Exercises for Beam Instrumentation and Diagnostics JUAS 2020

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1 Calculation of currents

A facility is built of a LINAC and a synchrotron for protons or heavy ions.

1.1 LINAC

The average current is 0.1 mA of protons. The pulsed LINAC has a macro-pulse length of 1 ms and a macro-pulse period of 20 ms. The acceleration is done with a rf-frequency of 100 MHz. The particles are distributed 10 % in rf phase around the reference particle in longitudinal direction. The transverse beam size is $2x = 1$ cm assuming a rectangular shape .

Assumption for all parts of this exercise to simplify the calculations: Homogeneous charge distribution with hard edges i.e. a rectangular shape. The beta-function β is constant everywhere and therefore the transverse beam size $x = \sqrt{\beta \cdot \epsilon}$ changes only due to the variation of the emittance ϵ.

Calculate the pulse current, the number of particles within one macro-pulse and within one bunch. What is the particle density at 10 MeV kinetic energy? Compare this density to that of the residual gas assuming a pressure of 10^{-7} mbar $= 10^{-5}$ Pa ($\simeq 10^{-7}$ torr). Calculate the average distances between the protons.

Hint: Use ideal gas theorem $p = nk_BT$, with n particle density, $T = 300$ K temperature and $k_B = 1.4 \cdot 10^{-23}$ J/K the Boltzmann-constant.

1.2 Synchrotron

With a kinetic energy of 10 MeV the particles are injected into a synchrotron with 220 m circumference using multi-turn injection of 100 μs length and 100 % efficiency. The final kinetic energy after acceleration is 1 GeV. How many particles are injected, what is the electrical current at injection and extraction? Give also the revolution frequency.

Due to the multi-turn injection, the horizontal emittance is enlarged by a factor of 25. Calculate the density at injection and extraction for a de-bunched beam, assuming the conservation of the normalized emittance $\epsilon_{norm} = \beta\gamma\epsilon$. Compare the particle density to the residual gas density for a pressure of 10^{-10} mbar. What are the average distances between the protons?

1.3 Extraction

a) Slow extraction: The stored particles are extracted within 10 s. Calculate the current and the mean distance between the particles in the transfer line. What is their density? Compare its density to the residual gas density for a pressure of 10^{-9} mbar.

b) Fast extraction: For a transfer to an other synchrotron, all particles are collected within one bunch of 100 ns length. Give the current of this bunch within a transfer line.

1.4 Changes for an ion accelerator

Assuming the same accelerator facility as above but for an Uranium beam. The kinetic energies are the same by changing the unit from MeV to MeV/u (mass unit u= 938 MeV).

At the ion source U^{4+} is generated; at 1 MeV/u it is stripped to U^{28+}; at 10 MeV/u again it is stripped to U^{70+} and after extraction to the bare nuclei U^{92+}. For the LINAC the electrical current stay the same as for protons. What are the differences?

2 Methods of current measurements at proton accelerators

For the below mentioned proton accelerators and beam parameters an appropriate method for current measurement should be chosen; give the main argument for your choice:
a) Behind the ion source with an energy of 100 keV, a beam current of 100 mA and a pulse duration of 1 ms.
b) Behind the ion source with the same parameter as under **a)** but a current of 10 nA.
c) Behind a proton LINAC with an energy of 100 MeV, a beam current of 100 mA and a pulse duration of 1 ms.
d) The permanent monitoring during the 1 s long acceleration within a synchrotron from an energy of 100 MeV to 1 GeV and a current of the circulating beam of about 100 mA.
e) The circulating current within a synchrotron after acceleration and de-bunching (i.e. a continous current)
f) In a transport line behind a synchrotron where 10^{12} protons are extracted within 1 μs.
g) The same parameters as under **f)** but with a duration of 10 s using 'slow extraction' mode.

3 Transformer for a pulsed LINAC

A macro-pulse of 1 ms length has to be measured by a current transformer with an allowed droop of 3 % and a upper cut-off frequency of $f_{high} = 100$ kHz (why is this bandwidth needed?). The core has the size: inner radius $r_i = 30$ mm and outer radius $r_o = 60$ mm and length in beam direction of 4 cm. The permeability of the torus is $\mu_r = 10^5$ and $\mu_0 = 4\pi \cdot 10^{-7}$ Vs/Am. Calculate the droop time τ_{droop} and the corresponding lower cut-off frequency f_{low}.

Passive transformer: The load resistor of a passive transformer is $R = 1$ kΩ, having a temperature of 300 K. The loss resistivity is $R_L = 10$ Ω. Calculate the required number of windings for the given droop, the sensitivity (beam current to voltage conversion [V/A]) and the detection threshold for a signal-to-noise of $S/N = 1$ for pure thermal contributions. Use the thermal noise voltage $U_{eff} = \sqrt{4k_B T R \Delta f}$ with Boltzmann-constant $k_B = 1.4 \cdot 10^{-23}$ J/K, temperature $T = 300$ K and bandwidth $\Delta f = f_{high} - f_{low}$. What is the corresponding minimal detectable beam current?

Active transformer: Calculated for the same the properties of an active transformer of same size with a open-loop gain of $A = 10^6$, a feedback resistor $R_f = 1$ MΩ and the same loss resistivity $R_L = 10$ Ω.

Fast passive transformer: Passive transformers are normally used for short bunch measurements as transferred between synchrotrons. They have a load resistor of $R = 50$ Ω for smooth signal transmission. For an allowed droop of 3 % per μs and a bandwidth of 100 MHz calculate the same properties as above. Because the permeability is frequency dependent $\mu_r \propto 1/f$ for $f > 100$ kHz, take an average value of $\mu_r \simeq 10^3$.

4 Slow extraction current measurement

Assuming the stored number of particles from exercise 1 at a kinetic energy of 1 GeV/u and 1 s extraction time for proton ($1.25 \cdot 10^{12}$ stored) and Uranium ($1.8 \cdot 10^{10}$ stored) case. A current measurement is done with a 0.5 cm long ionization chamber, filled with Argon at atmospheric pressure. The energy loss of protons is

$$\frac{dE_p}{\rho dx} = 0.00144 \frac{\text{MeV}}{\text{mg/cm}^2}.$$

The scaling for heavy ions in this energy range is $dE/dx \propto Z^2$. For Argon the W-value is 26.3 eV and the mass is 40 amu. (Avogadro-number $N_A = 6.0 \cdot 10^{23}$ mol^{-1} and volume of

1 mol gas is 22.4 l.)
Calculate the secondary currents for both ions. (The nuclear charge of Uranium is $Z = 92$ and mass $A = 238$.)

What is the secondary current using a secondary electron monitor (SEM) with the energy loss in aluminium for protons of

$$\frac{dE_p}{\rho dx} = 0.00177 \frac{\text{MeV}}{\text{mg/cm}^2}$$

and the secondary electron yield $Y = 27.4\ e^-/(\text{MeV/mg/cm}^2)$. What is the secondary current for the proton beam? What is the secondary current for the Uranium beam using the scaling $dE/dx \propto Z^2$?

5 Beam power at a LINAC

Calculate the absorbed power for a beam delivered by a LINAC with proton or Uranium $^{238}U^{28+}$ at 10 MeV/u stopped in a water cooled Faraday cup. The electrical macro-pulse current for both ions is 2 mA and the pulse length is 1 ms with a repetition rate of 50 Hz. What is the average power? What is the required flow of cooling water for a maximal increase of the water temperature of 60 ^{o}C. The specific heat of water is $c = 1\text{cal/gK} = 4.2\text{J/gK}$.
What is the peak power?

6 Material destruction for intense beams

A beam of 1 MeV/u Uranium $^{238}U^{4+}$ with a macro-pulse current of $I_{macro} = 10$ mA and duration of $t_{macro} = 200\ \mu$s is stopped on a copper plate e.g. inside a Faraday Cup. The penetration depth is $R_{Cu} = 9.5\ \mu$m. Assume a round beam of radius r with a constant beam density. Estimate the minimal possible beam spot to avoid melting of the stopper material. The required constants for copper are given in Table 1.

What does change, if such a beam hits a tungsten surface of a high power Faraday cup? The range in tungsten is $R_W = 7.4\ \mu$m; for further material constants see Table 1.

What does change, if the beam hit the vacuum pipe made of iron? The range in iron is $R_{Fe} = 9.7\ \mu$m; other material constants are given in Table 1.

Having a beam of radius $r_{beam} = 5$ mm. What is the maximum allowed pulse length to avoid melting?

What is a rough scaling for the minimal radius as a function of beam energy?

Calculate the same properties for a 1 MeV proton beam. The ranges are $R_{Cu} = 6.7\ \mu$m, $R_W = 5.38\ \mu$m and $R_{Fe} = 6.4\ \mu$m.

	Range R_{mat}	density ρ	melting T_{melt}	sp. heat c	melt. heat a_{melt}
Copper Cu	9.5 μm	8.96 g/cm^3	1083 ^{o}C	0.44 J/gK	210 J/g
Tungsten W	7.4 μm	19.25 g/cm^3	3380 ^{o}C	0.13 J/gK	192 J/g
Iron Fe	9.7 μm	7.87 g/cm^3	1535 ^{o}C	0.45 J/gK	275 J/g

Table 1: Range of the ions and material constants required for the calculations: Range R for 1 MeV/u Uranium, density ρ, melting temperature T_{melt}, specific heat c and melting heat a_{melt}.

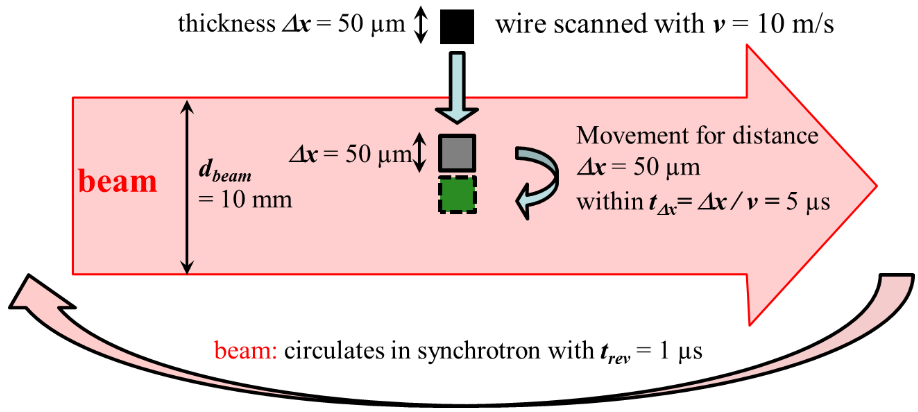

Figure 1: Simplified scheme of a flying wire measurement in a synchrotron.

7 Transverse profile by flying wire scanner

In a synchrotron a beam of protons or Uranium at an energy of 1 GeV/u is stored. A thin wire is scanned through the beam to monitor the transverse profile. The beam diameter is Ø 10 mm and the revolution time is 1.0 μs. The wire is made of Graphite with 50×50 μm and it is scanned with a velocity of $v = 10$ m/s. The energy loss in Graphite of protons is $dE/dx = 4.29$ MeV/cm and for Uranium of mass 238 amu is $dE/dx = 35640$ MeV/cm. The number of stored particles are for protons $N_{stored} = 10^{12}$ and for Uranium $N_{stored} = 10^9$.

Assumption to simplify the calculation: The beam and the wire have a rectangular cross section and the particles doesn't make betatron oscillations.

Calculate the energy loss per passage of the ion through the wire and the calculate the ratio with respect to the ion's kinetic energy.

Calculate the average number of the ion's passages trough the wire. *Hint:* Calculate first the time it take for the wire passage to pass a length of 50 μm (i.e. the wire thickness)! What is the total energy loss for an individual ion after these passages? Do you think, the particles can be used for further acceleration inside the synchrotron?

What is the total heat energy absorbed by the wire? Does the wire withstand the energy deposition without melting assuming a maximal power rating of 1 W/mm?

What happens, if not fully stripped Uranium is stored?

8 Signal estimation for an ionization profile monitor

8.1 LINAC

The transverse profile from a LINAC beam of 1 mA Ar^{10+} and pulse length 1 ms at energy 1.4 MeV/u should be measured with an ionization profile monitor. The length of the collecting strips are 100 mm. The residual gas contain 90 % N_2 and 10 % H_2 at a pressure of 10^{-7} mbar. Give the resulting voltage after a current-to-voltage converter with the sensitivity of 10 nA/V.

Assumption: Take a rectangular beam cross section and assume, that 10 % of the beam is covered by one strip.

The needed quantities are: Density at *normal* pressure for H_2 is $\rho_H = 0.09 \cdot 10^{-3}$ g/cm^3 and for N_2 it is $\rho_N = 1.25 \cdot 10^{-3}$ g/cm^3. The energy loss for H_2 is $dE_H/\rho dx = 66$ MeV/(mg/cm^2) and for N_2 it is $dE_N/\rho dx = 22$ MeV/(mg/cm^2). As a first approximation assume, that the average energy for the production of one e^--ion pair is 36 eV.

8.2 Synchrotron

Assume, that the beam is injected to a synchrotron with 10 fold multi-turn efficiency with respect to the current. The residual gas contains now only H_2 and has a pressure of 10^{-10} mbar. Calculate the current per strip for this case measured within 1 ms. How one can get a higher secondary current, without changing the vacuum pressure?

9 Signal estimation for broad-band BPM

The beam position in a proton synchrotron has to be measured by a shoe-box type pick-up. This pick-up has a length $l = 20$ cm, a distance from the beam centre to the plates is $a = 10$ cm and a total capacity of $C = 100$ pF, as used for the calculation of Fig. 5.3 and 5.4 in the lecture notes. The beam has a velocity of $\beta = 50$ % and a bunch length of $\sigma = 100$ ns. For the position read-out assume a linear response $U_\Delta/U_\Sigma = x/a$ as a function of displacement x. Calculate the transfer impedance Z_t of a half-cylindrical plate using a 1 MΩ termination. Give the sum voltage for a peak value of the beam current of $I_{beam} = 1$ A. What is the difference voltage for a $x = 1$ mm displacement in this case? What are the corresponding values for U_Σ and U_Δ for a 50 Ω termination assuming $Z_t(50\Omega) = Z_t(1\mathrm{M}\Omega)/20$, corresponding to a frequency of 1 MHz as shown in Fig. 5.3?

Remark: In principle, for the transfer impedance the Fourier-transformation is required (Why?). For a Gaussian function in time with standard deviation σ_t the Fourier-transformation is a Gaussian function (for positive frequencies centred at $f = 0$) having a standard deviation $\sigma_f = \frac{1}{2\pi\sigma_t}$.

What is the thermal noise at the $R = 50$ Ω and $R = 1$ MΩ amplifier input-impedance for a bandwidth of $\Delta f = 100$ MHz? Use the thermal noise voltage $U_{eff} = \sqrt{4k_BTR\Delta f}$ with the temperature $T = 300$ K and Boltzmann-constant $k_B = 1.4 \cdot 10^{-23}$ J/K. What is the minimal beam current for 1 mm displacement for a signal-to-noise ratio $S/N = 2$ using only this thermal noise? (For a real amplifier the noise contribution is at least a factor 2 bigger.)

For longer bunches the transfer impedance has a scaling $Z_t \propto \omega$ for the 50 Ω termination. At which bunch length the position sensitivity of the 1 MΩ and the 50 Ω terminations are equal?

How can the position sensitivity be improved and what is the main reason?

10 Profile measurement with a scraper

(from previous examination)

Inside a synchrotron the transverse beam profile is measured sometimes destructively using a scraper. It consist of a plate moved in the beam path, see Fig. 2. For the profile determination the scraped position is drawn on the horizontal axis, i.e. its position is the independent variable. The time needed to path the beam is orders of magnitude longer (typically 1 s) than the revolution time. During the measurement no refilling of the synchrotron occurs.

Discuss this method:

a) What is the primary *beam* quantity to be measured?

b) Propose a method to determine this quantity directly or indirectly and give the reason for your choice.

c) How is the beam *profile* generated from the measured primary signal and the scraper position? Make a principle drawing for the position dependence of the primary signal versus the position and the deduced profile.

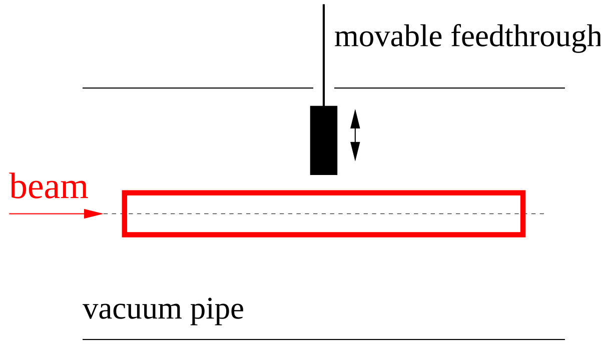

Figure 2: Scheme of a profile measurement at a synchrotron using a scraper.

d) Why it makes only sense to go up to the middle of the beam?
e) This method can also be used at a LINAC. What is the change of the primary signal as a function of the scraper position? What one has to do for getting the full transverse profile?

11 Beam energy measurement by time-of-flight (from previous examination)

At a proton LINAC the coarse final energy is known to be between 5 MeV and 10 MeV. The accelerator is running at 100 MHz. The distance of the two pick-ups used for the time-of-flight measurement is 4 m.
a) To what limits one has to know the energy to get an unique solution?
b) A third pick-up should be installed for the coarse energy determination. What is the maximal distance to get a unique solution?

12 Beam diagnostics design for a proton facility

The task is the design of beam diagnostics for a proton facility built of a LINAC, a booster-synchrotron and a high energy synchrotron as depicted in Fig. 3. The facility is outlined below, giving only the very basic parameters. It is a realistic task to ask for actual facts and numbers required for a adequate layout of the beam diagnostics devices.
The basic parameters for the facility are:

1. The proton source is located on a high voltage platform with $U = 100$ kV i.e. the energy of the protons are 100 keV. The maximal current is $I = 50$ mA, but also lower current down to 100 nA are sometimes needed for the commissioning of the facility.
2. A low energy LINAC (e.g. RFQ) is used for the acceleration to $E_{kin} = 3$ MeV. It is driven in a pulsed mode with maximal pulse length of $t_{pulse} = 1$ ms and $f_{rep} = 1$ Hz repetition rate. The accelerating frequency is $f_{acc} = 300$ MHz. The transverse beam size is in the order of 1 cm.
3. In the second part of the LINAC the particles are accelerated to $E_{kin} = 100$ MeV.
4. The $E_{kin} = 100$ MeV proton beam is injected in a synchrotron of $l = 200$ m circumference which delivers a maximal final kinetic energy of $E_{kin} = 1$ GeV. A time of $t_{cycle} = 1$ s is needed for acceleration from 100 MeV to 1 GeV. The acceleration frequency is in between 2.6 MHz $< f_{acc} <$ 5.3 MHz resulting in 4 circulating bunches. (The beam velocity at injection is $v/c = 43$ % and at extraction it is $v/c = 87$ % which corresponds to a time for one revolution from $t_{rev} = 1.6$ μs to $t_{rev} = 0.76$ μs.)

The bunch length is $\sigma_{bunch} = 20^o$ of the acceleration frequency (one standard deviation). The horizontal and vertical tunes are $Q_x = 4.28$ and $Q_y = 3.28$, respectively. At maximum 10^{12} protons can be stored in this synchrotron.

5. The beam is extracted towards the High Energy Beam Transport HEBT line of 300 m length. Three different location are foreseen to be delivered with beam, either by slow extraction within 1 s or by single-turn extraction within 0.76 μs to a fixed target as well as a transfer to a 2^{nd} synchrotron.
6. The beam can be injected to a second synchrotron of 2200 m length for further acceleration from 1 GeV to 100 GeV. The protons from the 1^{st} synchrotron are injected by a so called 'bunch-to-bucket' transfer i.e. 10 successive fillings are used to accumulate 10^{13} protons in the 2^{nd} synchrotron before acceleration is started. The acceleration takes $t_{cycle} = 1$ s and then the beam is extracted to a target. The resulting bunch length is $\sigma_{bunch} = 20^o$ of the acceleration frequency (one standard deviation). The tunes are $Q_x = 20.28$ and $Q_y = 18.28$, respectively.

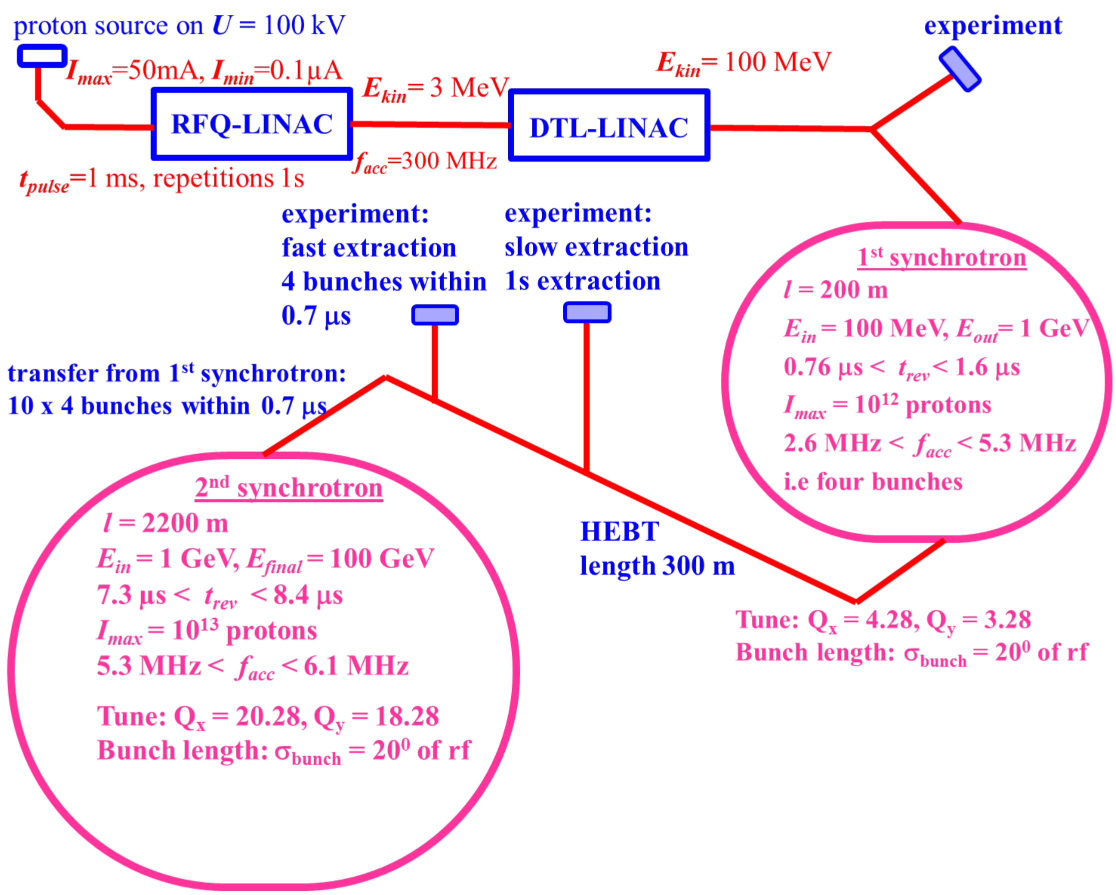

Figure 3: The simplified facility for beam diagnostics design.