Fermions and the spontaneous breaking of scale invariance

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Scale invariance

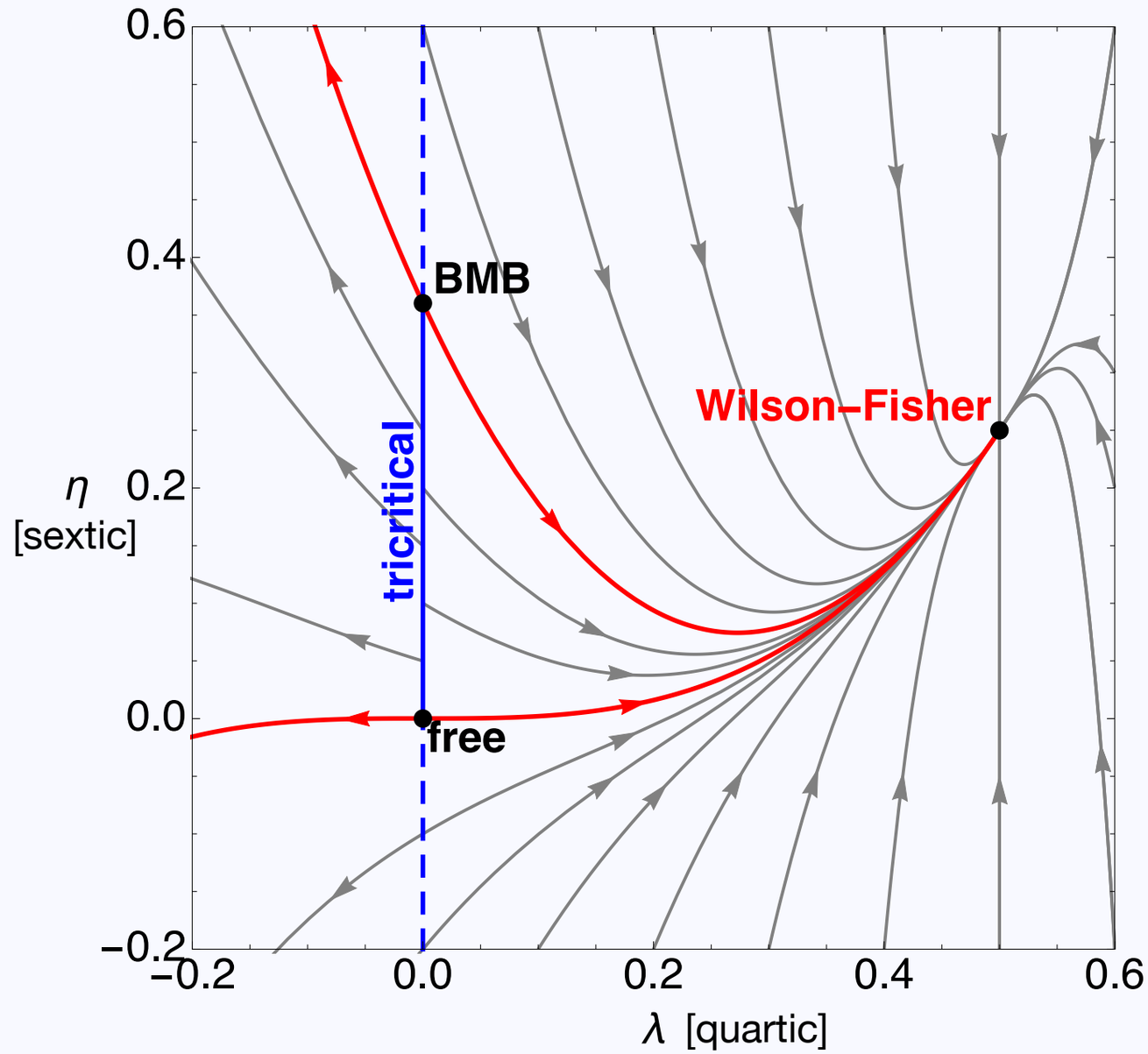
Consider a QFT with a mass M…

- decompose $M = k \cdot m$
- fluctuations induce running: $\partial_t m = \beta(m)$
- RG fixed point: $\beta(m_*) = 0$
- M_* vanishes when $k \to 0$ for any ***finite*** m_*

$[t = \ln k/\Lambda]$

Scalar field theory

- *O(N)*-symmetric with classical ϕ^6 term in $d = 3$, $N \to \infty$
- effective action: $\Gamma_k = \int \mathrm{d}^d x \left\{ \frac{1}{2} (\partial\phi)^2 + U_k(\phi^2) \right\}$ $\left[\phi^2 \equiv \sum_{i=1}^{N} \phi_i \phi_i \right]$
- Phase structure:
  - Wilson-Fisher (IR)
  - *line* of tricritical FPs (UV)
  - line ends at BMB FP



BMB phenomenon

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Spontaneous Breaking of Scale Invariance and the Ultraviolet Fixed Point in O(N)-Symmetric (ϕ_3^6) Theory

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(Received 28 June 1983)

At large N, the $\eta\vec{\phi}^6$ theory is shown to possess a nontrivial ultraviolet fixed point. A new phase is found where asymptotic scale invariance is spontaneously broken and a dynamical mass is generated through dimensional transmutation. At the tricritical limit, the spontane-

see also:
[*David, Kessler, Neuberger, PRL '85*]
[*Omid, Semenoff, Wijewardhana, 1605.00750*]
[*Litim, Marchais, Mati, 1702.05749*]

SUSY:
[*Bardeen, Higashijima, Moshe, Nucl.Phys.B '85*]
[*Heilmann et al., 1208.5389*]

***O(N)* plus Chern-Simons:**
[*Bardeen, Moshe, 1402.4196*]

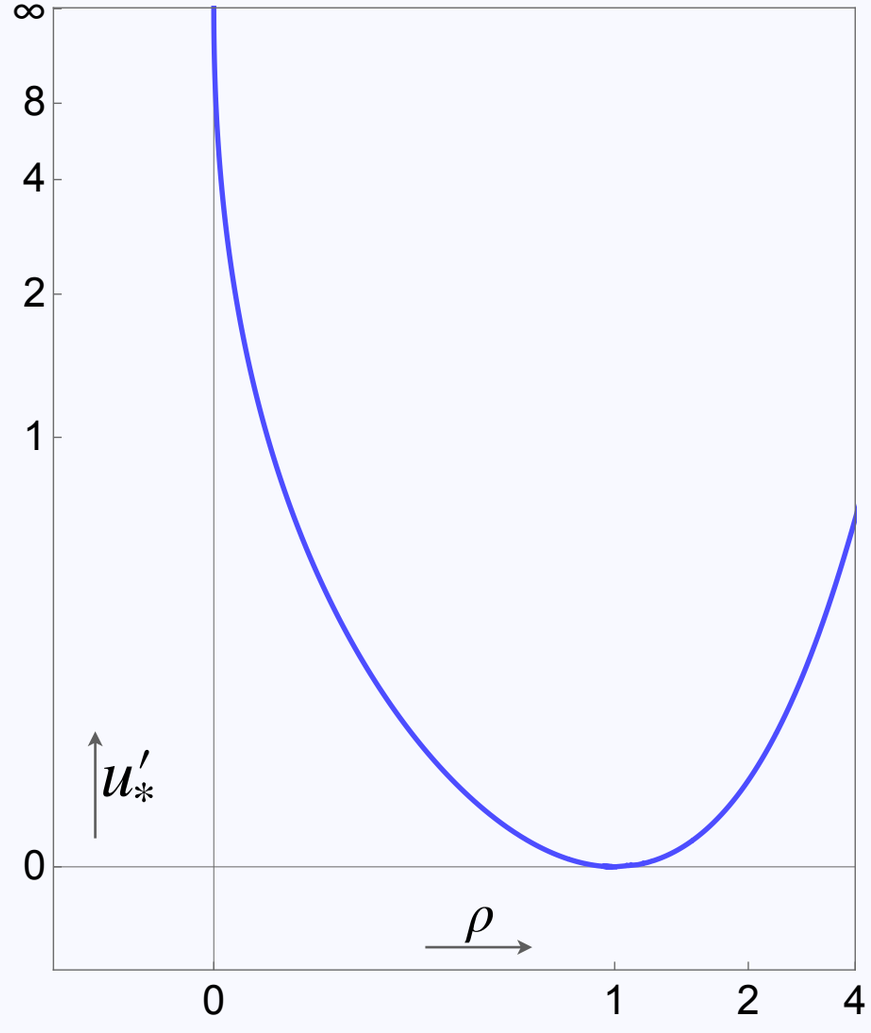
- pole in derivative of BMB effective potential

$$u_*' \sim \frac{1}{\sqrt{\rho}}, \quad \rho \to 0^+$$

- but $u_*'(0) = m_*^2 \;\Rightarrow\; \lim_{k\to 0^+} k \cdot m_* \neq 0$

- *spontaneous breaking of scale invariance*

[dimensionless variables: $u \sim U_k$, $\rho \sim \phi^2$]



Gross-Neveu

- classical action in d euclidean dimensions $\left[a \in \{1,\ldots,N_{\mathrm{f}}\}\right]$

$$S = \int \mathrm{d}^d x \left\{ \bar{\psi}_a \not{\partial} \psi_a + \frac{1}{2} G \left(\bar{\psi}_a \psi_a\right)^2 \right\}$$

- discrete “chiral” symmetry: $\psi \to \gamma^5 \psi, \;\; \bar{\psi} \to -\bar{\psi}\gamma^5$

- four-fermion coupling: $[G] = 2 - d$

- *asymptotically…* ***free*** in $d = 2$ [*Gross, Neveu, PRD ’74*]

  safe in $d = 3$ [*Gawędzki, Kupiainen, Nucl.Phys.B ’85*] [*Rosenstein, Warr, Park, PRL ‘88*] [*de Calan et al., PRL ’91*]

Gross-Neveu

- classical action in d euclidean dimensions $\left[a \in \{1,\ldots,N_{\mathrm{f}}\}\right]$

$$S = \int \mathrm{d}^d x \left\{ \bar{\psi}_a \not{\partial} \psi_a + \frac{1}{2} G \left(\bar{\psi}_a \psi_a\right)^2 + \frac{1}{3!} H \left(\bar{\psi}_a \psi_a\right)^3 \right\}$$

- discrete “chiral” symmetry: $\psi \to \gamma^5 \psi, \;\; \bar{\psi} \to -\bar{\psi}\gamma^5$ $[H = 0]$

- four-fermion coupling: $[G] = 2 - d$

- *asymptotically…* ***free*** in $d = 2$

  safe in $d = 3$

[*Gross, Neveu, PRD ’74*]

[*Gawędzki, Kupiainen, Nucl.Phys.B ’85*]
[*Rosenstein, Warr, Park, PRL ‘88*]
[*de Calan et al., PRL ’91*]

Phase structure

$$d = 3,\ N_\mathrm{f} \to \infty$$

dimensionless couplings:

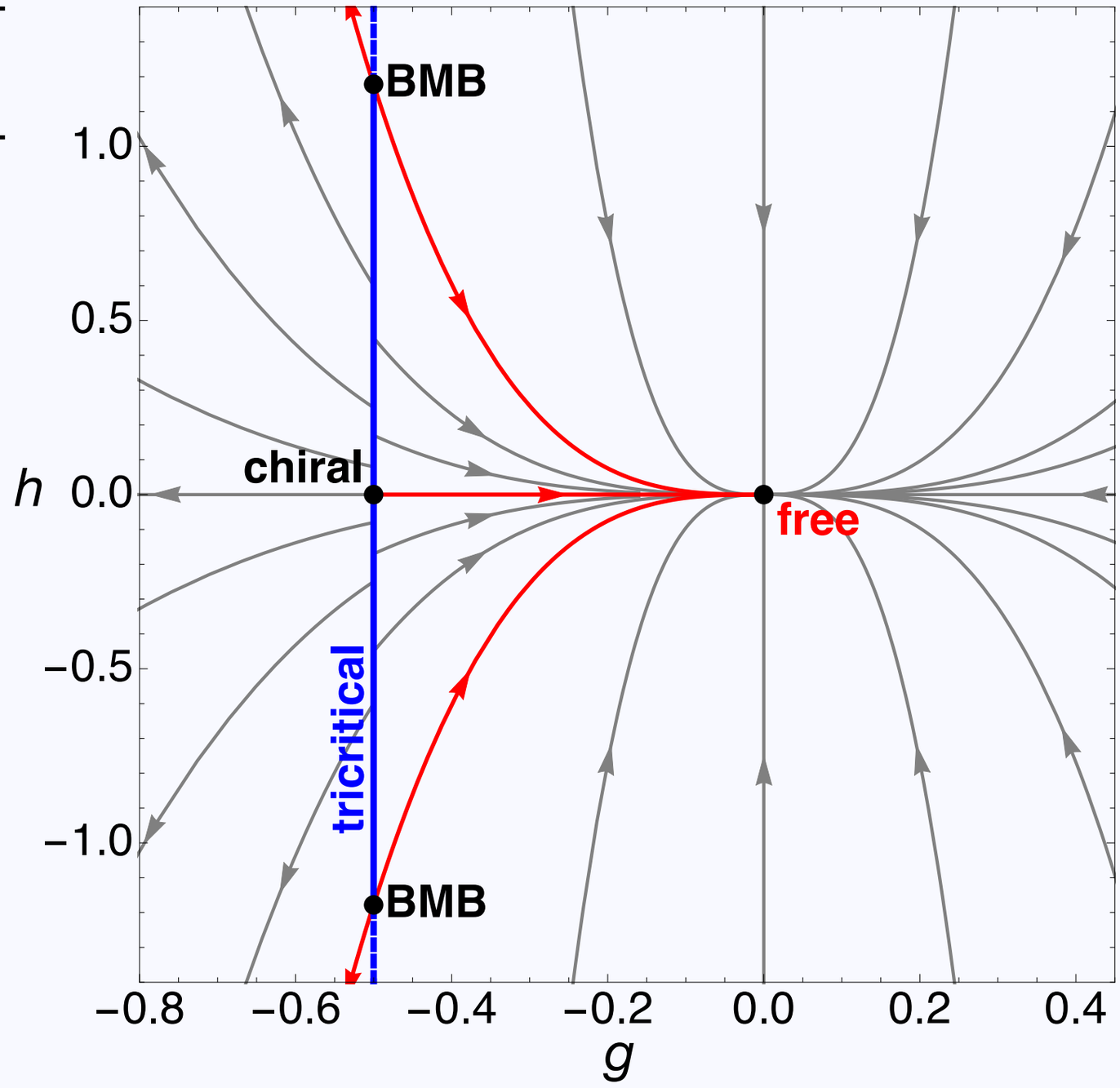
- g ~ four-fermion
- h ~ six-fermion

free in IR:

- Wilson-Fisher scaling

tricritical line in UV:

- chirally symmetric centre
- BMB-type endpoints
- free boson scaling



chiral FP well studied:

[*Rosenstein, Warr, Park, PRL '89*]
[*de Calan et al., PRL '91*]
[*Braun, Gies, Scherer, 1011.1456*]
[*Jakovác, Patkós, 1306.2660*]
[*Jakovác, Patkós, Pósfay, 1406.3195*]
[*Ihrig, Mihaila, Scherer, 1806.04977*]

BMB fixed point

- fermionic effective potential $V_k = V_k(\bar{\psi} \cdot \psi)$

  $v \sim V_k,\ z \sim \bar{\psi}\psi$

  $v'(0) = m$

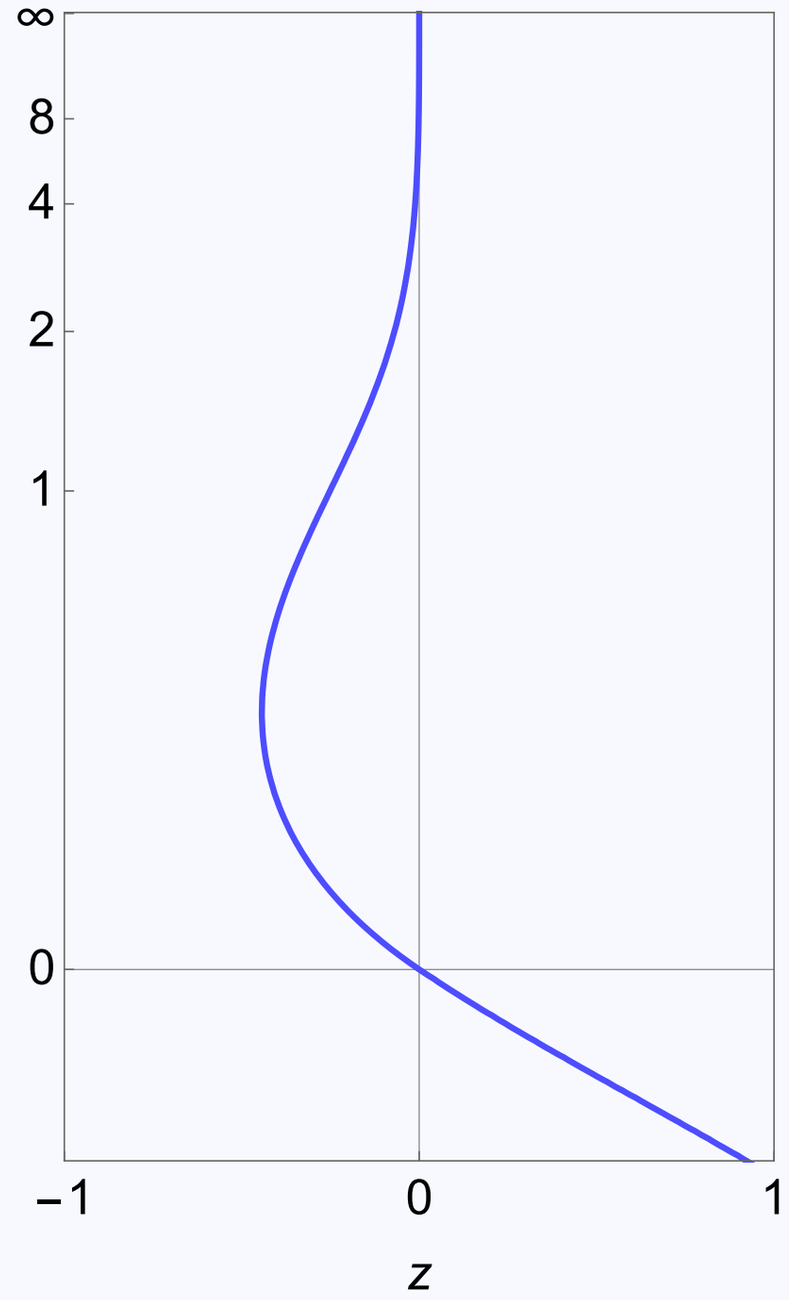
- derivative of BMB potential

$$v'_* \sim (-z)^{-1/3}, \quad z \to 0^-$$

- exponents ν, α violate hyperscaling law identically to scalar case

$$[d\nu = 2 - \alpha]$$

$$\nu_{\mathrm{BMB}} = \frac{1}{3} \qquad \alpha_{\mathrm{BMB}} = \frac{2}{3}$$



O(N):
[*Bander, Bardeen, Moshe, PRL '83*]
[*David, Kessler, Neuberger, PRL '85*]

GN:
[*CCH, Litim, 191X.XXXX*]

Outlook

- explicit breaking of discrete symmetry leads to fermionic BMB phenomenon

- critical exponents consistent with boson-fermion duality
  see, e.g.:
  [*Maldacena, Zhiboedov, 1204.3882*]
  [*Aharony, Jain, Minwalla, 1808.03317*]

- finite-N: what is the fate of the fermionic BMB?