Measurement of longitudinal parameter:

- Definition of longitudinal phase space
- Proton LINAC: Determination of mean energy
- Determination of longitudinal emittance
- Bunch length measurement for non-relativistic beams
- Bunch length measurement for relativistic beams
- Summary

Longitudinal ↔ transverse correspondences:

- position relative to rf ↔ transverse center-of-mass
- bunch structure in time ↔ transverse profile in horizontal and vertical direction
- momentum or energy spread ↔ transverse divergence
- longitudinal emittance ↔ transverse emittance.

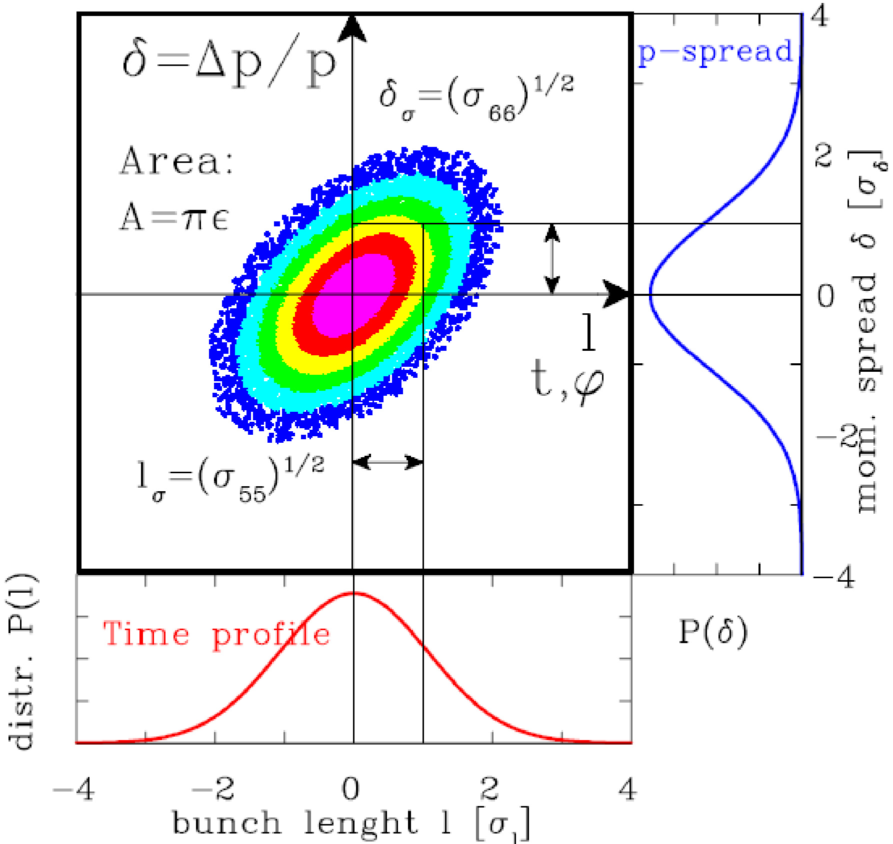
The longitudinal dynamics is described by the longitudinal emittance as given by:

- Spread of the bunches ***l***
  in time, length ***or*** rf-phase.
- Momentum spread ***δ = Δp/p***,
  or energy spread ***ΔW/W***

$$\Rightarrow \quad \varepsilon_{long} = \frac{1}{\pi}\int_A dl \cdot d\delta$$

The normalized value is preserved:

$$\varepsilon_{long}^{norm} = \beta\gamma \cdot \varepsilon_{long}$$



Discussed devices:

- Pick-ups for bunch length and emittance.
- Special detectors (low E_{kin} protons), streak cameras & ele.-optical modulation (e^-)

The Bunch Position measured by a Pick-Up

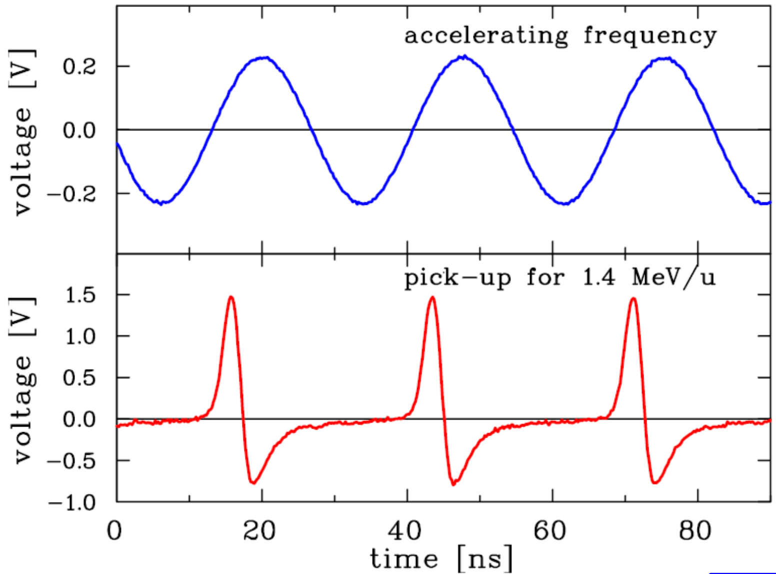
The ***bunch position*** is given relative to the accelerating rf.

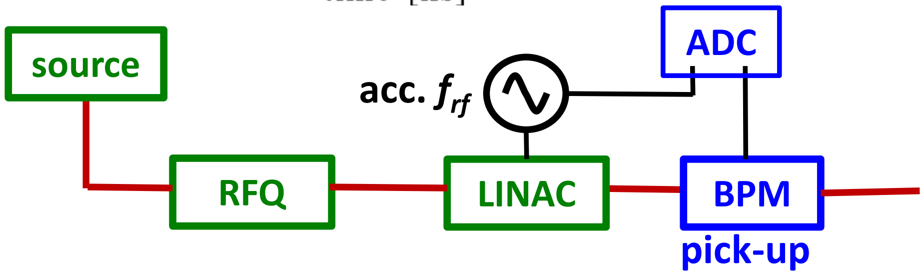
e.g. φ_{ref}=-30° inside a rf cavity

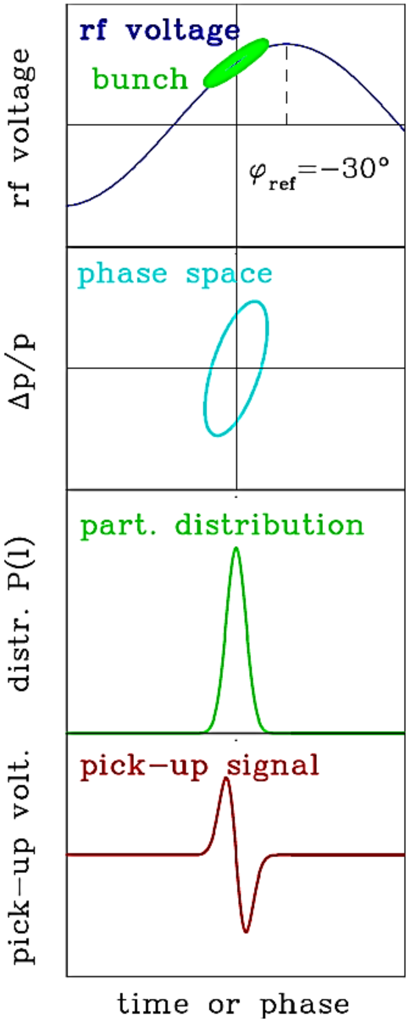
must be well aligned for optimal acceleration

Transverse correspondence: Beam position

Example: Pick-up signal for f_{rf}=36 MHz rf at GSI-LINAC:







Outline:

- Definition of longitudinal phase space
- Proton LINAC: Determination of mean energy

  used for alignment of cavities phase and amplitude
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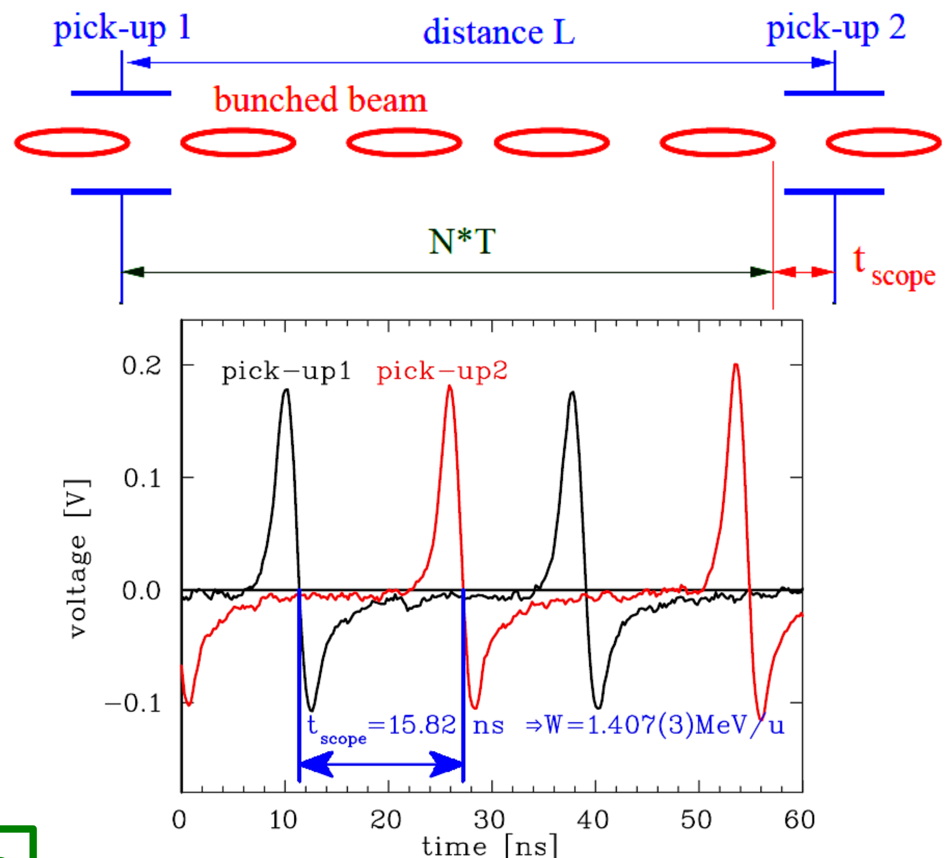
The energy delivered by a LINAC is sensitive to the mechanics, rf-phase and amplitude.

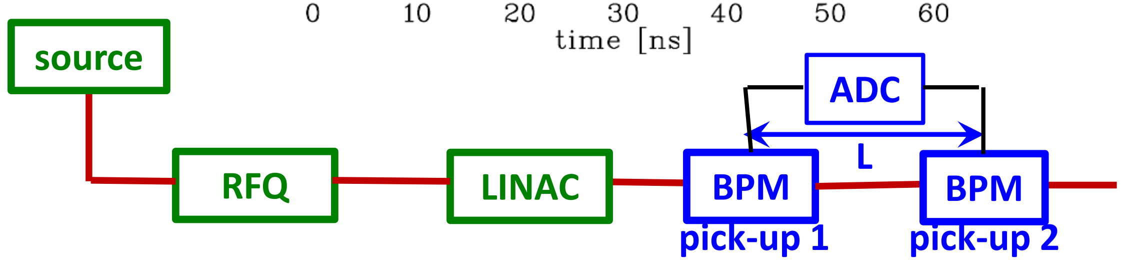
For non-relativistic energies at proton LINACs time-of-flight (TOF) with two pick-ups is used:

$$\beta c = \frac{L}{NT + t_{scope}}$$

$\rightarrow$ the velocity β is measured.



Example: Time-of-flight signal from two pick-ups at 1.4 MeV/u:
The reading is t_{scope}= 15.82(5) ns with
f_{rf} = 36.136 MHz $\Leftrightarrow$ T = 27.673 ns
L = 1.629(1) m & N = 3
$\Rightarrow$ β = 0.05497(7)
$\Leftrightarrow$ E_{kin} =1.407(3) MeV/u
The accuracy is typically 0.1 %
i.e. comparable to $\Delta p/p$



Precision of TOF Measurement for non-relativistic Energy

The precision of TOF is given by the accuracy in time and distance reading:

$$\frac{\Delta\beta}{\beta} = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta t}{NT + t_{\mathrm{scope}}}\right)^2}$$

Accuracy of scope reading $\boldsymbol{\Delta t} \approx$100 ps, uncertainty in distance $\boldsymbol{\Delta L} \approx$ 1 mm.

Example: GSI-LINAC: $\boldsymbol{L}$ = 3.25 m and $\boldsymbol{f_{rf}}$ = 36 MHz:

Location (LINAC module name)	unit	RFQ	IH1	IH2	AL4
Output energy W	MeV/u	0.12	0.75	1.4	11.4
Velocity β	%	1.6	4.0	5.5	15.5
Total time-of-flight t_{ToF}	ns	677	271	197	70
Bunch spacing β c/f_{rf}	cm	13	33	45	129
Resolution $\Delta E/E$	%	0.07	0.10	0.12	0.22

- The accuracy is typically 0.1 % (same order of magnitude as beam's $\boldsymbol{\Delta p/p}$)
- The length has to be matched to the velocity
- Due to the distance of $\approx$ 3 m, different solutions for the # of bunches ***N*** are possible

$\rightarrow$ A third pick-up has to be installed closed by, to get an unique solution.

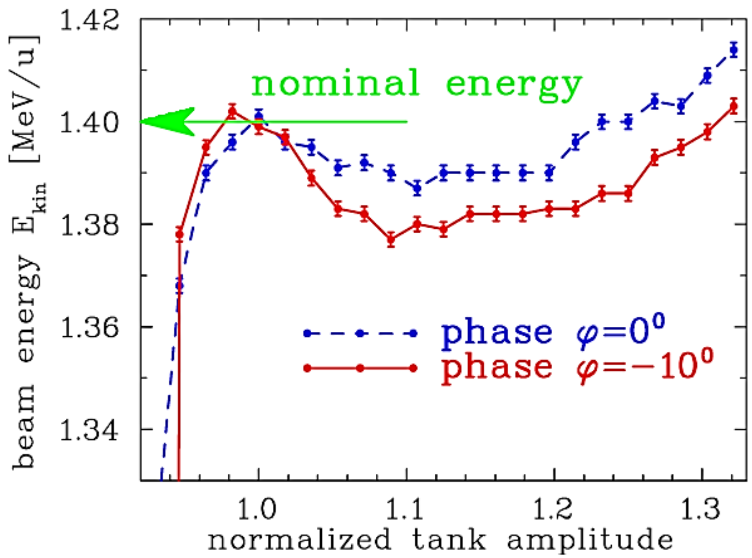
Cavity Alignment using a TOF Measurement

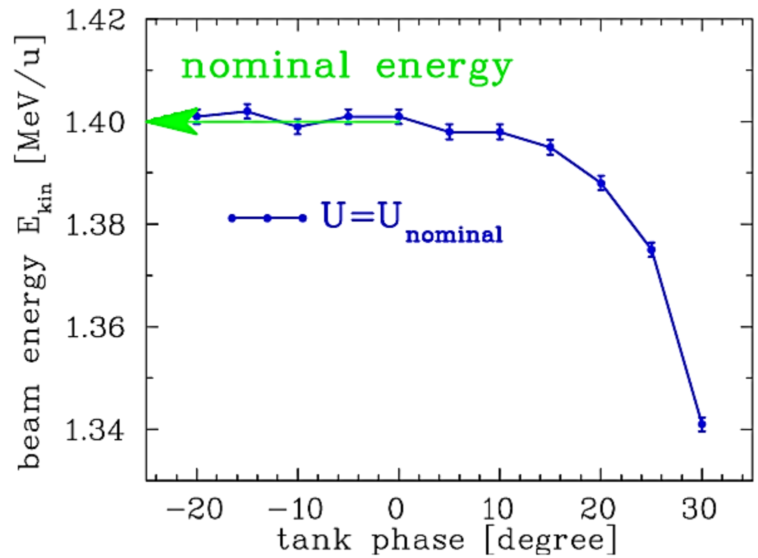
The mean energy is important for the matching between LINAC module.

It depends on phase and amplitude of the rf wave inside the cavities.

Example: Energy at GSI LINAC (nominal energy 1.400 MeV/u):

(distance between pick-ups: $\boldsymbol{L}$ = 1.97 m $\Rightarrow$ $\boldsymbol{N}$ = 4 bunches)





➢***Proton LINACs:*** Amplitude and phase should be carefully aligned by precise TOF

➢***Electron LINACs***: Due to relativistic velocity, TOF is not applicable.

Outline:

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6-dim Phase Space for Accelerators

The particle trajectory is described with the 6-dim vector $x^t = (x, x', y, y', l, \delta)$

For linear beam behavior the 6x6 transport matrix R is used:

Transformation from location s_0 to s_1 for a single particle is:

$$\vec{x}(s_1) = \mathrm{R} \cdot \vec{x}(s_0)$$

$$\vec{x}(s_1) = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & \cdots & \cdots & \cdots & \cdots \\ R_{31} & \cdots & R_{33} & R_{34} & \cdots & \cdots \\ R_{41} & \cdots & R_{43} & R_{44} & \cdots & \cdots \\ R_{51} & \cdots & \cdots & \cdots & R_{55} & R_{56} \\ R_{61} & \cdots & \cdots & \cdots & R_{65} & R_{66} \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{pmatrix}$$

Envelope i.e. emittance defined by beam matrix:

$$\sigma(s_1) = \mathrm{R} \cdot \sigma(s_0) \cdot \mathrm{R}^T$$

R separates in 3 matrices only **<u>if</u>** the transverse and longitudinal planes do **<u>not</u>** couple,

e.g. no dispersion ***D = -R₁₆ = 0***

The longitudinal beam matrix **σ** is **<u>then</u>** a 2 x 2 matrix

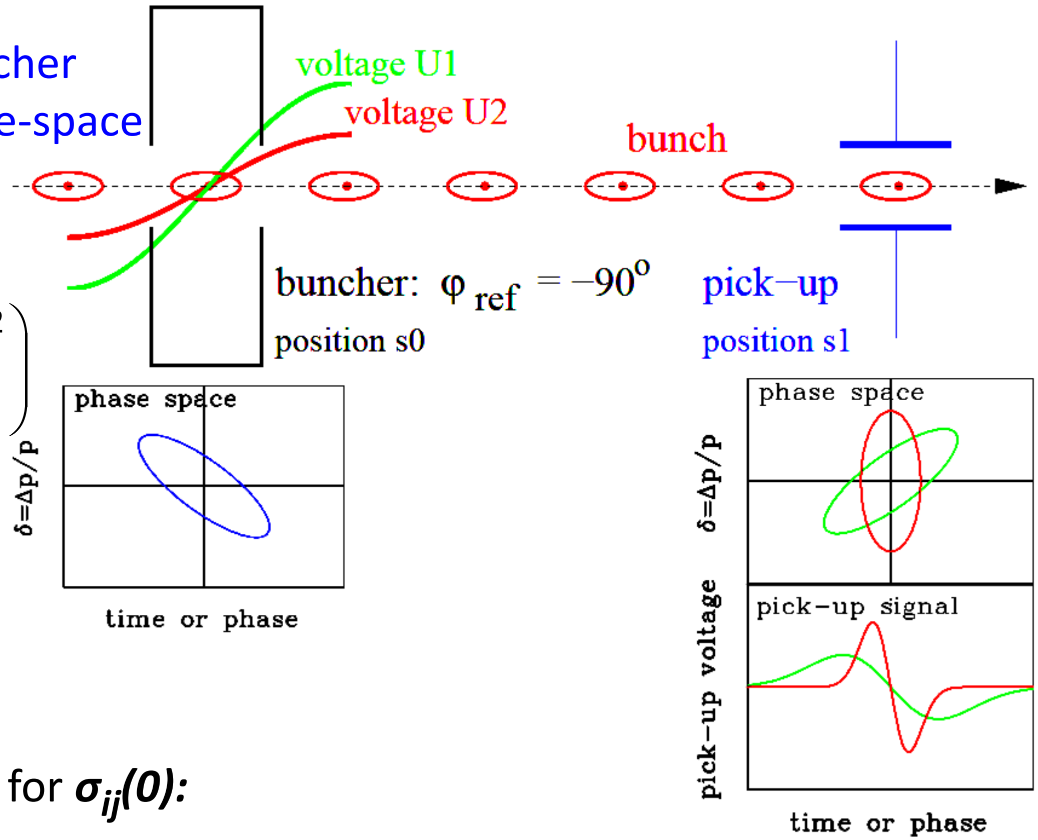
with bunch length $l_{rms} = \sqrt{\sigma_{55}}$ & momentum spread $\frac{\Delta p}{p} = \delta_{rms} = \sqrt{\sigma_{66}}$

Longitudinal focusing:

Variation of the bunch shape by a rf-buncher

$\rightarrow$ components 5 and 6 from 6-dim phase-space

Transversal corres.: Quadrupole variation



- Transfer matrix of buncher & drift:

$$R_{buncher} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}, \quad R_{drift} = \begin{pmatrix} 1 & L/\gamma^2 \\ 0 & 1 \end{pmatrix}$$

with focal length: $1/f = \dfrac{2\pi f_{rf}}{Apv^2} \cdot U$

- Variation of buncher amplitude ***U***

  $\Rightarrow$ different bunch width at s_1:

  beam matrix $\Delta t^2_{max} = \sigma_{55}(1, f)$

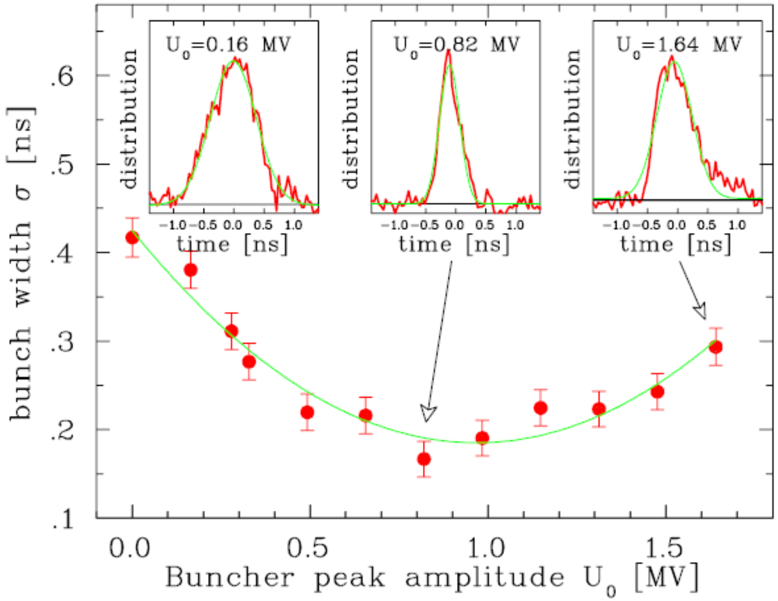
- System of redundant linear equations for $\sigma_{ij}(0)$:

$$\begin{aligned}
\sigma_{55}(1, f_1) &= R^2_{55}(f_1)\cdot\sigma_{55}(0) + 2R_{55}(f_1)R_{56}(f_1)\cdot\sigma_{56}(0) + R^2_{56}(f_1)\cdot\sigma_{66}(0) && \text{focusing } f_1 \\
&\vdots \\
\sigma_{55}(n, f_n) &= R^2_{55}(f_n)\cdot\sigma_{55}(0) + 2R_{55}(f_n)R_{56}(f_n)\cdot\sigma_{56}(0) + R^2_{56}(f_n)\cdot\sigma_{66}(0) && \text{focusing } f_n
\end{aligned}$$

Result of a longitudinal Emittance Measurement

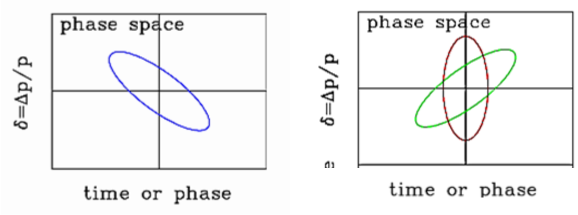
Example GSI LINAC: Voltage variation at buncher for 11.4 MeV/u Ni^{14+} beam, 31 m drift:

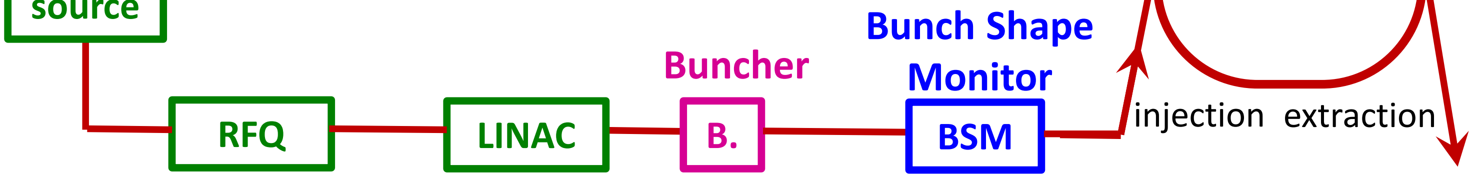
- ➢ The structure of short bunches can be determined with special monitor
- ➢ This example: The resolution is better than 50 ps or 2° for 108 MHz
- ➢ Typical bunch length at proton LINACs: $\sigma_{bunch} \approx$ 10 to 300 ps
- ➢ Determination of longitudinal emittance possible



Application for synchrotron injection:

Shaping of longitudinal phase space by buncher
i.e. long bunches ⇔ low momentum spread
to match to the synchrotron long acceptance

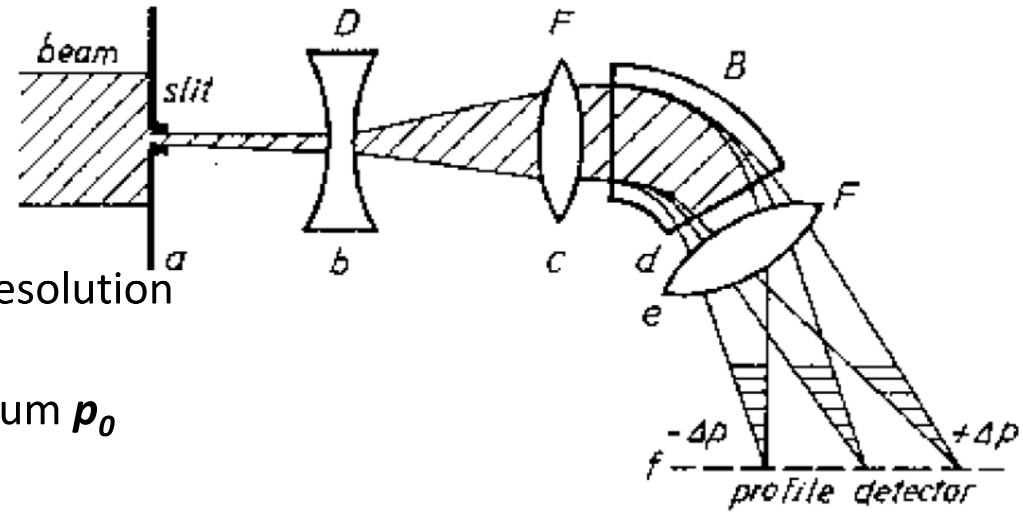




Measurement of Energy Spread by magnetic Spectrometer

Transfer line: The mom. spread $\boldsymbol{\delta = \Delta p/p}$ can be determined by a magnetic spectrometer: via dispersion, the momentum is shifted to a spatial distance.

An appropriate optic must b e chosen to separate the transverse and longitudinal parameters

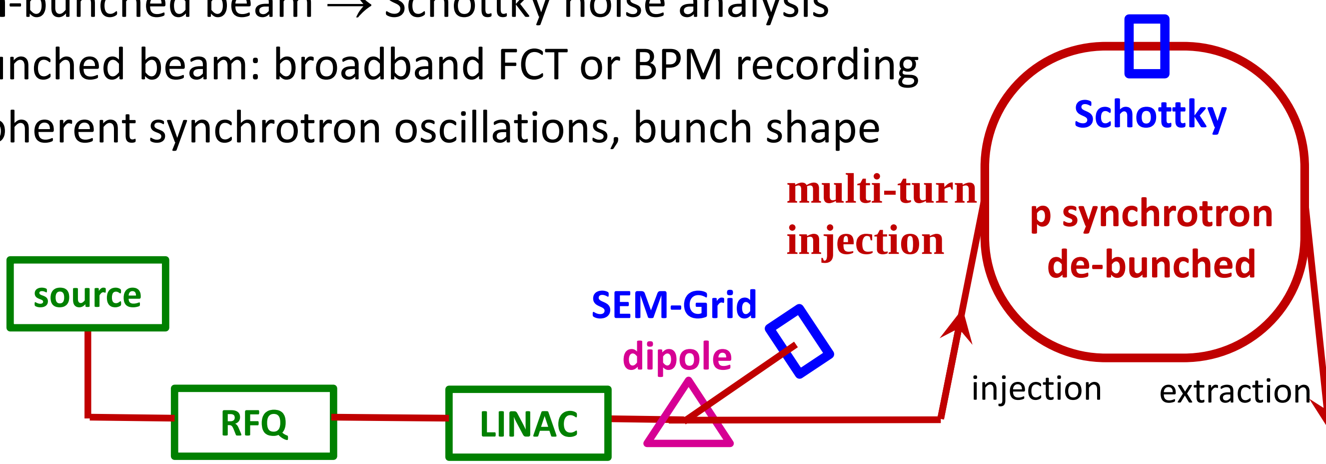


However, a synchrotron is a very high resolution spectrometer

Goal: Measurement of central momentum $\boldsymbol{p_0}$

and momentum spread $\Delta \boldsymbol{p} / \boldsymbol{p_0}$

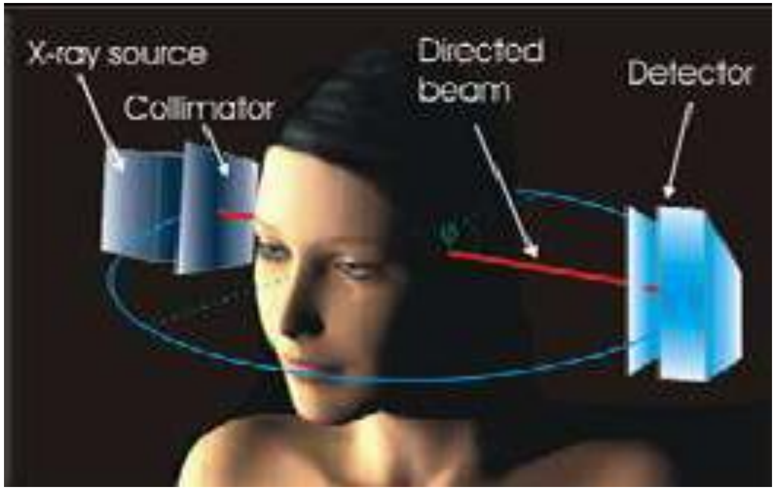
- **un**-bunched beam $\rightarrow$ Schottky noise analysis
- bunched beam: broadband FCT or BPM recording
  coherent synchrotron oscillations, bunch shape

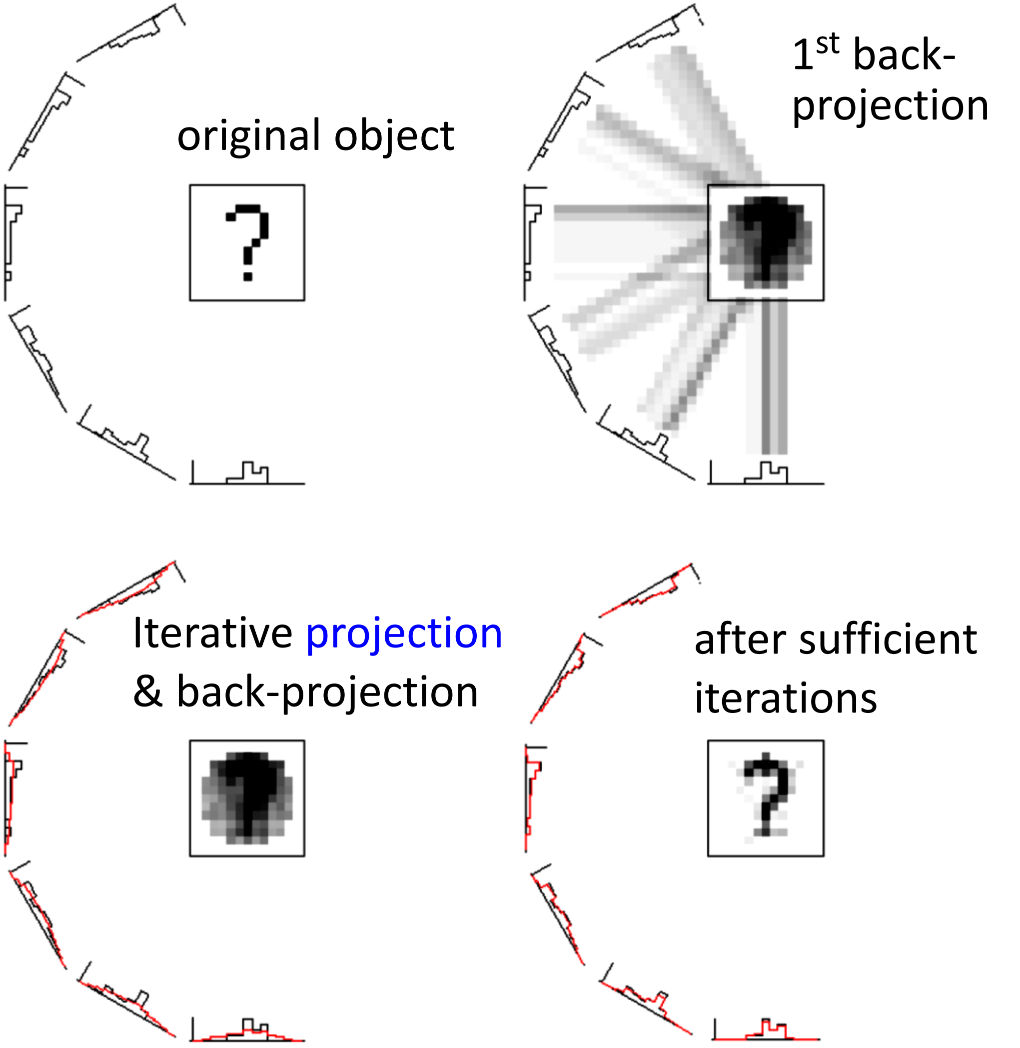


Longitudinal Emittance using tomographic Reconstruction

Tomography is medical image method

Tomography:
2-dim reconstruction of
sufficient 1-dim projections





Algebraic back projection:

Iterative process by redistributing the 2-dim image and considering the differences to the previous iteration step.

Longitudinal Emittance using tomographic Reconstruction

Tomography is medical image method

Tomography:

2-dim reconstruction of
sufficient 1-dim projections

Application at accelerators:

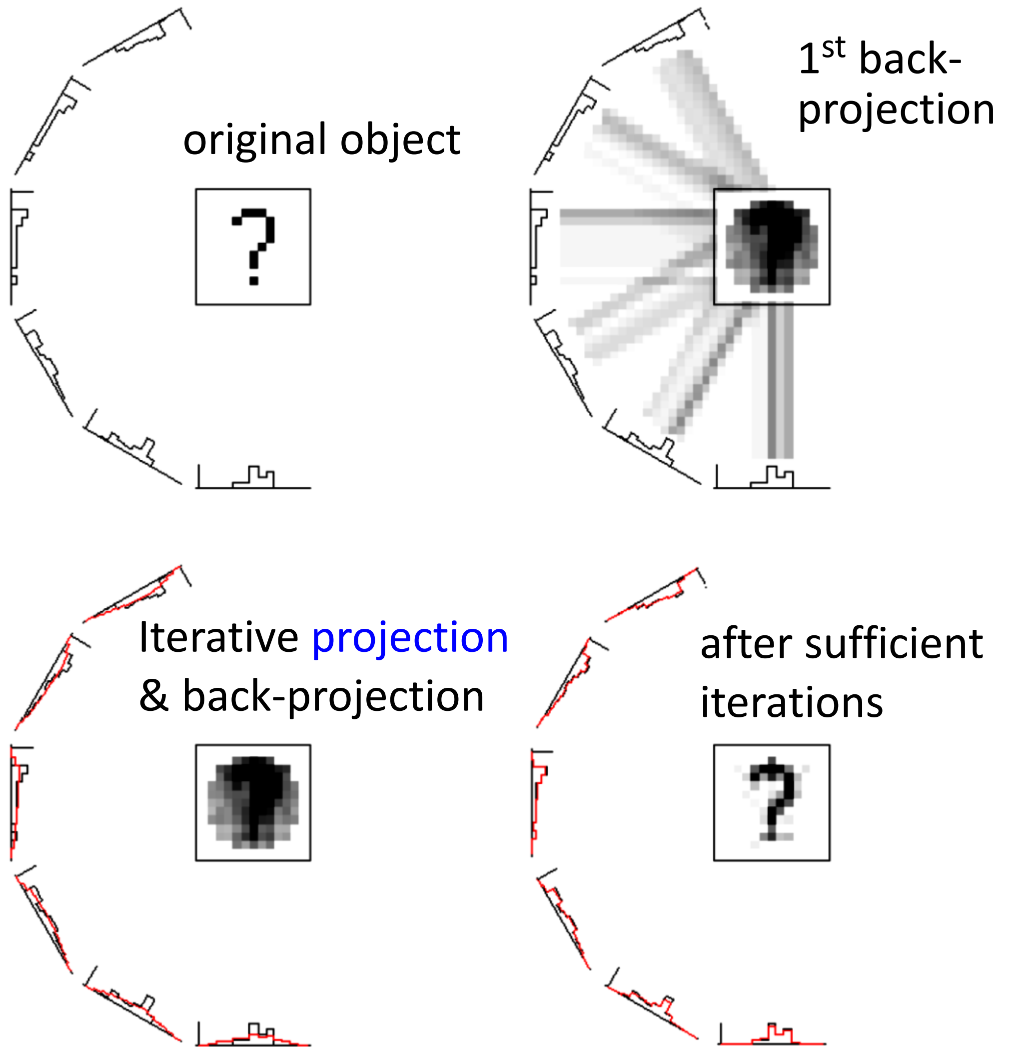
Longitudinal emittance
evolution in synchrotrons.

Bunch observation:

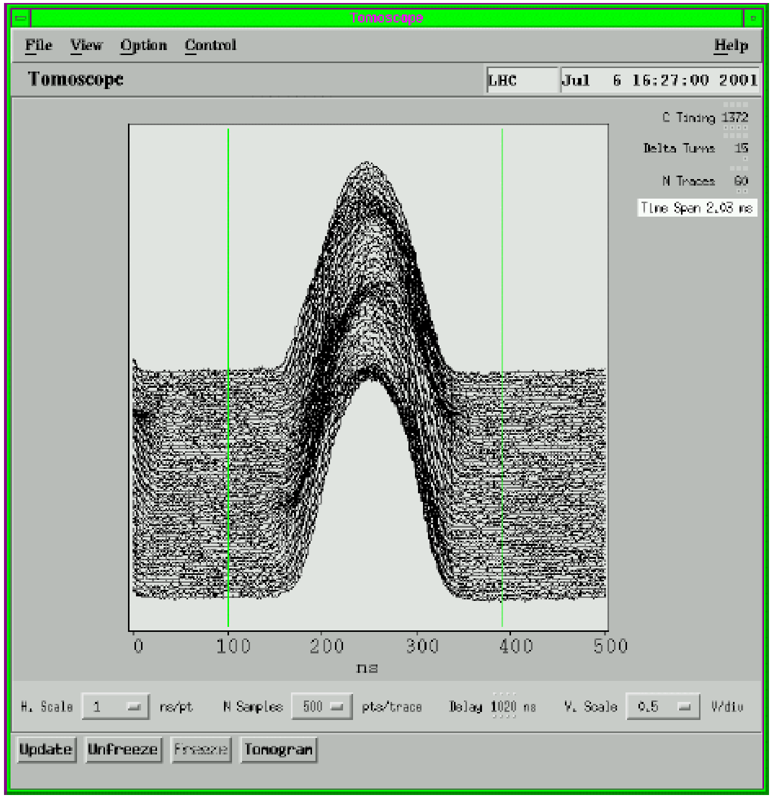
Each revolution,
the bunch shape changes a bit
due to synchrotron oscillations.
Fulfilled condition: $f_{synch} << f_{rev}$.

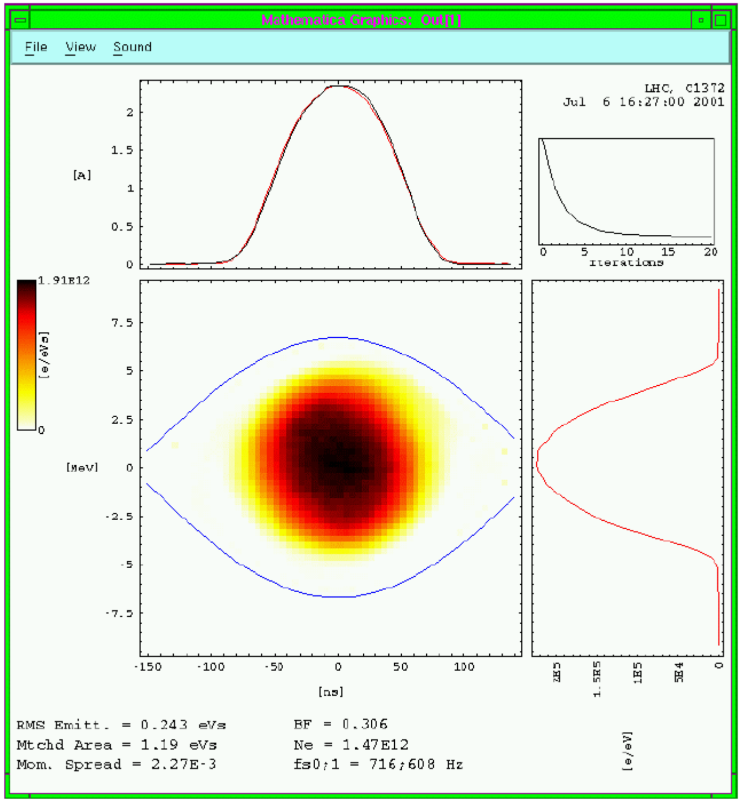
Algebraic back projection:

Iterative process by redistributing
the 2-dim image and considering the
differences to the previous iteration step.



Bunches from 500 turns at the CERN PS and the phase space for the first time slice, measured with a wall current monitor:

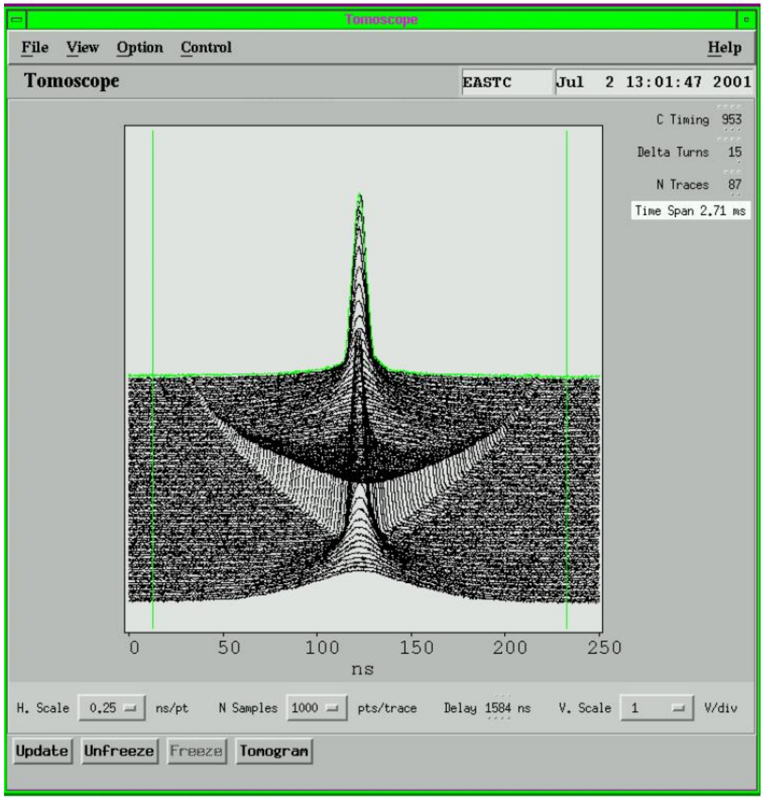


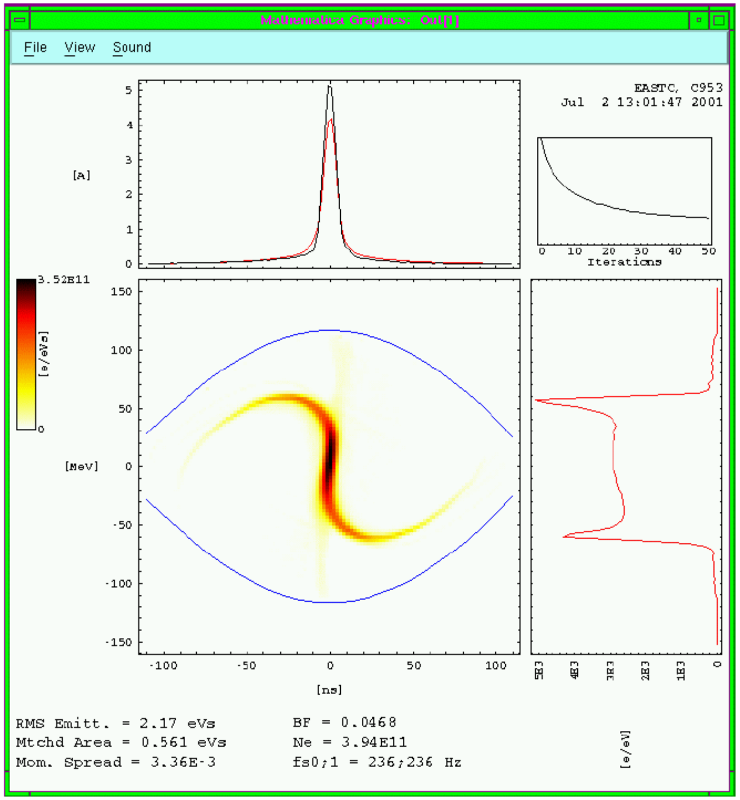


Typical bucket filling. Important knowledge for bunch ’gymnastics’.

Results of tomographic Reconstruction at a Synchrotron II

Bunches from 500 turns at the CERN PS and the phase space for the first time slice, measured with a wall current monitor:





Mismatched bunch shown oscillations and filamentation due to 'bunch-rotation'.

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- **Bunch length measurement for non-relativistic beams**

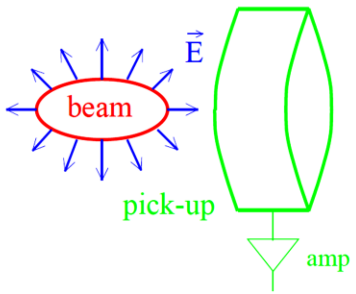
  Determination of particle arrival
- **Bunch length measurement for relativistic beams**
- **Summary**

Bunch Structure at low E_{kin}: Not possible with Pick-Ups

Pick-ups are used for:
- precise for bunch-center relative to rf
- course image of bunch shape

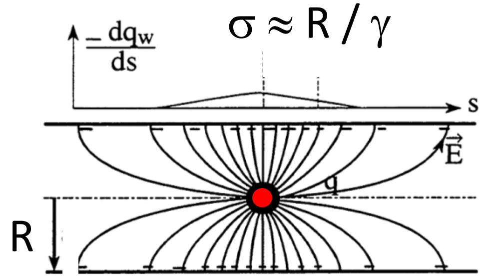
But:

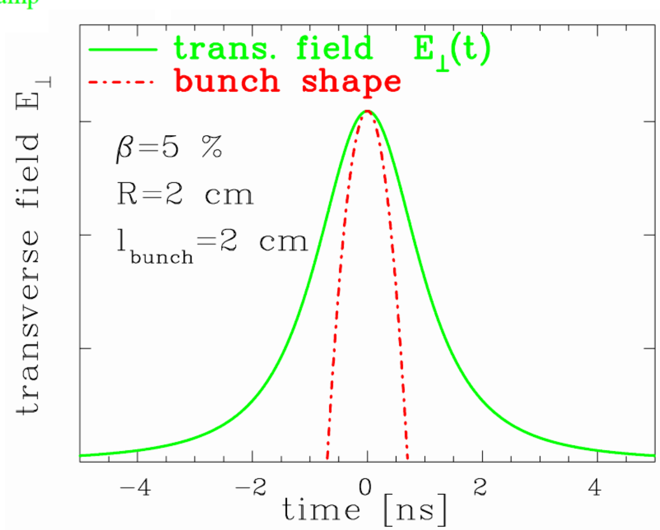
For $\beta \ll 1 \rightarrow$ long. E-field significantly modified:



ampl.

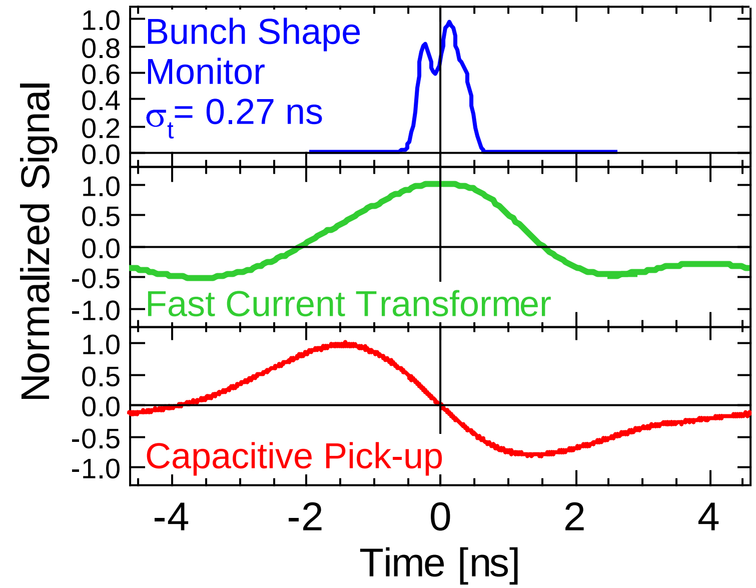
$\sigma \approx R / \gamma$





Example: Comparison pick-up – particle counter:

Ar beam of 1.4 MeV/u (β = 5.5%) , f_{rf} = 108 MHz



1/ f_{rf} = 9.2 ns

$\Rightarrow$ the pick-up signal is insensitive to bunch 'fine-structure'

Low Velocity Effect: General Consideration

Lorentz transformation of single point-like charge:

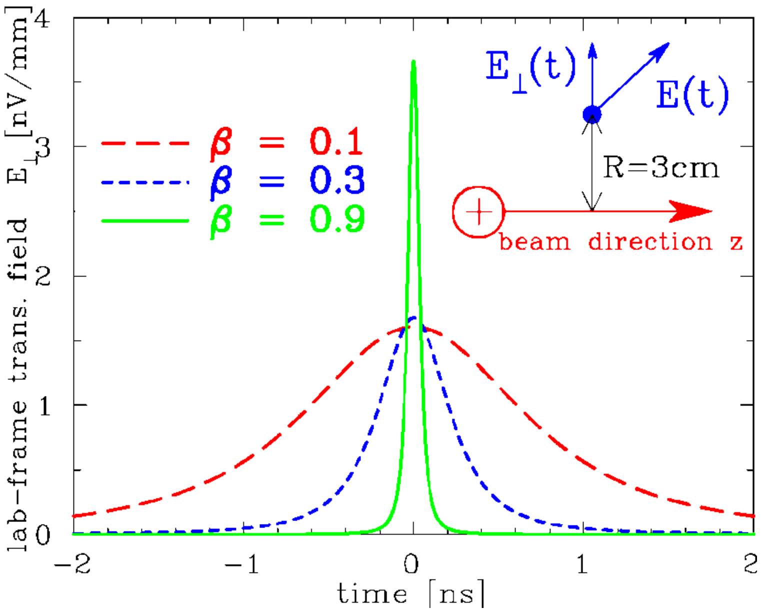
Lorentz boost ***and*** transformation of time: $E_\perp(t) = \gamma \cdot E'_\perp(t')$ and $t \rightarrow t'$

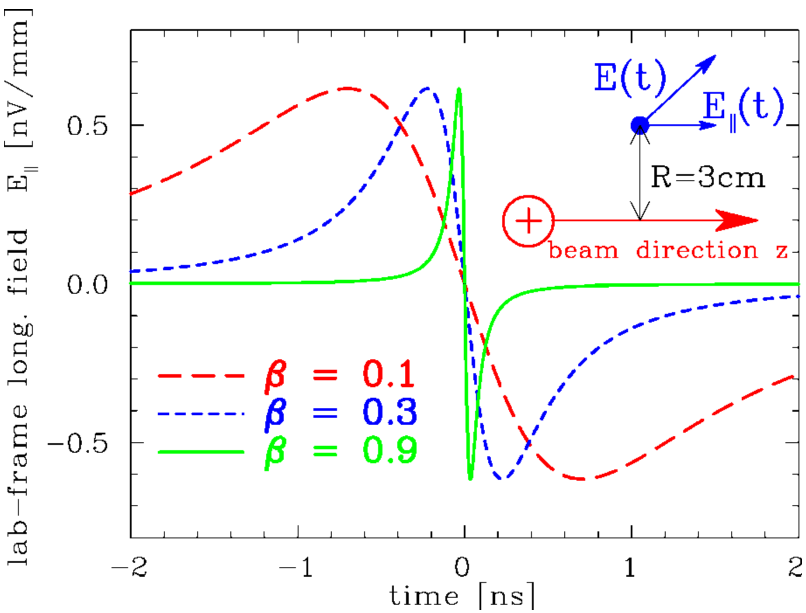
Trans. $\boldsymbol{E_\perp}$ lab.-frame of a point charge:

$$E_\perp(t) = \frac{e}{4\pi\varepsilon_0} \cdot \frac{\gamma R}{\left[R^2 + (\gamma\beta ct)^2\right]^{3/2}}$$

Long. $\boldsymbol{E_{||}}$ lab.-frame of a point charge:

$$E_\parallel(t) = -\frac{e}{4\pi\varepsilon_0} \cdot \frac{\gamma\beta ct}{\left[R^2 + (\gamma\beta ct)^2\right]^{3/2}}$$





Broadband coaxial Faraday Cups for Bunch Structure

The bunch structure can be observed with cups, having a bandwidth up to several GHz.

Bandwidth and rise time: BW [GHz] = **0.3/t_{rise}**[ns]

Impedance of a

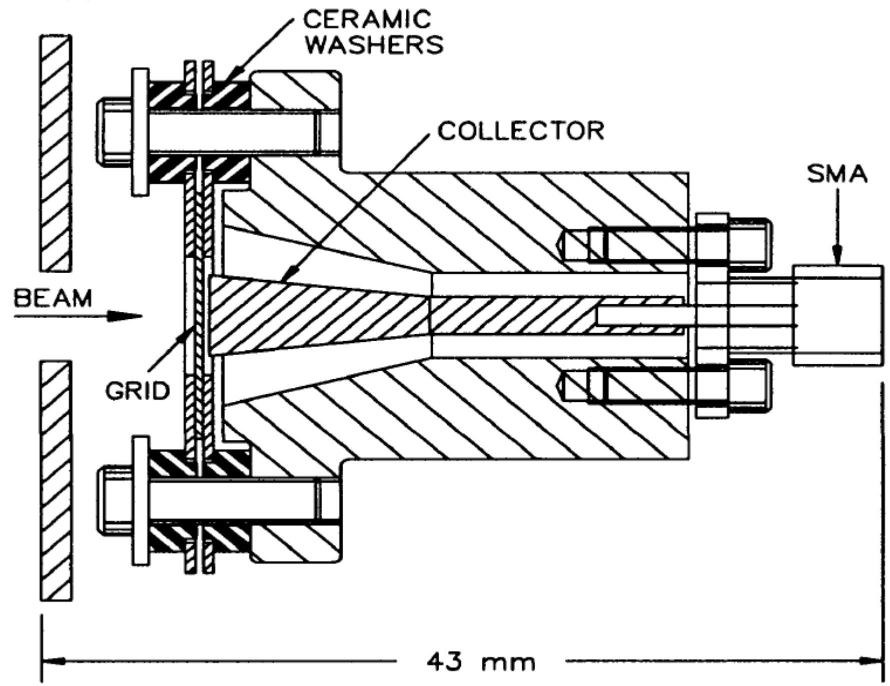
coaxial transmission line:

$$Z_0 = \frac{Z_c}{2\pi} \cdot \ln \frac{r_{\text{shield}}}{r_{\text{coll}}}$$

$$\text{with} \;\; Z_c = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}}$$

$$\text{for vacuum} \;\; Z_C = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\,\Omega$$

$\rightarrow$ impedance matching to prevent for reflections



Voltage reflection: $\rho_V = \dfrac{Z - Z_0}{Z + Z_0}$

Voltage Standing Wave Ratio: $\text{VSWR} = \dfrac{Z}{Z_0} = \dfrac{1+\rho_V}{1-\rho_V}$

***Z* = *Z*$_0$**: no reflection. ***Z* = 0** $\Rightarrow$ **ρ_V** = −1: short circuit. ***Z* = ∞** $\Rightarrow$ **ρ_V** = 1: open circuit.

Realization of a Broadband coaxial Faraday Cup

Bunch Structure using secondary Electrons for low E_{kin} Protons

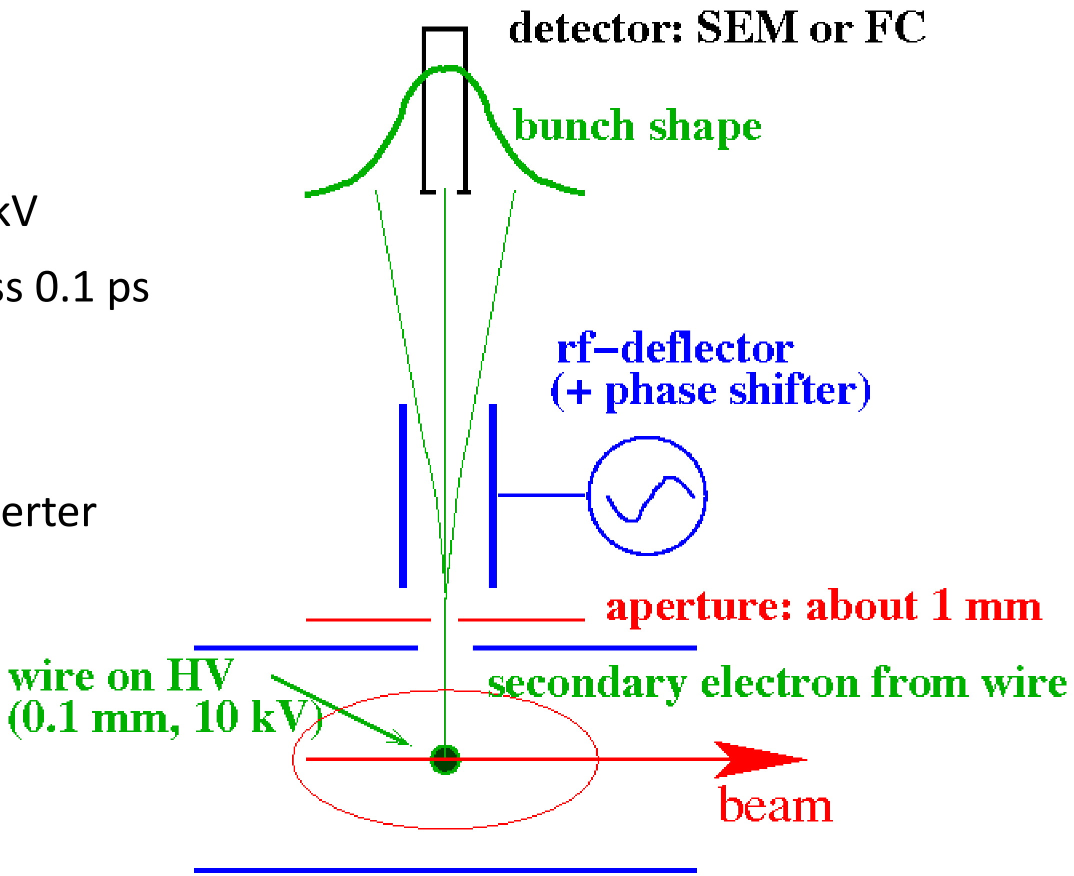
Secondary e⁻ liberated from a wire carrying the time information.

→ Bunch Shape Monitor (BSM)

Working principle:

- insertion of a 0.1 mm wire at ≈ 10 kV
- emission of secondary e⁻ within less 0.1 ps
- secondary e⁻ are accelerated
- toward an rf-deflector
- rf-deflector as 'time-to-space' converter
- detector with a thin slit
- slow shift of the phase
- resolution ≈1° < 10 ps
- Measurements are comparable

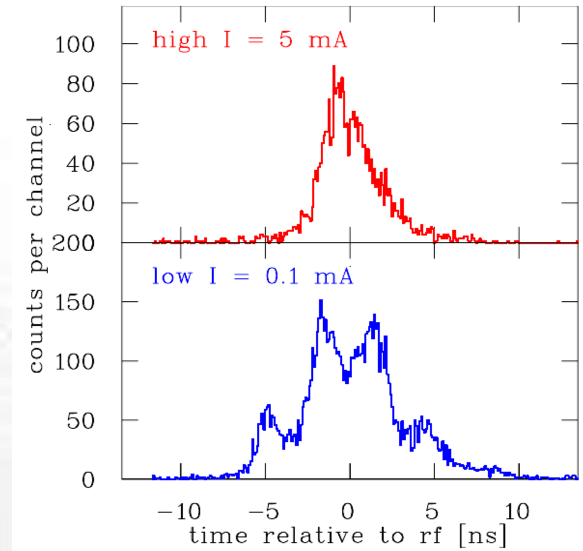
  to that obtained with particle detectors.

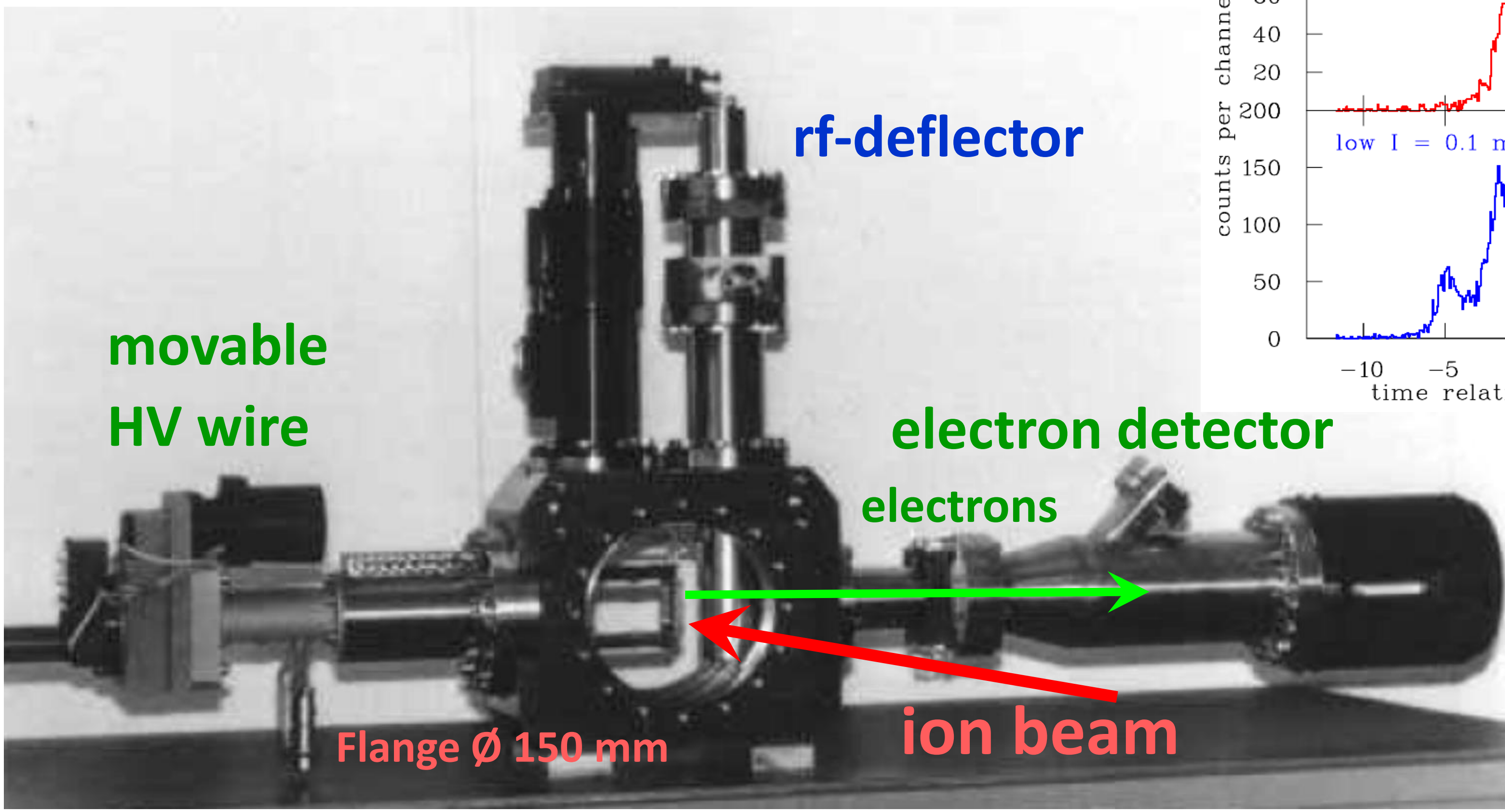


SEM: secondary electron multiplier

Realization of Bunch Shape Monitor at CERN LINAC2

Example: The bunch shape at 120 keV/u for 120 keV/u:





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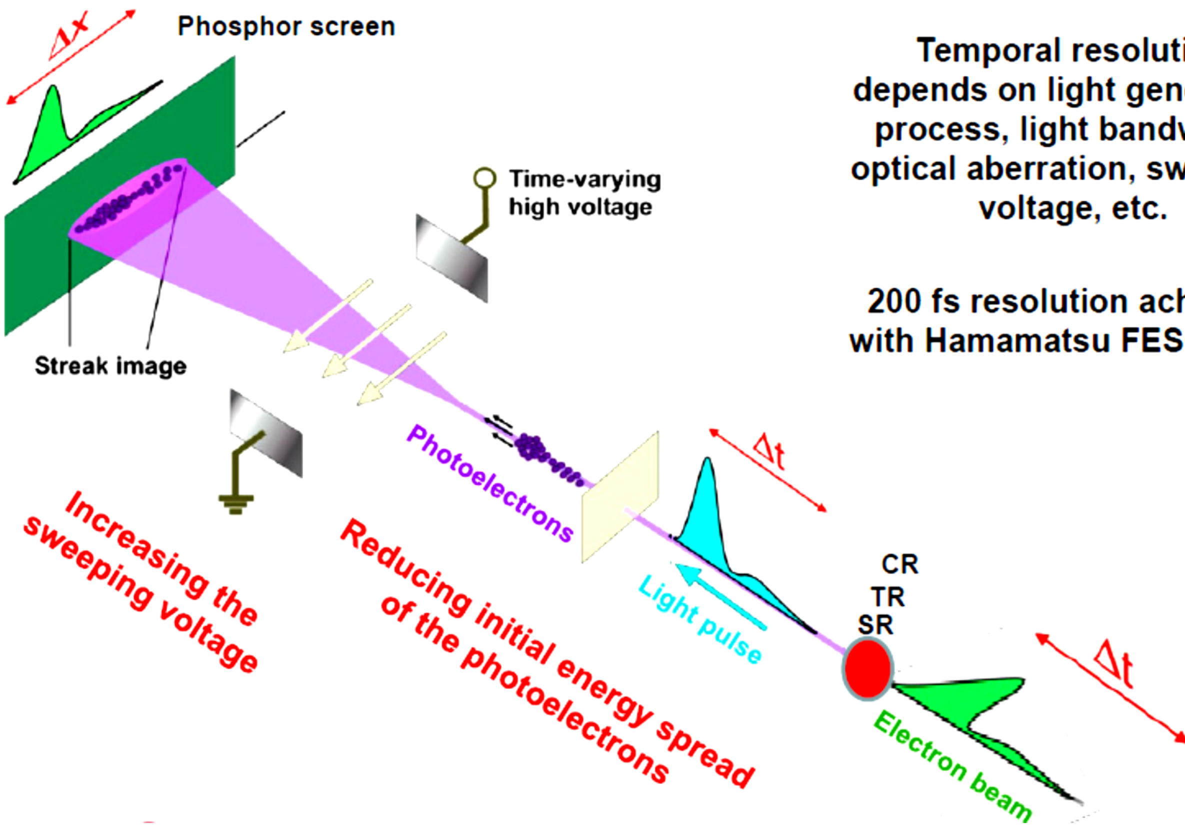
  Synchrotron light monitor and electro-optical modulation of a laser beam
- **Summary**

Bunch Length Measurement for relativistic e$^-$

Electron bunches are too short (σ_t < 300 ps) to be covered by the bandwidth of pick-ups (f < 1 GHz $\Leftrightarrow$ t_{rise} >300 ps) for structure determination.

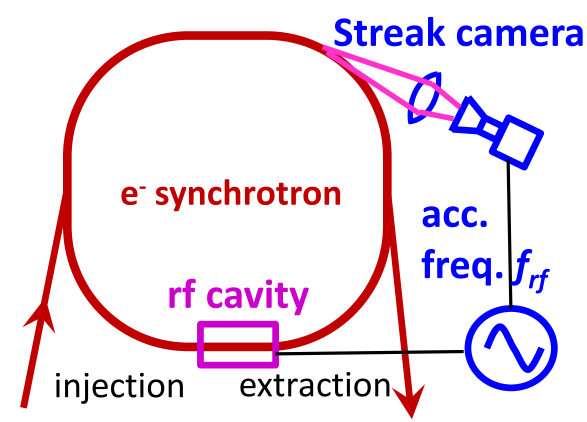
$\rightarrow$ Time resolved observation of synchr. light with a streak camera: Resolution $\approx$ 1 ps.



Temporal resolution depends on light generation process, light bandwidth, optical aberration, sweeping voltage, etc.

200 fs resolution achieved with Hamamatsu FESCA-200

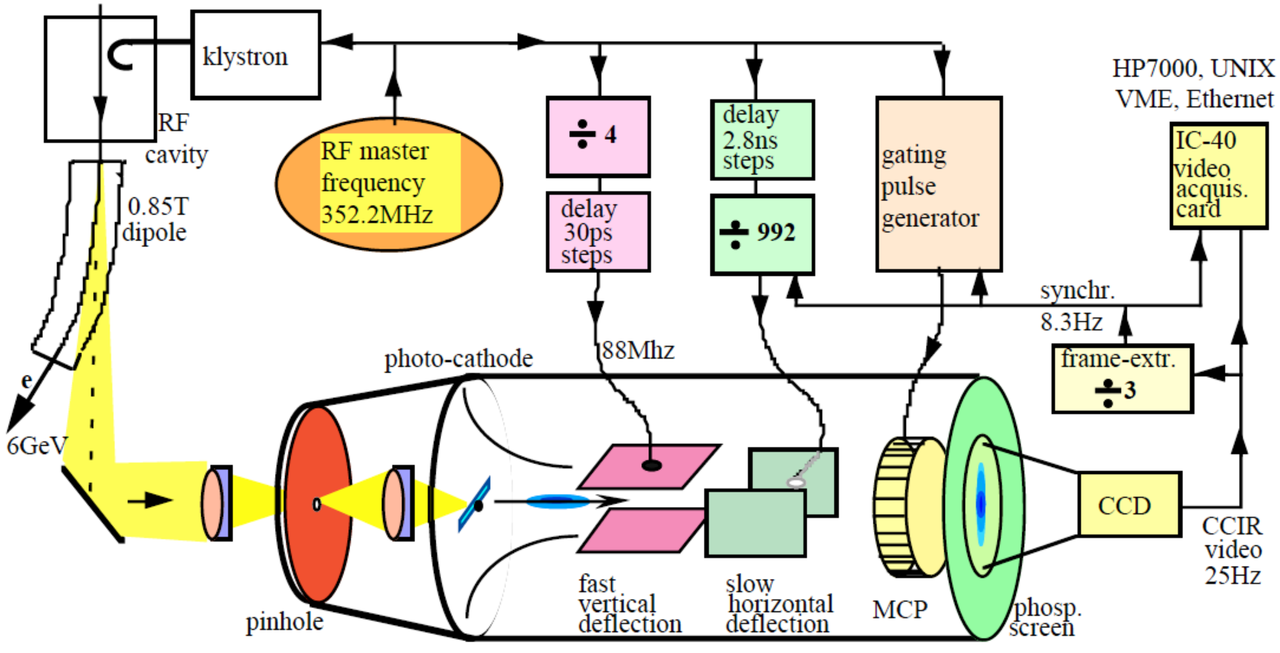
From D. Xiang, IPAC'12

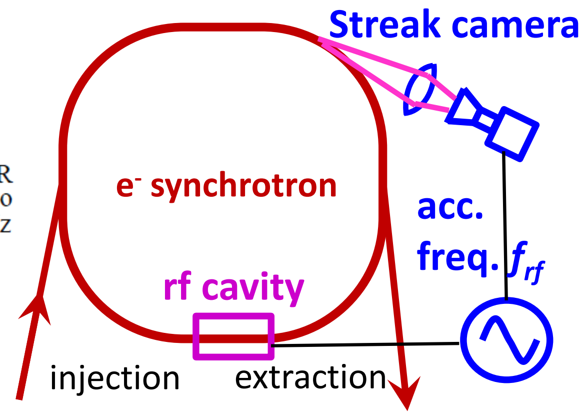


Electron bunches are too short (σ_t < 100 ps) to be covered by the bandwidth of pick-ups (f < 3 GHz ⇔ t_{rise} > 100 ps) for structure determination.

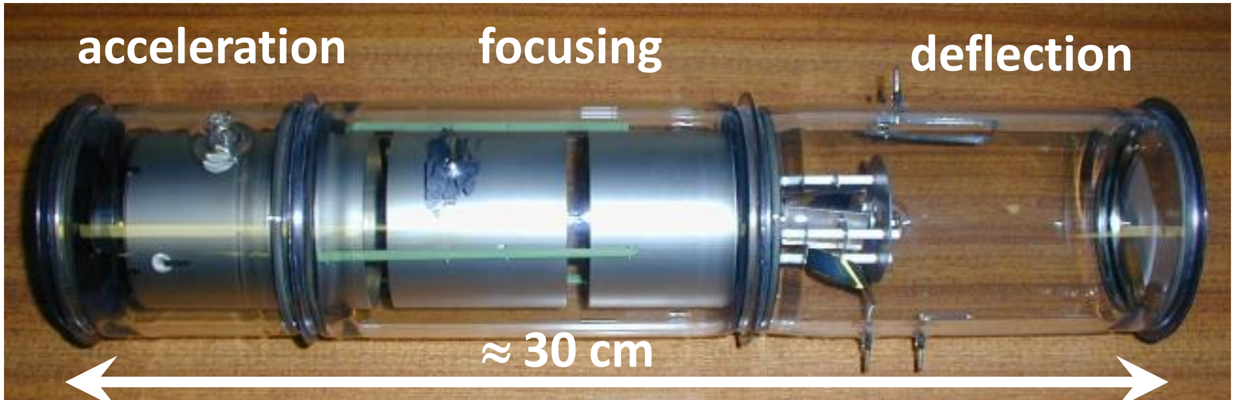
→ Time resolved observation of synchr. light with a streak camera: Resolution ≈ 1 ps.

Scheme of a streak camera:



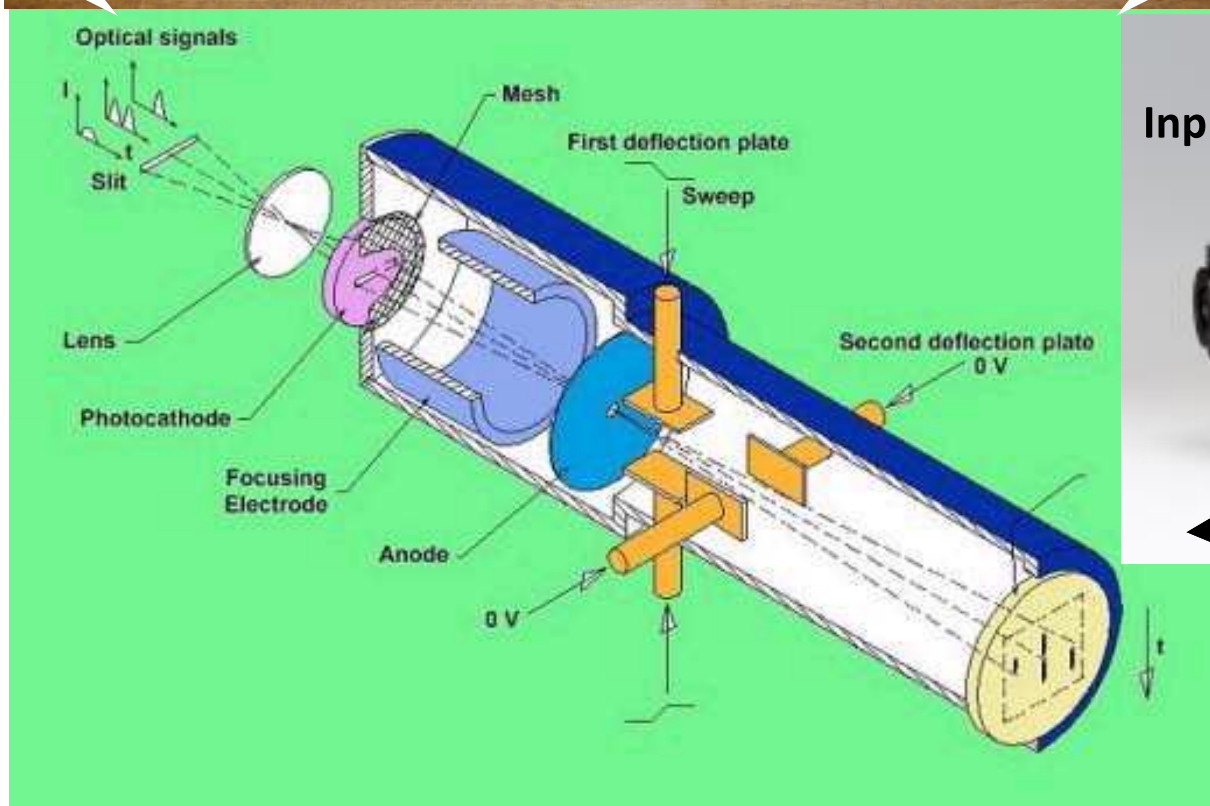


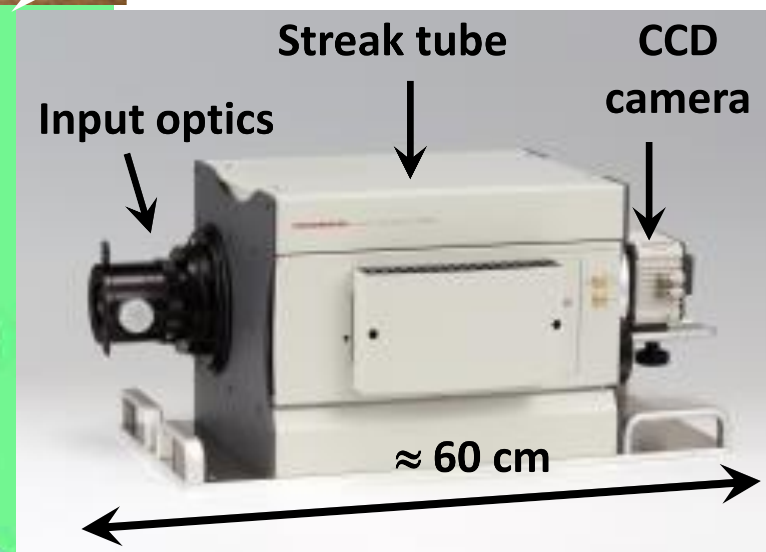
Technical Realization of Streak Camera



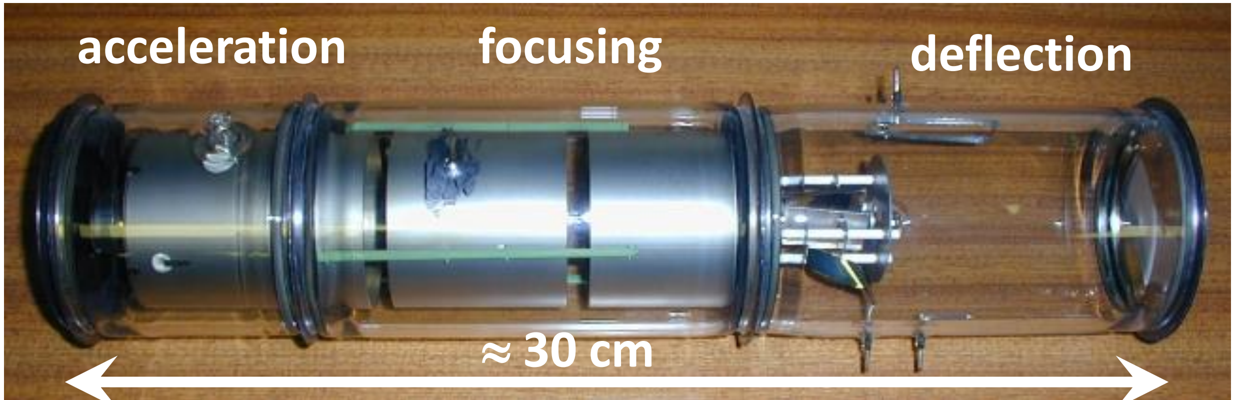
Hardware of a streak camera

Time resolution down to 0.5 ps:



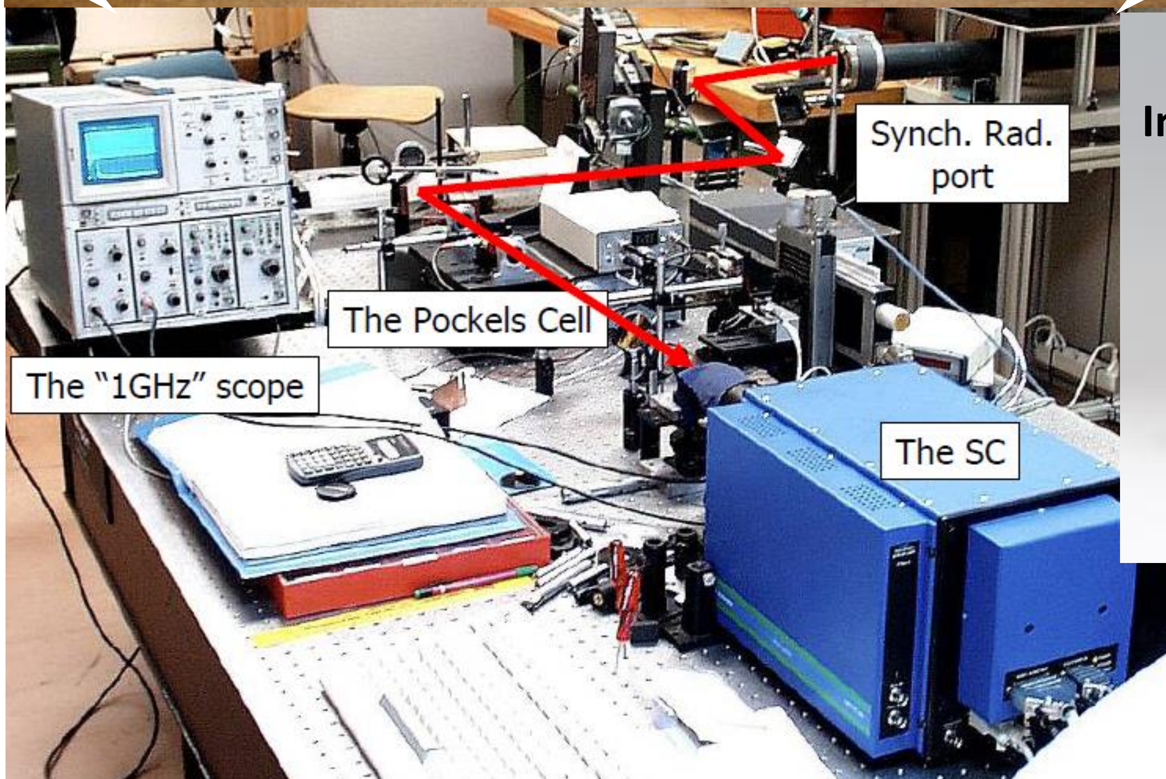


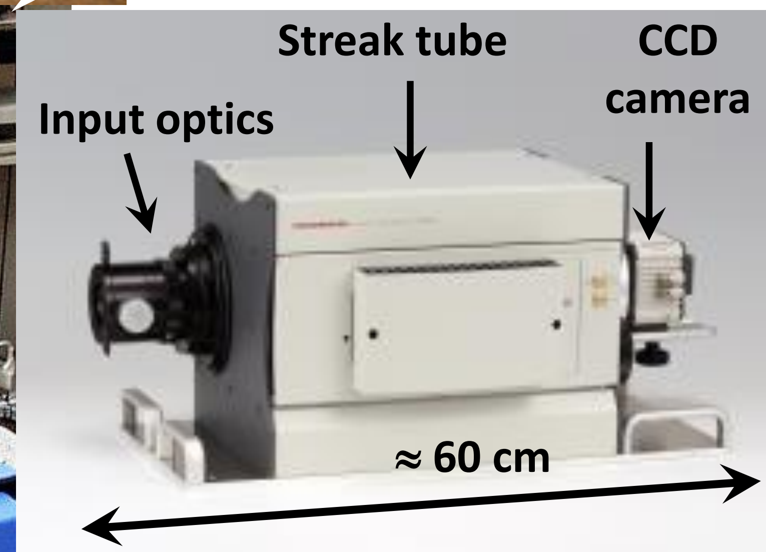
Technical Realization of Streak Camera



Hardware of a streak camera

Time resolution down to 0.5 ps:





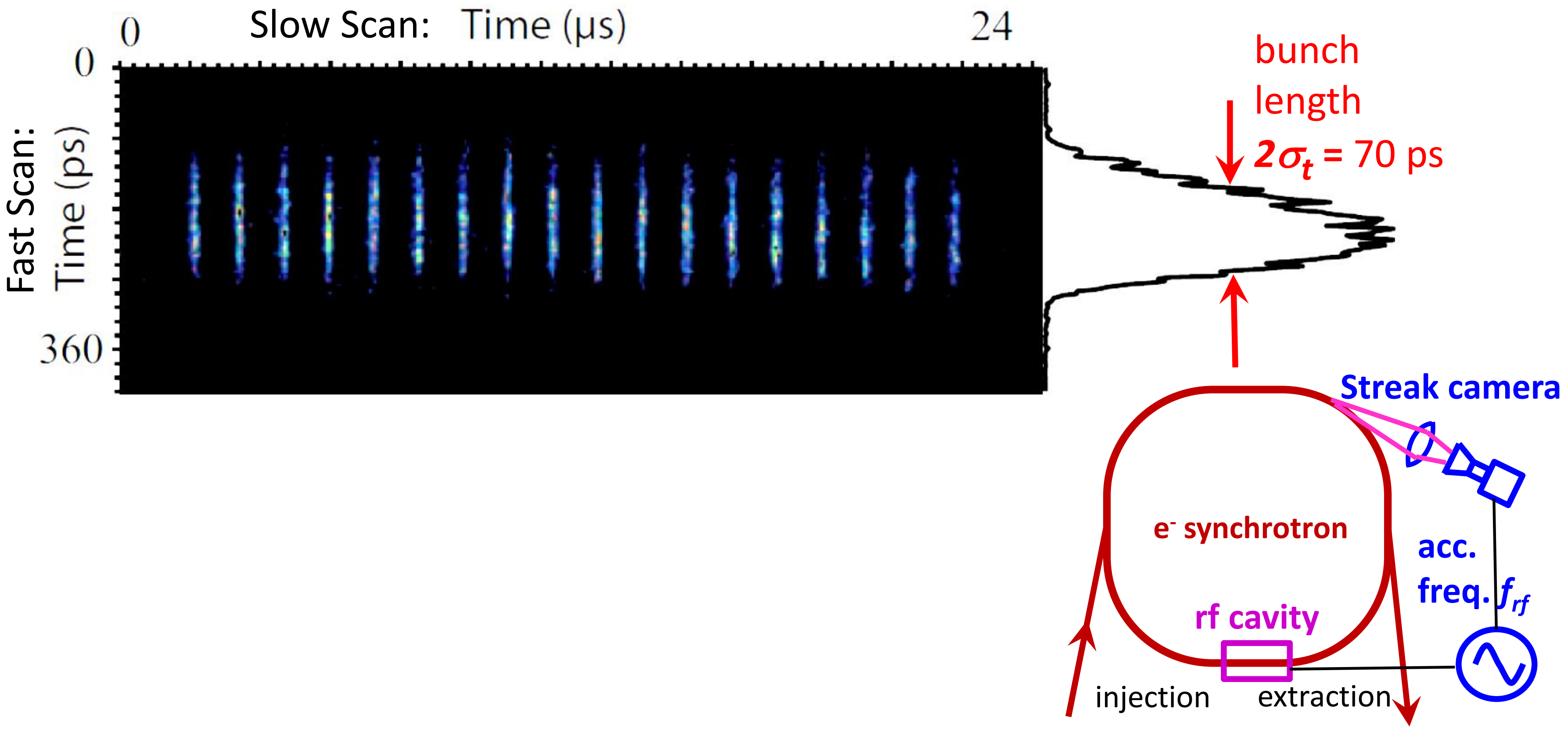
Results of Bunch Length Measurement by a Streak Camera

The streak camera delivers a fast scan in vertical direction (here 360 ps full scale)

and a slower scan in horizontal direction (24 µs).

Example: Bunch length at the synchrotron light source SOLEIL for U_{rf} = 2 MV

for slow direction 24 µs and scaling for fast scan 360 ps: measure σ_t = 35 ps.



Courtesy of M. Labat et al., DIPAC'07

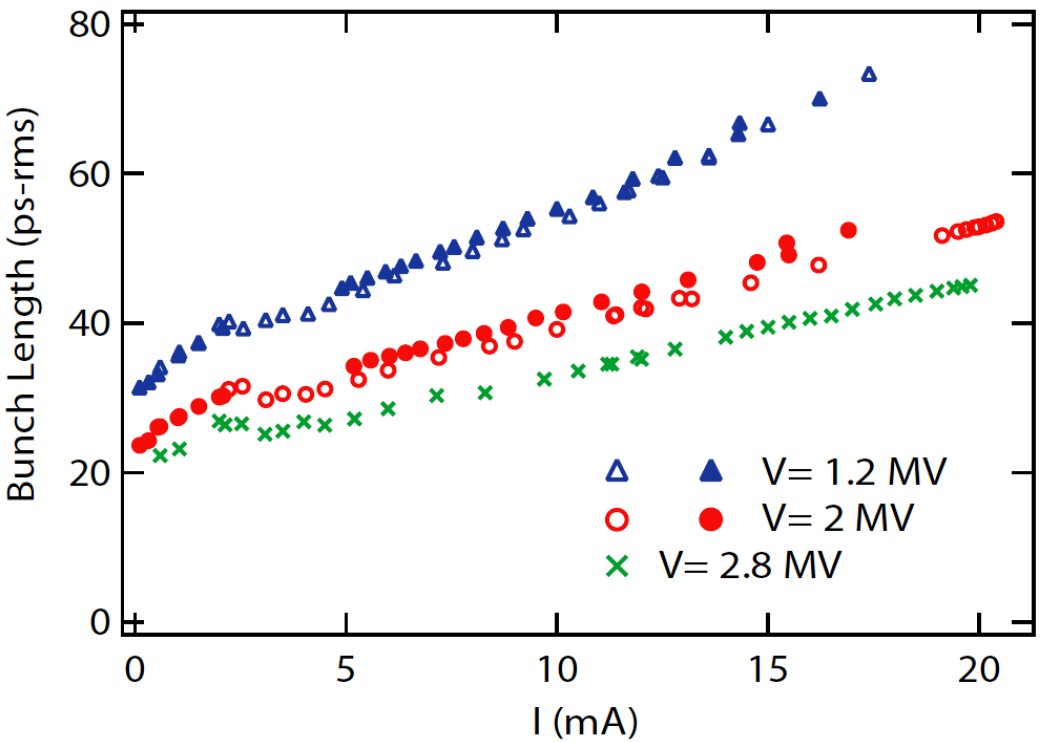
The Importance of Bunch Length by Streak Camera

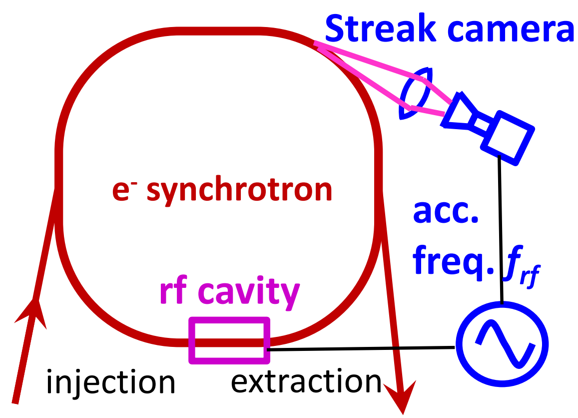
Short bunches are desired by the synchrotron light users for time resolved spectroscopy.

The bunch focusing is changed by the rf-amplitude.

Example: Bunch length σ_t as a function of stored current

(space-charge de-focusing, impedance broadening) for different rf-amplitudes at SOLEIL:





The Artist View of a Streak Camera

FARADAY CUP 1998

Purpose. To recognize and encourage innovative achievements in the field of accelerator beam instrumentation.

Award. The Faraday Cup Award consists of a US$ 5000 prize and a certificate to be presented at the next Beam Instrumentation Workshop. Winners participating in the BIW will be given a $1000 travel allowance.

Eligibility. Nominations are open to contributors of all nations regardless of the geographical location at which the work was done.
The Award goes normally to one person, but may be shared by recipients having contributed to the same accomplishment. It will normally be awarded to scientists in the early stage of their career. Nominations of candidates shall remain active for 2 competitions.

Establishment and support. The Award was established in 1991 with the support of the Beam Instrumentation Workshop Organizing Committee.

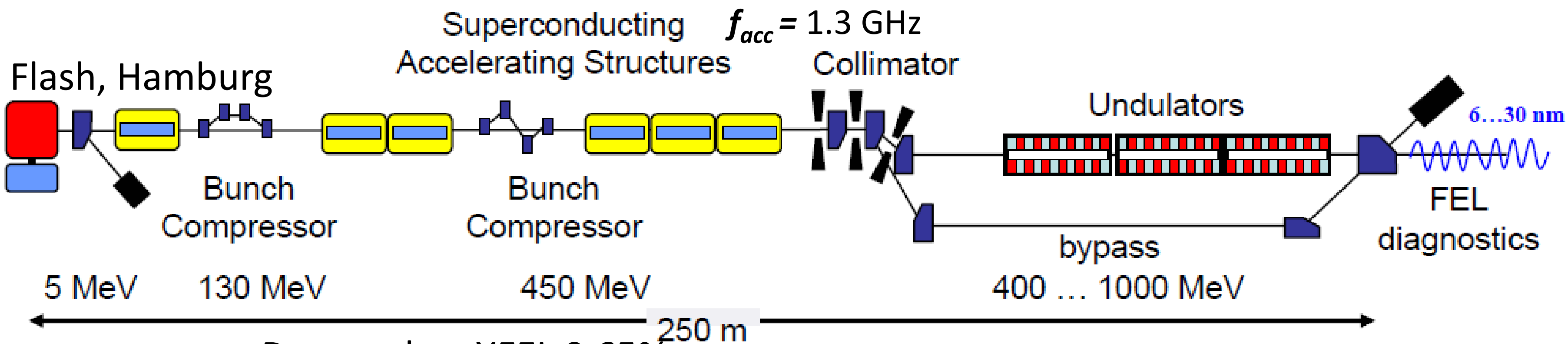
Rules. The Faraday Cup shall be awarded for an outstanding contribution to the development of an innovative beam diagnostics instrument of proven workability. The Faraday Cup is only awarded for published contribution and delivered performance -as opposed to theoretical performance. Rules are available on request.

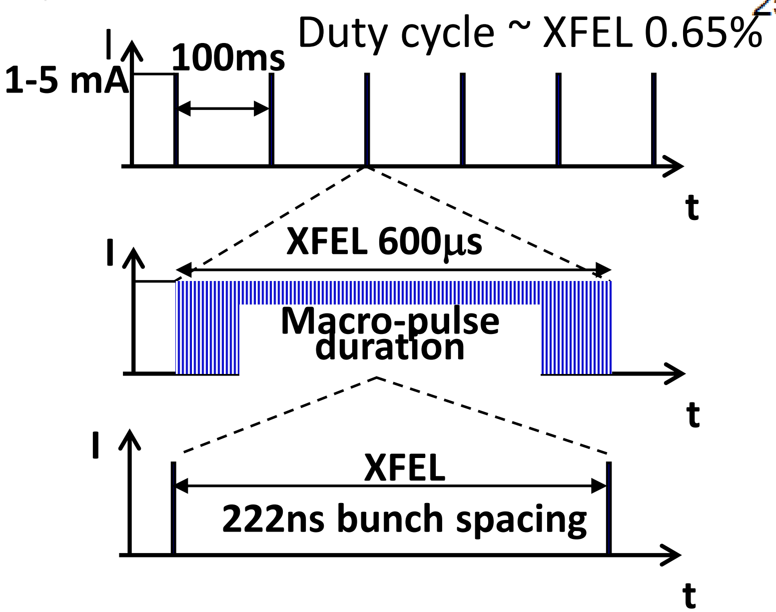
Award Committee. The Beam Instrumentation Workshop Organizing Committee.

Nominations. The nomination package shall include the name of the candidate, relevant publications, a statement outlining his/her personal contribution and that of others, two letters from co-workers familiar with the candidate and his contribution. Two master copies suitable for photocopying of this package must be submitted not later than the 15th of November 1997 to Steven Smith c/o BIW'98 Secretariat, SLAC, Stanford University, Stanford CA 94305-4085, U.S.A..

4th Generation Light Sources: LINAC based, single pass with large energy loss

$E_{electron} \approx 1 \ldots 18$ GeV, **coherent** light from undulator, E_γ < 1000 keV, temporally short pulse



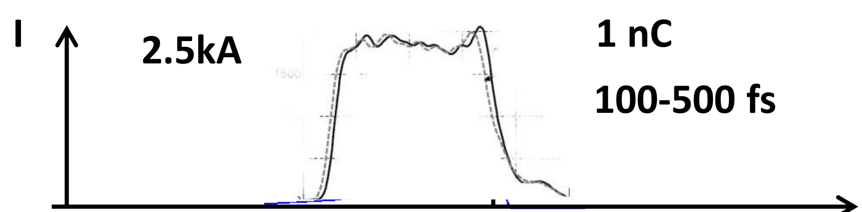


Goal: Short bunches with **high** number of particles

→ short, intense laser pulses for electron generation

Requirement: Position stability ⇒ resolution < 1 µm

Single bunch duration < 1 ps



Bunch Length Measurement by electro-optical Method

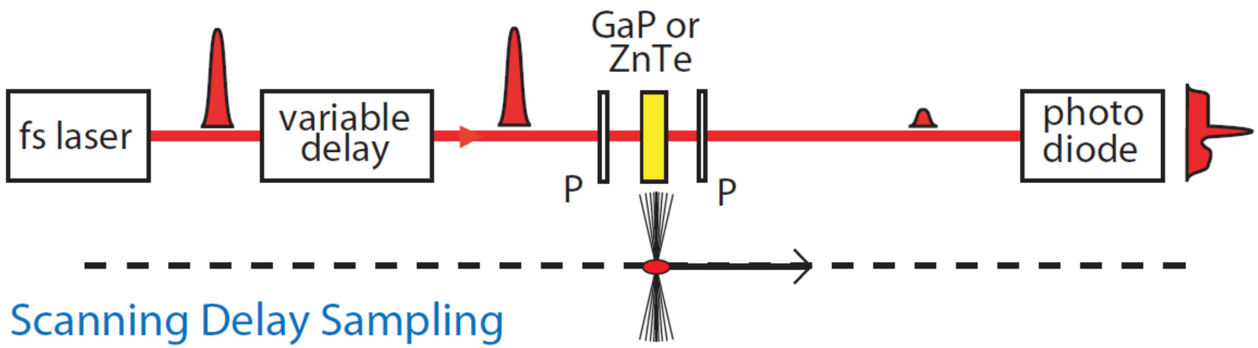
FELs → bunch length below 1 ps is achieved, i.e. below the resolution of streak camera

➢ Short laser pulses with $\boldsymbol{t} \approx$ **10 fs** and electro-optical modulator

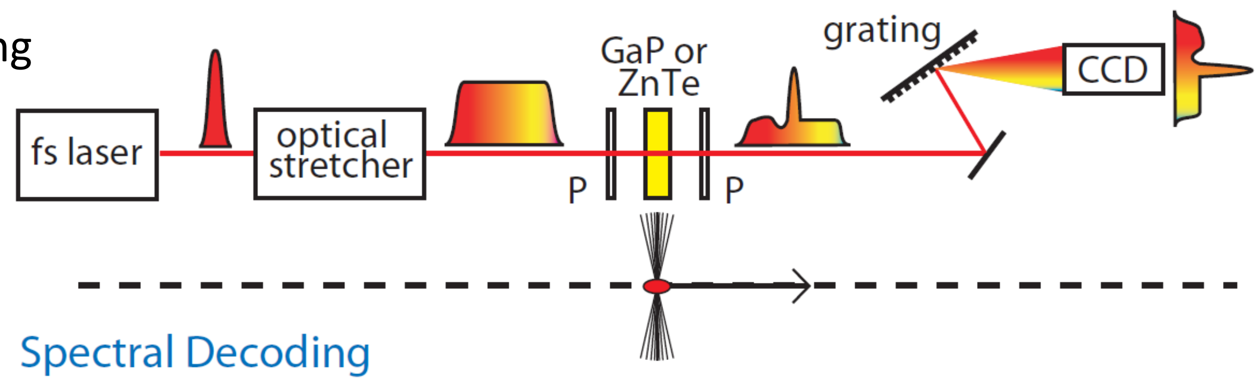
Electro optical modulator: birefringent, rotation angle depends on external electric field

Relativistic electron bunches: transverse field $E_{\perp,\, lab} = \gamma E_{\perp,\, rest}$ carries the time information.

Measurement by **scanning**
Short laser pulses $\boldsymbol{t}$ **< 10 fs**
and delay line
⇒ scanning method

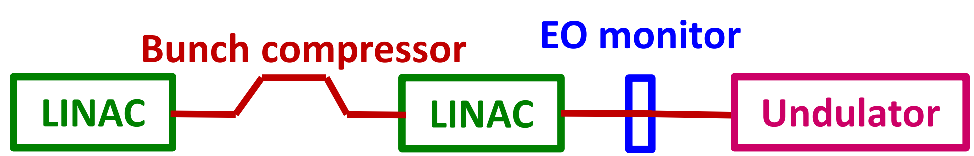


Measurement spectral decoding
Short laser pulses $\boldsymbol{t}$ **< 10 fs**
has broad frequency spectrum
and delay line
⇒ **single shot** determination



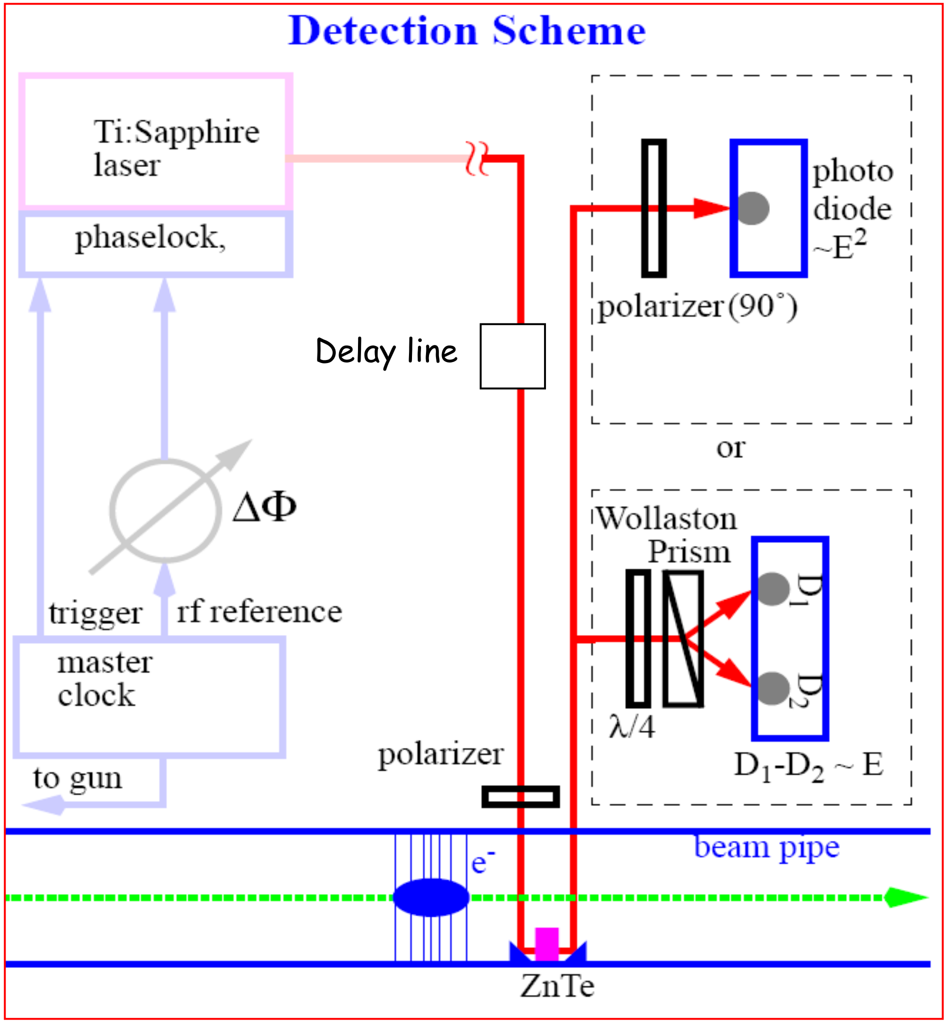
Further methods used!

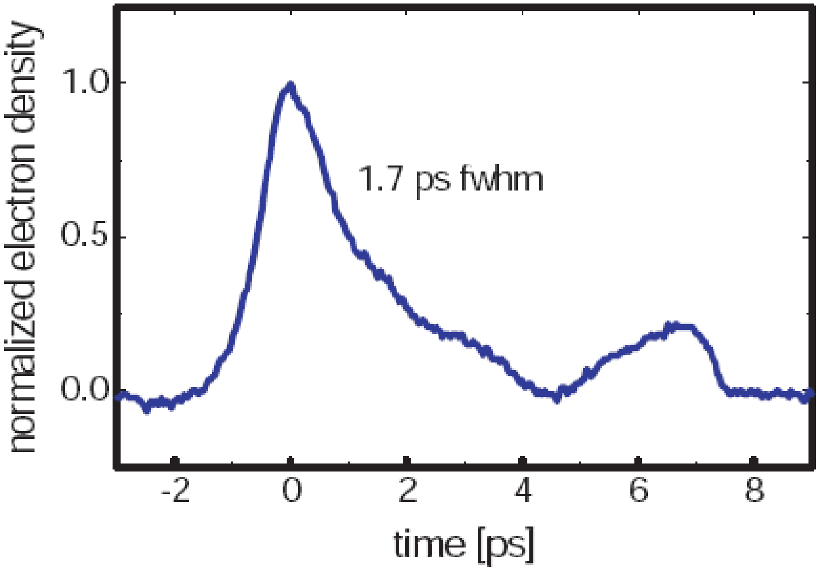
From S.P.Jamison et al., EPAC 2006



Realization of EOS Scanning

Setup of a scanning EOS method.

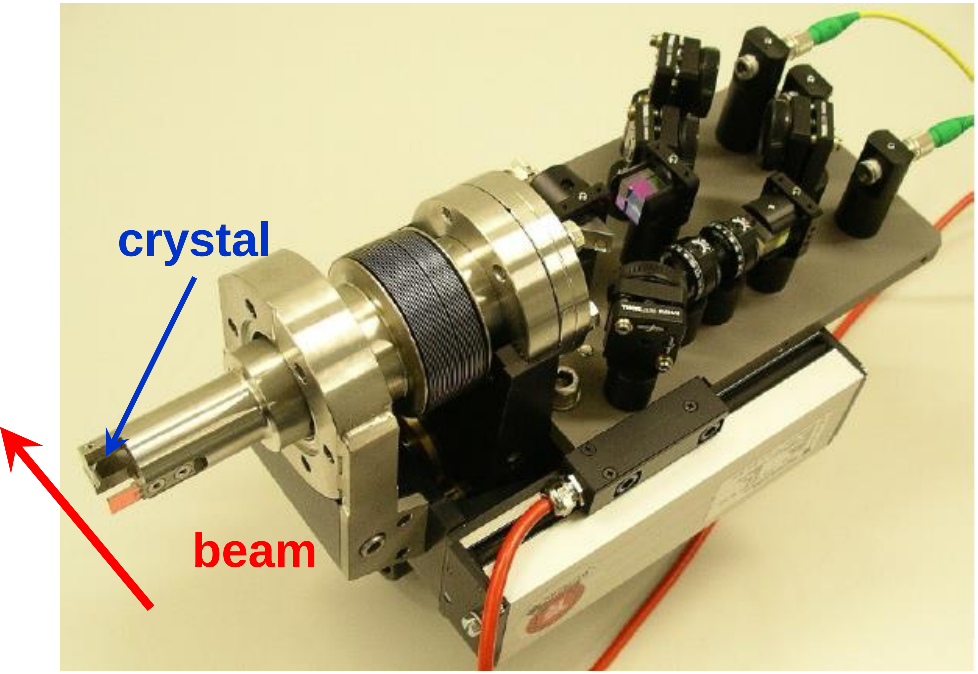


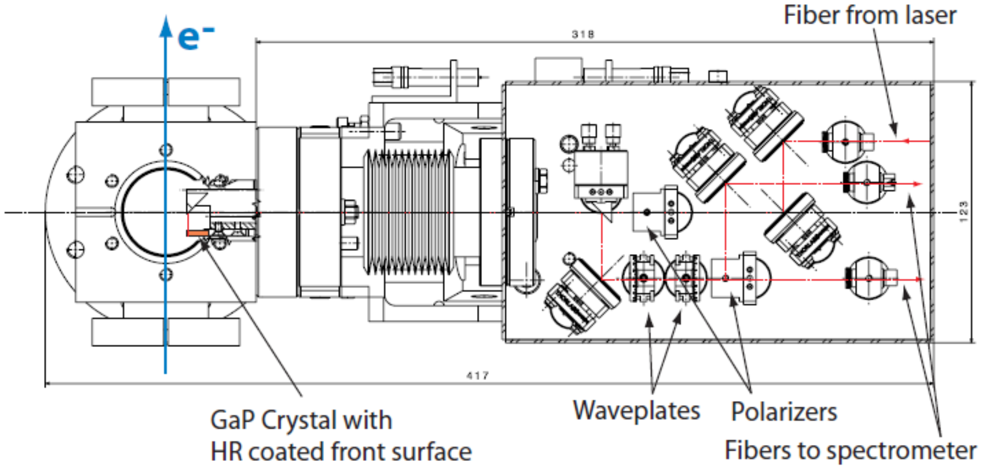


Using 12fs pulses from Ti:Al_2O_3 laser at 800nm and ZnTe crystal 0.5mm thick with a e^- - beam 46MeV of 200pC

X. Yan *et al*, Phys. Rev. Lett. 85, 3404 (2000)

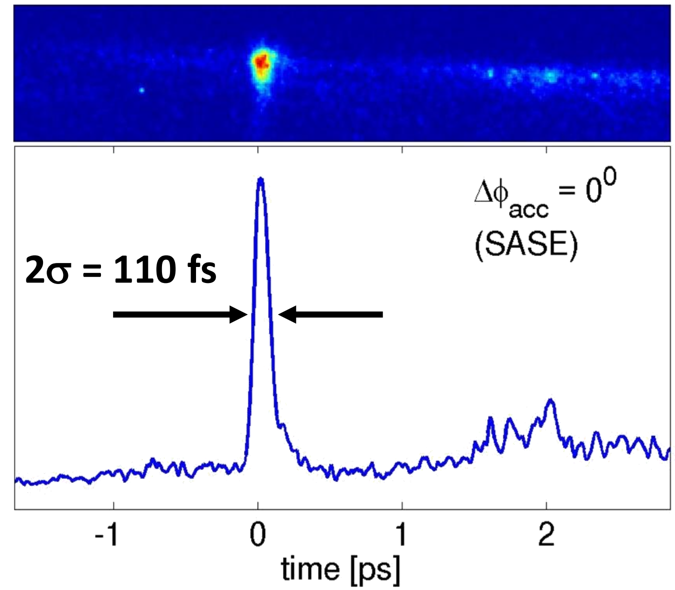
Hardware of a compact EOS Scanning Setup





Example: Bunch length at FLASH

100 fs bunch duration = 30 µm length

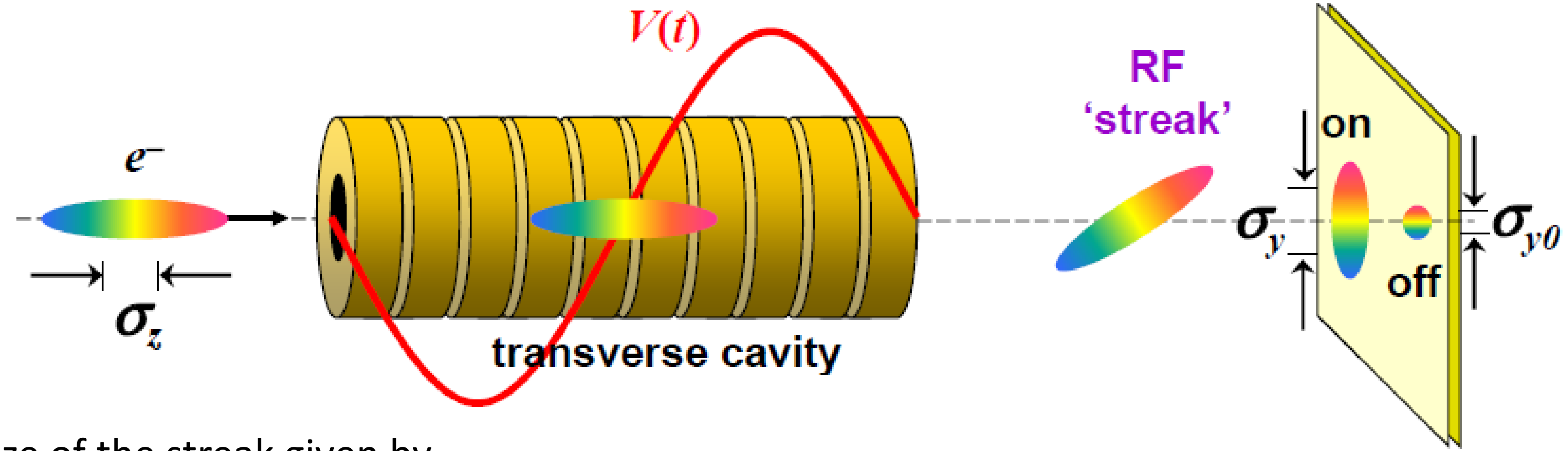


Steffen et al, DIPAC 2009

Steffen et al., Phys. Rev. AB 12, 032802 (2009)

Bunch Length by rf-Deflection: Principle

Transversal deflection of the bunch i.e. time-to-space conversion

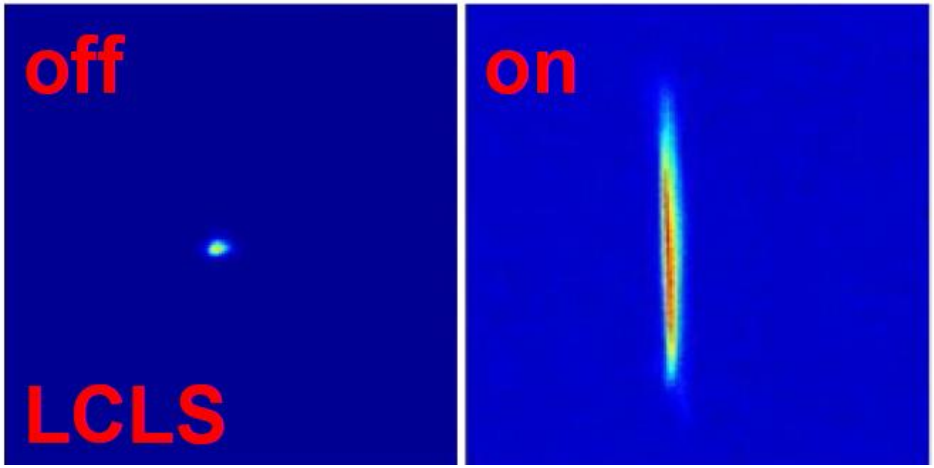


Size of the streak given by

$$\sigma_y = \sqrt{\sigma_{y0}^2 + R_{35} \cdot k \cdot \sigma_z^2}$$

k is determined by the rf-power

$$k = \frac{2\pi e \cdot U_{rf}}{\lambda_{rf} E}$$



From D. Xiang, IPAC'12

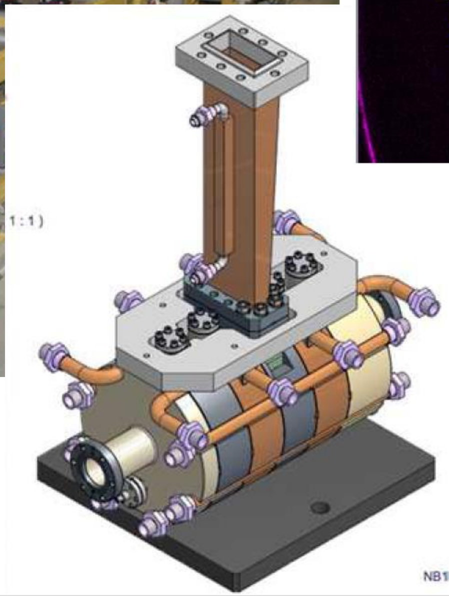
Bunch Length by rf-Deflection: Hardware

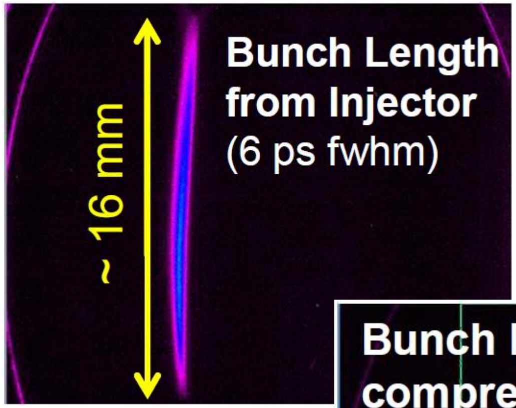
Transversal deflection of the bunch

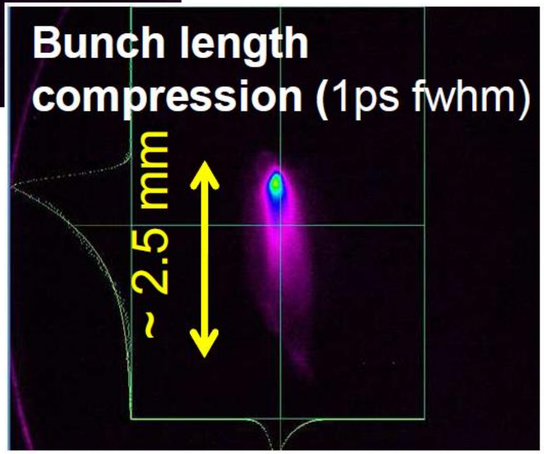
i.e. time-to-space conversion

Example: Cavity at FERMI, Trieste, Italy

Beam energy	320 MeV
Typical beam size	0.2 mm
Length	0.5 m
Frequency	2.998 GHz
Max. rf power	5 MW
Total trans. volt.	4.9 MV
Time resolution	70 fs







From M. Veronese, BIW’12

Summary of longitudinal Measurements

Longitudinal ↔ transverse correspondences:

- position relative to rf ↔ transverse center-of-mass
- bunch structure in time ↔ transverse profile in space
- momentum or energy spread ↔ transverse divergence.

Determination uses:

Broadband pick-ups:
- position relative to rf, mean energy
- emittance at transfer lines or synchrotron via tomography

assumption: bunches longer than pick-up.

Particle detectors:
- TOF or secondary e^- from wire

→ for non-relativistic proton beams

reason: ***E***-field does not reflect bunch shape.

Streak cameras:
- time resolved monitoring of synchrotron radiation

→ for relativistic e^--beams, $t_{bunch} < 1$ ns

reason: too short bunches for rf electronics.

Laser scanning:
- Electro-optical modulation of short laser pulse

→ very high time resolution

Beam deflection:
- Transverse deflection of primary beam

→ very high time resolution, but most expensive 'device'.

Backup slides

Bunch length measurement by electro-optical methods

For Free Electron Lasers $\rightarrow$ bunch length below 1 ps is achieved

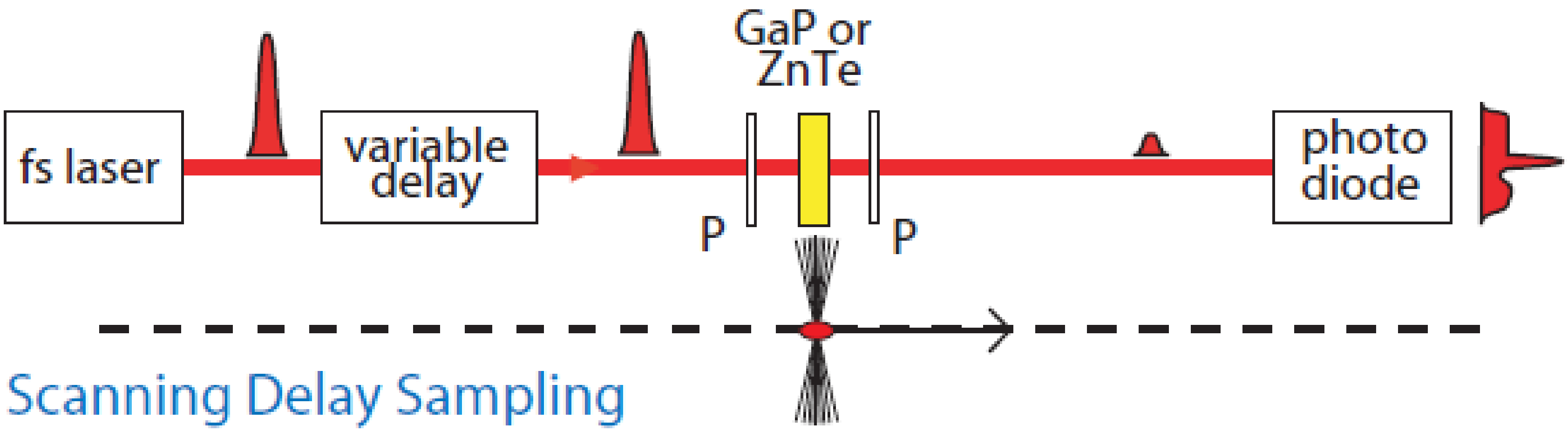
$\rightarrow$ below resolution of streak camera

$\rightarrow$ short laser pulses with $t \approx$ **10 fs** and electro-optical modulator

Electro optical modulator: birefringent, rotation angle depends on external electric field

Relativistic electron bunches: transverse field $E_{\perp, lab} = \gamma E_{\perp, rest}$ carries the time information

Scanning of delay between bunch and laser $\rightarrow$ time profile after several pulses.



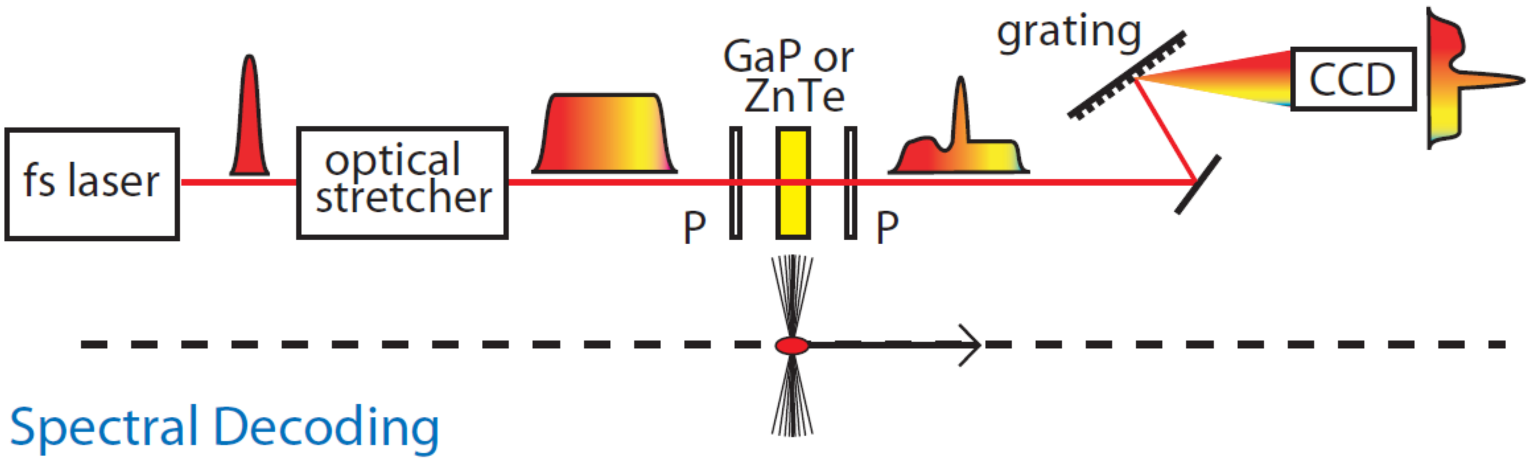
From S.P.Jamison et al., EPAC 2006

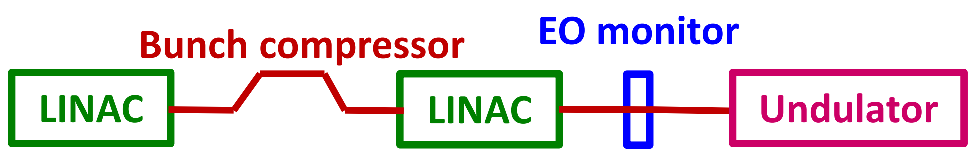
Bunch Length Measurement by electro-optical Method

For Free Electron Lasers → bunch length below 1 ps is achieved

Short laser pulse ⇔ broad frequency spectrum (property of Fourier transformation)

Optical stretcher: Separation of colors by different path length ⇒ **single-shot observation**





Courtesy S.P.Jamison et al., EPAC 2006