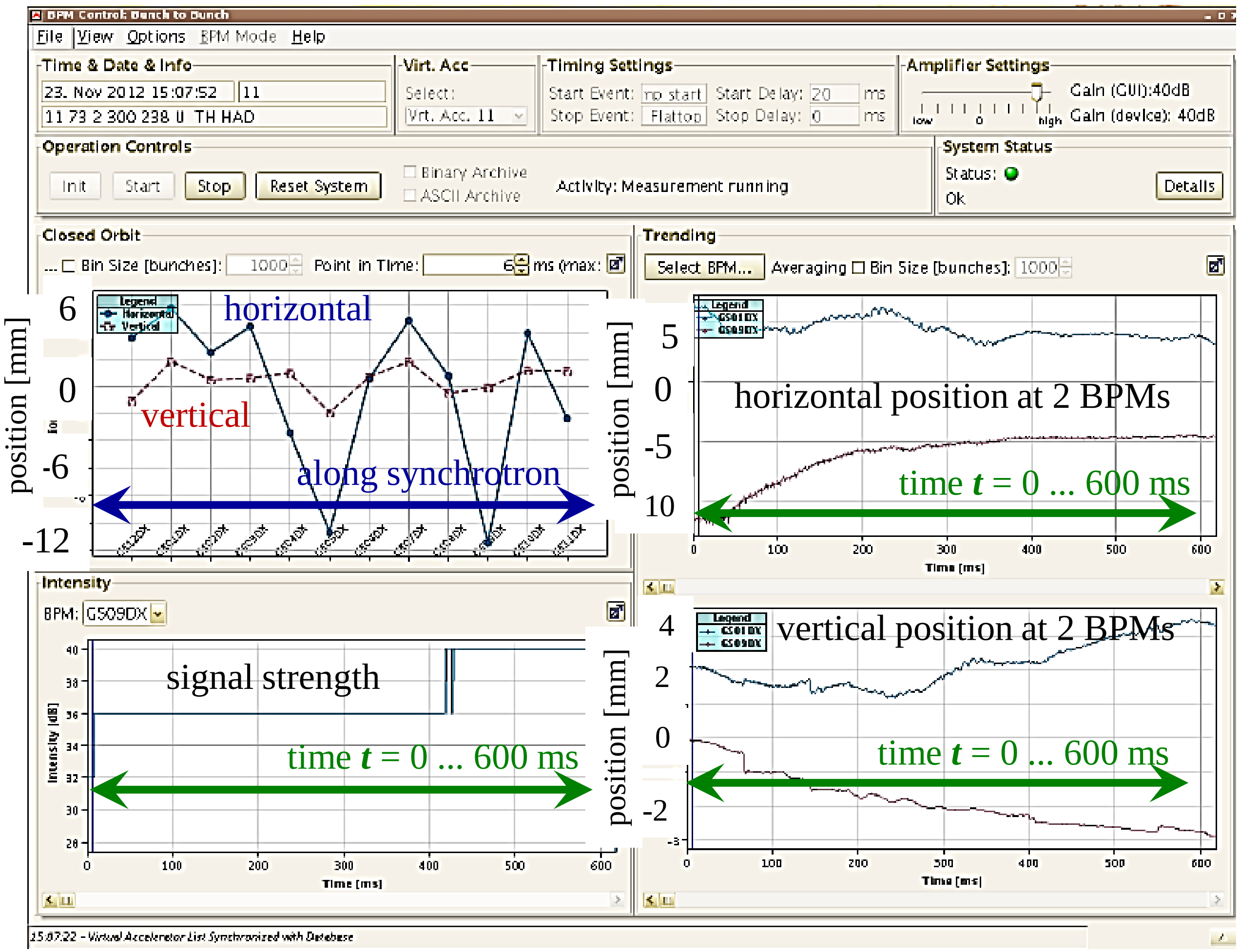
Tune values at LHC: Q_h = 64.3, Q_v = 59.3

Close Orbit Measurement with BPMs

Single bunch position averaged over 1000 bunches → closed orbit with ms time steps.

It differs from ideal orbit by misalignments of the beam or components.

Example: GSI-synchrotron at two BPM locations, 1000 turn average during acceleration:



Closed orbit:

Beam position averaged over many turns (i.e. betatron oscillations).

The result is the basic tool for alignment & stabilization

Remark as a role of thumb:

Number of BPMs within a synchrotron: $N_{BPM} \approx 4 \cdot Q$

Relation BPMs ↔ tune due to close orbit stabilization feedback

(justification outside of the scope of this lecture)

Closed Orbit Feedback: Typical Noise Sources

Beam movement:

Short term (min to 10 ms):

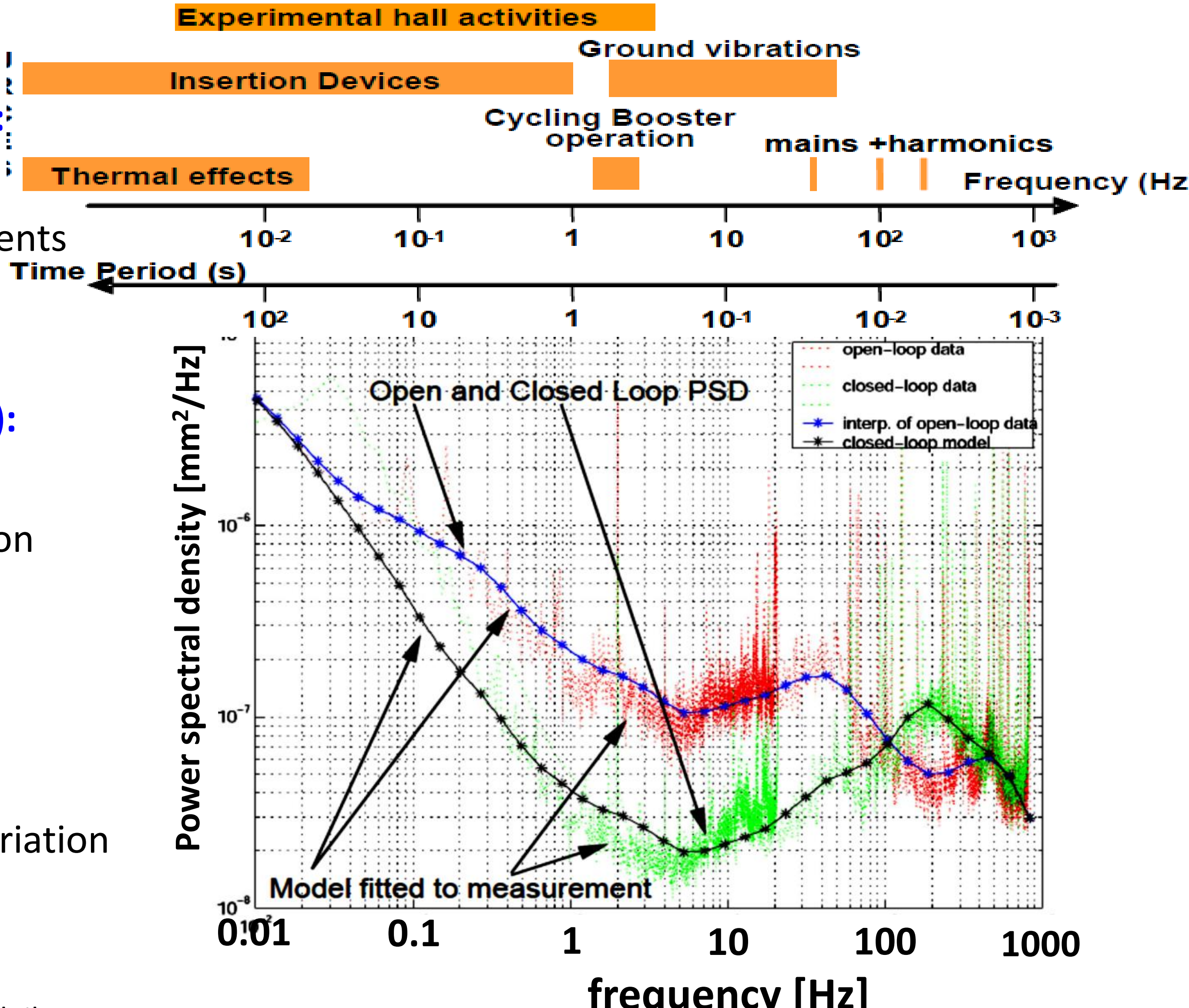
- Traffic
- Machine (crane) movements
- Water & vacuum pumps
- 50 Hz main power net

Medium term (day to min):

- Movement of chambers due to heating by radiation
- Day-night variation
- tide, moon cycle

Lang term (> days):

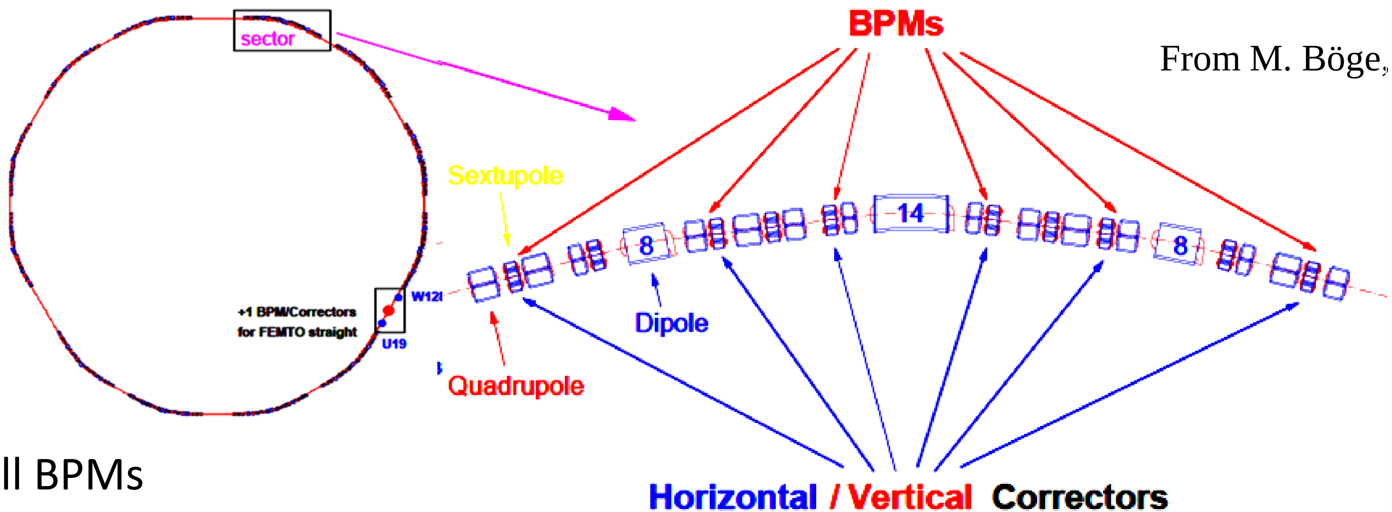
- Ground settlement
- Seasons, temperature variation



From M. Böge, PSI, N. Hubert, Soleil

Orbit feedback: Synchrotron light source → spatial stability of light beam

Example from SLS-Synchrotron at Villigen, Swiss:



From M. Böge, PSI

Procedure:

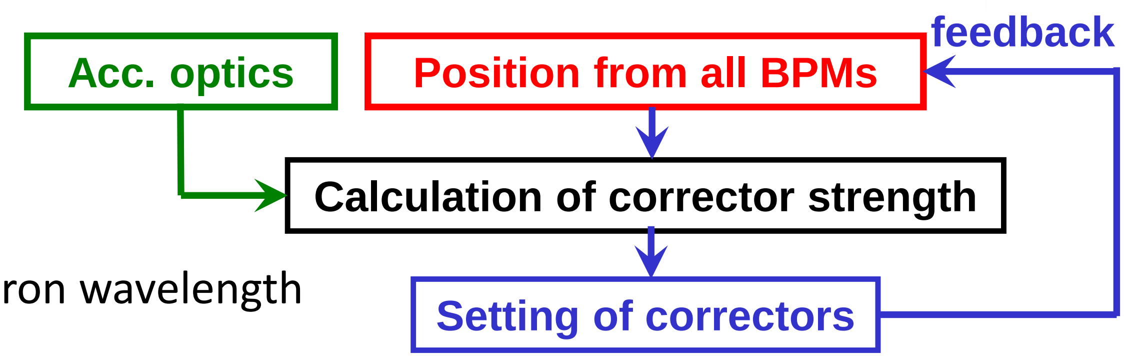
1. Position from all BPMs

2. Calculation of corrector setting via Orbit Response Matrix

3. Digital feedback loop

⇒ regulation time down to 10 ms

⇒ Role od thumb: ≈ 4 BPMs per betatron wavelength

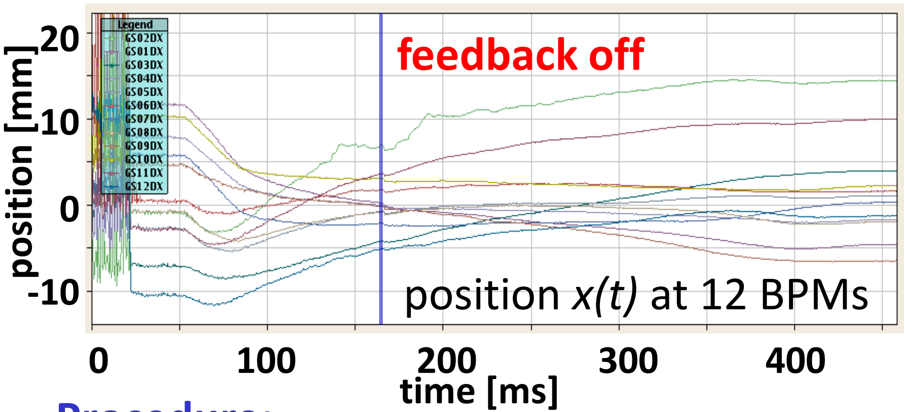


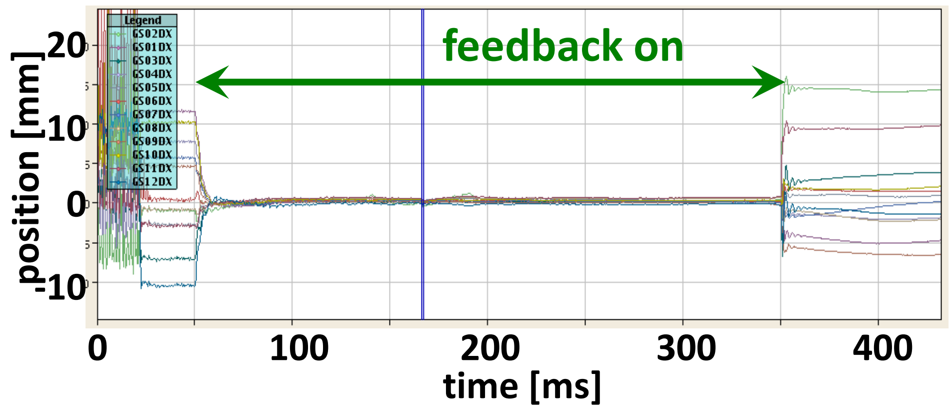
Uncorrected orbit: typ. $\langle x^2 \rangle_{rms} \approx 1$ mm

Corrected orbit: $\langle x^2 \rangle_{rms} \approx 1$ µm up to 100 Hz bandwidth!

Orbit feedback:

Example: 12 beam positions at GSI-SIS during ramping from 8.6 to 500 MeV/u for Ar^{18+}





Procedure:

1. Position from all 12 BPMs

2. Calculation of corrector setting on fast (FPGA-based) electronics

3. Submission to corrector magnets

4. New position measurement

⇒ regulation time down to 10 ms

Role of thumb:

Movement related to tune i.e. ‘natural oscillations by periodic focusing

To determine the ‘sine-like’ oscillation 4 BPMs per oscillation are required

⇒ 4 BPMs per tune value (but detailed investigation required to determine the # of BPMs

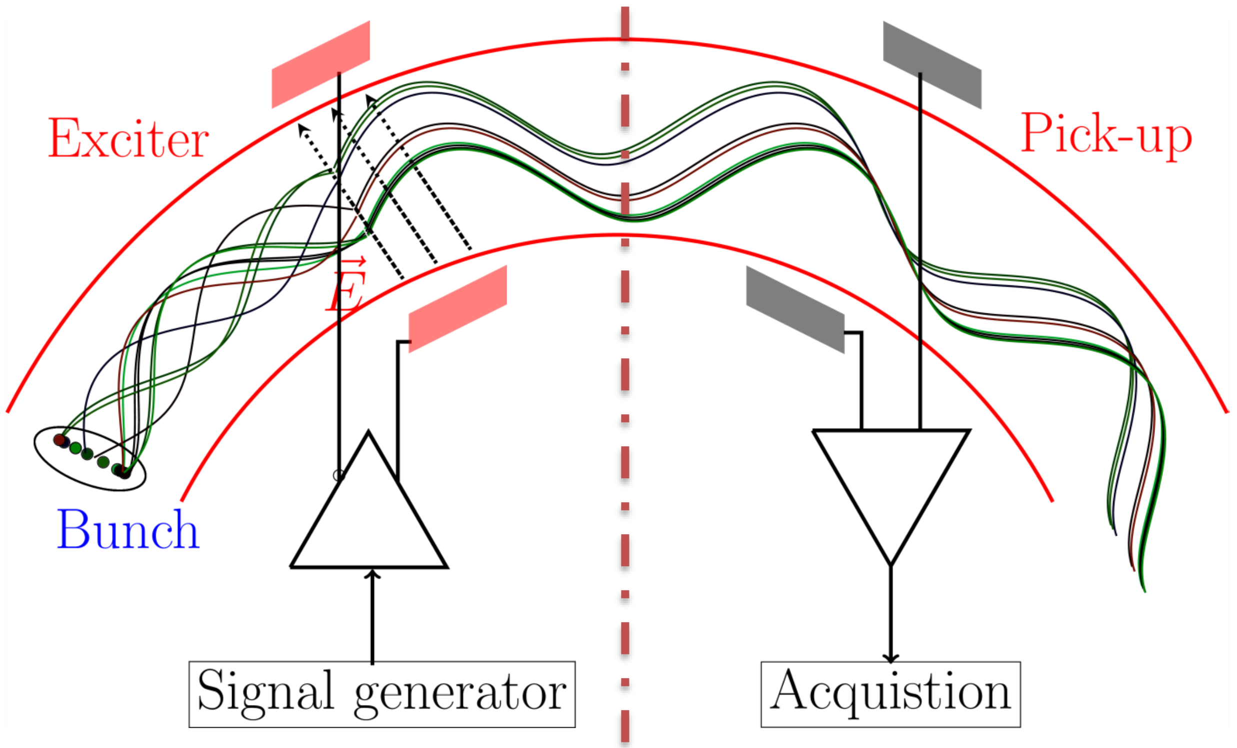
Tune Measurement: General Considerations

Coherent excitations are required for the detection by a BPM

Beam particle's ***in-coherent*** motion ⇒ center-of-mass stays constant

Excitation of **all** particles by rf ⇒ ***coherent*** motion

⇒ center-of-mass variation turn-by-turn



Graphics by R. Singh, GSI

Tune Measurement: General Considerations

The tune Q is the number of betatron oscillations per turn.

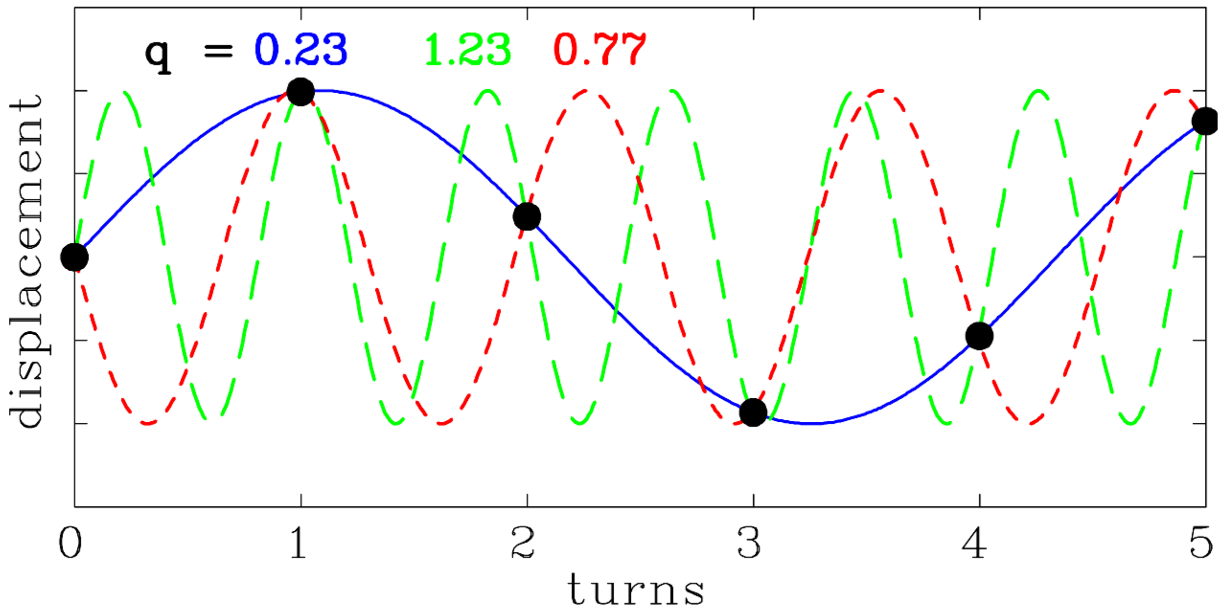
The betatron frequency is $f_\beta = Q \cdot f_0$.

Measurement: excitation of ***coherent*** betatron oscillations + position from one BPM.

From a measurement one gets only the non-integer part q of Q with $Q = n \pm q$.

Moreover, only $0 < q < 0.5$ is the unique result.

Example: Tune measurement for six turns with the three lowest frequency fits:

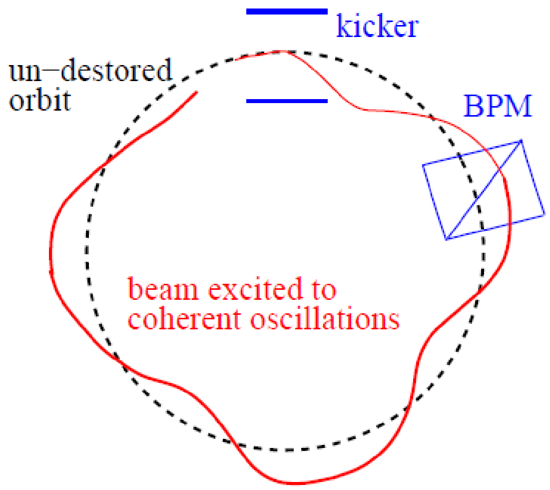


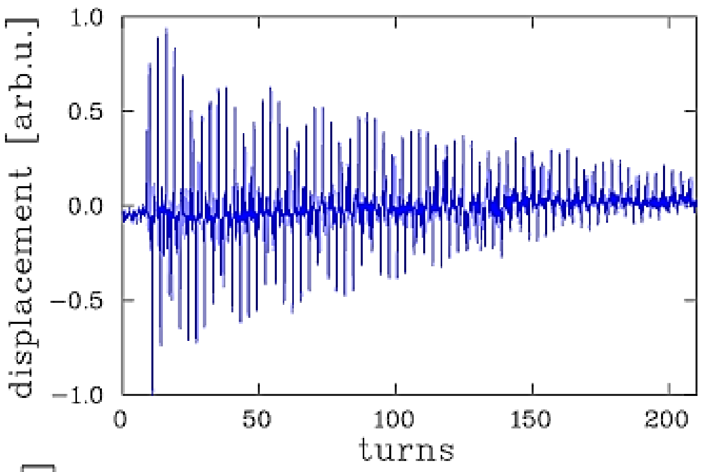
To distinguish for $q < 0.5$ or $q > 0.5$:

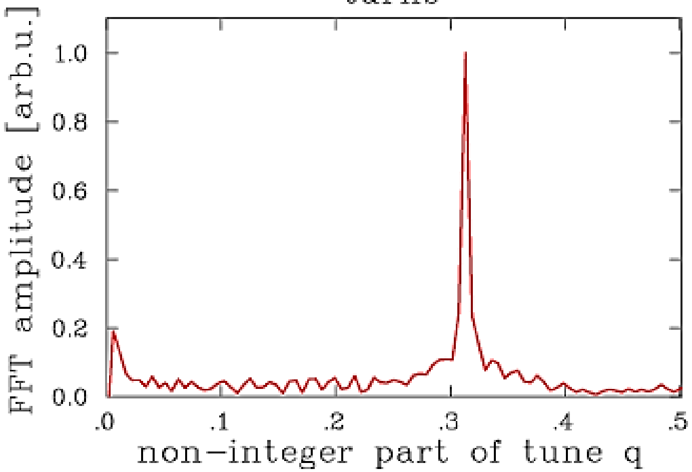
Changing the tune slightly, the direction of q shift differs.

Tune Measurement: The Kick-Method in Time Domain

The beam is excited to
coherent betatron oscillation
→ the beam position measured
each revolution ('turn-by-turn')
→ Fourier Trans. gives the non-integer tune ***q***.
Short kick compared to revolution.





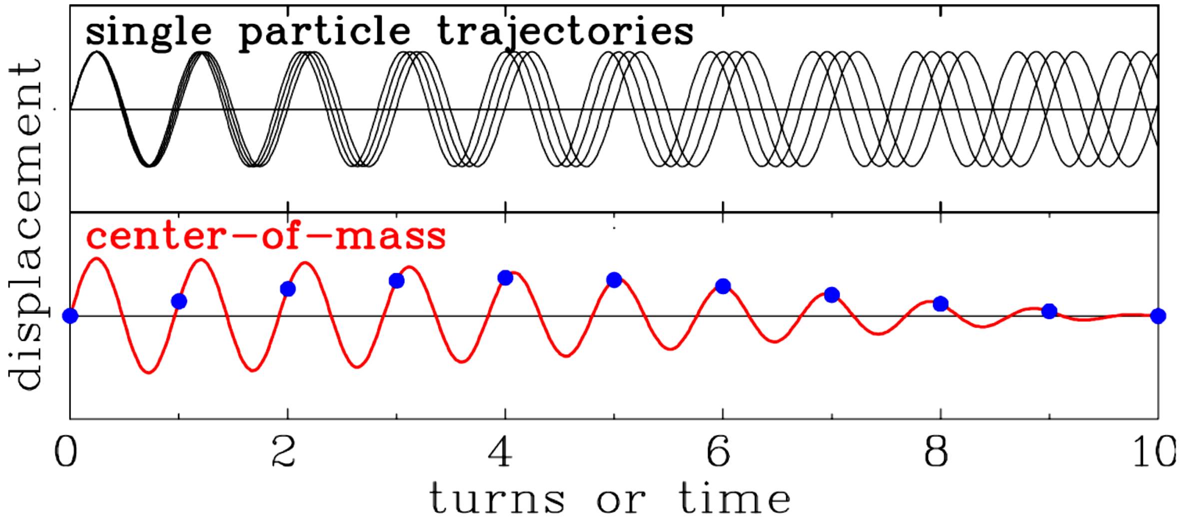


The de-coherence time limits the **resolution**:
N non-zero samples
⇒ General limit of discrete FFT: $$\Delta q > \frac{1}{2N}$$

N = 200 turn ⇒ **Δ*q*** > 0.003 as resolution
(tune spreads are typically **Δ*q*** ≈ 0.001!)

Tune Measurement: De-Coherence Time

The particles are excited to betatron oscillations, but due to the spread in the betatron frequency, they getting out of phase ('Landau damping'):



Scheme of the individual trajectories of four particles after a kick (top) and the resulting ***coherent*** signal as measured by a pick-up (bottom).

$\Rightarrow$ Kick excitation leads to limited resolution

Remark: The tune spread is much lower for a real machine.

Instead of one kick, the beam can be excited by a sweep of a sine wave, called 'chirp'

→ **B**eam **T**ransfer **F**unction (BTF) Measurement

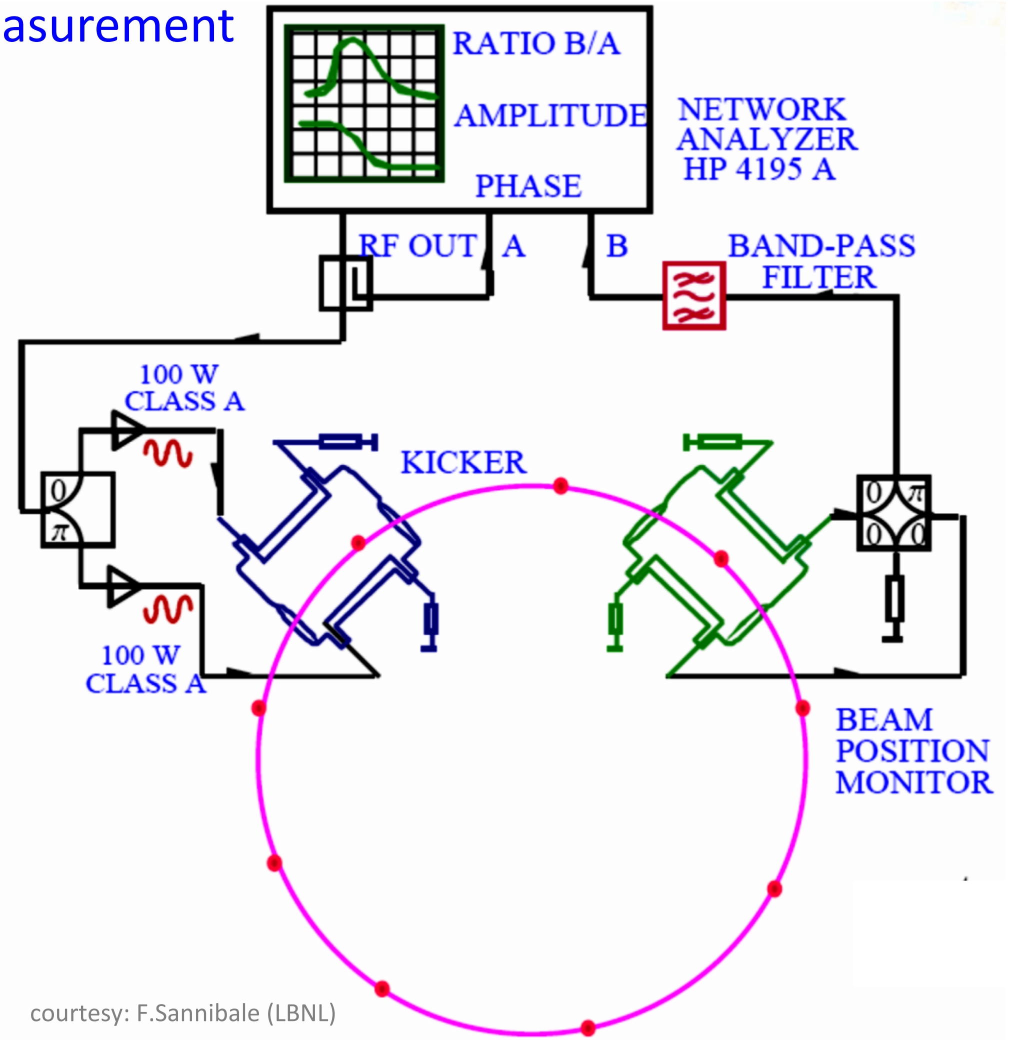
as the velocity response to a kick

Prinziple:

Beam acts like a driven oscillator!

Using a network analyzer:

- RF OUT is feed to the beam by a kicker (reversed powered as a BPM)
- The position is measured at one BPM
- Network analyzer: amplitude and phase of the response
- Sweep time up to seconds due to de-coherence time per band
- resolution in tune: up to 10^{-4}

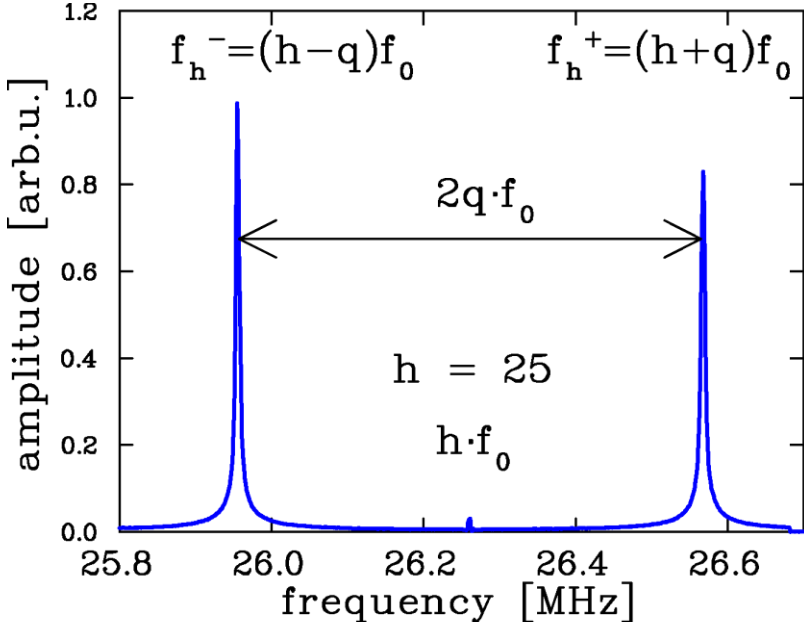


courtesy: F.Sannibale (LBNL)

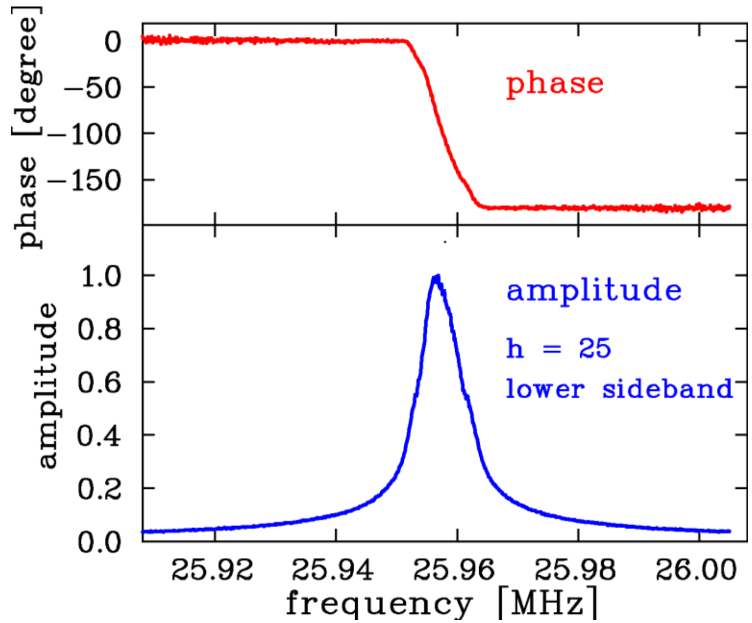
Tune Measurement: Result for BTF Measurement

BTF measurement at the GSI synchrotron, recorded at the 25th harmonics.

A wide scan with both sidebands at h=25th-harmonics:



*A detailed scan for the **lower** sideband → beam acts like a driven oscillator:*



From the position of the sidebands ***q*** = 0.306 is determined. From the width ***Δf/f*** $\approx 5 \cdot 10^{-4}$ the tune spread can be calculated via $\Delta f_h^- = \eta \frac{\Delta p}{p} \cdot h f_0 \left(h - q + \frac{\xi}{\eta} Q \right)$

Advantage: High resolution for tune and tune spread (also for de-bunched beams)

Disadvantage: Long sweep time (up to several seconds).

Tune Measurement: *Gentle* Excitation with Wideband Noise

Instead of a sine wave, noise with adequate bandwidth can be applied

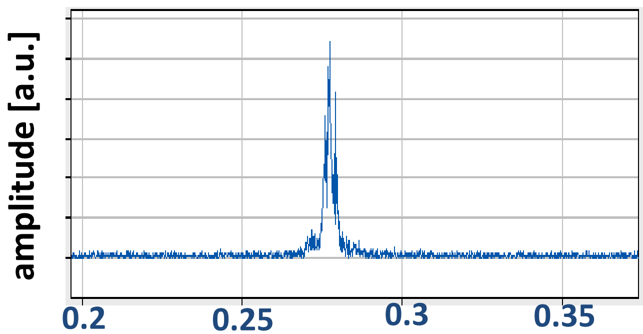
→ beam picks out its resonance frequency:

- broadband excitation with white noise of ≈ 10 kHz bandwidth
- turn-by-turn position measurement by fast ADC
- Fourier transformation of the recorded data

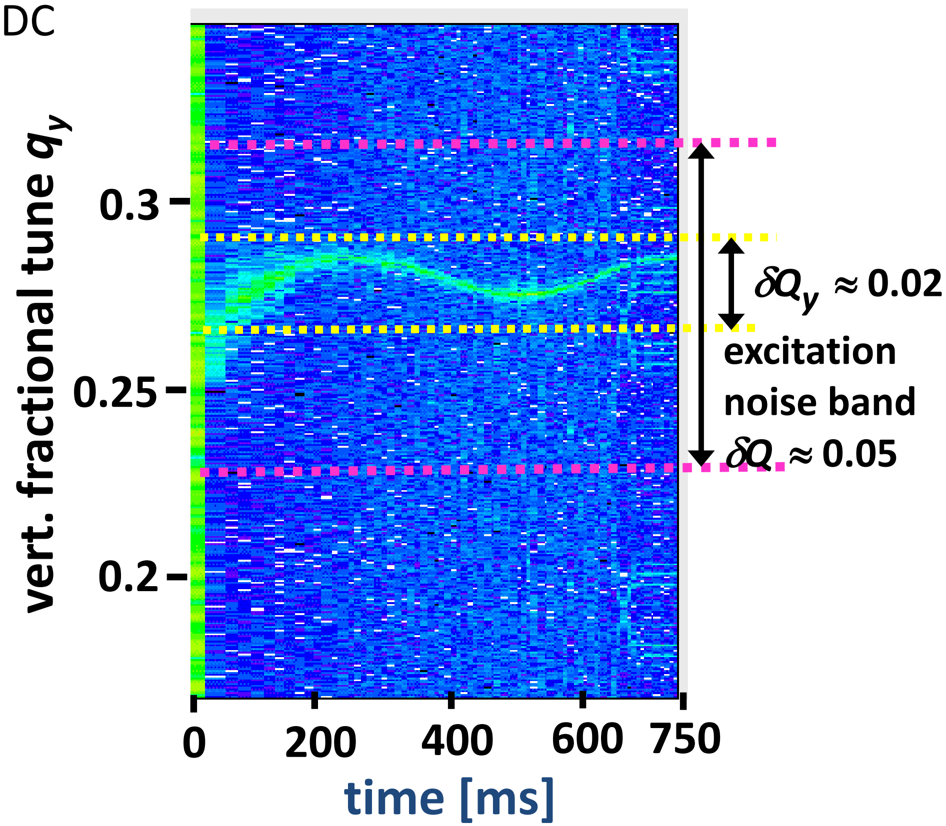
⇒ Continues monitoring with low disturbance

Example: Vertical tune within 4096 turn duration ≃ 15 ms at GSI synchrotron 11 → 300 MeV/u in 0.7 s

vertical tune at fixed time ≈ 15ms



vertical tune versus time



Advantage:
Fast scan with good time resolution

Disadvantage: Lower precision

The position of dipoles and quadrupoles

- give the linear lattice functions
- at injection point ***D* = 0** is favored
- chromatic correction with sextupoles,

Definition of dispersion ***D(s)***:

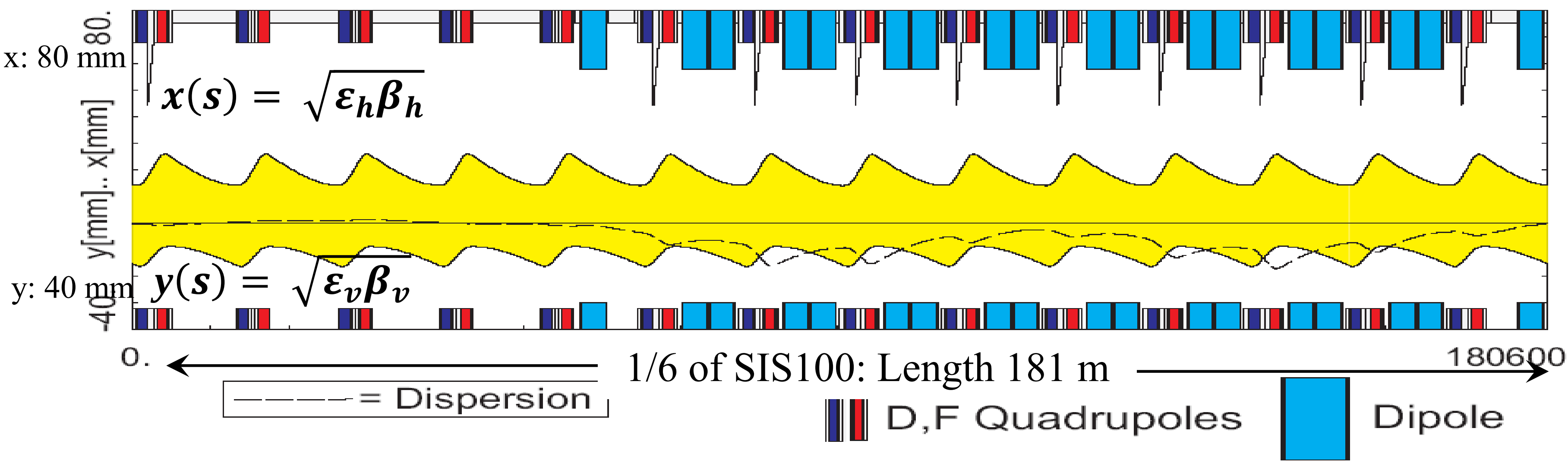
$$x_D(s) = D(s) \cdot \Delta p/p_0$$

Definition of chromaticity ξ per turn:

$$\Delta Q/Q_0 = \xi \cdot \Delta p/p_0$$

Example: GSI SIS100 ion synchrotron

Length C [m]		1086
Energy E_{kin} [GeV/u]		0.2 → 2
Tune $Q_{h/v}$	h/v	18.84 / 18.73
Max. dispersion $\|D\|$ [m]		1.73
Max. β–function $\beta_{h/v}$ [m]	h/v	19.6 / 19.6
Natural chromaticity $\xi_{h/v}$	h/v	-1.19 / -1.20
Injected emittance $\varepsilon_{h/v}$ [mm mrad]	h/v	35 / 15
Injected mom. spread $\Delta p/p_0$ [%]		0.05



β-Function Measurement from Bunch-by-Bunch BPM Data

Excitation of **coherent** betatron oscillations: From the position deviation ***x_{ik}*** at the BPM ***i*** and turn ***k*** the ***β***-function ***$\beta(s_i)$*** can be evaluated.

The position reading is: ($\hat{x}_i$ amplitude, μ_i phase at i, Q tune, s_0 reference location)

$$x_{ik} = \hat{x}_i \cdot \cos(2\pi Qk + \mu_i) = \hat{x}_0 \cdot \sqrt{\beta(s_i)/\beta(s_0)} \cdot \cos(2\pi Qk + \mu_i)$$

$\rightarrow$ a turn-by-turn position reading at many location (4 per unit of tune) is required.

The ratio of β-functions at different location:

$$\frac{\beta(s_i)}{\beta(s_0)} = \left(\frac{\hat{x}_i}{\hat{x}_0}\right)^2$$

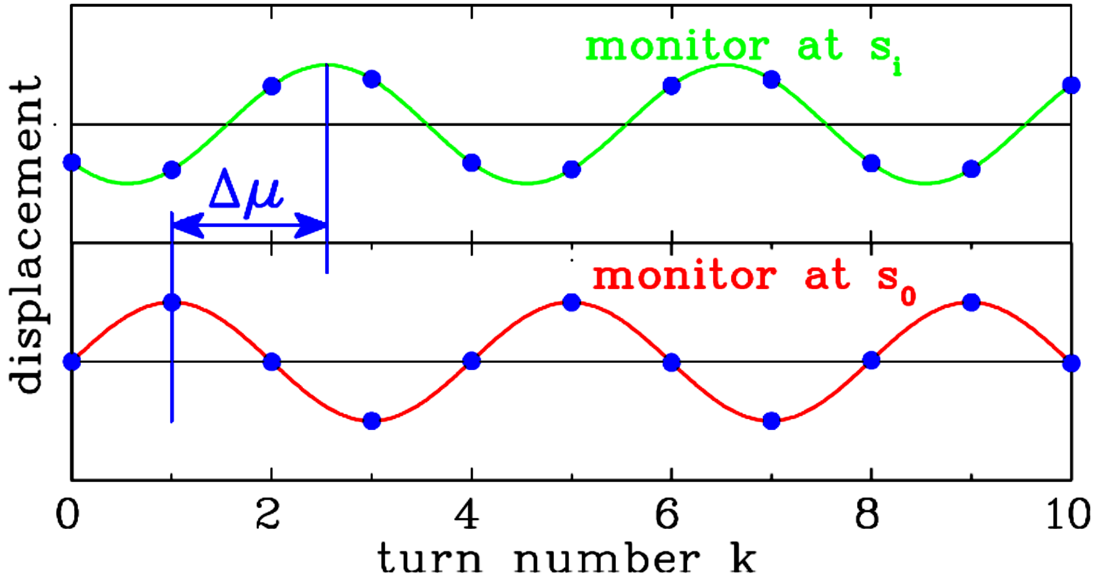
The phase advance is:

$$\Delta\mu = \mu_i - \mu_0$$

Without absolute calibration,

β-function is more precise:

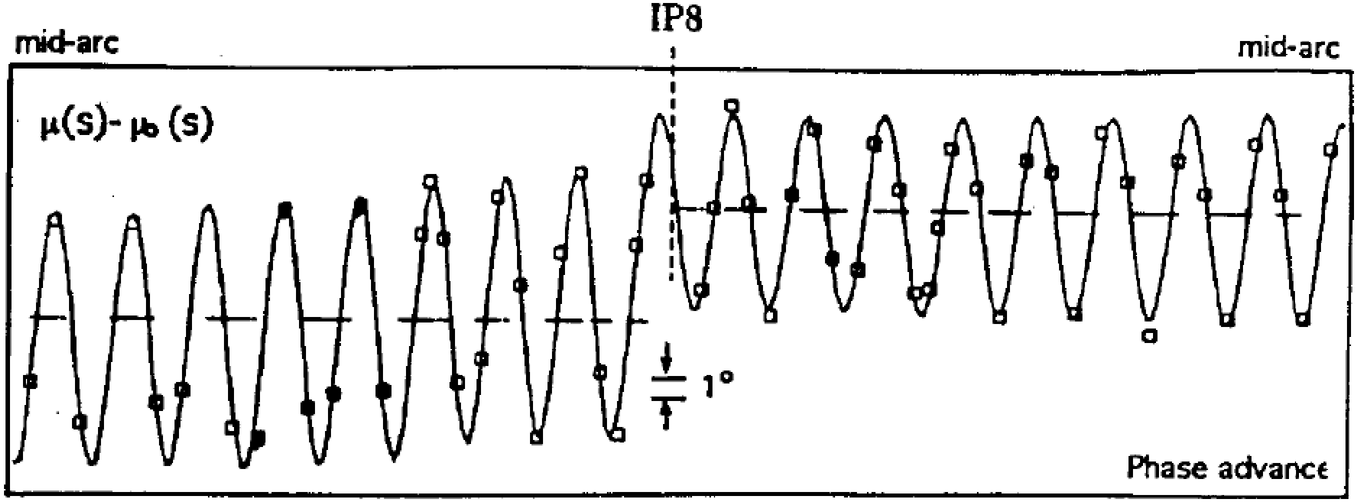
$$\Delta\mu = \int_{S0}^{Si} \frac{ds}{\beta(s)}$$



Excitation of **coherent** betatron oscillations: From the position deviation ***x**_ik_* at the BPM ***i*** and turn ***k*** the betatron phase is measured.

$$\Delta\mu(s_i) = \int_{S0}^{Si} \frac{ds}{\beta(s)}$$

Example: Phase advance $\mu(s)$ compared to the expected $\mu_0(s)$ by optics calculation e.g. MADX at each BPM at CERN's at LEP (e^+ - e^- collider with 27 km, previously in LHC tunnel)



From J. Borer et al, EPAC'92

Result:

- Model does not describes the reality completely, corrections required
- At interaction point IP (detector location) an additional phase shift is originated
- Alignment by correction dipoles (steerer), quadrupoles or sextupoles.

Excitation of **coherent** betatron oscillations: From the position deviation ***x_{ik}*** at the BPM ***i*** and turn ***k*** the beta-function can be determined

$$\Delta\mu(s_i) = \int_{S0}^{Si} \frac{ds}{\beta(s)}$$

*Example: Measured **$\beta(s)$*** compared to the expected ***$\beta_0(s)$ and*** normalized for each BPM at BNL for RHIC (proton-proton or ions circular collider with 3.8 km length)

Result:

- Model does not describes the reality completely
- Corrections executed
- Increase of the luminosity

Remark:

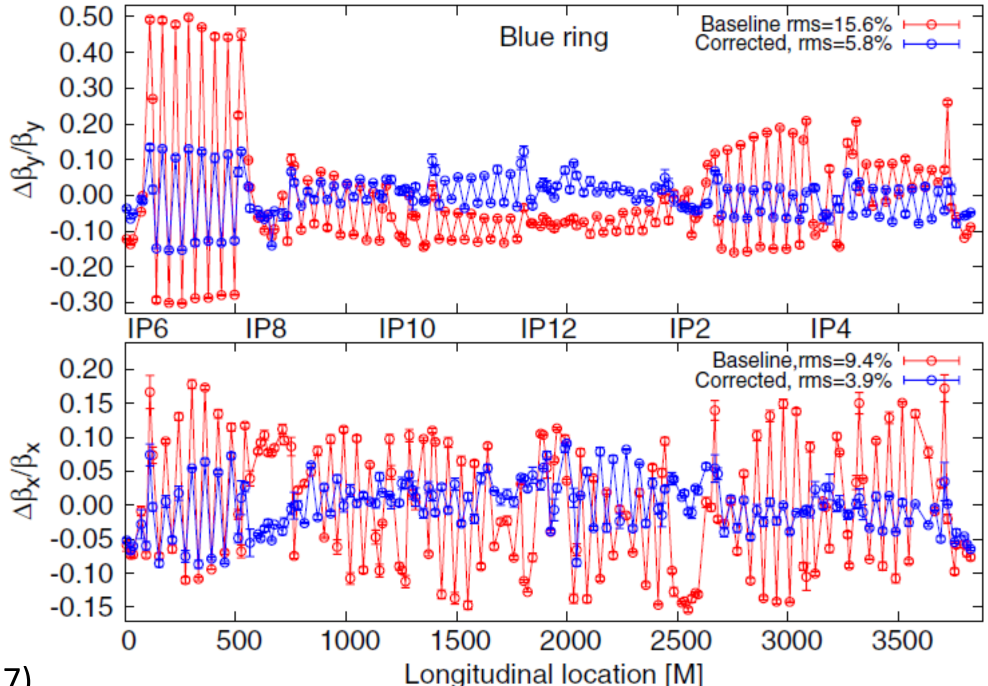
Measurement accuracy depends on

- BPM accuracy
- Numerical evaluation method

See e.g.:

R. Tomas et al., Phys. Rev. Acc. Beams **20,** 054801 (2017)

A. Wegscheider et al., Phys. Rev. Acc. Beams **20,** 111002 (2017)



From X. Shen et al.,
Phys. Rev. Acc. Beams **16**, 111001 (2013)

Dispersion Measurement from Closed Orbit Data

Dispersion $D(s_i)$: Change of momentum ***p*** by detuned rf-cavity

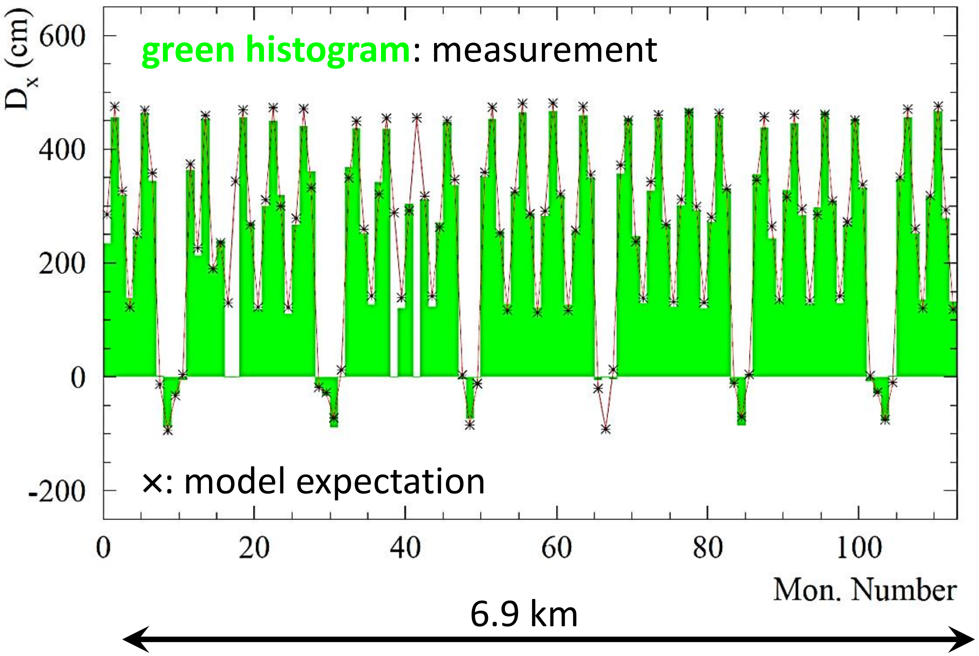
→ Position reading at one location $x_i = D(s_i) \cdot \frac{\Delta p}{p}$:

→ Result from plot of x_i as a function of $\Delta p/p$ ⇒ slope is local dispersion $D(s_i)$

Example: Dispersion measurement ***D(s)*** at BPMs at CERN SPS

Theory-experiment correspondence after correction of

- BPM calibration
- quadrupole calibration



From J. Wenninger: CAS on BD, CERN-2009-005 & J. Wenninger CERN-AB-2004-009

Dispersion $D(s_i)$: Excitation of coherent betatron oscillations and change of momentum ***p*** by detuned rf-cavity:

→ Position reading at one location: $$x_i = D(s_i) \cdot \frac{\Delta p}{p}$$

→ Result from plot of x_i as a function of ***Δp/p*** ⇒ slope is local dispersion $D(s_i)$.

Chromaticity ξ: Excitation of coherent betatron oscillations and change of momentum ***p*** by detuned rf-cavity:

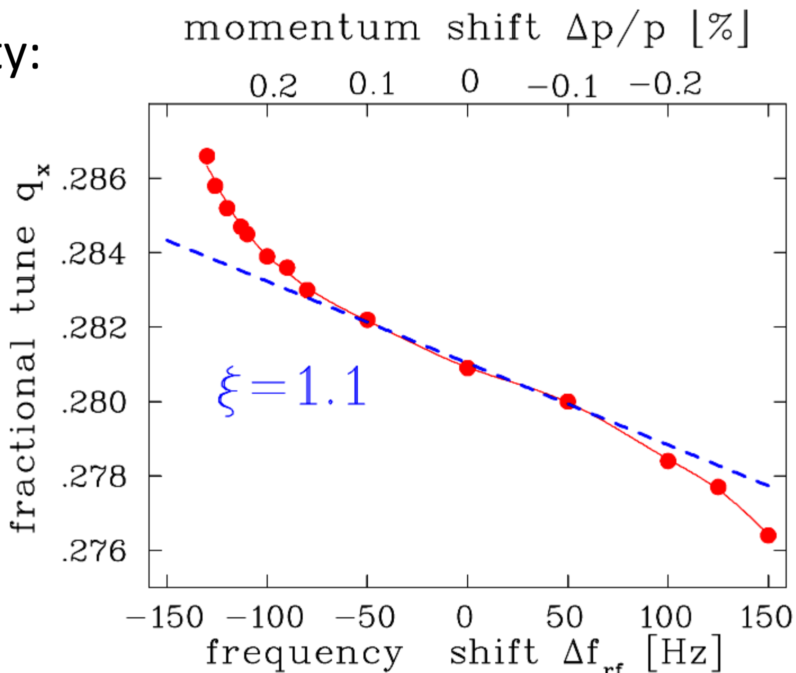
→ Tune measurement

(kick-method, BTF, noise excitation):

$$\frac{\Delta Q}{Q} = \xi \cdot \frac{\Delta p}{p}$$

Plot of ***ΔQ/Q*** as a function of ***Δp/p***

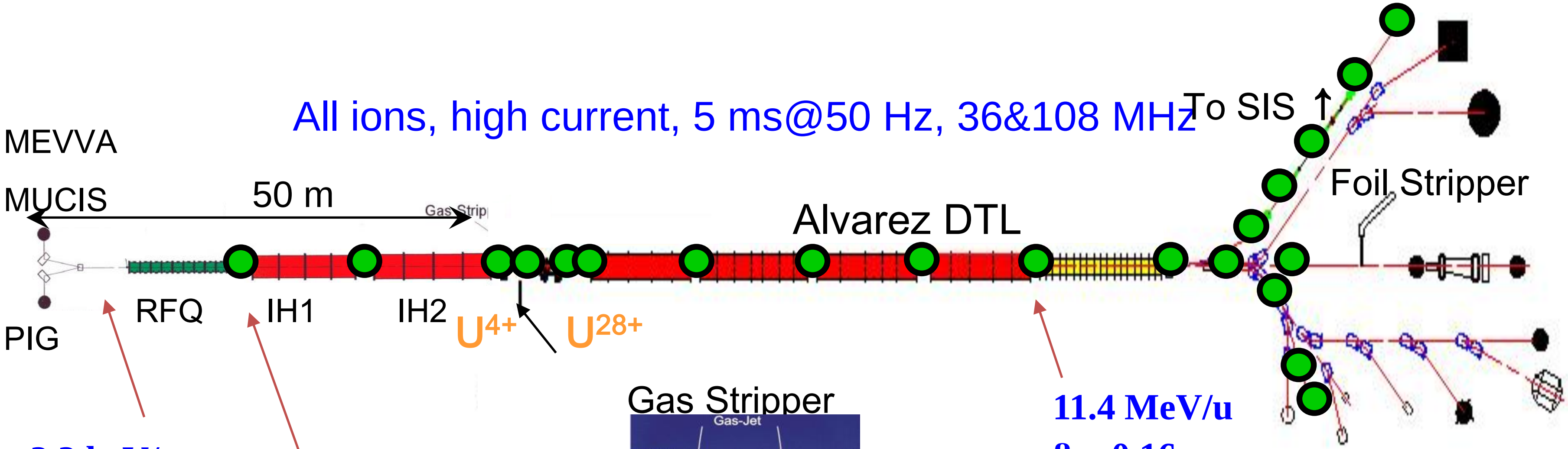
⇒ slope is dispersion ***ξ***.

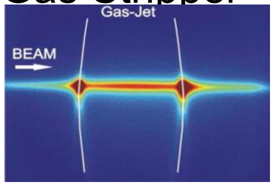


Measurement at LEP

BPM: Capacitive type, for position and time-of-flight

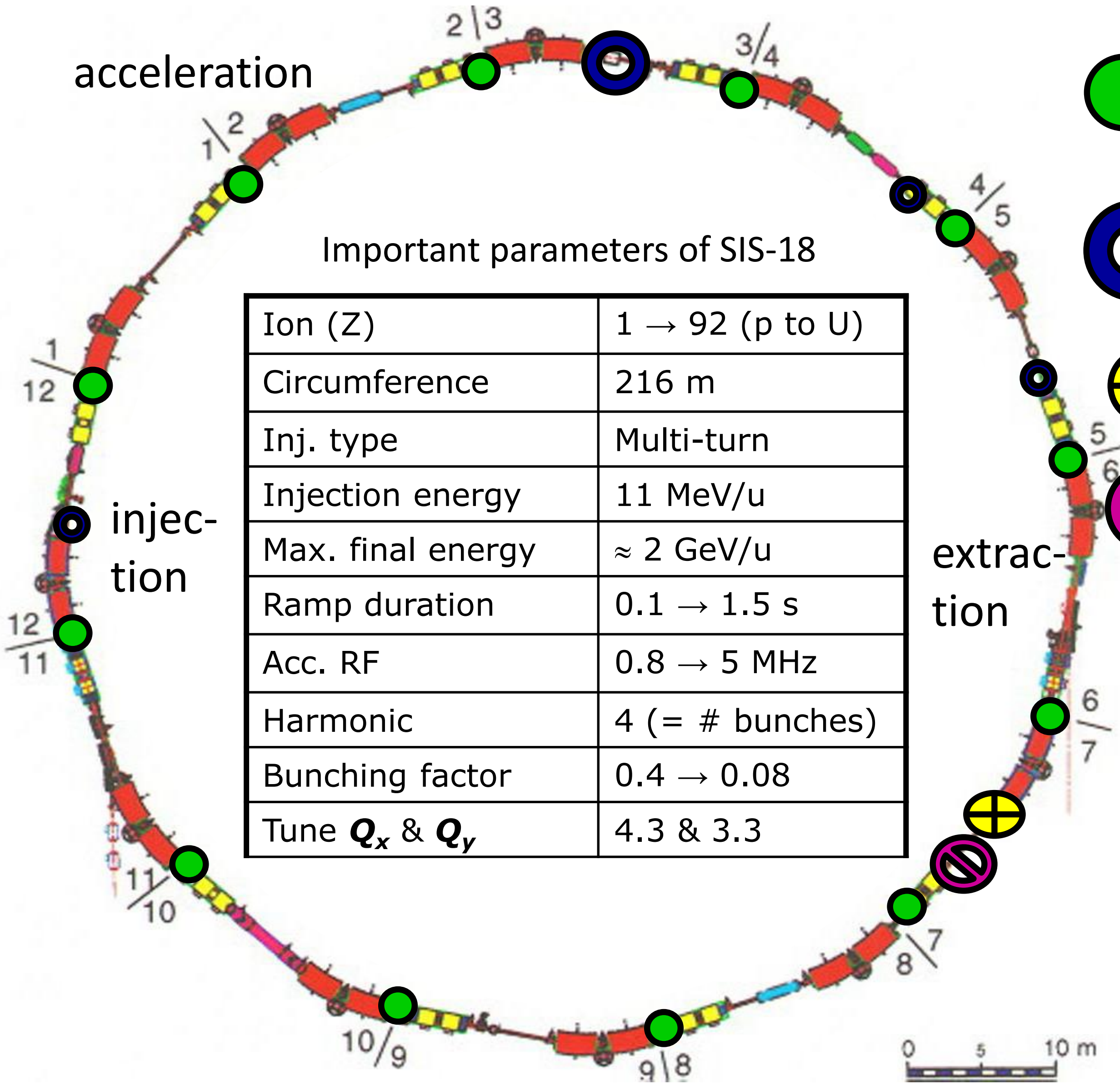
total 25 device





Constructed in the 70th, Upgrade 1999,
further upgrades in preparation

Appendix GSI Ion Synchrotron: Position, tune etc. Measurement



Important parameters of SIS-18

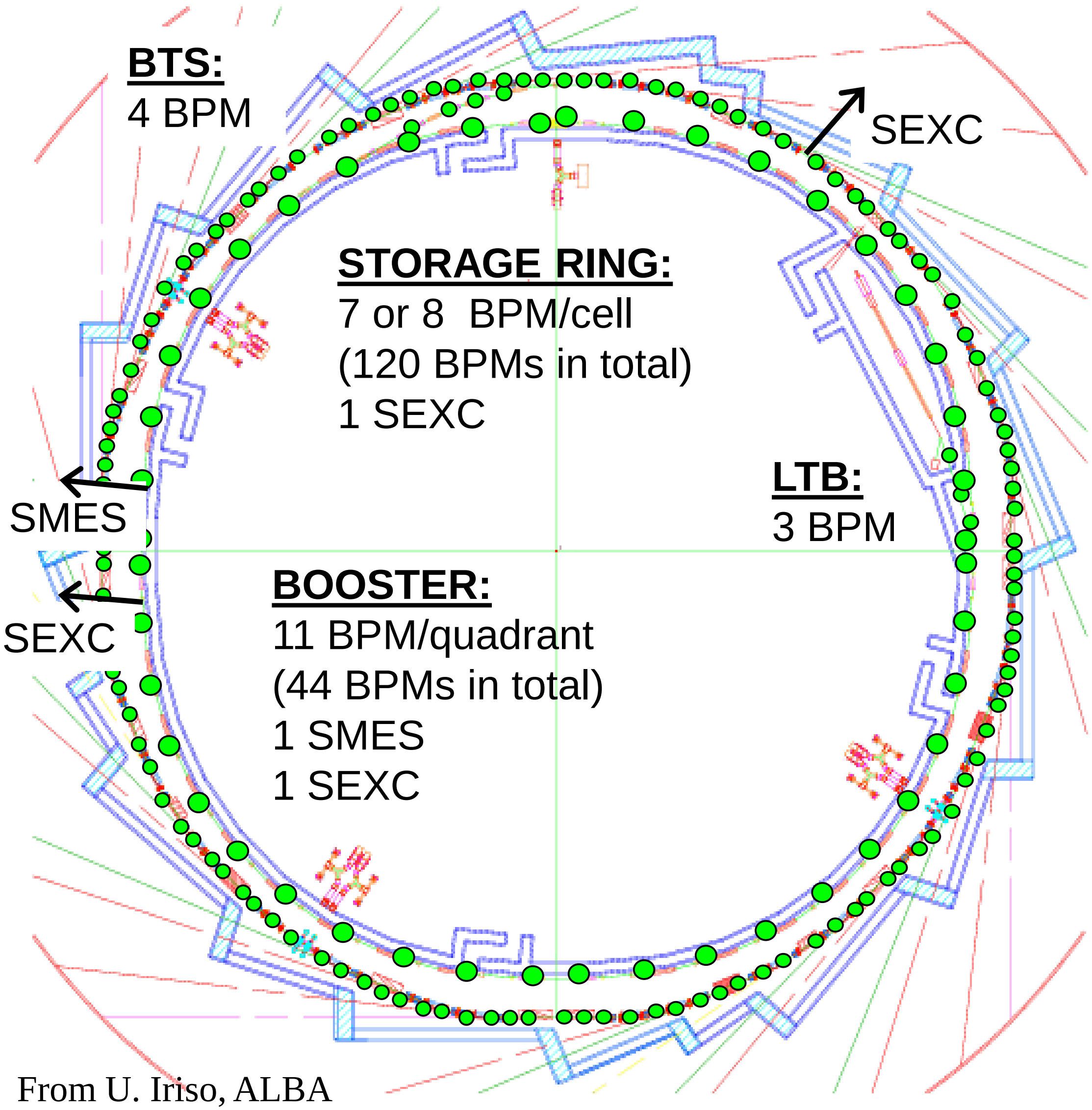
Ion (Z)	1 → 92 (p to U)
Circumference	216 m
Inj. type	Multi-turn
Injection energy	11 MeV/u
Max. final energy	≈ 2 GeV/u
Ramp duration	0.1 → 1.5 s
Acc. RF	0.8 → 5 MHz
Harmonic	4 (= # bunches)
Bunching factor	0.4 → 0.08
Tune Q_x & Q_y	4.3 & 3.3

BPM: position 1 μs to s
12 devices

BPM: for tune measurement
1 device

Exciter: for tune excitation

Schottky pick-up:
Δp/p, Q, ξ ....
1 device



From U. Iriso, ALBA

Beam position:

Center of mass

- Many locations!
- Frequent operating tool
- For position stabilization i.e. closed orbit feedback

Abbreviation:

Meas. Stripline → SMES ↑
Exc. Stripline → SEXC
Button BPMs → BPM ●

Summary Pick-Ups for bunched Beams

The electric field is monitored for bunched beams using rf-technologies ('frequency domain'). Beside transfromers they are the most often used instruments!

Differentiated or proportional signal: rf-bandwidth $\leftrightarrow$ beam parameters

Proton synchrotron: 1 to 100 MHz, mostly 1 MΩ $\rightarrow$ proportional shape

LINAC, e−-synchrotron: 0.1 to 3 GHz, 50 Ω $\rightarrow$ differentiated shape

Important quantity: transfer impedance $Z_t(\omega, \beta)$.

Types of capacitive pick-ups:

Shoe-box (p-synch.), button (p-LINAC, e−-LINAC and synch.)

Remark: Stripline BPM as traveling wave devices are frequently used

Position reading: difference signal of four pick-up plates (BPM):

- **Non-intercepting** reading of center-of-mass → online measurement and control
  slow reading $\rightarrow$ closed orbit, ***fast bunch-by-bunch*** $\rightarrow$ trajectory
- Excitation of ***coherent betatron oscillations*** and response measurement
  excitation by short kick, white noise or sine-wave (BTF)
  $\rightarrow$ tune q, chromaticity ξ, dispersion D etc.

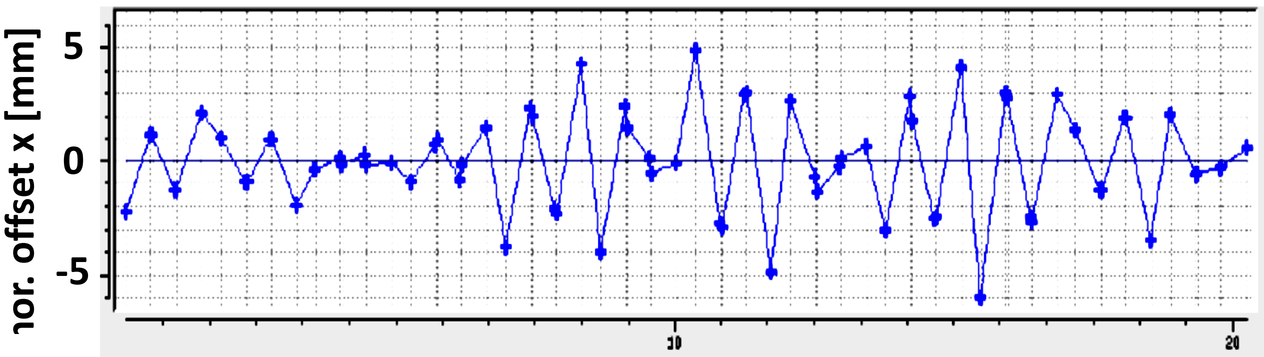
Backup slides

Closed Orbit Feedback: Results

Orbit feedback: Compensating variations of different kind, goal: $\Delta x \approx 0.1 \cdot \sigma_x$

Synchrotron light source → spatial stability of light beam

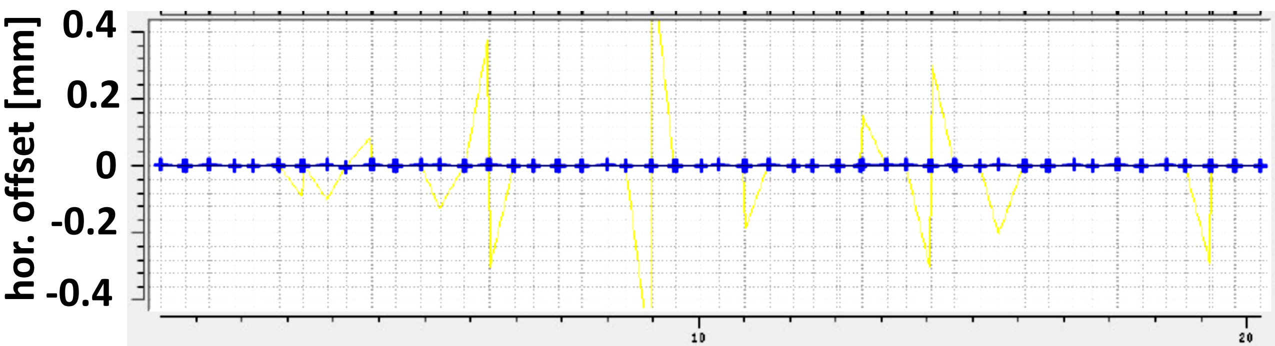
Example from SLS-Synchrotron at Villigen, Swiss:



path along synchrotron [betatron phase]

Uncorrected orbit:

Beam offset and oscillation here $\langle x^2 \rangle_{rms}$ = 2.3 mm



path along synchrotron [betatron-phase]

Corrected orbit:

Beam dynamically corrected here $\langle x^2 \rangle_{rms}$ = 1 µm !

From M. Böge, PSI