Joint Universities Accelerator School

JUAS 2020

Archamps, France, 2. – 4. March 2020

Normal-conducting accelerator magnets
Lecture 1: Basic principles

Thomas Zickler

CERN

Scope of the lectures

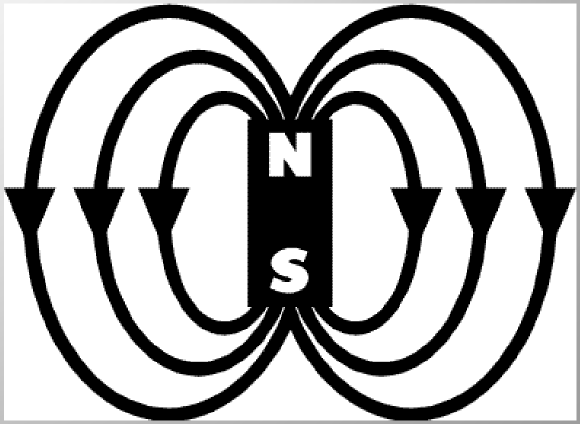
Overview of electro-magnet technology as used in particle accelerators considering *normal-conducting, iron-dominated* electro-magnets (generally restricted to direct current situations)

Main goal is to:

- create a fundmental understanding in accelerator magnet technology
- provide a guidebook with practical instructions how to start with the design of a conventional accelerator magnet
- focus on applied and practical design aspects using 'real' examples
- introduce finite element codes for practical magnet design
- present an outlook into magnet manufacturing, testing and measurements

Not covered:

– permanent magnet technology
– superconducting technology



Literature

- CAS proceedings, Fifth General Accelerator Physics Course, University of Jyväskylä, Finland, September 1992, CERN Yellow Report 94-01
- International Conference on Magnet Technology, Conference proceedings
- Iron Dominated Electromagnets, J. T. Tanabe, World Scientific Publishing, 2005
- Magnetic Field for Transporting Charged Beams, G. Parzen, BNL publication, 1976
- Magnete, G. Schnell, Thiemig Verlag, 1973 (German)
- Field Computation for Accelerator Magnets: Analytical and Numerical Methods for Electromagnetic Design and Optimization, S. Russenschuck, Wiley-VCH, 2010
- Practical Definitions & Formulae for Normal Conducting Magnets, D. Tommasini, Sept. 2011
- CAS proceedings, Magnetic measurements and alignment, Montreux, Switzerland, March 1992, CERN Yellow Report 92-05
- CAS proceedings, Measurement and alignment of accelerator and detector magnets, Anacapri, Italy, April 1997, CERN Yellow Report 98-05
- The Physics of Particle Accelerators: An Introduction, K. Wille, Oxford University Press, 2000
- CAS proceedings, Magnets, Bruges, Belgium, June 2009, CERN Yellow Report 2010-004

… and there will be a Special CAS on Magnet Design and Magnetic Measurements in autumn 2020 (see: https://cas.web.cern.ch/)

Acknowledgements

Many thanks …

… to all my colleagues who contributed to this lecture, in particular L.Bottura, M.Buzio, B.Langenbeck, N.Marks, A.Milanese, S.Russenschuck, D.Schoerling, C.Siedler, S.Sgobba, D.Tommasini, A.Vorozhtsov

Program (1)

Lecture 1 — Monday 2.3. (10:45 – 12:15)

Introduction & Basic principles

- Why do we need magnets?
- Basic principles and concepts
- Magnet types in accelerators

Lecture 2 — Monday 2.3. (14:00 – 15:00)

Magnet production, tests and measurements

- Magnetic materials
- Manufacturing techniques
- Assembly & tests

Lecture 3 — Monday 2.3. (15:00 – 16:00)

Analytical design

- What do we need to know before starting?
- Yoke design
- Coil dimensioning
- Cooling layout
- Cost estimation and optimization

Program (2)

Lecture 4 — Tuesday 3.3. (15:00 – 16:00)

Applied numerical design

- Building a basic 2D finite-element model
- Interpretation of results
- Typical application examples

Tutorial — Tuesday 3.3. (16:15 – ???)

Case study (part 1)

- Students are invited to design and specify a ‚real‘ magnet
- Analytical magnet design with pencil & paper

Mini-workshop — Wednesday, 4.3. (9:00 – 12:00)

Case study (part 2)

- Computer work
- Numerical magnet design

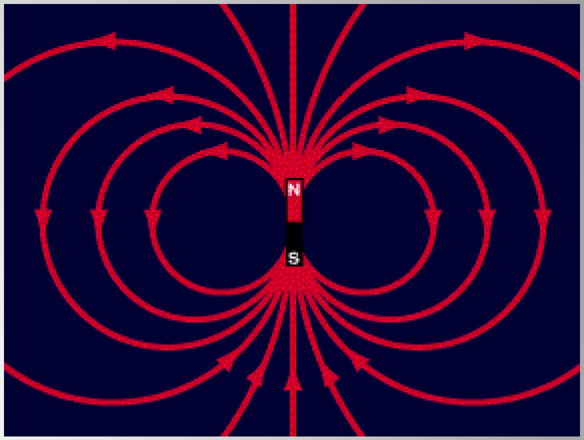
Exam (together with SC magnets) — Thursday, 19.3. (9:00 – 10:30)

Lecture 1: Basic principles

- Why do we need magnets?
- Magnetic circuit
- Basic principles and concepts
- Field description
- Magnet types and applications

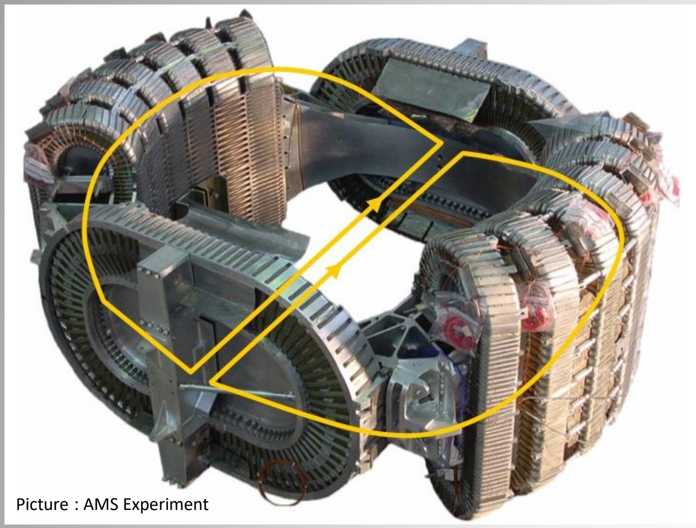
Magnetic units

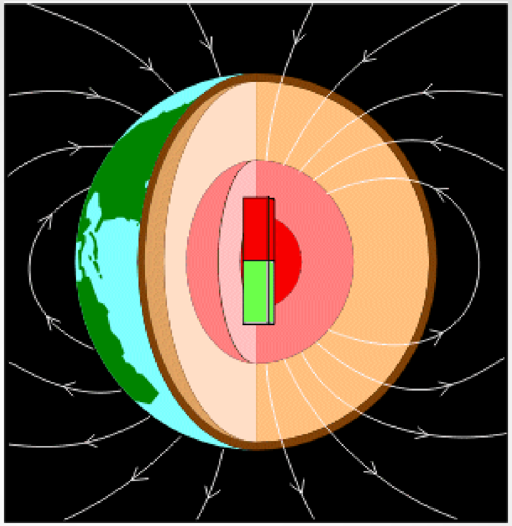
IEEE defines the following units:

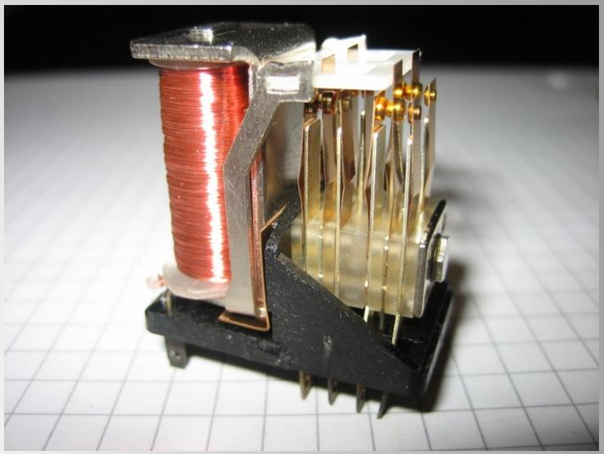


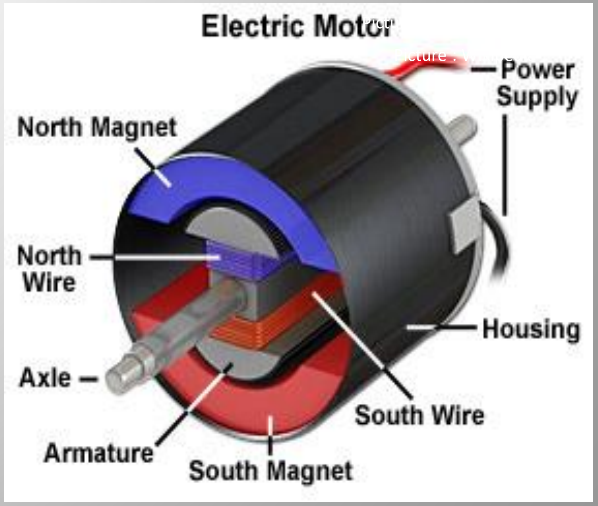
- Magnetic field:
  - H (vector) [A/m]
  - the magnetizing force produced by electric currents
- Electro-motive force:
  - e.m.f. or U [V or (kg m^2)/(A s^3)]
  - here: voltage generated by a time varying magnetic field
- Magnetic flux density or magnetic induction:
  - B (vector) [T or kg/(A s^2)]
  - the density of magnetic flux driven through a medium by the magnetic field
  - Note: induction is frequently referred to as "Magnetic Field“
  - H, B and μ relates by: $B = \mu H$
- Permeability:
  - $\mu = \mu_0 \, \mu_r$
  - permeability of free space $\mu_0 = 4 \pi \, 10^{-7}$ [V s/A m]
  - relative permeability μ_r (dimensionless): $\mu_{\text{air}} = 1$; $\mu_{\text{iron}} > 1000$ (not saturated)
- Magnetic flux:
  - ϕ [Wb or (kg m^2)/(A s^2)]
  - surface integral of the flux density component perpendicular trough a surface

Magnets everywhere…

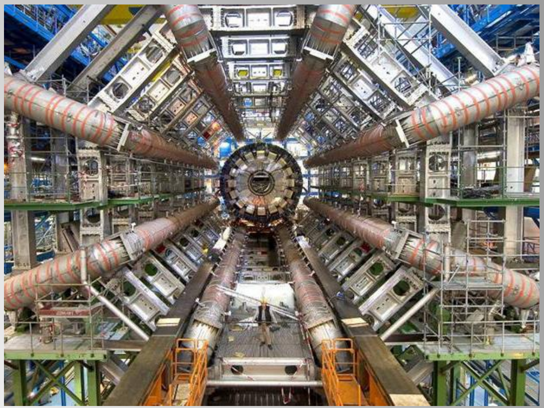

Picture : AMS Experiment

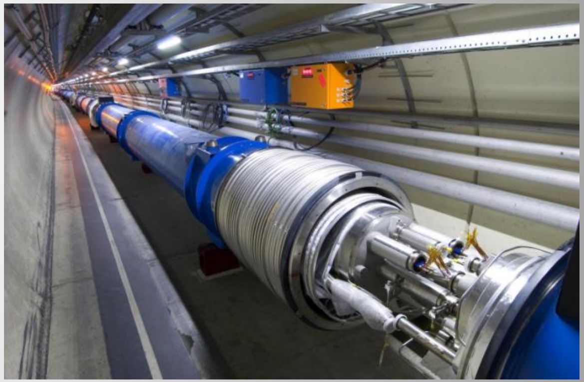


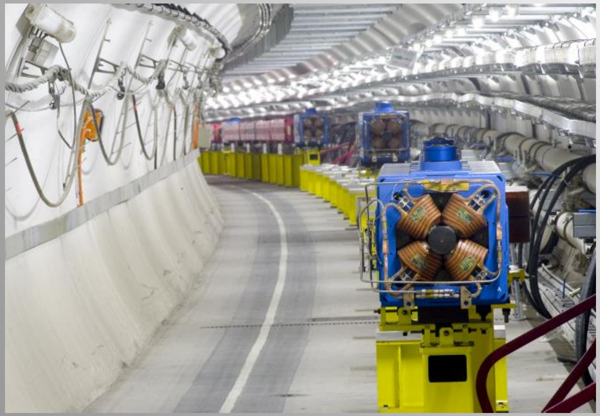




Magnets at CERN



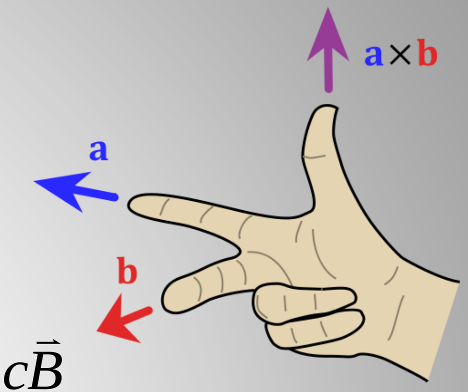


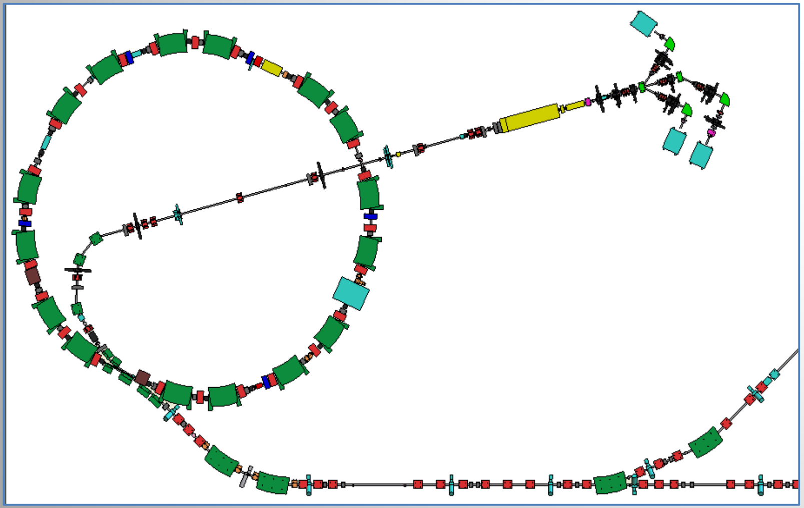


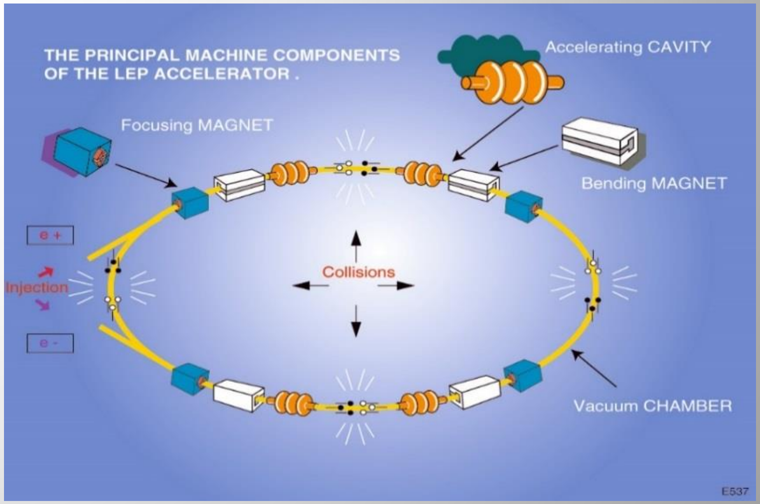
Normal-conducting magnets:
4800 magnets (50 000 tonnes) are installed in the CERN accelerator complex
Superconducting magnets:
10 000 magnets (50 000 tonnes) mainly in LHC
Permanent magnets:
150 magnets (4 tonnes) in Linacs & EA

Why do we need magnets?

- Interaction with the beam
  - guide the beam to keep it on the orbit
  - focus and shape the beam
- Lorentz's force: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$
  - for relativistic particles this effect is equivalent if $\vec{E} = c\vec{B}$
  - if $B = 1$ T then $E = 3 \cdot 10^8$ V/m(!)



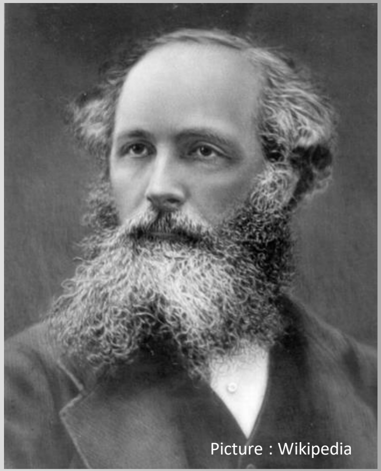




- Permanent magnets provide (in general) only constant magnetic fields
- Electro-magnets can provide adjustable magnetic fields

Maxwell's equations

In 1873, Maxwell published "Treatise on Electricity and Magnetism" in which he summarized the discoveries of Coulomb, Oersted, Ampere, Faraday, et. al. in four mathematical equations:


Picture : Wikipedia

Gauss' law for electricity:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \oint_{\partial V} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

Gauss' law of flux conservation:

$$\nabla \cdot \vec{B} = 0 \qquad \oint_{\partial V} \vec{B} \cdot d\vec{A} = 0$$

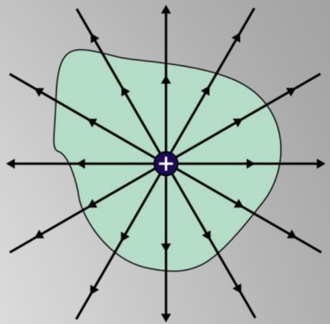
Faraday's law of induction:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \oint_{\partial A} \vec{E} \cdot d\vec{s} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_A \vec{B} \cdot d\vec{A}$$

Ampere's circuital law:

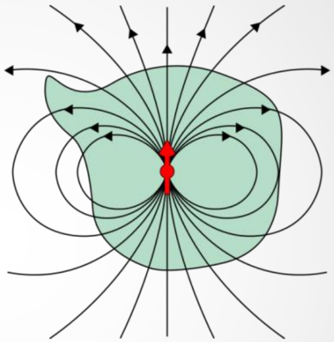
$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \oint_{\partial A} \vec{B} \cdot d\vec{s} = \int_A \mu_0 \vec{J} \cdot d\vec{A} + \frac{d}{dt}\int_A \mu_0 \varepsilon_0 \vec{E} \cdot d\vec{A}$$

Maxwell‘s equations



Gauss‘ law for electricity:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$



Gauss‘ law of flux conservation:

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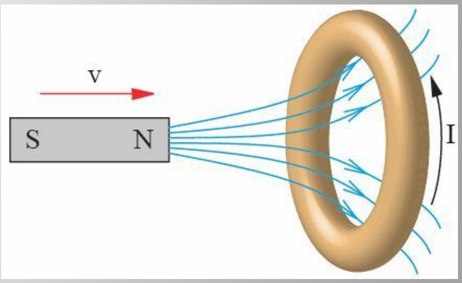
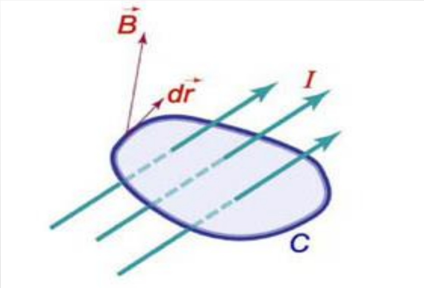
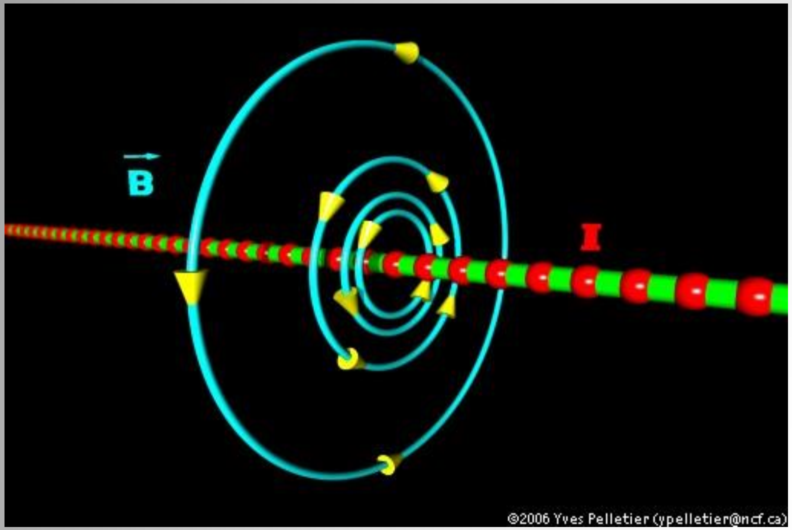


Ampere‘s law:

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$



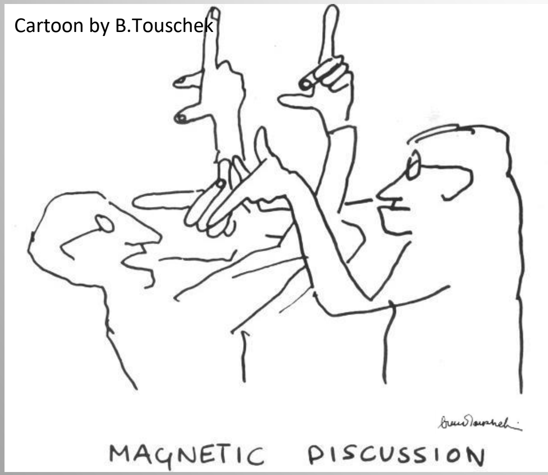
Producing the field



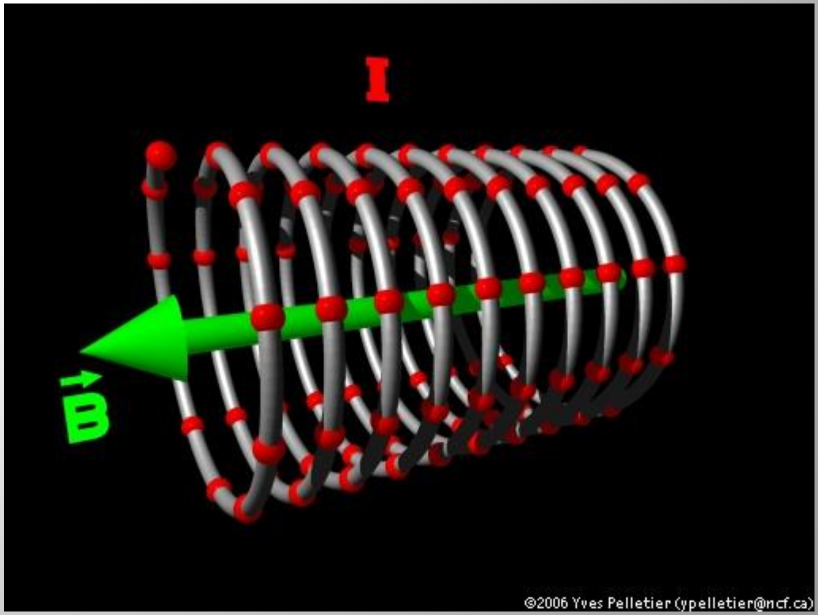
Maxwell & Ampere:

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

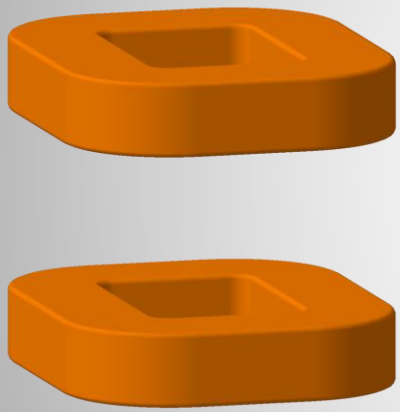
„An electrical current is surrounded by a magnetic field“

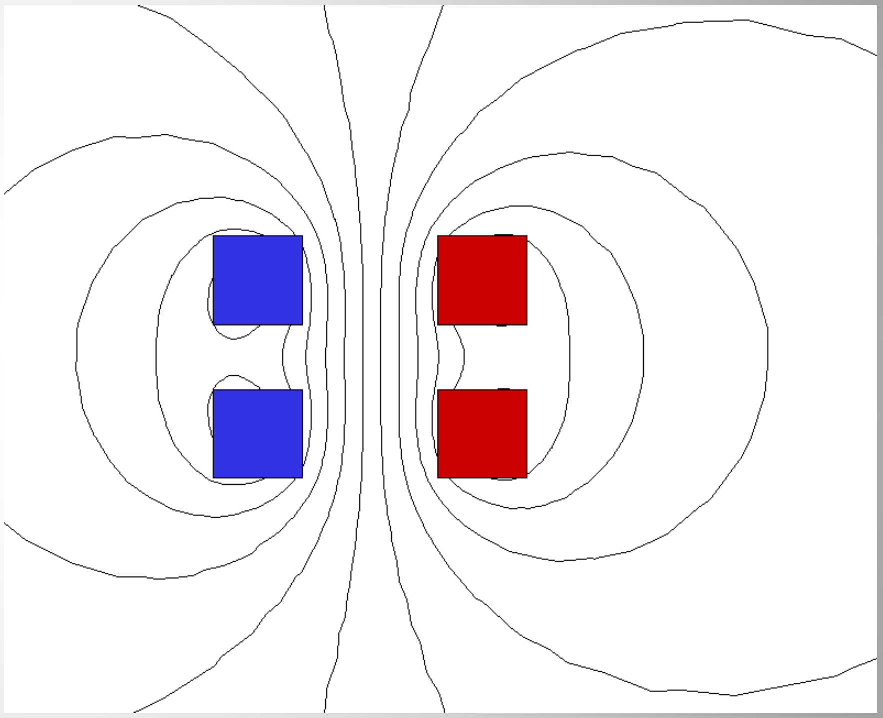


„*Right hand rule*“ applies



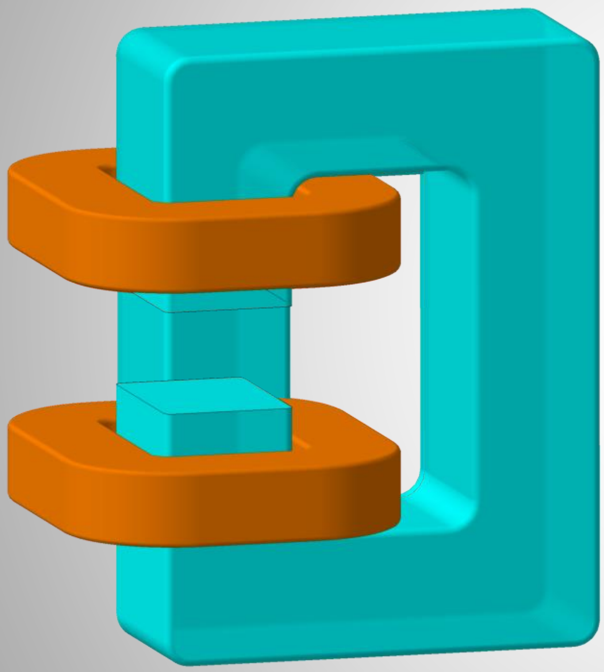
Magnetic circuit

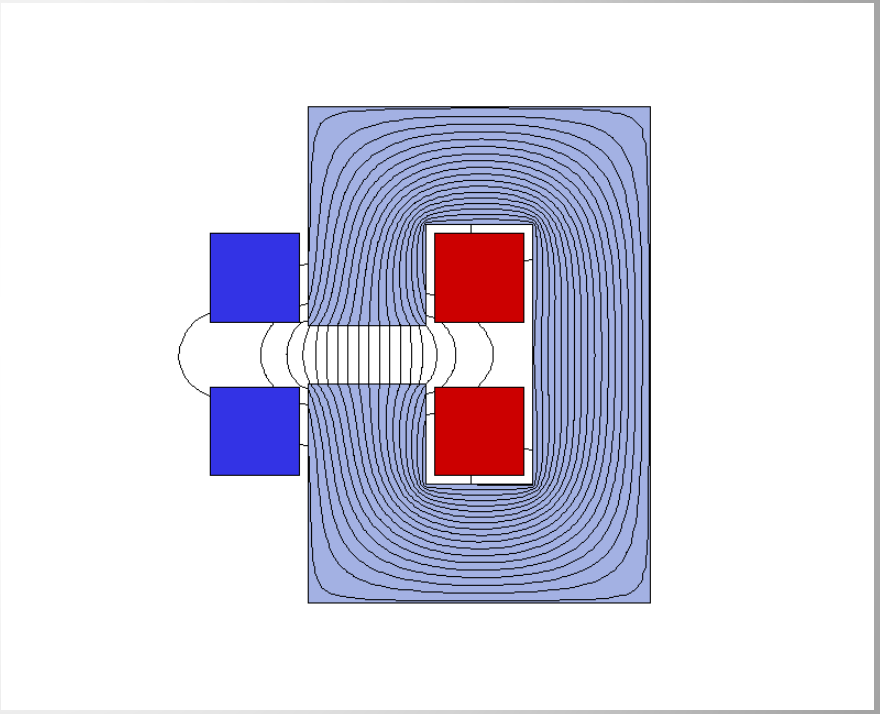




Flux lines represent the magnetic field
Coil colors indicate the current direction

Magnetic circuit



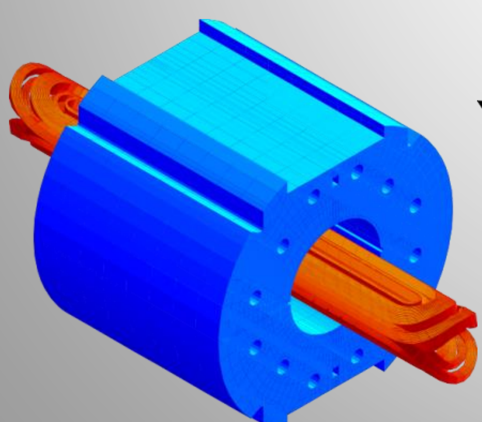


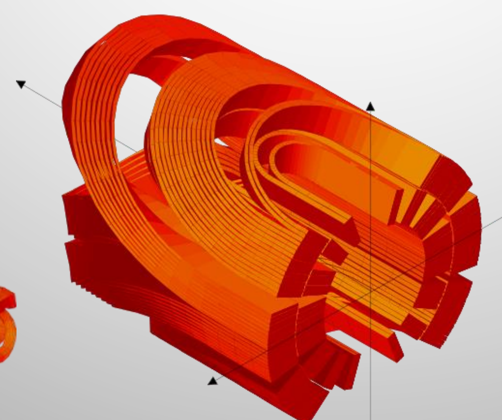
Coils hold the electrical current
Iron holds the magnetic flux

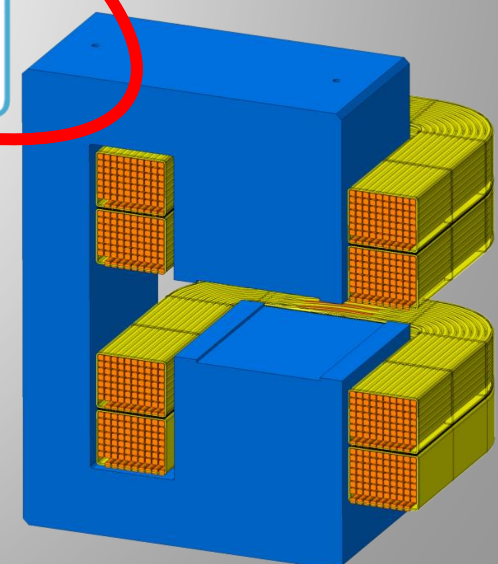
→ *"iron-dominated magnet"*

Magnet technologies

- Magnets
  - Electro-magnets
    - Superconducting
      - **Coil dominated** $B < 11$ T
      - Iron dominated $B < 2$ T
    - Normal-conducting
      - Coil dominated $B < 1$ T
      - **Iron dominated** $B < 2$ T
  - Permanent magnets



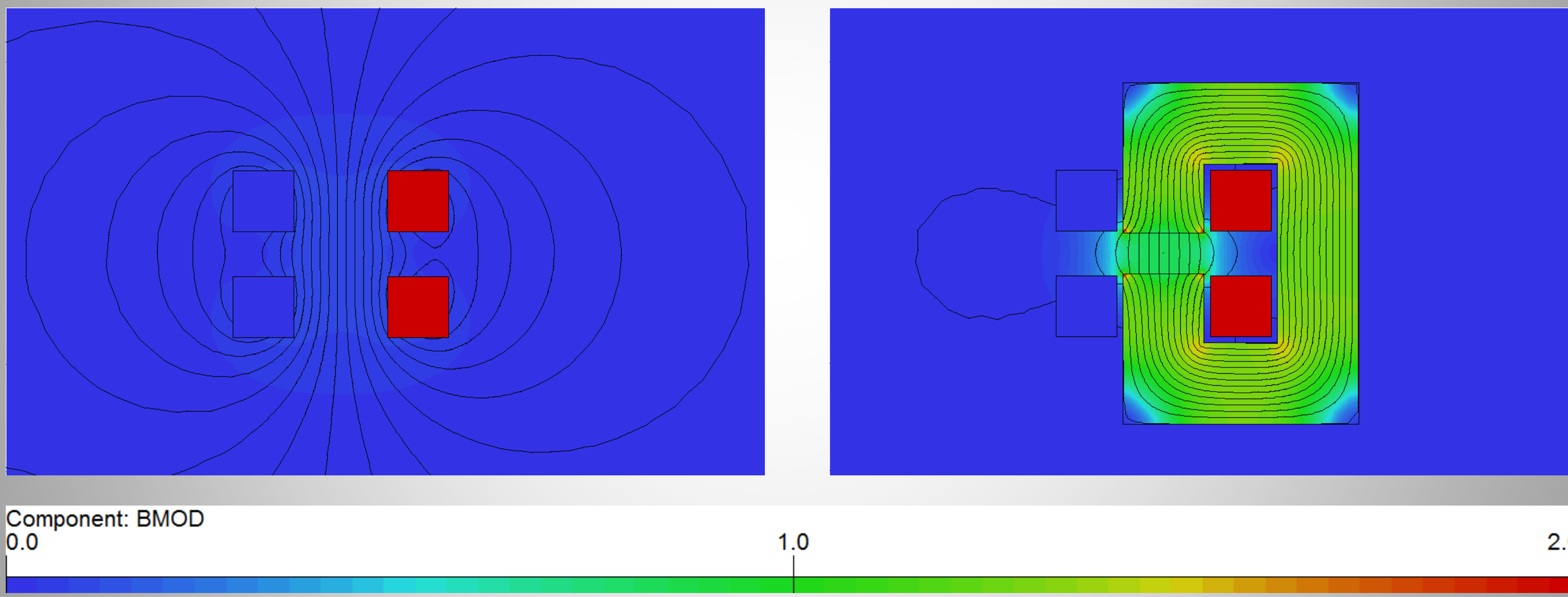




Magnetic circuit

$I = 32$ kA
$B_{centre} = 0.09$ T

$I = 32$ kA
$B_{centre} = 0.80$ T

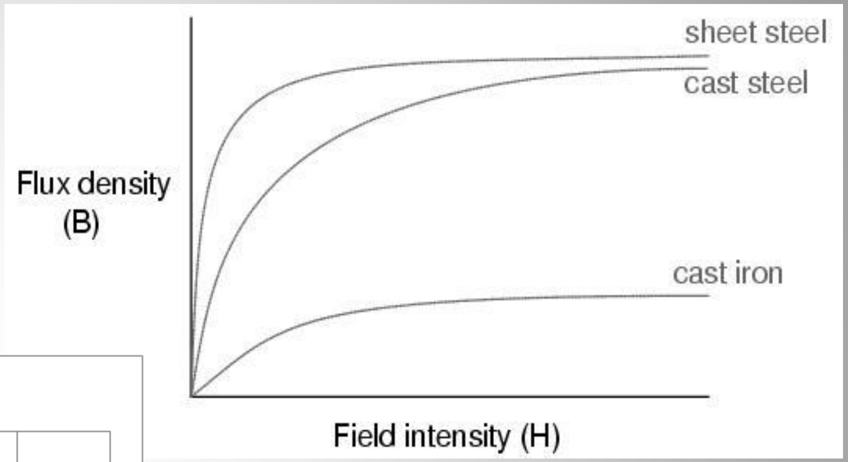


The presence of a magnetic circuit can increase the flux density in the magnet aperture by factors

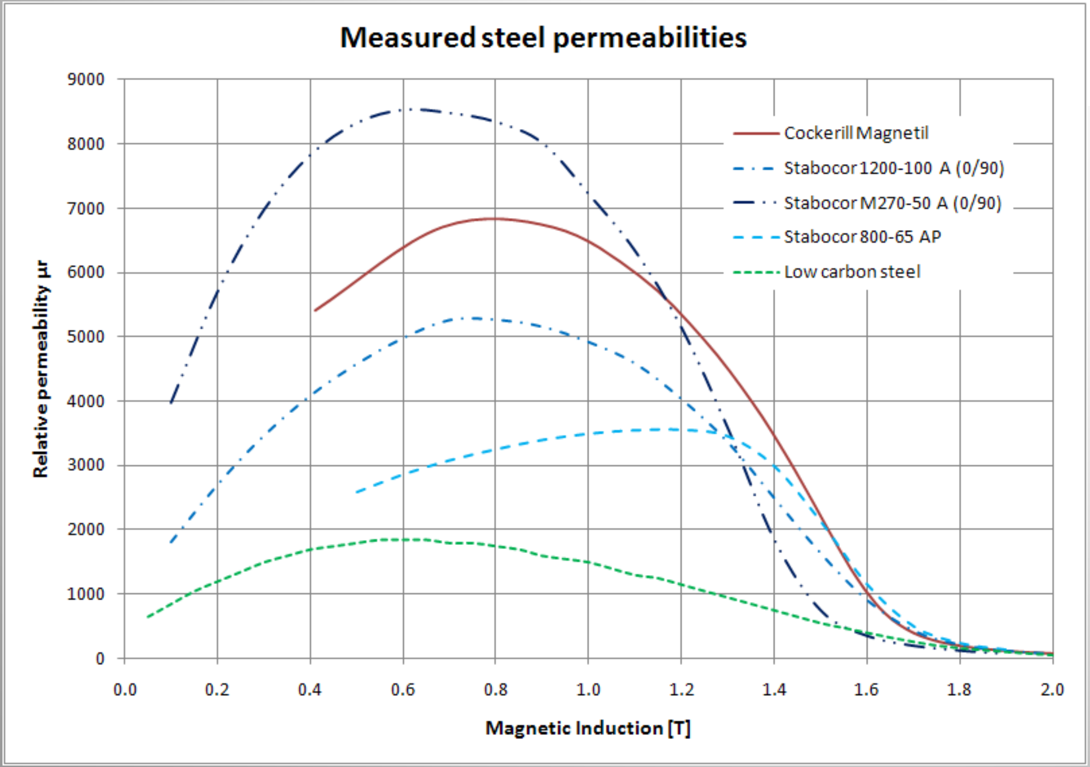
Note: the asymmetric field distribution is an artifact from the FE-mesh

Permeability

$$\vec{B} = \mu\vec{H} \qquad \mu = \mu_0\mu_r$$



Permeability: correlation between magnetic field strength H and magnetic flux density B



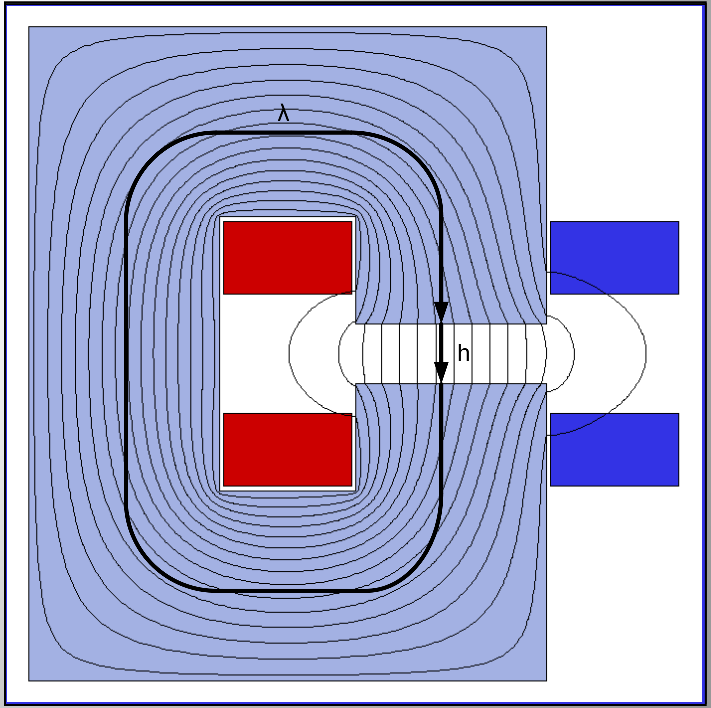
Ferro-magnetic materials: high permeability (μ_r>>1), but not constant

Excitation current in a dipole

Ampere's law $\oint \vec{H} \cdot d\vec{l} = NI$ and $\vec{B} = \mu \vec{H}$

leads to $$NI = \oint \frac{\vec{B}}{\mu} \cdot d\vec{l} = \int_{gap} \frac{\vec{B}}{\mu_{air}} \cdot d\vec{l} + \int_{yoke} \frac{\vec{B}}{\mu_{iron}} \cdot d\vec{l} = \frac{Bh}{\mu_{air}} + \frac{B\lambda}{\mu_{iron}}$$

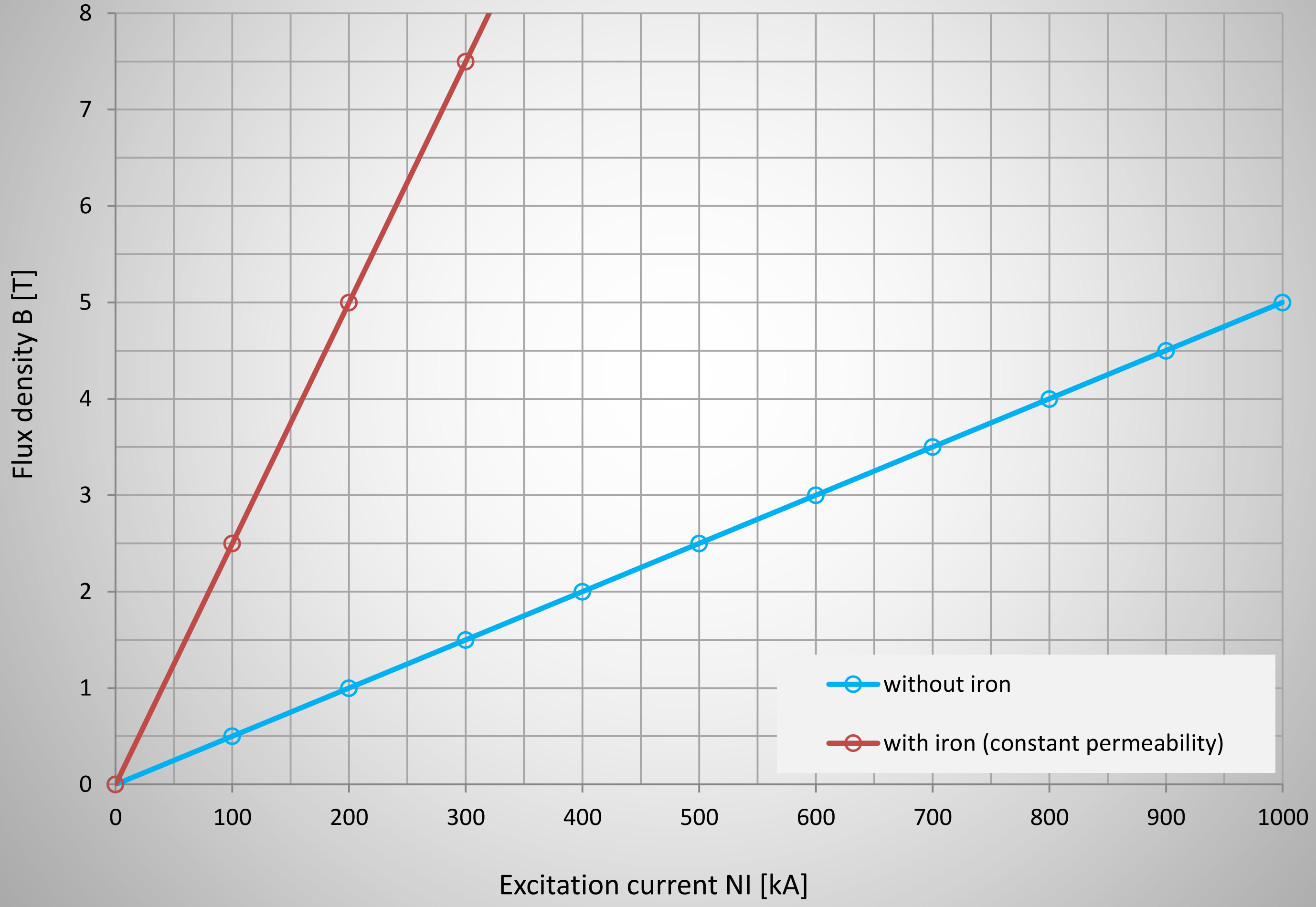
assuming, that B is constant along the path.



If the iron is not saturated: $$\frac{h}{\mu_{air}} >> \frac{\lambda}{\mu_{iron}}$$

then: $$NI_{(per\ pole)} \approx \frac{Bh}{2\mu_0}$$

Transfer function



Reluctance and saturation

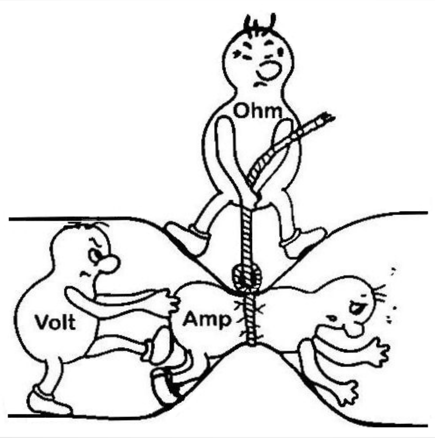
Similar to electrical circuits, one can define the 'resistance' of a magnetic circuit, called 'reluctance':

Ohm's law:

$$R_E = \frac{U}{I} = \frac{l_E}{A_E \sigma}$$

Hopkinson's law:

$$R_M = \frac{NI}{\Phi} = \frac{l_M}{A_M \mu_r \mu_0}$$



- Voltage drop U [V]
- Resistance R_E [Ω]
- Current I [A]
- El. conductivity σ [S/m]
- Conductor length l_E [m]
- Conductor cross section A_E [m^2]

- Magneto-motive force NI [A]
- Reluctance R_M [A/Vs]
- Magnetic flux Φ [Wb]
- Permeability μ [Vs/Am]
- Flux path length in iron l_M [m]
- Iron cross section A_M [m^2] (perpendicular to flux)

…but: μ_{iron} is in general not constant!

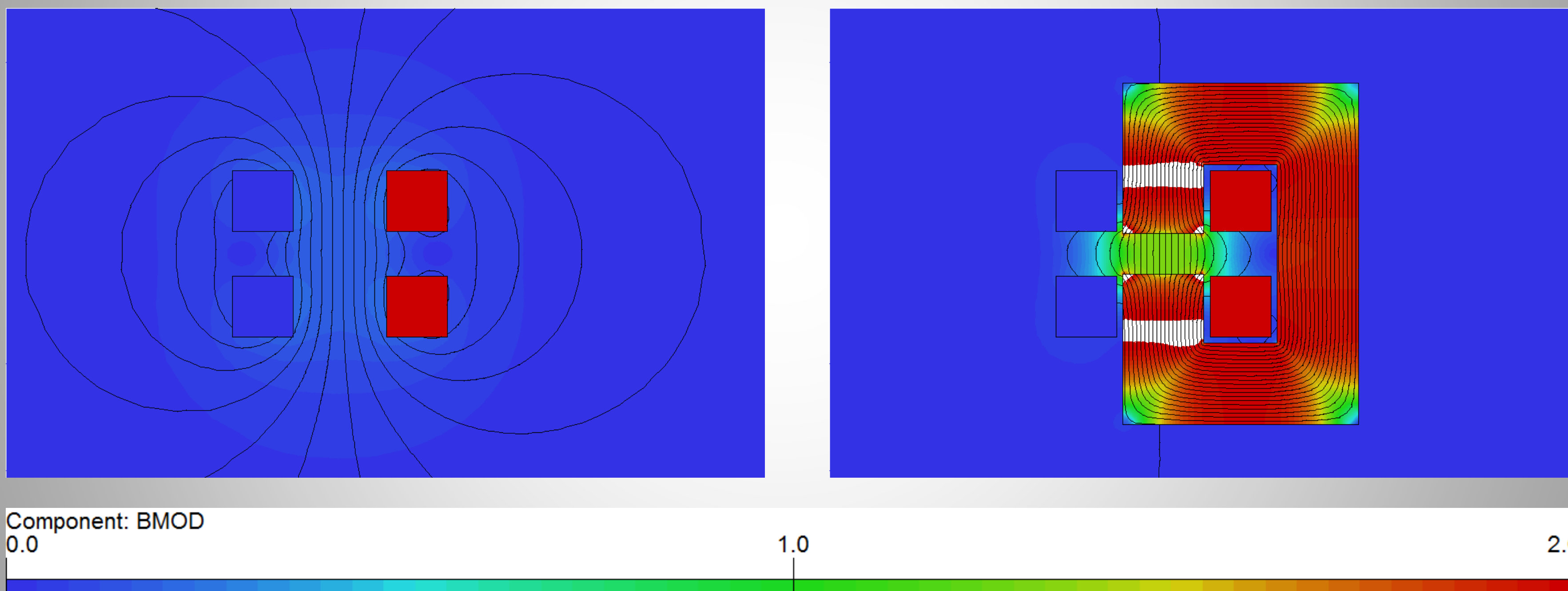
Reluctance and saturation

$I = 32$ kA
$B_{centre} = 0.09$ T

$I = 64$ kA
$B_{centre} = 0.18$ T

$I = 32$ kA
$B_{centre} = 0.80$ T

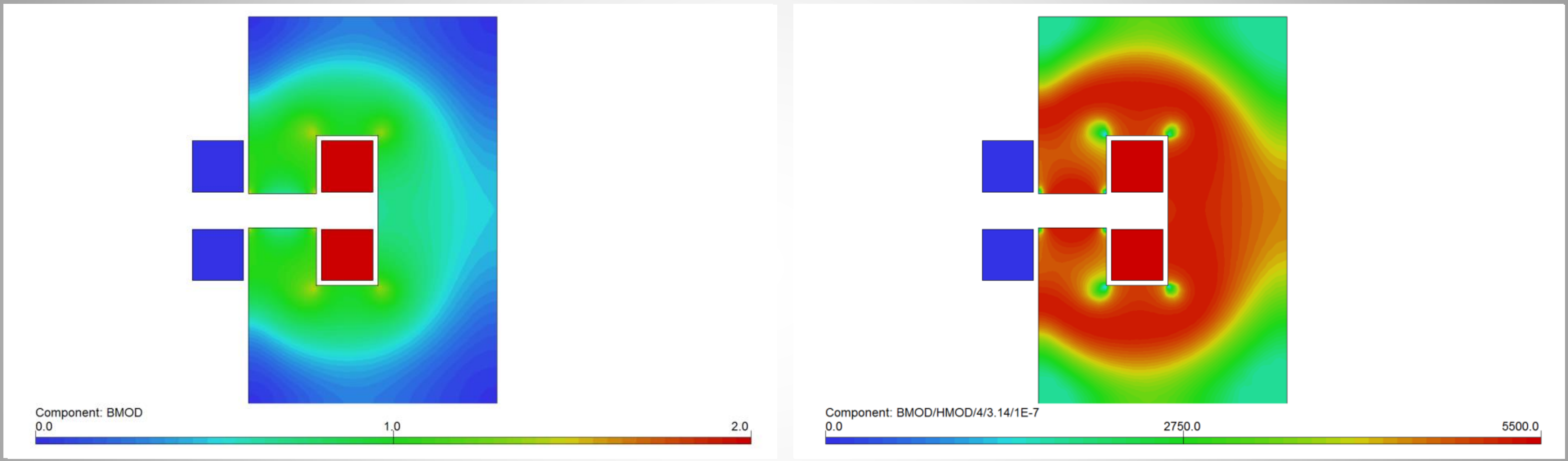
$I = 64$ kA
$B_{centre} = 1.30$ T



Increase of B above 1.5 T in iron requires non-proportional increase of H

Iron saturation (small μ_{iron}) leads to inefficiencies

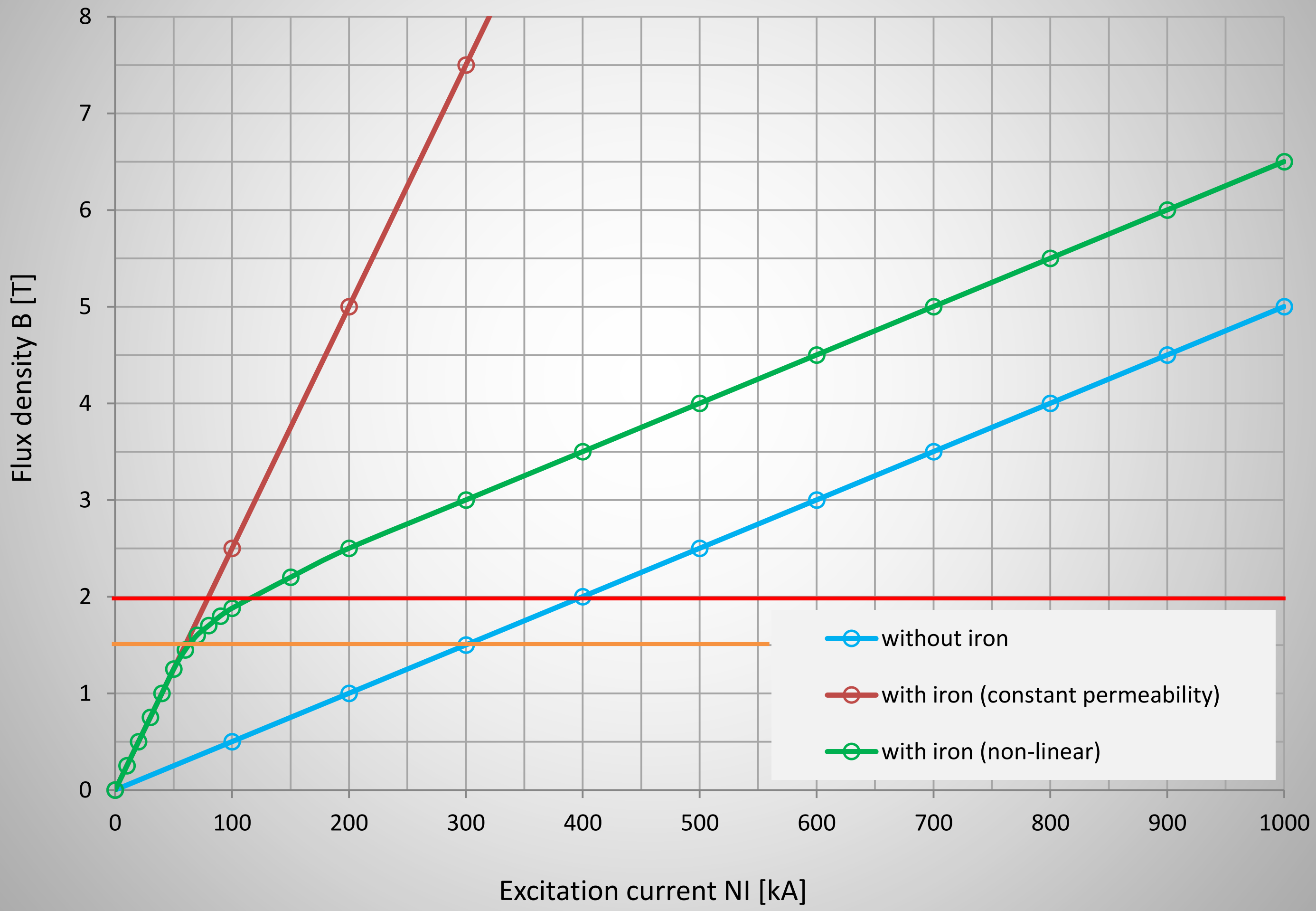
Reluctance and saturation



Keep yoke reluctance small by providing sufficient iron cross-section!

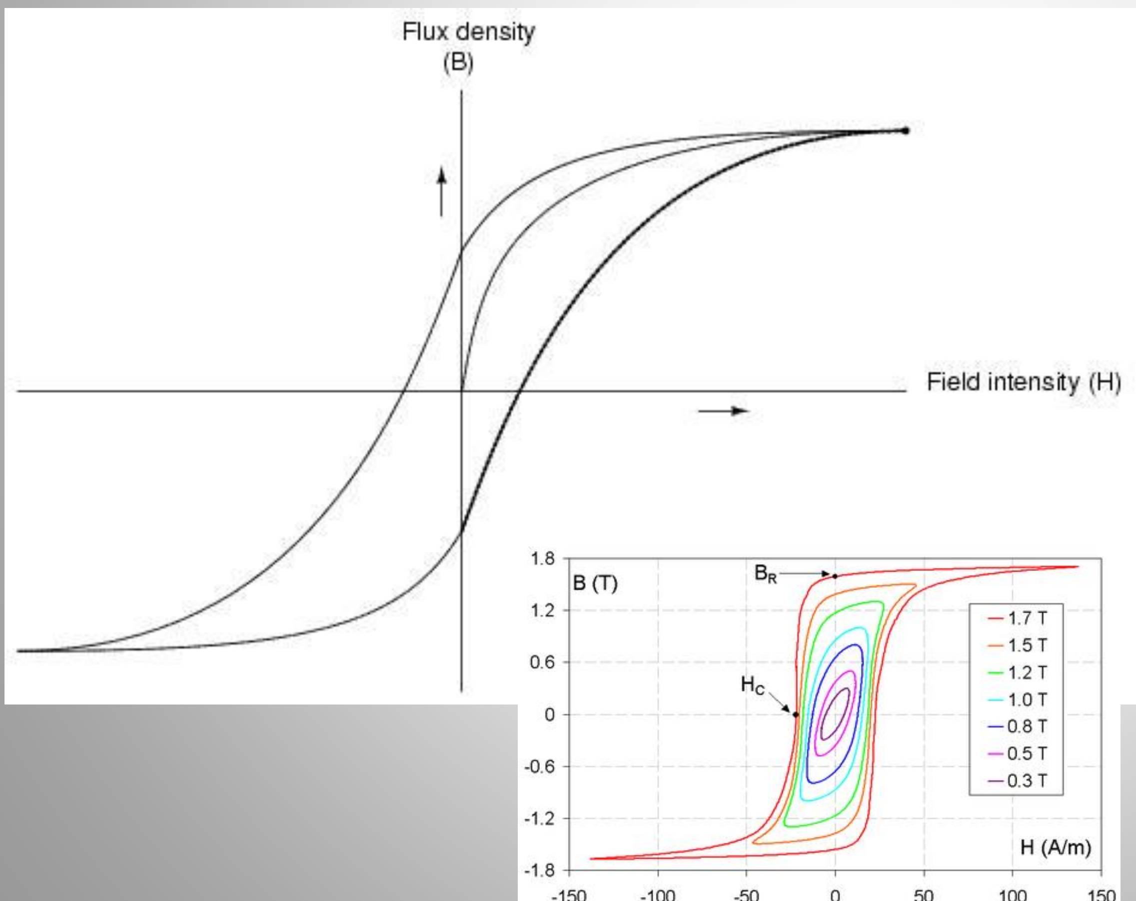
Reluctance and saturation

$$\vec{B} = \mu_0 \vec{H} + \vec{J} = \mu_0 \mu_r \vec{H}$$



Steel hysteresis

Flux density $B(H)$ as a function of the field strength is different, when increasing and decreasing excitation

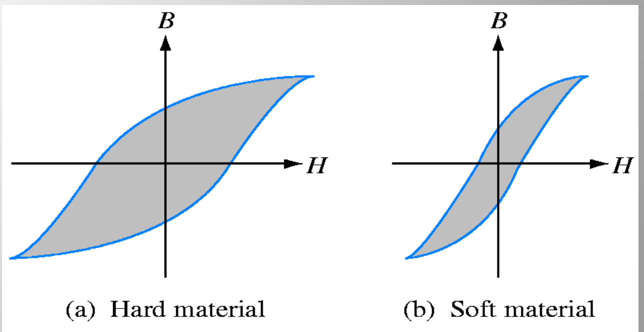


Remanent field (Retentivity):

$H = 0 \rightarrow B = B_r > 0$

Coercivity or coercive force:

$B = 0 \rightarrow H = H_c < 0$



$H_c > 1000$ A/m $H_c < 1000$ A/m

Residual field in a magnet

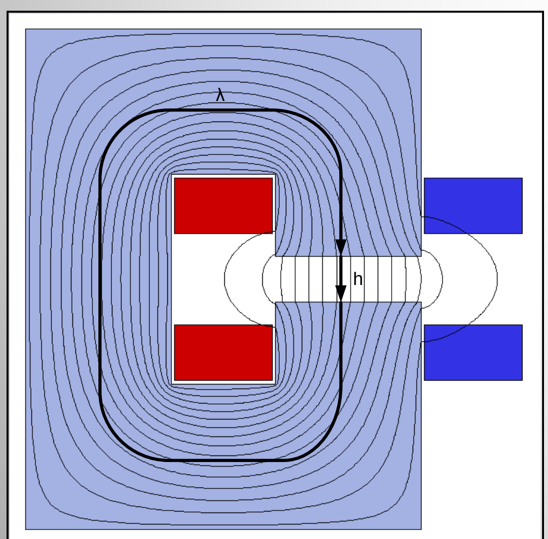
In a continuous ferro-magnetic core (transformer) the residual field is determined by the remanent field B_r

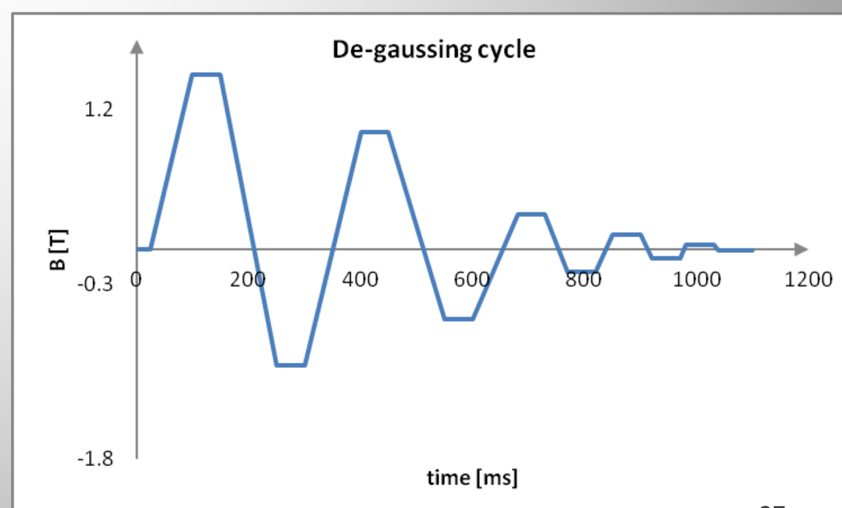
In a magnet core (gap), the residual field B_{res} is determined by the coercivity H_c

Assuming the coil current $I = 0$:

$$\oint \vec{H} \cdot \vec{dl} = \int_{gap} \vec{H}_{gap} \cdot \vec{dl} + \int_{yoke} \vec{H}_c \cdot \vec{dl} = 0$$

$$B_{res} = -\mu_0 H_c \frac{\lambda}{h}$$



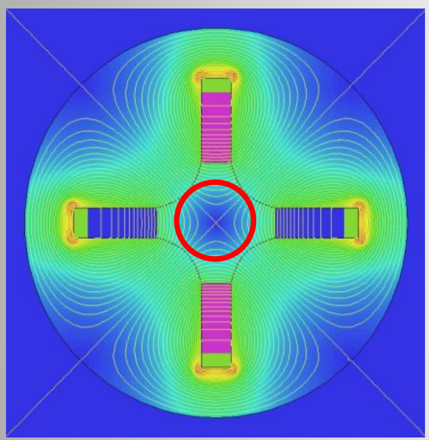


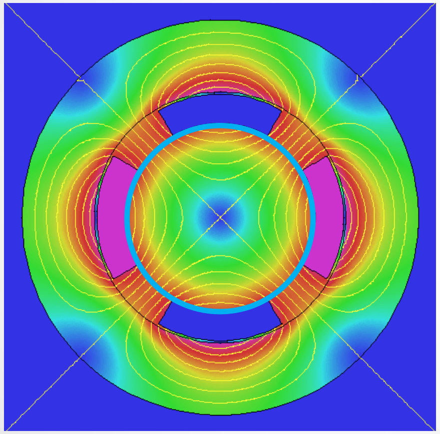
Demagnetization cycle!

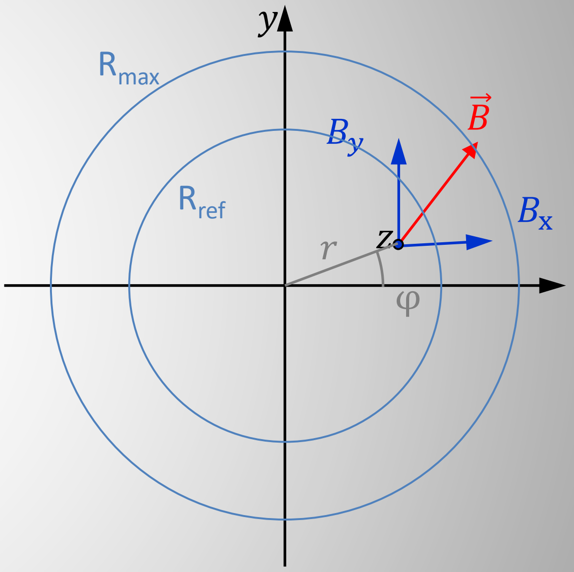
Field description

How can we conveniently describe the field in the aperture?

- at any point (in 2D) $z = x + iy = re^{i\varphi}$
- for any field configuration
- regardless of the magnet technology







Solution: multipole expansion, describing the field within a circle of validity with scalar coefficients

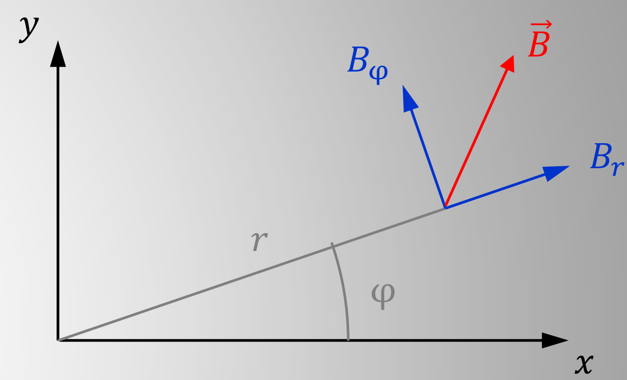
$$B_y(z) + iB_x(z) = \sum_{n=1}^{\infty} (B_n + iA_n) \left(\frac{z}{R_{ref}} \right)^{n-1}$$

Field description

For radial and tangential components of the field the series contains sin and cos terms (Fourier decomposition):

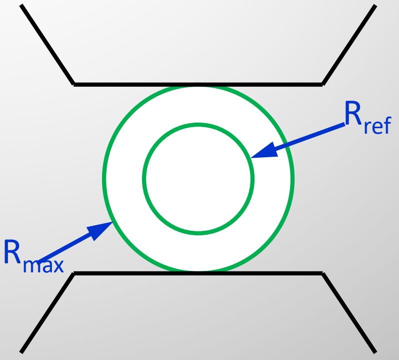
$$B_r(r,\varphi) = \sum_{n=1}^{\infty} \left(\frac{r}{R_{ref}}\right)^{n-1} [B_n \sin(n\varphi) + A_n \cos(n\varphi)]$$

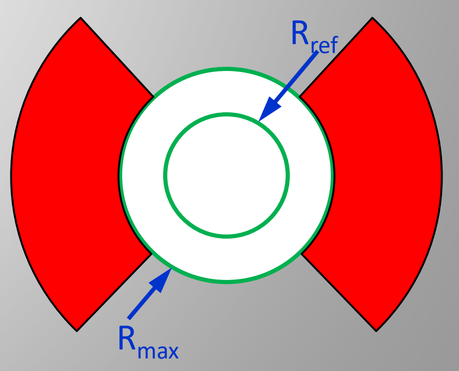
$$B_\varphi(r,\varphi) = \sum_{n=1}^{\infty} \left(\frac{r}{R_{ref}}\right)^{n-1} [B_n \cos(n\varphi) - A_n \sin(n\varphi)]$$



This 2D decomposition holds only in a region of space:

- without magnetic materials ($\mu_r = 1$)
- without currents
- when B_z is constant

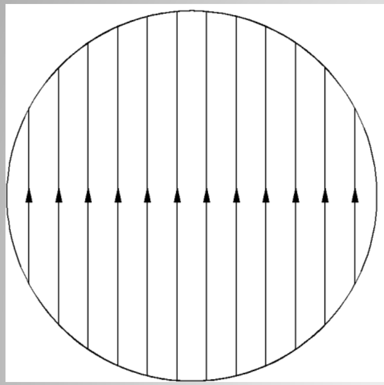




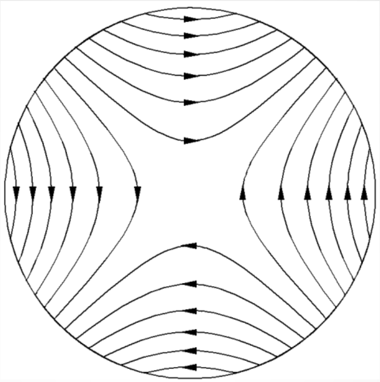
Field description

Each multipole term has a corresponding magnet type:

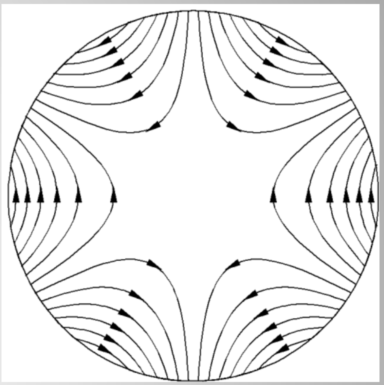
B_1: normal dipole



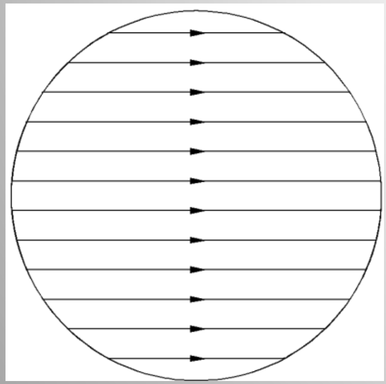
B_2: normal quadrupole



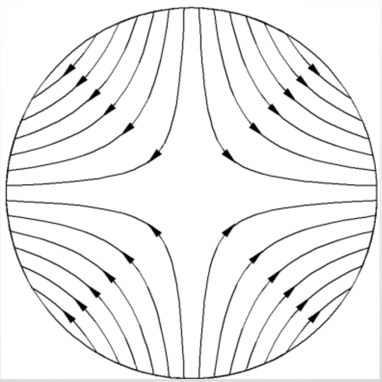
B_3: normal sextupole



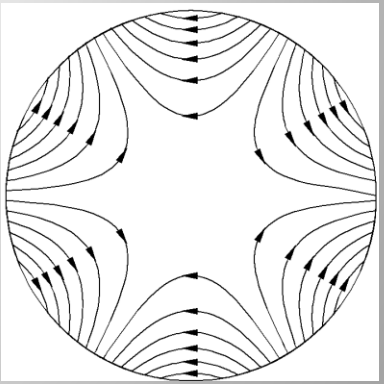
A_1: skew dipole



A_2: skew quadrupole



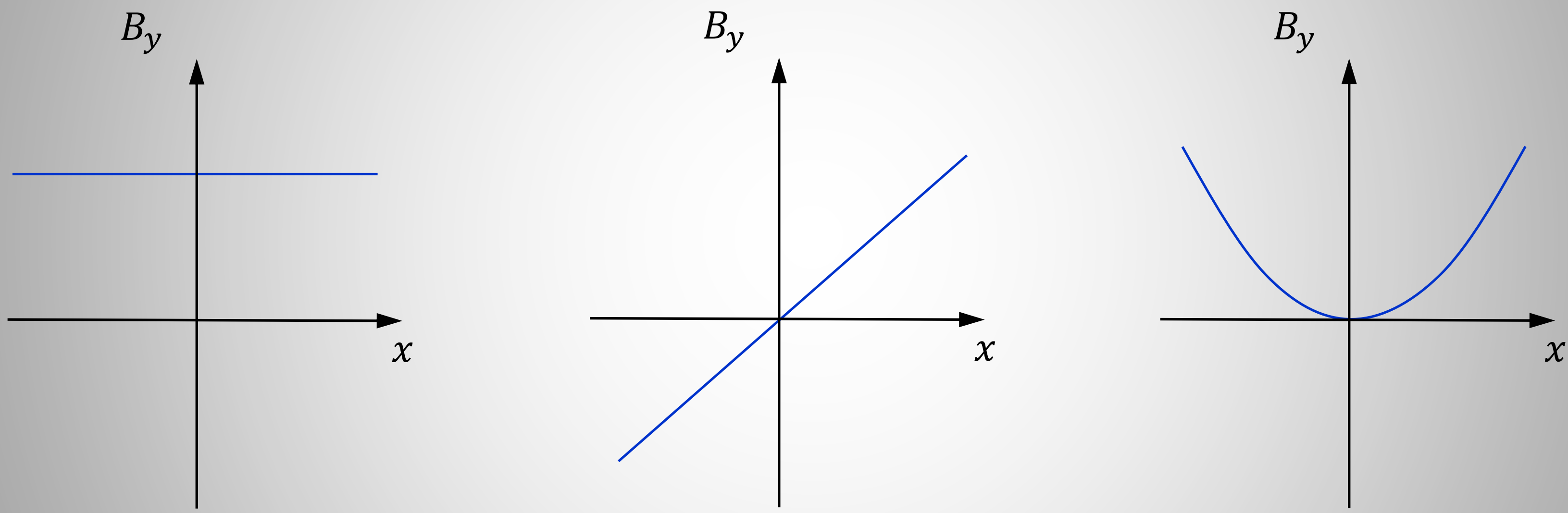
A_3: skew sextupole



Vector equipotential lines are flux lines. $\vec{B}$ is tangential to the flux lines point by point.
Scalar equipotential lines are orthogonal to the vector equipotential lines. They define the boundary conditions for shaping the field.

Field description

Field expansion along x: $B_y(x) = \sum_{n=1}^{\infty} B_n \left(\frac{x}{R_{ref}}\right)^{n-1} = B_1 + B_2 \frac{x}{R_{ref}} + B_3 \frac{x^2}{R_{ref}^2} + \cdots$



B_1: dipole B_2: quadrupole B_3: sextupole

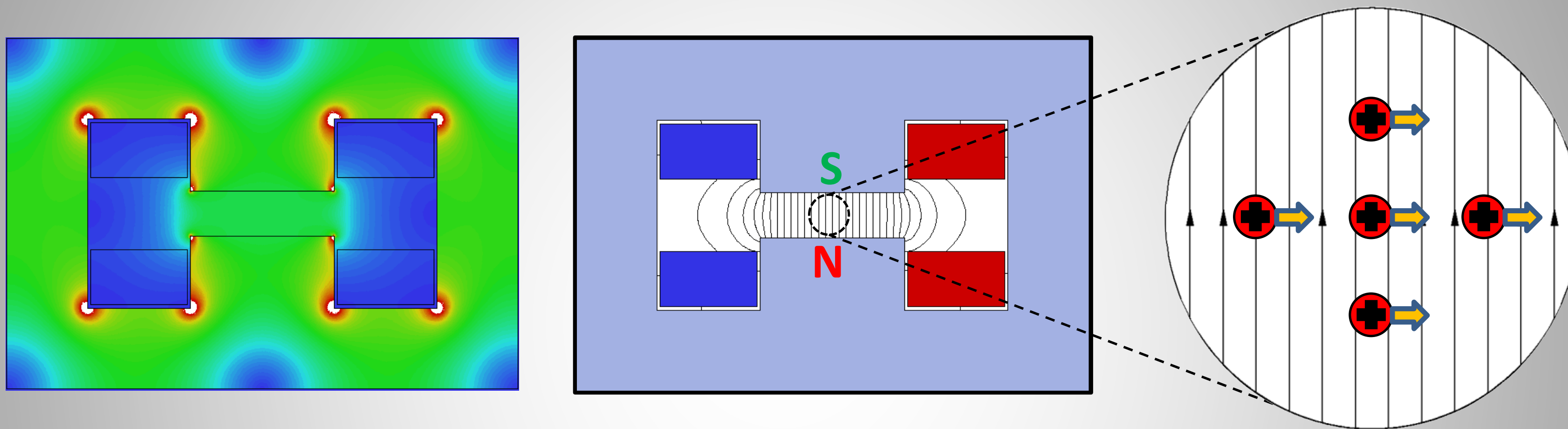
$$G = \frac{B_2}{R_{ref}} = \frac{\partial B_y}{\partial x}$$

The field profile in the horizontal plane follows a polynomial expansion

The ideal poles for each magnet type are lines of constant scalar potential

Dipole

Purpose: bend or steer the particle beam



Equation for normal (non-skew) ideal (infinite) poles:

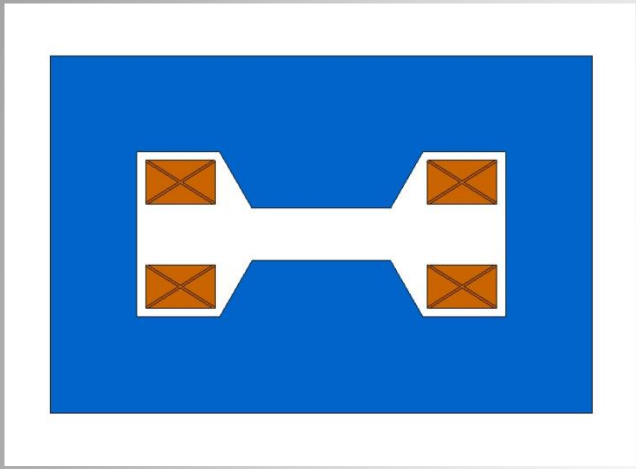
$$y = \pm h/2 \quad (\rightarrow \text{straight line with } h = \text{gap height})$$

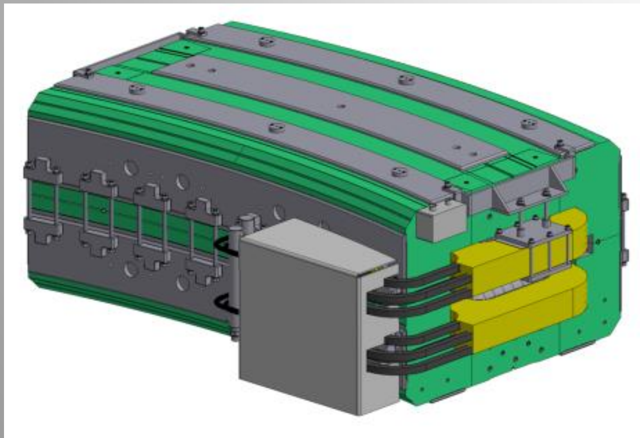
Magnetic flux density: $B_x = 0$; $B_y = B_1 = \text{const.}$

Applications: synchrotrons, transfer lines, spectrometry, beam scanning

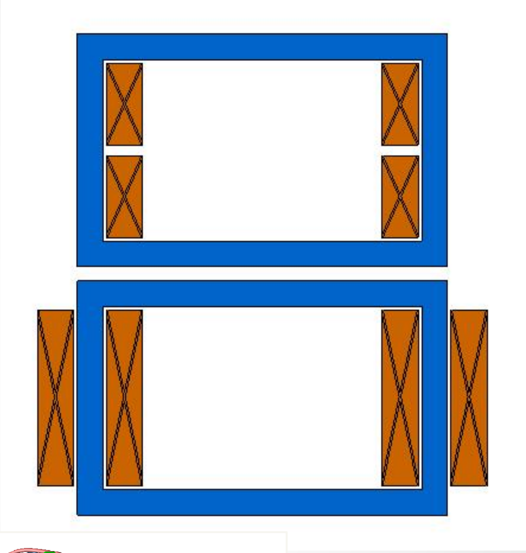
Dipole types

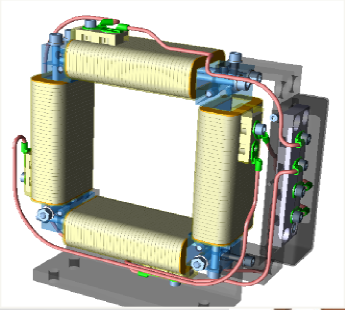
H-Shape

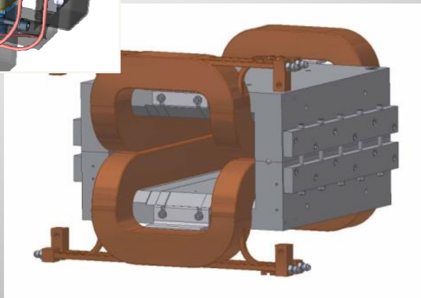




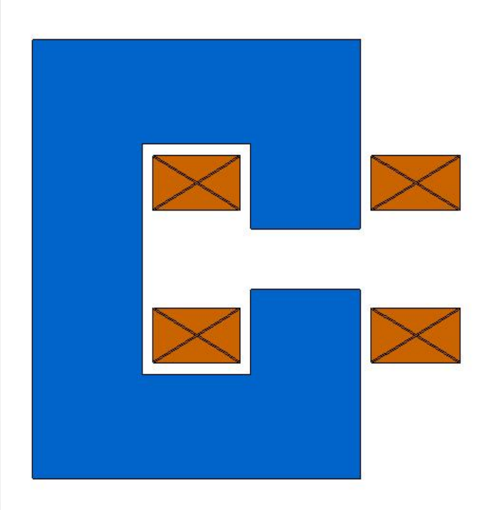
O-Shape

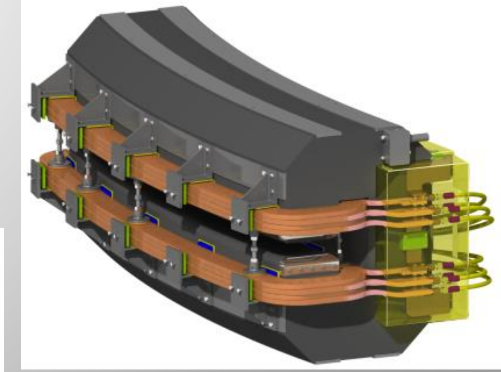






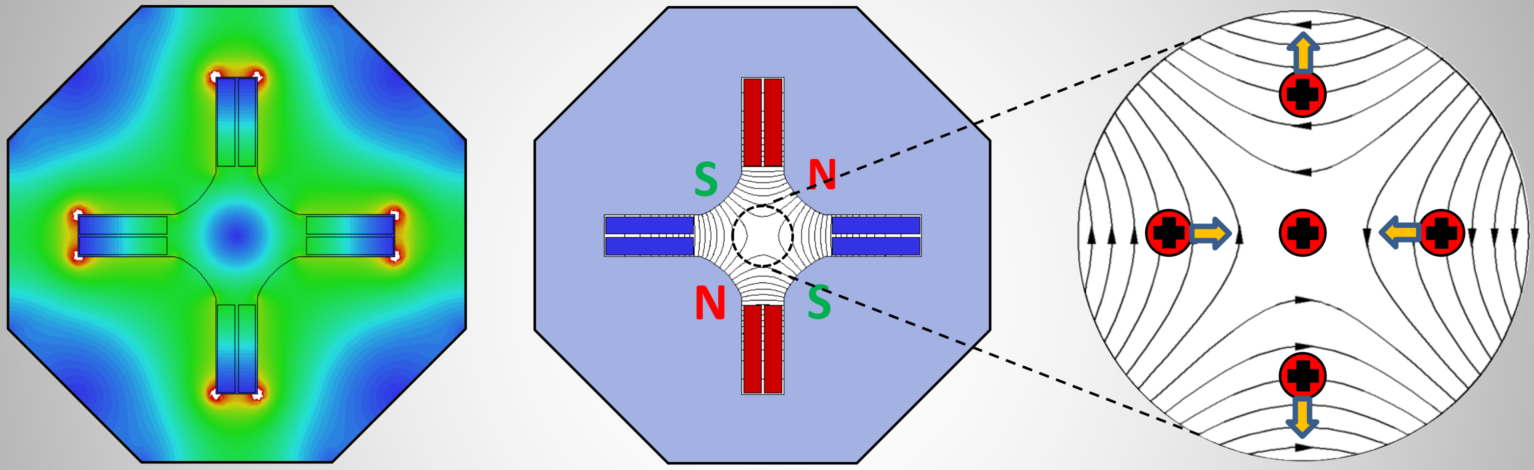
C-Shape





Quadrupole

Purpose: focusing the beam (horizontally focused beam is vertically defocused)



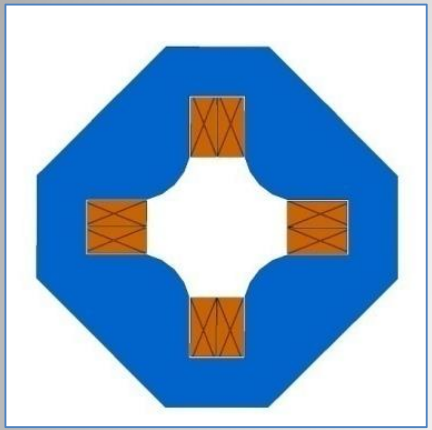
Equation for normal (non-skew) ideal (infinite) poles:

$2xy = \pm r^2$ (→ hyperbola with r = aperture radius)

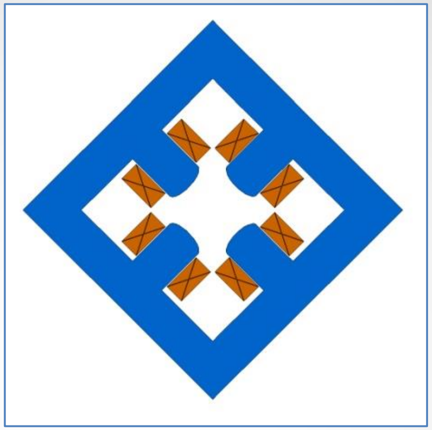
Magnetic flux density: $B_x = \frac{B_2}{R_{ref}} y$; $B_y = \frac{B_2}{R_{ref}} x$

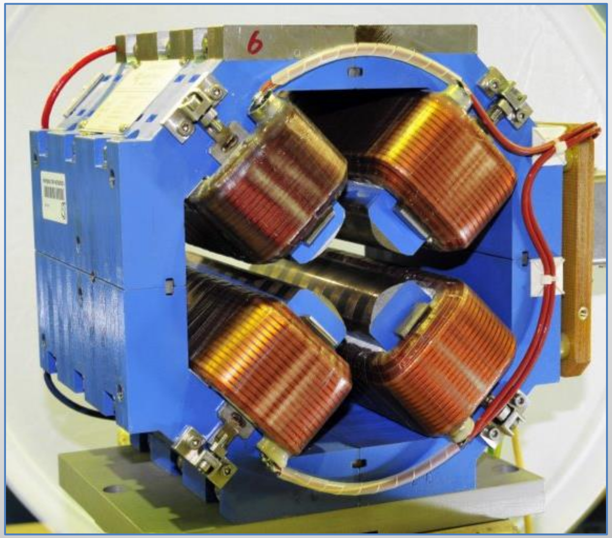
Quadrupole types

Standard quadrupole I

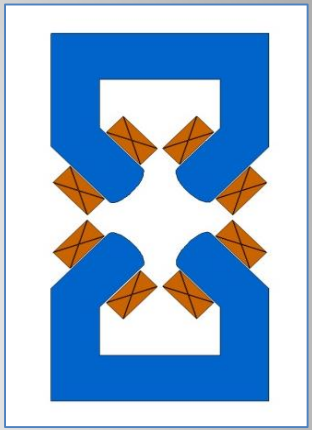


Standard quadrupole II



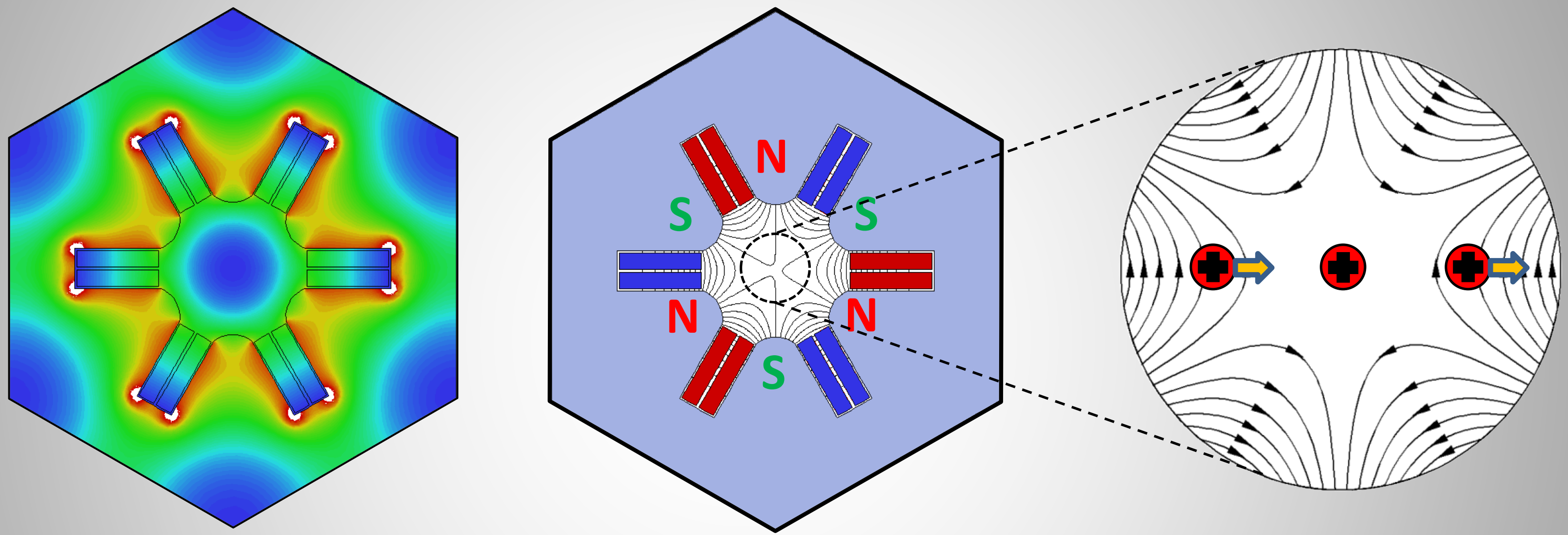


Collins or Figure-of-Eight



Sextupole

Purpose: correct chromatic aberrations of ‘off-momentum’ particles



Equation for normal (non-skew) ideal (infinite) poles:

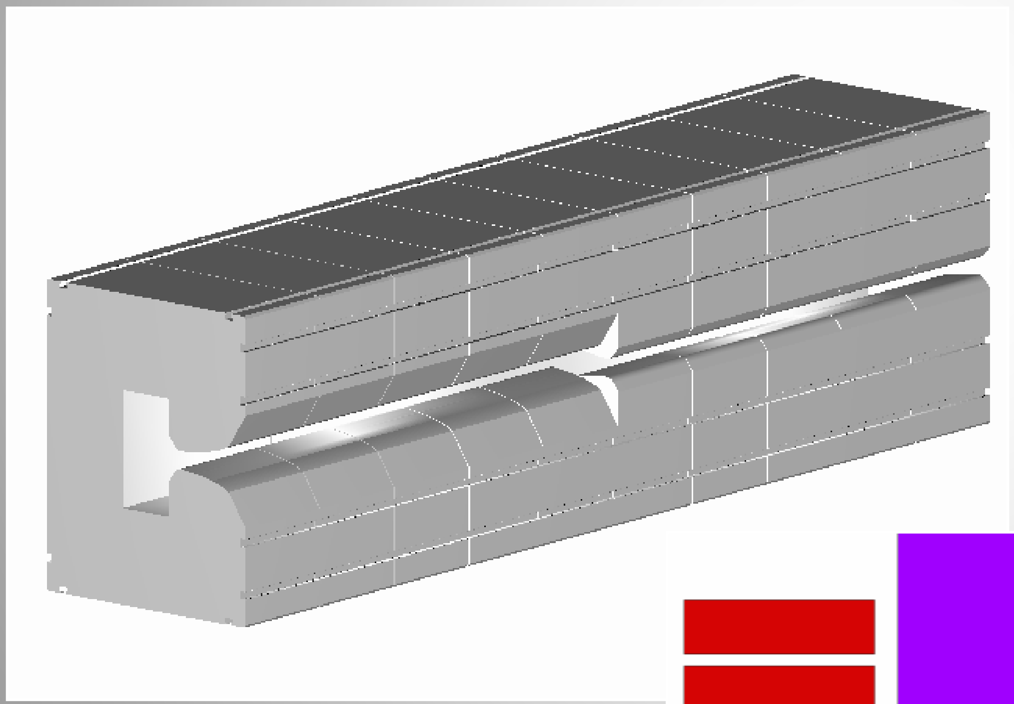
$$3x^2y - y^3 = \pm r^3 \text{ (with } r = \text{aperture radius)}$$

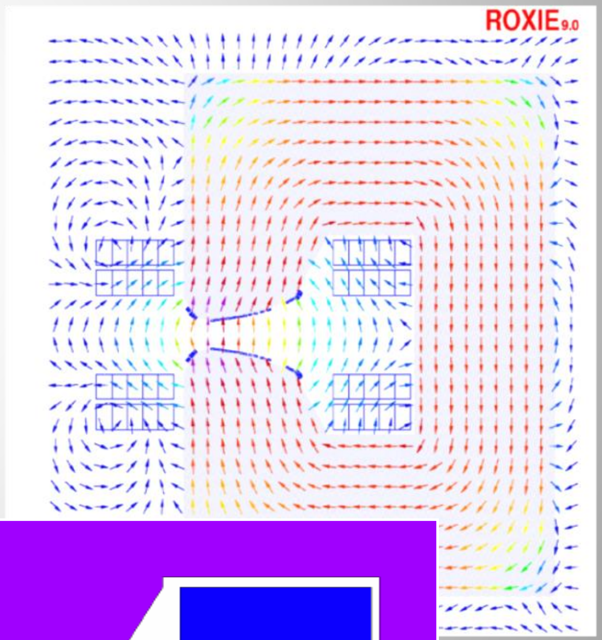
Magnetic flux density: $B_x = \frac{B_3}{R_{ref}^2} xy$; $B_y = \frac{B_3}{R_{ref}^2}(x^2 - y^2)$

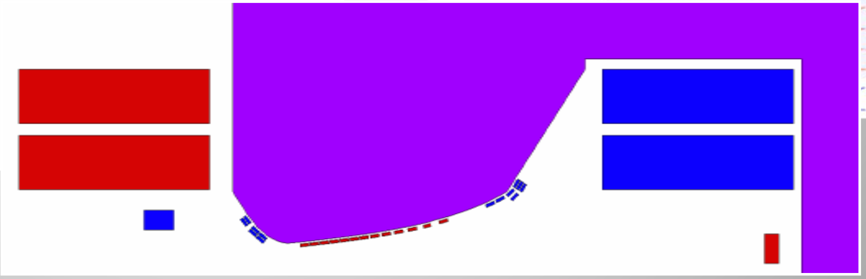
Combined function magnets

Functions generated by pole shape (sum a scalar potentials):

- Amplitudes cannot be varied independently
- Dipole and quadrupole: PS main magnet (PFW, Fo8…)



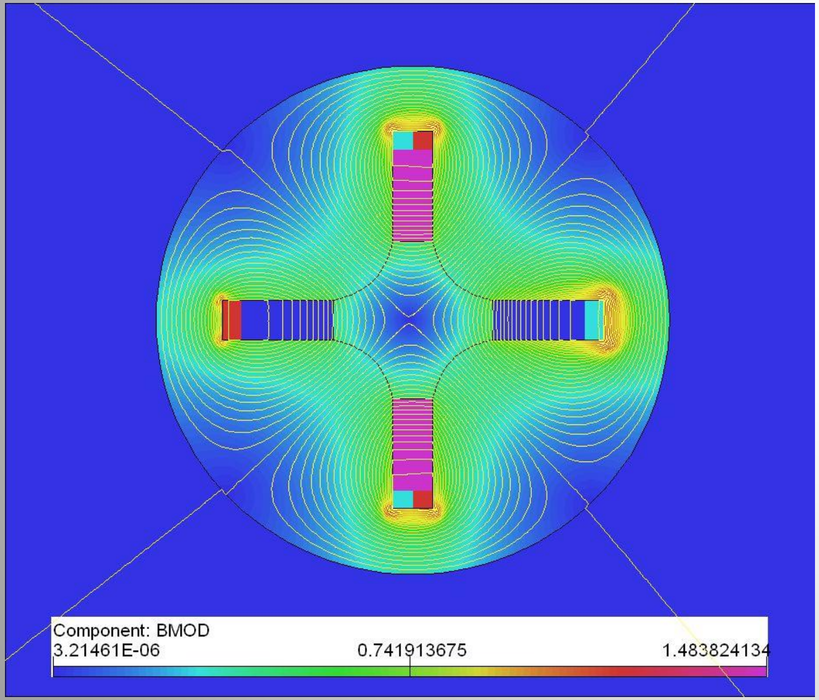


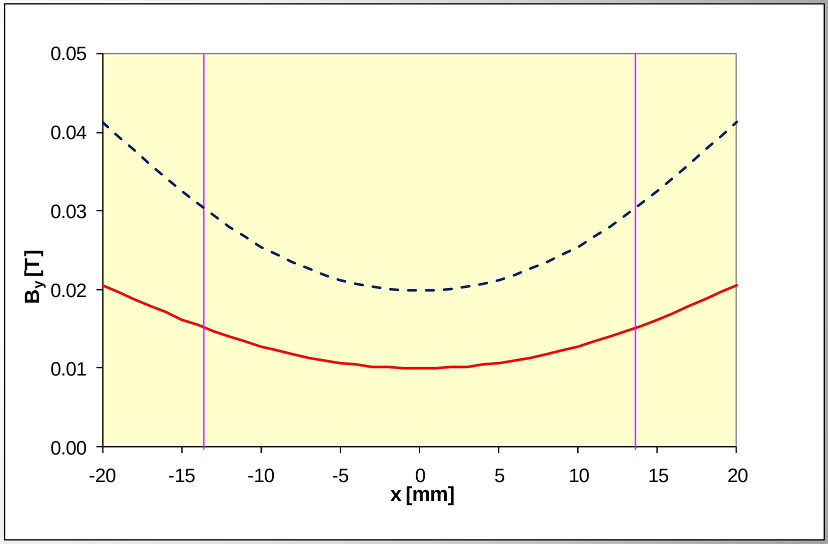


Combined function magnets

Functions generated by individual coils:

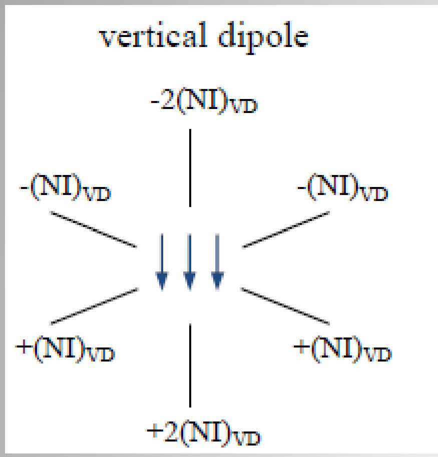
Amplitudes can be varied independently

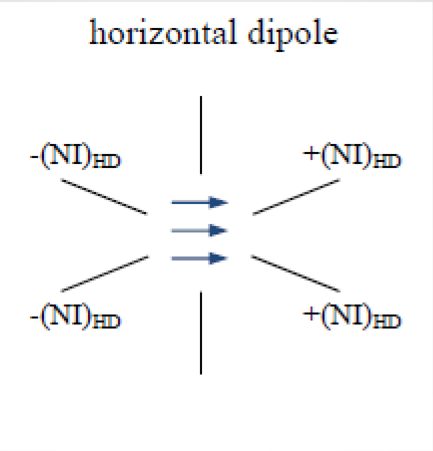


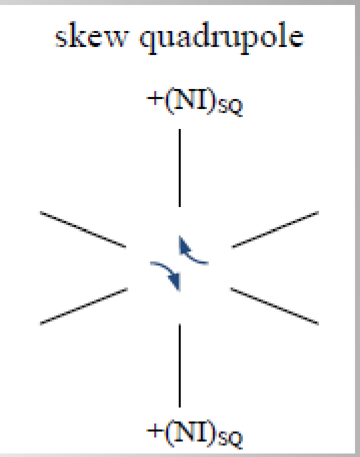


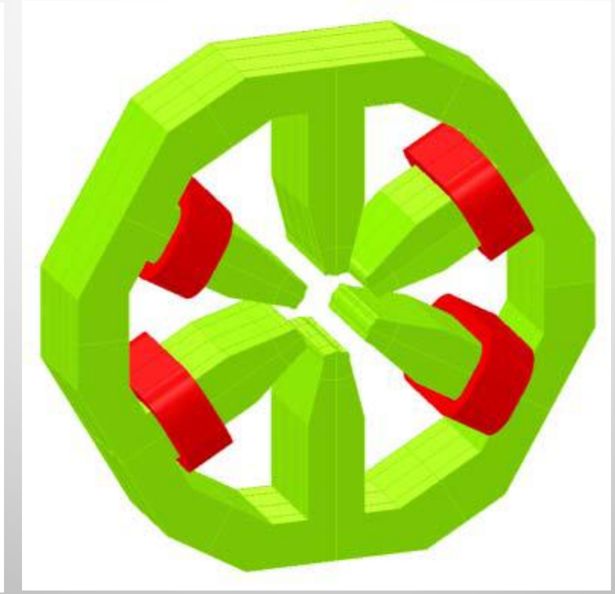
Quadrupole and corrector dipole (strong sextupole component in dipole field)

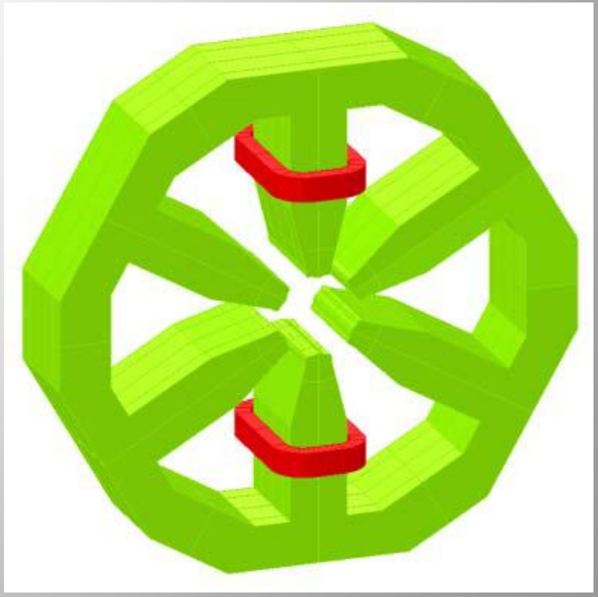
Combined function magnets







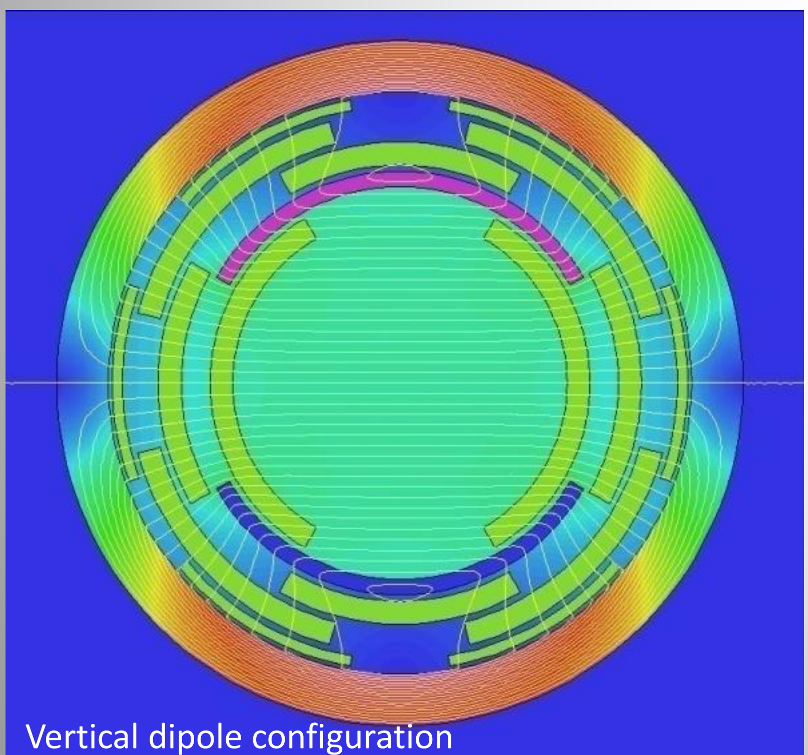




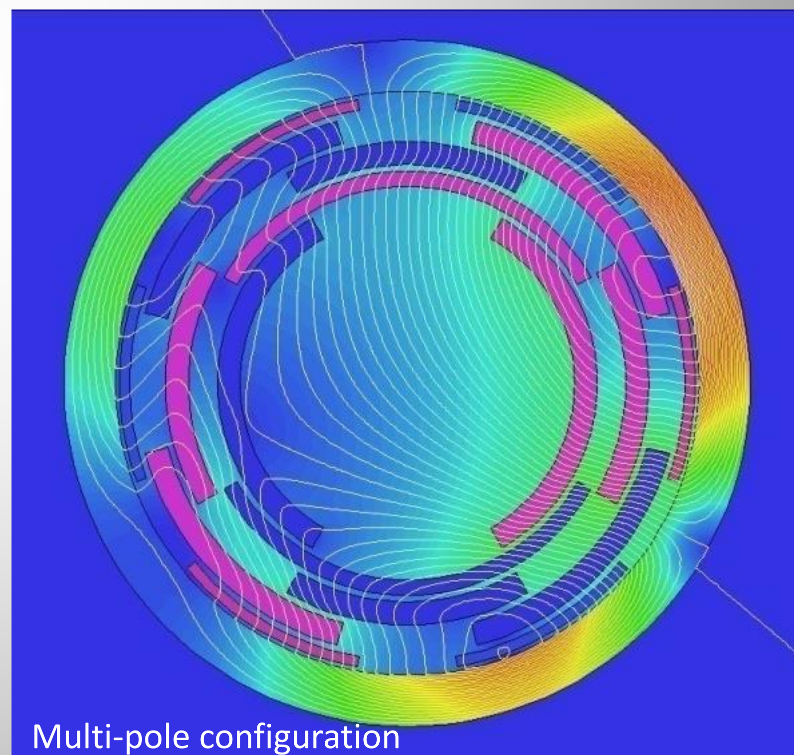
Courtesy of A. Milanese

Coil dominated magnets

- Nested multi-pole corrector (moderate field levels)
- Iron for shielding only
- Field determined by current distribution

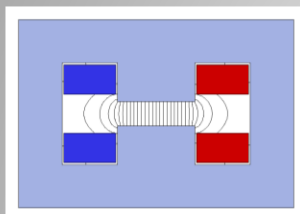
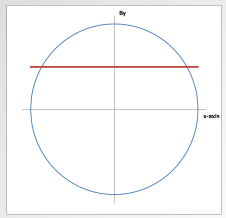
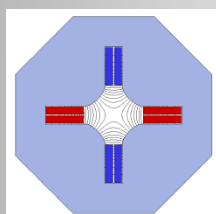
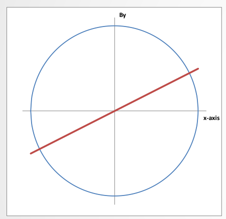
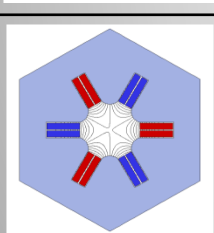
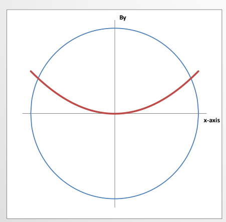
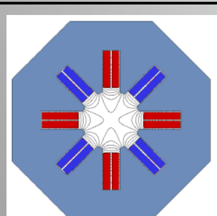
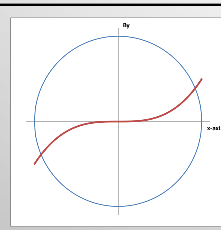


Vertical dipole configuration



Multi-pole configuration

Magnet types

Pole shape	Field distribution	Pole equation	B_x, B_y
		$y = \pm r$	$B_x = 0$ $B_y = B_1 = \text{const.}$
		$2xy = \pm r^2$	$B_x = \frac{B_2}{R_{ref}} y$ $B_y = \frac{B_2}{R_{ref}} x$
		$3x^2y - y^3 = \pm r^3$	$B_x = \frac{B_3}{R_{ref}^2} xy$ $B_y = \frac{B_3}{R_{ref}^2}(x^2 - y^2)$
		$4(x^3y - xy^3) = \pm r^4$	$B_x = \frac{B_4}{R_{ref}^3}(3x^2y - y^3)$ $B_y = \frac{B_4}{6R_{ref}^3}(x^3 - 3xy^2)$

Summary

- Magnets are needed to guide and shape particle beams
- Coils carry the electrical current, the iron yoke carries the magnetic flux
- Magnetic steel is characterized by its relative permeability μ_r and its coercivity H_c
- Iron saturation influences the efficiency of the magnetic circuit and has to be taken into account in the design
- The 2D (magnetic) vector field can be expressed as a series of multipole coefficients
- Different magnet types for different functions