Joint Universities Accelerator School

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Normal-conducting accelerator magnets
Lecture 3: Analytical design

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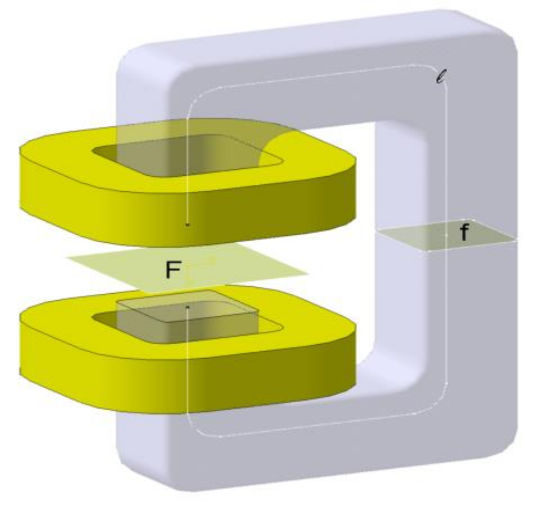
CERN

Lecture 3: Analytical design

- Goals in magnet design
- What do we need to know before starting?
- Defining the requirements & constraints
- Deriving the magnet main parameters
- Coil design and cooling
- Cost estimates and optimization

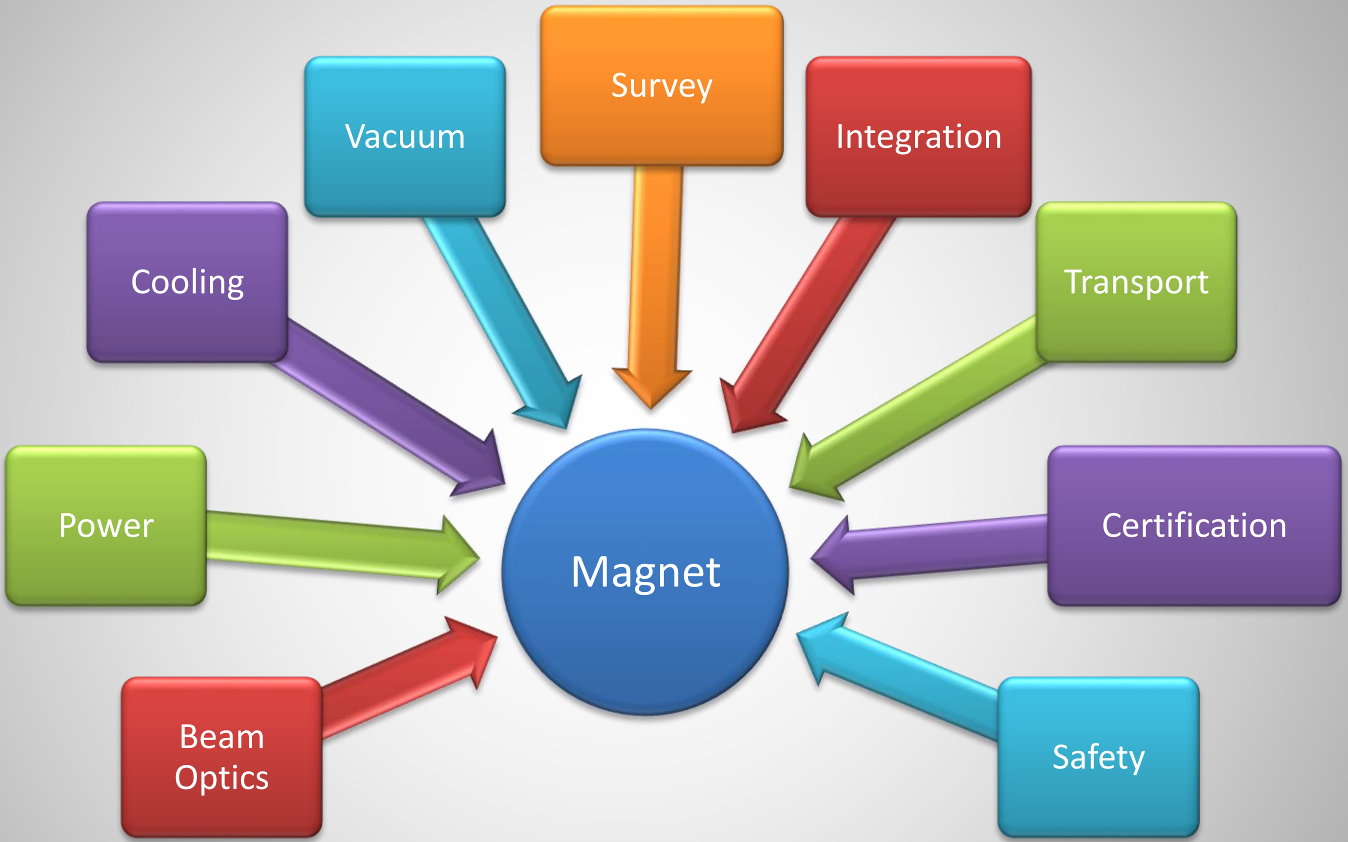


Goals in magnet design

The goal is to produce a product just good enough to perform reliably with a sufficient safety factor at the lowest cost and on time.

- Good enough:
  - Obvious parameters are clearly specified, but tolerance difficult to define
  - Tight tolerances lead to increased costs
- Reliability:
  - Get MTBF high and MTTR reasonably low
  - Reliability is usually unknown for new design
  - Requires experience to search for a compromise between extreme caution and extreme risk (expert review)
- Safety factor:
  - Allows operating a device under more demanding condition as initially foreseen
  - To be negotiated between the project engineer and the management
  - Avoid inserting safety factors a multiple levels (costs!)

Magnet interfaces



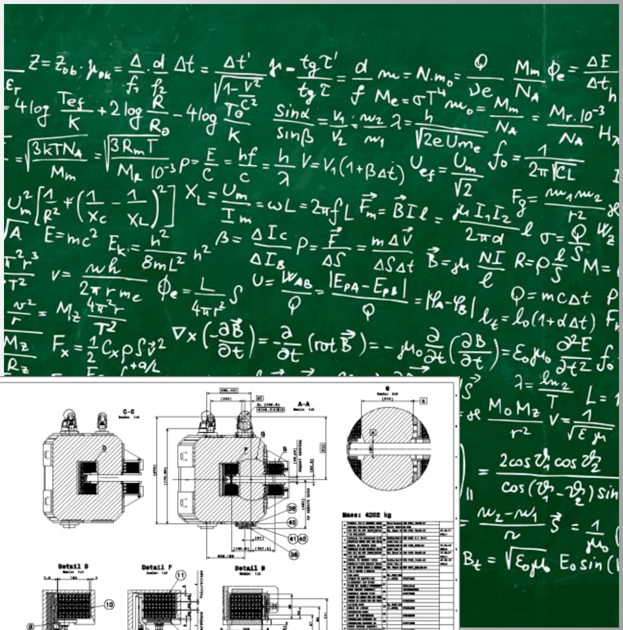
A magnet is not a stand-alone device!

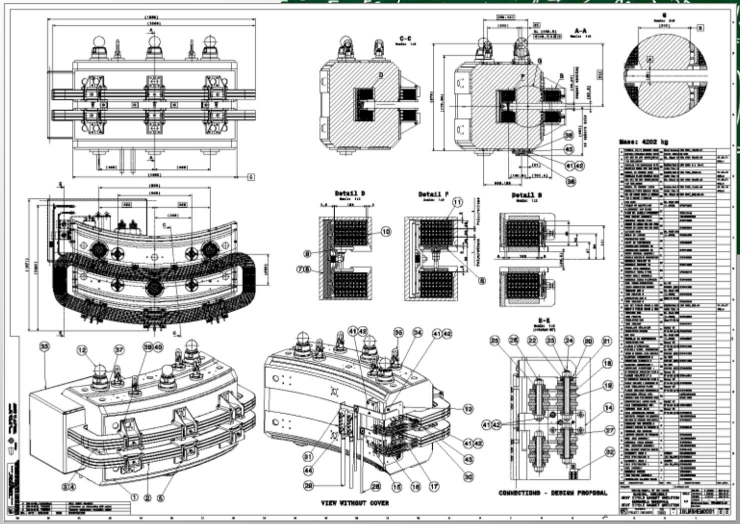
Design process

Electro-magnetic design is an iterative process:

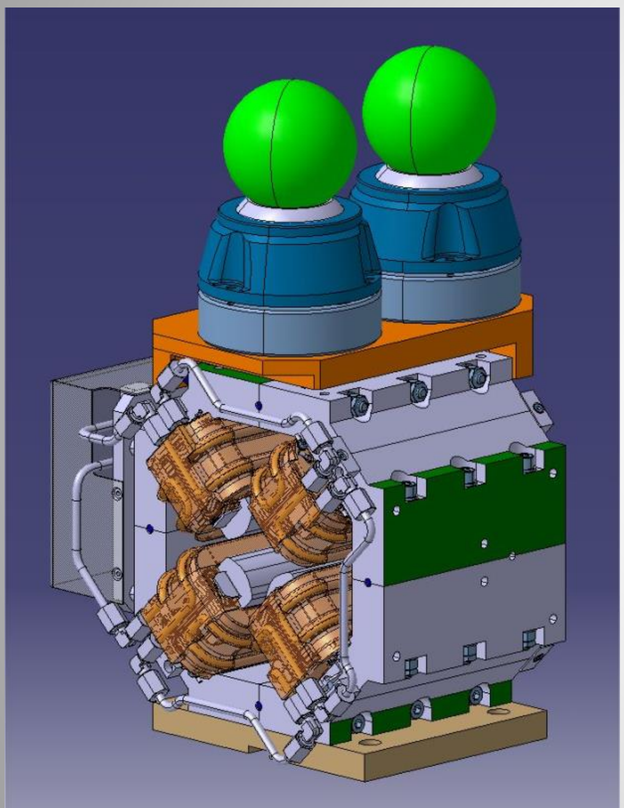
Collect input data → Analytical design → Numerical 2D/3D simulations → Mechanical design → Drawings & specifications

- Field strength (gradient) and magnetic length
- Integrated field strength (gradient)
- Aperture and ‚good field region'
- Field quality:
  - field homogeneity
  - maximum allowed multi-pole errors
  - settling time (time constant)
- Operation mode: continous, cycled
- Electrical parameters
- Mechanical dimensions
- Cooling
- Environmental aspects





Magnet Components



Alignment targets

Yoke

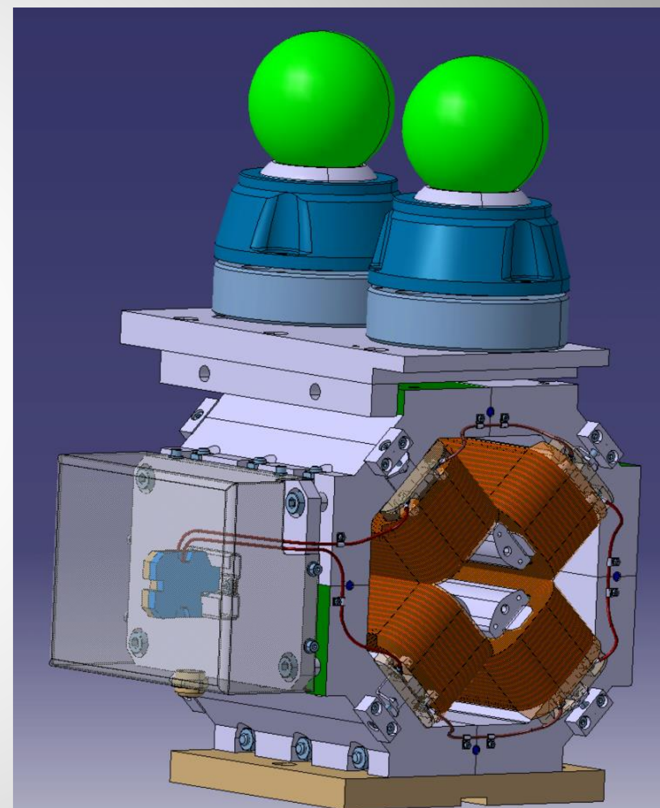
Coils

Sensors

Cooling circuit

Connections

Support



Beam rigidity

Beam rigidity ($B\rho$) [Tm]: $$(B\rho) = \frac{p}{q} = \frac{1}{qc}\sqrt{T^2 + 2T\,E_0}$$

p: particle momentum [kg m/s]

q: particle charge [Coulombs]

c: speed of light [m/s]

T: kinetic beam energy [eV]

E_0: particle rest mass energy [eV]
(0.51 MeV for electrons, 938 MeV for protons)

" …resistance of the particle beam against a change of direction when applying a bending force…"

Magnetic induction

Dipole bending field B [T]:

$$B = \frac{(B\rho)}{r_M}$$

B: Flux density or magnetic induction (vector) [T]

r_M: magnet bending radius [m]

Quadrupole field gradient B' [T/m]:

$$B' = (B\rho)k$$

k: quadrupole strength [m^{-2}]

Sextupole differential gradient B'' [T/m^2]:

$$B'' = (B\rho)m$$

m: sextupole strength [m^{-3}]

Excitation current in a dipole

Ampere's law $\oint \vec{H} \cdot d\vec{l} = NI$ and $\vec{B} = \mu \vec{H}$ with $\mu = \mu_0 \mu_r$

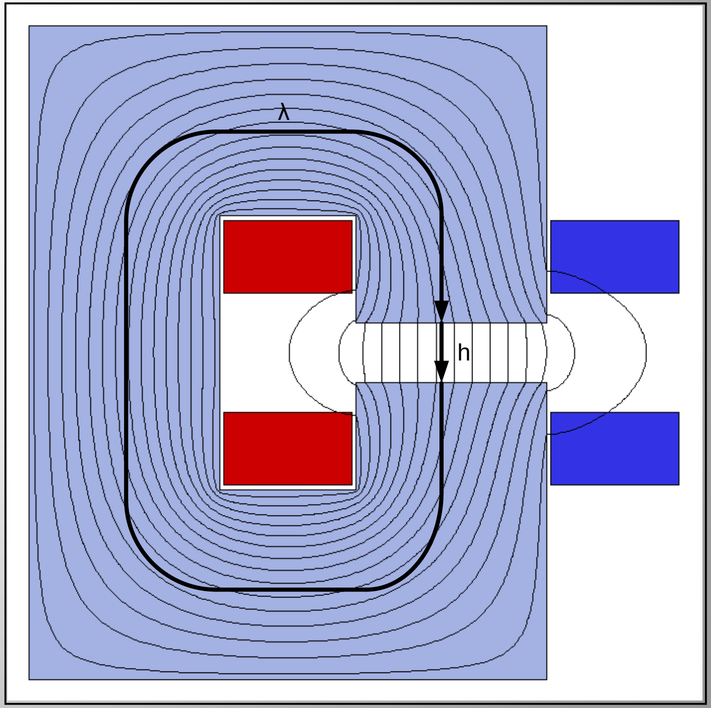
leads to $NI = \oint \frac{\vec{B}}{\mu} \cdot d\vec{l} = \int_{gap} \frac{\vec{B}}{\mu_{air}} \cdot d\vec{l} + \int_{yoke} \frac{\vec{B}}{\mu_{iron}} \cdot d\vec{l} = \frac{Bh}{\mu_{air}} + \frac{B\lambda}{\mu_{iron}}$

assuming, that B is constant along the path

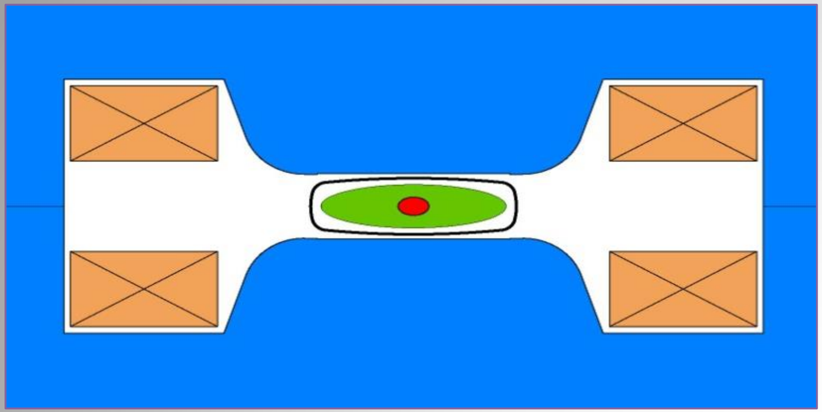
If the iron is not saturated: $\frac{h}{\mu_{air}} >> \frac{\lambda}{\mu_{iron}}$

then: $NI_{(per\ pole)} \approx \frac{Bh}{2\eta\mu_0}$

h: gap height [m]
η: efficiency (typically 95% - 99 %)



Aperture size



Aperture

- Good-field region
  - Max. beam size envelope (typical 3-sigma)
    - Lattice functions: beta functions and dispersion
    - Geometrical transverse emittances (energy depended)
    - Momentum spread

$$\sigma = \sqrt{\varepsilon \beta + \left(D \frac{\Delta p}{p} \right)^2}$$

  - Closed orbit distortions (few mm)
- Vacuum chamber thickness (0.5 – 5 mm)
- Installation and alignment margin (0 – 10 mm)

"…good-field region: central region around the theoretical beam trajectory where the field quality has to be within certain tolerances…"

Pole design

It is easy to derive perfect mathematical pole configurations for a specific field configuration

In practice poles are not ideal: finite width and end effects result in multi-pole errors disturbing the main field

The uniform field region is limited to a small fraction of the pole width

Estimate the size of the poles and calculate the resulting fields (numerically)

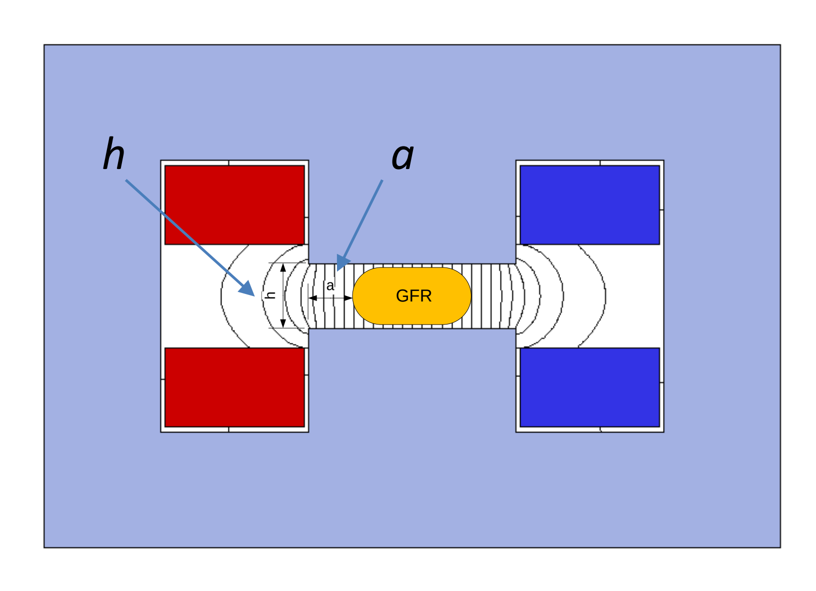
Better approach: calculate the necessary pole overhang for an un-optimized* design

$$x_{unoptimized} = 2\frac{a}{h} = -0.36\ln\frac{\Delta B}{B_0} - 0.90$$

x: pole overhang normalized to the gap

a: pole overhang: excess pole beyond the edge of the good field region to reach the required field uniformity

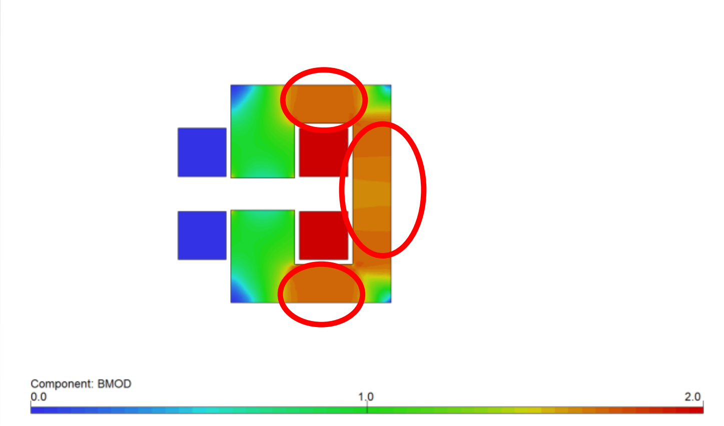
h: magnet gap



*) see Lecture 4 for corresponding formula using an optimized pole design

Yoke dimensioning

Avoid saturated parts in the yoke:

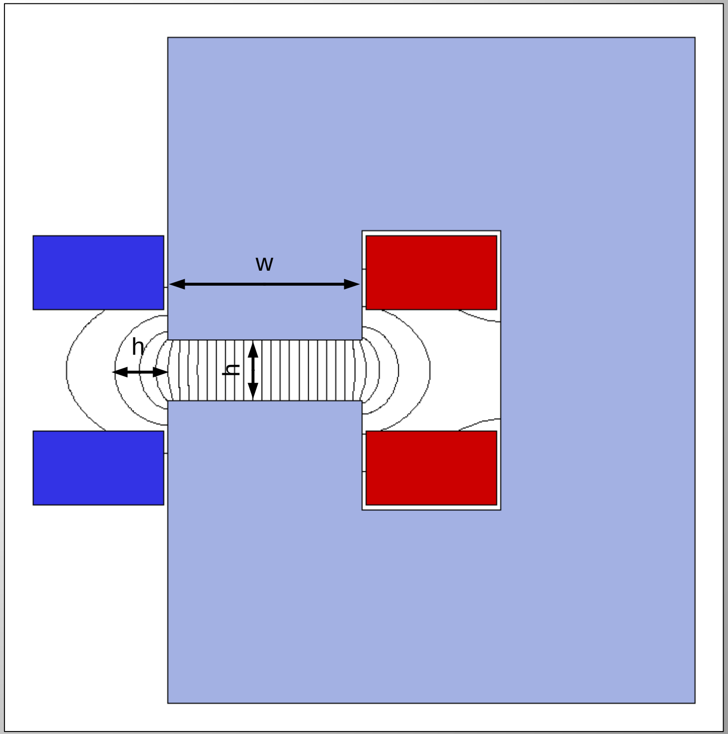


Total flux in the return yoke:

- includes the gap flux and stray flux

$$\Phi = \int_A B \cdot dA \approx B_{gap}(w+2h)l_{mag}$$

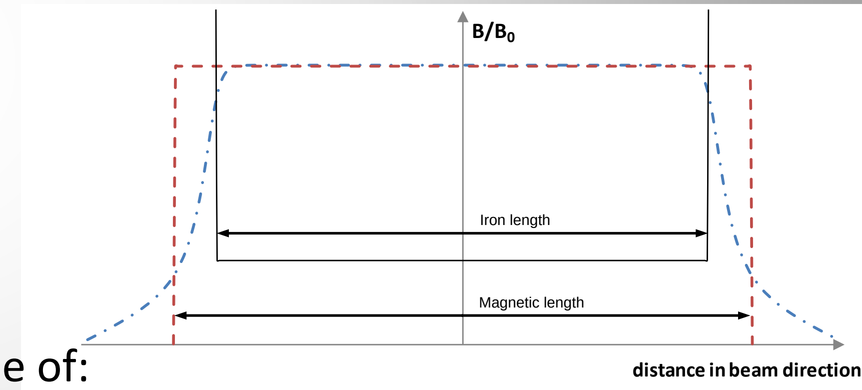
$$B_{leg} \cong B_{gap} \frac{w+2h}{w_{leg}}$$



Magnetic length

Coming from ∞, B increases towards the magnet center (stray flux)

Magnetic length: $$l_{mag} = \frac{\int_{-\infty}^{\infty} B(z) \cdot dz}{B_0}$$

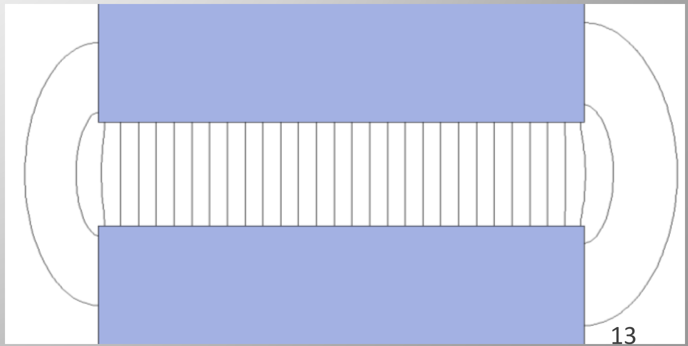


'Magnetic' length > iron length

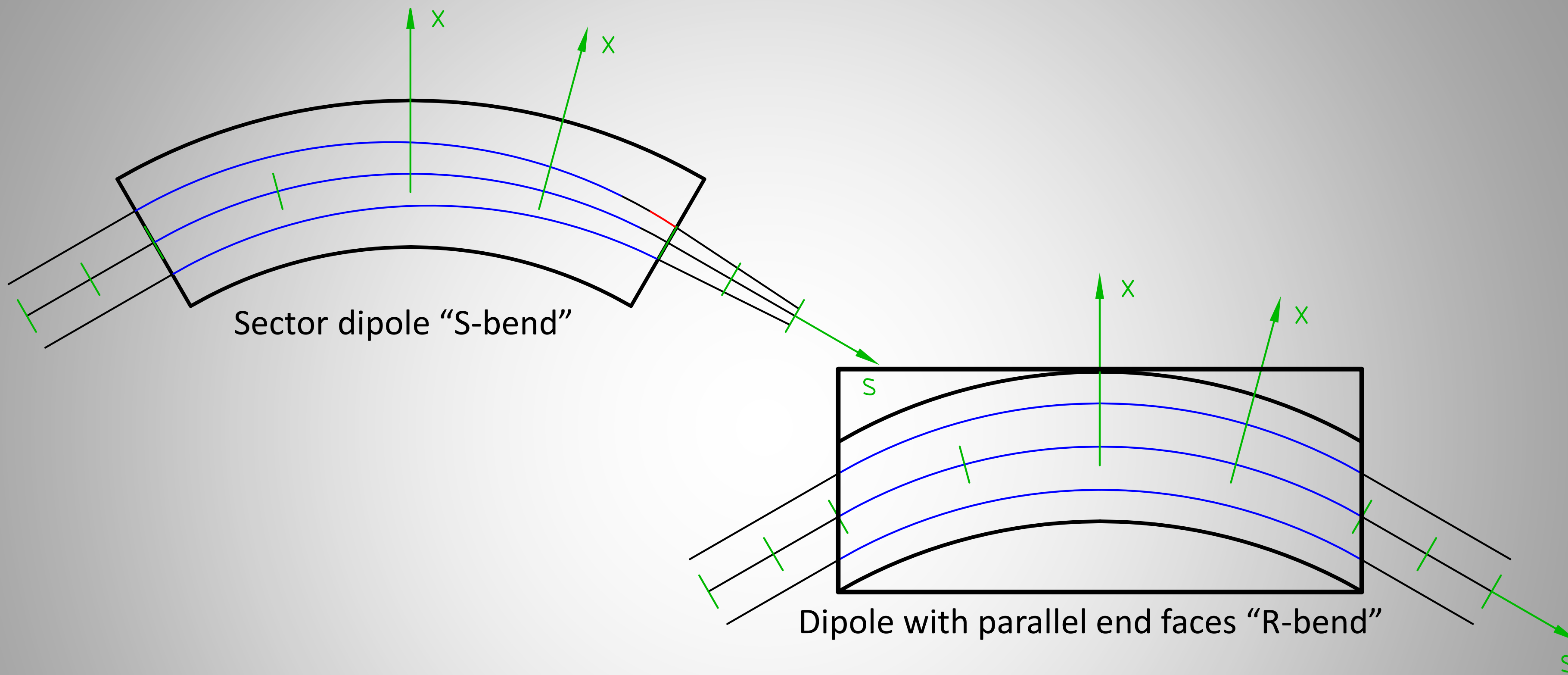
Approximation for a dipole: $$l_{mag} = l_{iron} + 2hk$$

Geometry specific constant k gets smaller in case of:

- pole length < gap height
- saturation
- precise determination only by measurements or 3D numerical calculations



Excursion: S-bend vs. R-bend



The two types are slightly different in terms of focusing:

- S-bend: focuses horizontally
- R-bend: no horizontal focusing, but small vertical defocusing at the edges

Note: the curvature has no effect, it is just for saving material, otherwise the pole would have to be wider ("*sagitta*").

Excitation current in a Quadrupole

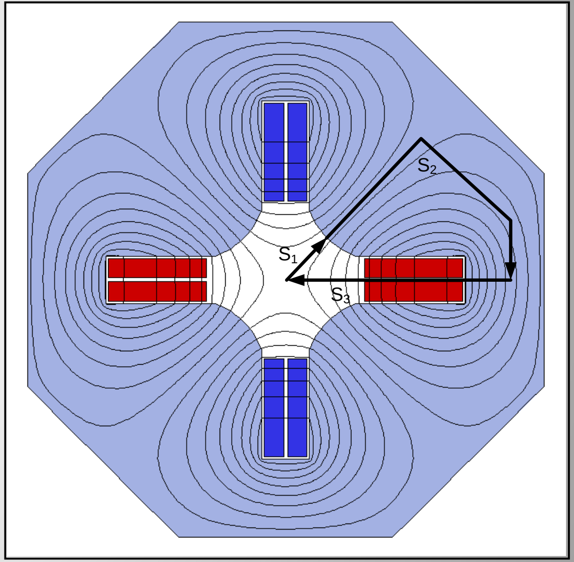
Choosing the shown integration path gives:

$$NI = \oint \vec{H} \cdot \vec{dl} = \int_{s1} \vec{H}_1 \cdot \vec{dl} + \int_{s2} \vec{H}_2 \cdot \vec{dl} + \int_{s3} \vec{H}_3 \cdot \vec{dl}$$

For a quadrupole, the gradient $B' = \frac{dB}{dr}$ is constant

and $B_y = B' x \qquad B_x = B' y$

Field modulus along s_1: $H(r) = \frac{B'}{\mu_0}\sqrt{x^2 + y^2} = \frac{B'}{\mu_0} r$



Neglecting H in s_2 because: $R_{M,s2} = \frac{s_2}{\mu_{iron}} << \frac{s_1}{\mu_{air}}$

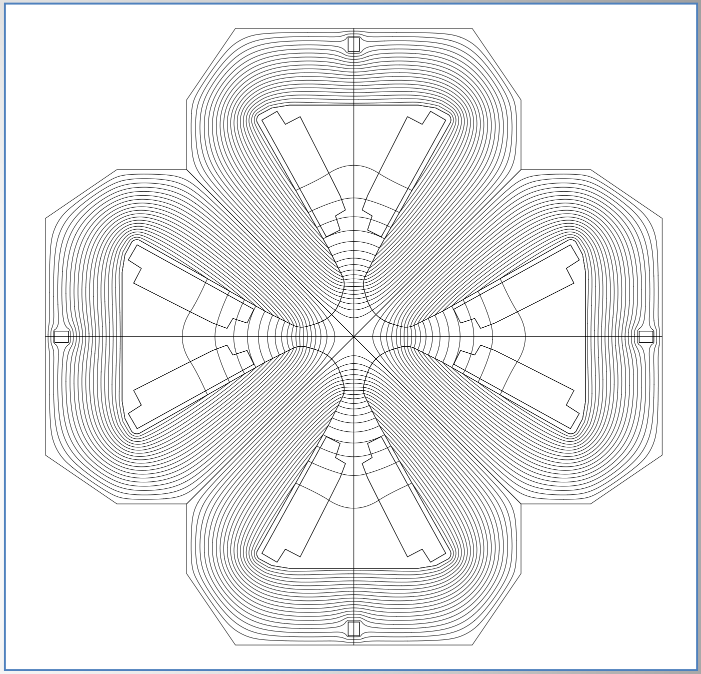
and along s_3: $\int_{s3} \vec{H}_3 \cdot \vec{dl} = 0$

Leads to: $NI \approx \int_0^R H(r)\, dr = \frac{B'}{\mu_0} \int_0^R r\, dr$

$$NI_{(per\ pole)} = \frac{B' r^2}{2\eta\mu_0}$$

Magnetic length

Magnetic length for a quadrupole:



$$l_{mag} = l_{iron} + 2r\,k$$

NI increases with the square of the quadrupole aperture:

$$NI \propto r^2 \qquad P \propto r^4$$

More difficult to accommodate the necessary Ampere-turns (= coil cross section)

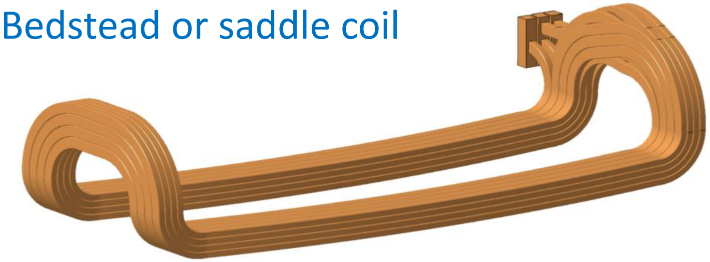
$\rightarrow$ truncating the hyperbola leads to a decrease in field quality

Coil design

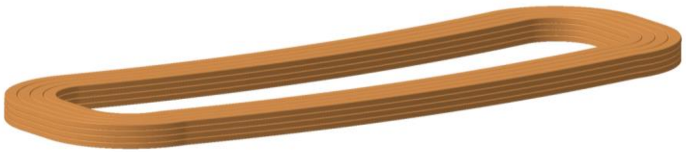
Ampere-turns NI are determined, but the current density j, the number of turns N and the coil cross section need to be defined

- Coil type selection
- Power requirements
- Cooling circuit computation
- Conductor selection
- Optimization

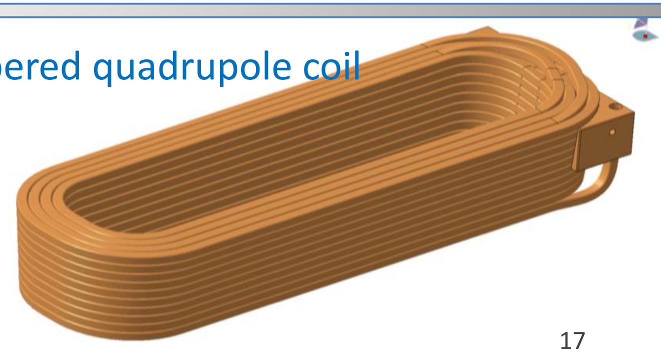
Bedstead or saddle coil



Racetrack coil



Tapered quadrupole coil



Current density

Assuming the magnet cross-section and the yoke length are known, one can estimate the total dissipated power per magnet:

$$P_{dipole} = \rho \frac{Bh}{\eta \mu_0} j l_{avg} 10^6 \qquad P_{quadrupole} = 2\rho \frac{B' r^2}{\eta \mu_0} j l_{avg} 10^6$$

- For a constant geometry, the power loss P is proportional to the current density j
- The current density j has a direct impact on coil size, coil cooling, power converter choice, operation costs and investment costs

j: current density [A/mm²]: $j = \frac{NI}{f_c A} = \frac{I}{a_{cond}}$

ρ: resistivity [Ωm] of coil conductor

l_{avg}: average turn length [m]; approximation: 2.5 $l_{iron} < l_{avg} <$ 3 l_{iron} for racetrack coils

a_{cond}: conductor cross section [mm²]

A: coil cross section [mm²]

f_c: filling factor = $\frac{\text{net conductor area}}{\text{coil cross section}}$

(includes geometric filling factor, insulation, cooling duct, edge rounding)

Note: If the magnet is not operated in dc, the rms power has to be considered.

Number of turns

The determined ampere-turns NI have to be divided into N and current I

Basic relations: $P_{magnet} \propto j$ $V_{magnet} \propto Nj$ $R_{magnet} \propto N^2 j$

Large N = low current = high voltage	Small N = high current = low voltage
• Small terminals	• Large terminals
• Small conductor cross-section	• Large conductor cross-section
• Thick insulation for coils and cables	• Thin insulation in coils and cables
• Less good filling factor in the coils	• Good filling factor in the coils
• Low power transmission loss	• High power transmission loss

The number of turns N are chosen to match the impedances of the power converter and connections

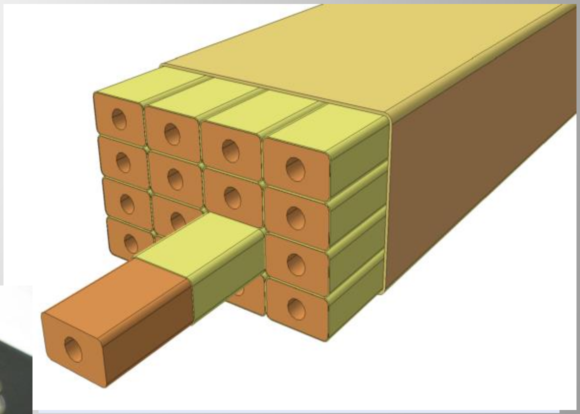
Attention when ramping the magnet: $V_{tot} = RI + L\frac{dI}{dt}$

Coil cooling

Air cooling by natural convection:

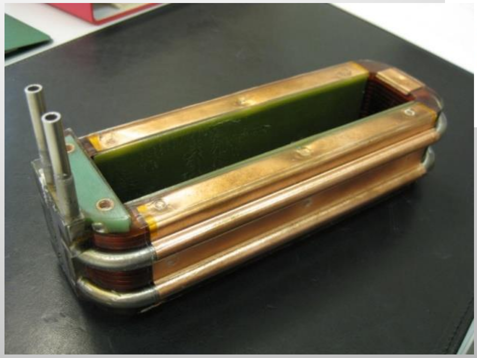
- Current density
  - $j < 2$ A/mm^2 for small, thin coils
- Cooling enhancement
  - Heat sink with enlarged radiation surface
  - Forced air flow (cooling fan)
- Only for magnets with limited strength (e.g. correctors)

Direct water cooling:



- Typical current density $j \leq 10$ A/mm^2
- Requires demineralized water (low conductivity) and hollow conductor profiles

Indirect water cooling:



- Current density $j \leq 3$ A/mm^2
- Tap water can be used

Direct water cooling

Practical recommendations and canonical values:

- Water cooling: 2 A/mm^2 ≤ j ≤ 10 A/mm^2
- Pressure drop: 1 ≤ Δp ≤ 10 bar (possible up to 20 bar)
- Low pressure drop might lead to more complex and expensive coil design
- Flow velocity should be high enough, so flow is turbulent
- Flow velocity u_{avg} ≤ 4 m/s to avoid erosion and vibrations
- Acceptable temperature rise: ΔT ≤ 30°C
- For advanced stability: ΔT ≤ 15°C

Assuming:

- Long, straight and smooth pipes without perturbations
- Turbulent flow = high Reynolds number (Re > 4000)
- Good heat transfer from conductor to cooling medium
- Temperature of inner conductor surface equal to coolant temperature
- Isothermal conductor cross section

Note: practical (non-SI) units are used in the following slides for convenience

Direct water cooling

Useful simplified formulas using water as cooling fluid:

Water flow Q [litre/min] necessary to remove power P: $Q_{water} = 14.3 \frac{P}{\Delta T} 10^{-3}$

P: dissipated power [W]

ΔT: temperature increase [°C]

Average water velocity u_{avg} [m/s] in a round tube: $u_{avg} = 16.67 \frac{Q}{A} = 66.67 \frac{Q}{\pi d^2}$

$A = \frac{\pi d^2}{4}$: bore cross-section [mm²]

d: hydraulic diameter [mm]

Pressure drop Δp [bar] : $\Delta p \approx 60\, l\, \frac{Q^{1.75}}{d^{4.75}}$ (from Blasius' law)

l: cooling circuit length [m]

Reynolds number Re []: $Re = d \frac{u_{avg}}{\nu} 10^{-3}$

Re: dimensionless quantity used to help predict similar flow patterns in different fluid flow situations

ν: kinematic viscosity of coolant is temperature depending, for simplification it is assumed to be constant ($6.58 \cdot 10^{-7}$ m²/s @ 40°C for water)

Cooling circuit design recipe

Already determined: current density j, power P, current I, number of turns N

1. Select number of layers m and number of turns per layer n
2. Round up N if necessary to get reasonable (integer) numbers for n and m
3. Define coil height c and coil width b: $A = bc = \frac{NI}{j\, f_c}$ (Aspect ratio $c : b$ between 1 : 1 and 1 : 2 and $0.6 \le f_c \le 0.8$)
4. Calculate average turn length $l_{avg} = pole\ perimeter + 4b$
5. The total length of cooling circuit $l = \frac{K_c N l_{avg}}{K_w}$ (start with single cooling circuit per coil)
6. Select ΔT, Δp and calculate cooling hole diameter $d = 0.5 \left(\frac{P}{\Delta T\, K_w}\right)^{0.368} \left(\frac{l}{\Delta p}\right)^{0.21}$
7. Change Δp or number of cooling circuits, if necessary
8. Determine conductor area $a = \frac{I_{nom}}{j} + \frac{d^2 \pi}{4} + r_{edge}(4 - \pi)$
9. Select conductor dimensions and insulation thickness
10. Verify if resulting coil dimensions, N, R, I, V, ΔT are still compatible with the initial requirements (if not, start new iteration)
11. Compute coolant velocity and coolant flow
12. Verify if Reynolds number is inside turbulent range ($Re > 4000$)

K_c: Number of coils

K_w: Number of cooling circuits per coil

Cooling circuit design

Number of cooling circuits per coil: $\Delta p \propto \frac{1}{K_W^3}$

→ Doubling the number of cooling circuits K_w reduces the pressure drop by a factor of eight for a constant flow

Diameter of cooling channel: $\Delta p \propto \frac{1}{d^5}$

→ Increasing the cooling channel by a small factor can reduce the required pressure drop significantly

Cost estimate

Production specific tooling:

10 to 20 k€/tooling

Material:

Steel sheets: 1.0 - 1.5 € /kg

Copper conductor: 10 to 20 € /kg

Yoke manufacturing:

Dipoles: 6 to 10 € /kg (> 1000 kg)

Quads/Sextupoles: 50 to 80 € /kg (> 200 kg)

Small magnets: up to 300 € /kg

Coil manufacturing:

Dipoles: 30 to 50 € /kg (> 200 kg)

Quads/Sextupoles: 65 to 80 € /kg (> 30 kg)

Small magnets: up to 300 € /kg

Contingency:

10 to 20 %

Magnet	***Magnet type***	***Dipole***	
	Number of magnets (incl. spares)	18	
	Total mass/magnet	8330	kg
Fixed costs	Design	14	kEuros
	Punching die	12	kEuros
	Stacking tool	15	kEuros
	Winding/molding tool	30	kEuros
Yoke	Yoke mass/magnet	7600	kg
	Used steel (incl. blends)/magnet	10000	kg
	Yoke manufacturing costs	8	Euros/kg
	Steel costs	1.5	Euros/kg
Coil	Coil mass/magnet	730	kg
	Coil manufacturing costs	50	Euros/kg
	Cooper costs (incl. insulation)	12	Euros/kg
Total costs	Total order mass	150	Tonnes
	Total fixed costs	71	kEuros
	Total Material costs	428	kEuros
	Total manufacturing costs	1751	kEuros
	Total magnet costs	2250	kEuros
	Contingency	20	%
	Total overall costs	**2700**	**kEuros**

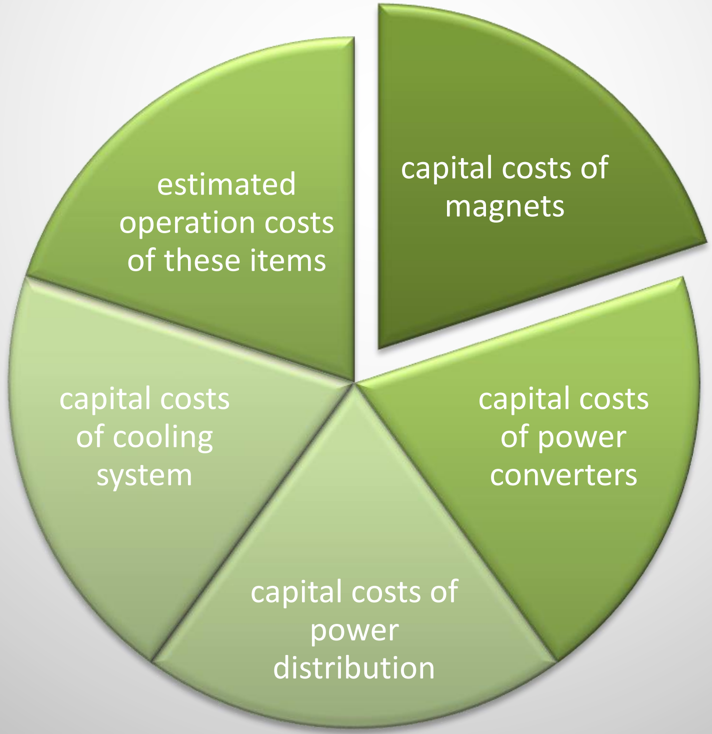
NOT included: magnetic design, supports, cables, water connections, alignment equipment, magnetic measurements, transport, installation
Prices for 2011

Cost optimization

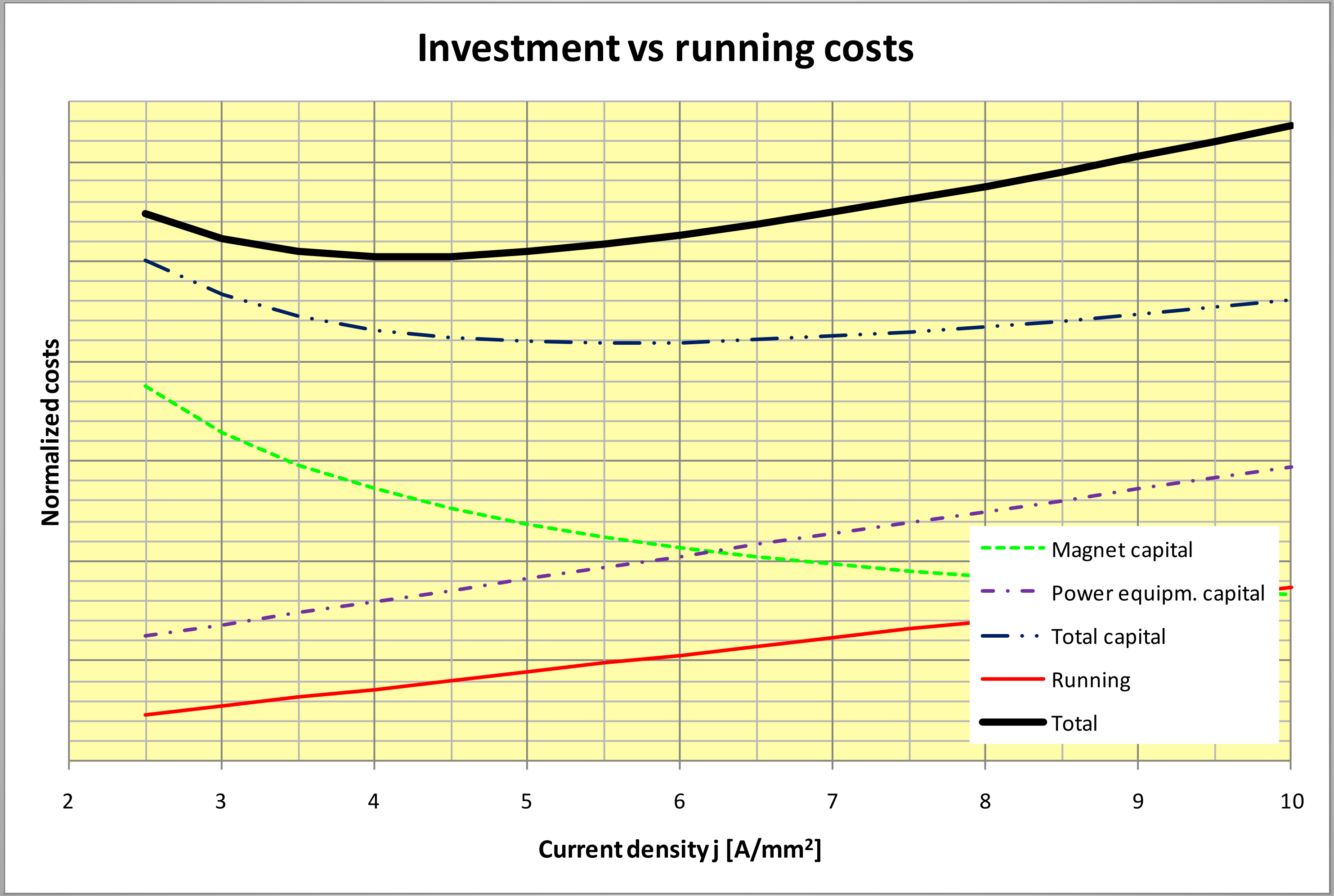
Focus on economic design!

Design goal: Minimum total costs over projected magnet lifetime by optimization of capital (investment) costs against running costs (power consumption)

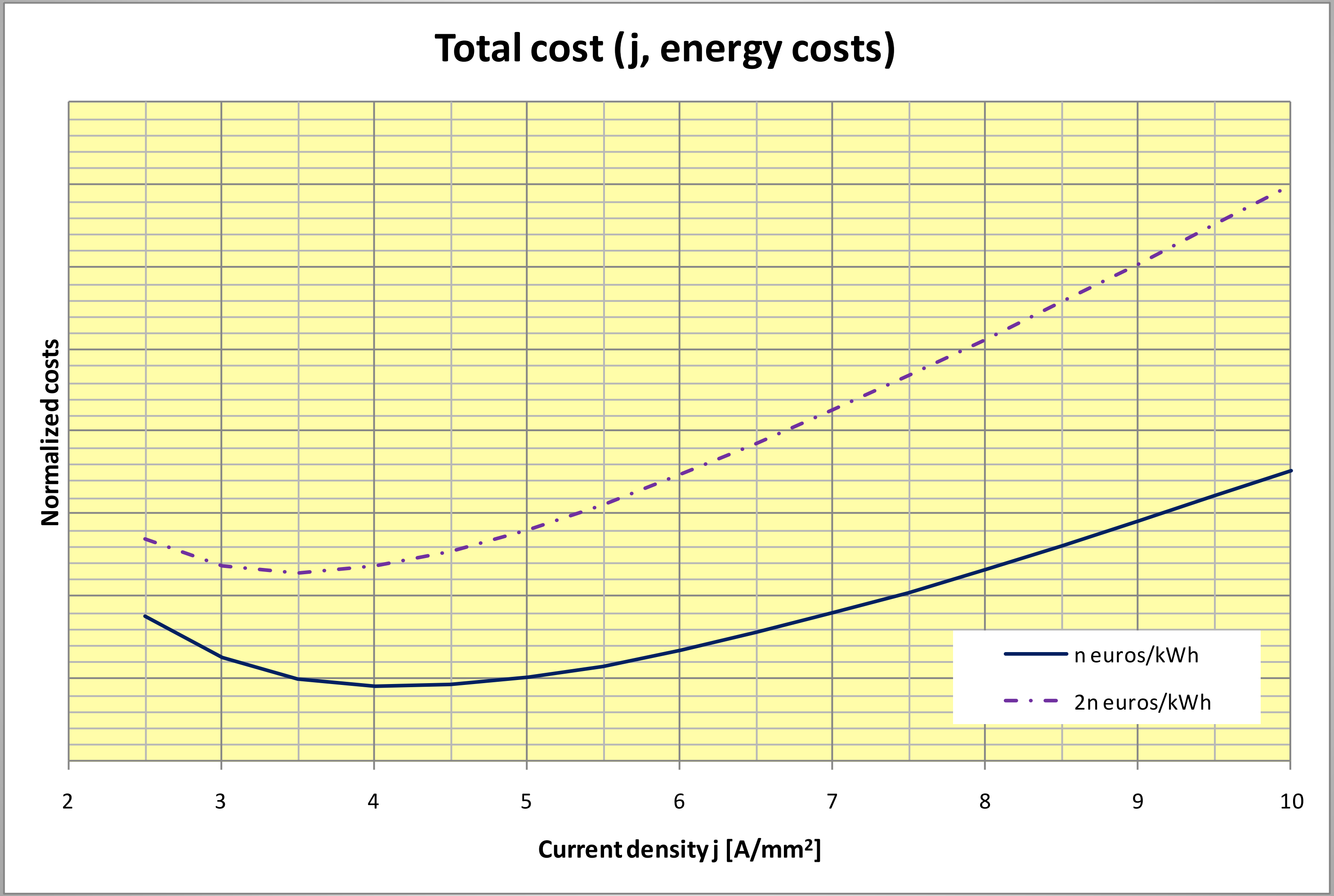
Total costs include:



Cost optimization



Cost optimization



Homework - Practical example

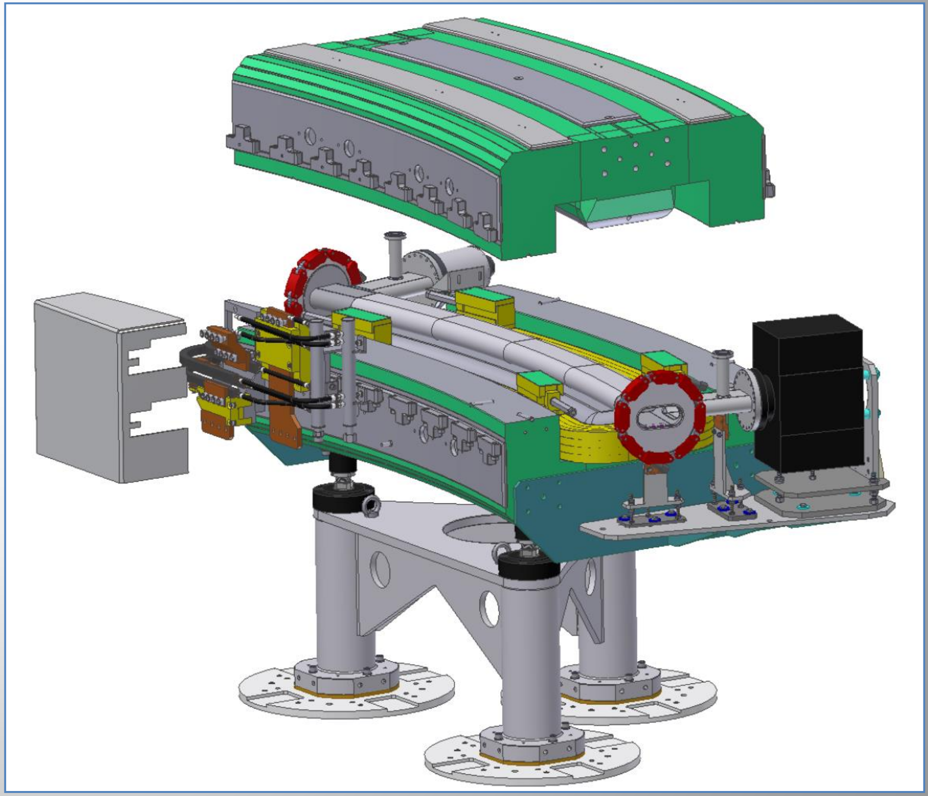
MedAustron: ion therapy facility near Vienna/Austria

ebg MedAustron

Providing beam energies from 120 to 400 MeV/u for carbon ions (C^{6+}) and from 60 to 220 MeV for protons

16 synchrotron bending magnets:

- Bending angle: 22.5°
- Bending radius: 4.231 m
- Field ramp rate: 3.75 T/s
- Max. current*: 3000 A
- Overall length: < 2 m
- Field quality: $\frac{\Delta\int B\cdot dl}{\int B\cdot dl} = 2\cdot 10^{-4}$



*) which can be delivered from the power converter

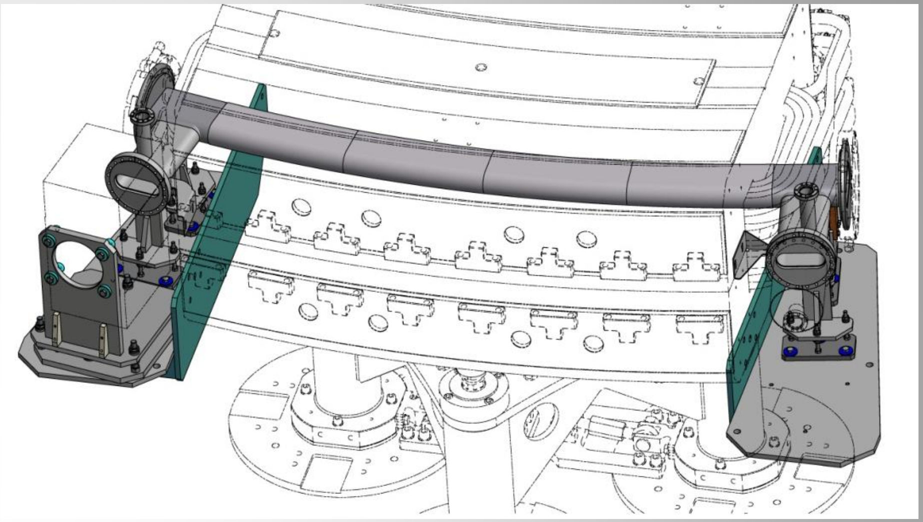
Homework - Practical example

Magnet aperture:

Horizontal GFR: ±60 mm

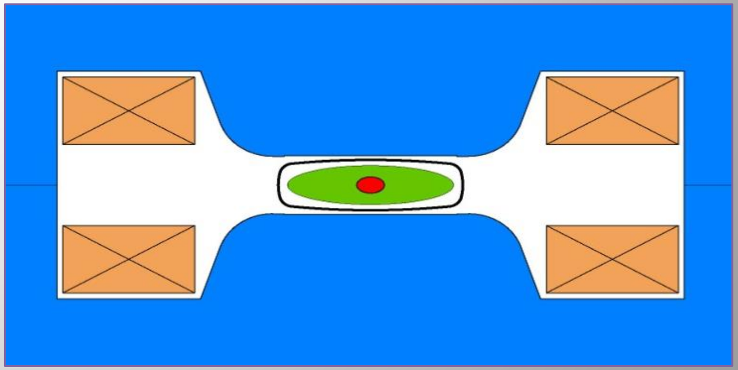
Vertical GFR: ±28 mm

Vacuum chamber thickness: 5 mm



Requested:

- Max. required B = ?
- Excitation current NI = ?
- Number of turns N (per pole) = ?



Summary

- Before starting the design, all input parameters, requirements, constraints and interfaces have to be known and well understood (prepare a checklist or functional specification!)
- Analytical design is necessary to derive the main parameters of the future magnet before entering into a detailed design using numerical methods
- Magnet design is an iterative process often requiring a high level of experience and/or educated guessing
- Critically review your final design and compare it with the initial requirements
- Cost optimization is an important design aspect, in particular in view of future energy costs