# Production of 't Hooft-Polyakov monopoles from magnetic fields

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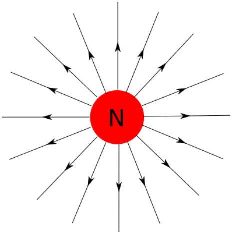
# The difficulty with monopoles

- Monopoles are strongly coupled: magnetic coupling

$$q_{\mathrm{m}} = \frac{2\pi n}{q_{\mathrm{e}}}, \;\; n \in \mathbb{Z} \qquad \text{(Dirac 1931)}$$

- Perturbative cross-sections e.g. Drell-Yan are not valid
- In order to do reliable calculations need a nonperturbative production mechanism

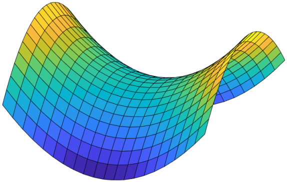


# Schwinger pair production

- EM field unstable to decay into charged particle-antiparticle pairs (Sauter 1931, Schwinger 1951)
- Critical field strength $E \sim \frac{m_{\rm e}^2}{e} \sim 10^{18}\,\mathrm{V\,m^{-1}}$. Currently unobserved but lasers are getting close!
- If monopoles exist, will be produced in a strong enough magnetic field; $B \sim \frac{m_{\rm m}^2}{q_{\rm m}}$
- Lack of observation of monopoles $\to$ lower bound on monopole masses

# Calculating Schwinger production rate

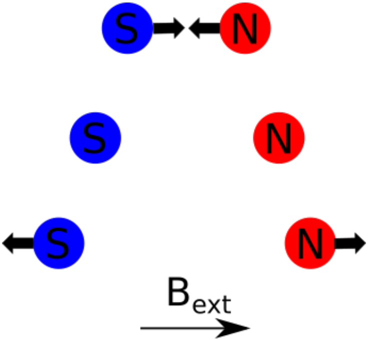
- Investigate Schwinger production rate at high temperature via a *sphaleron*
- Static, unstable solution to the equations of motion
- Represents the highest point along a path (in field configuration space) between the two vacua
- If fields can be excited to the sphaleron configuration, pair production can occur: rate $\propto \exp(-E_{\mathrm{sph}})$



*Sphaleros* — Ancient Greek: "likely to make one stumble"

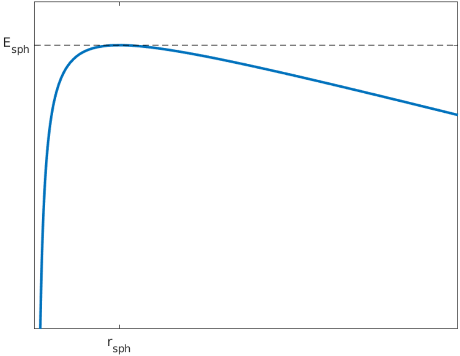
# Sphaleron for production of point monopoles

- Consider a monopole-antimonopole pair in a constant, homogenous magnetic field
- Unstable equilibrium where the attractive force between the poles balances the external force



# Sphaleron for production of point monopoles

$$E(r) = 2M - \frac{q_{\rm m}^2}{r} - q_{\rm m}Br$$




$$r_{\rm sph} = \sqrt{\frac{q_{\rm m}}{B}}, \qquad E_{\rm sph} = 2M - 2\sqrt{\frac{q_{\rm m}^3}{B}}.$$

# The need to move beyond pointlike monopoles

- Pointlike approximation valid if $r_{\rm sph} \gg r_{\rm m}$
- Strongest magnetic fields in the universe occur in heavy-ion collisions
- Our previous work shows that pointlike approximation breaks down if we consider fields relevant to heavy ions (Gould, Ho & Rajantie 2019).
- Must take internal structure into account

# SU(2) Georgi-Glashow theory

- SU(2) Gauge theory with adjoint scalar.
- Spontaneous symmetry breaking gives scalar mass $m_{\mathrm{s}} = 2\sqrt{\lambda}v$ and charged vector boson masses $m_{\mathrm{v}} = \sqrt{2}gv$.
- Remaining vector boson is a massless photon; magnetic field defined by projection onto this.

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \mathrm{Tr}(D_\mu\Phi D^\mu\Phi) - V(\Phi),$$

$$\begin{aligned} D_\mu\Phi^a &:= \partial_\mu\Phi^a + ig\varepsilon^{abc}A^b_\mu\Phi^c, \\ F^a_{\mu\nu} &:= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + ig\varepsilon^{abc}A^b_\mu A^c_\nu, \\ V(\Phi) &:= \lambda\left(\mathrm{Tr}(\Phi^2) - v^2\right)^2. \end{aligned}$$

# SU(2) Georgi-Glashow theory

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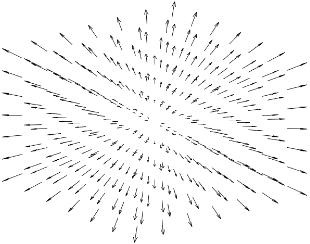


- Admits 't Hooft-Polyakov monopole solutions ('t Hooft 1981, Polyakov 1981) with 'hedgehog' scalar field configuration

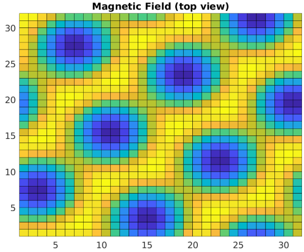
$$q_\mathrm{m} = \frac{4\pi}{g}, \qquad M = \frac{4\pi m_\mathrm{v}}{g} f\left(\frac{m_\mathrm{s}}{m_\mathrm{v}}\right); \qquad f(z) \sim 1.$$

# Important aside — Ambjørn-Olesen Instability

- In sufficiently strong magnetic fields, if a theory contains charged vector bosons, the vacuum is no longer stable (Ambjørn & Olesen 1988).
- Critical field strength

$$B_{\text{crit}} = \frac{m_{\text{v}}^2}{g}$$

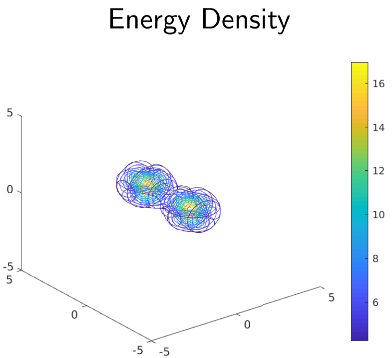
- Ambjørn and Olesen considered Georgi-Glashow theory without backreaction from scalar field and found a 'vortex lattice' solution.

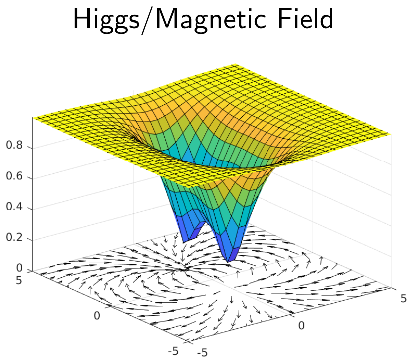


# Overview of our calculations

- Numerical calculations in discretised Georgi-Glashow model
- Searching for saddle points of energy density
- Modified gradient descent algorithm (Chigusa *et al.*, 2019). Lifts negative mode so saddle point can be found.
- Periodic boundary conditions quantise flux — allows us to impose external field.
- Using appropriate initial conditions we have been able to find the analogue of the pointlike sphaleron with solitonic monopoles.

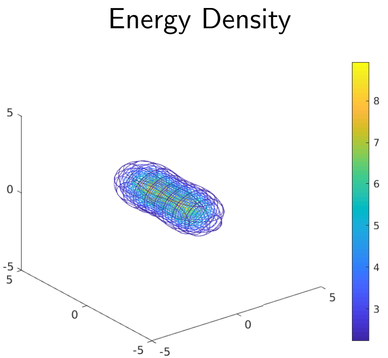
# Results: weak magnetic fields

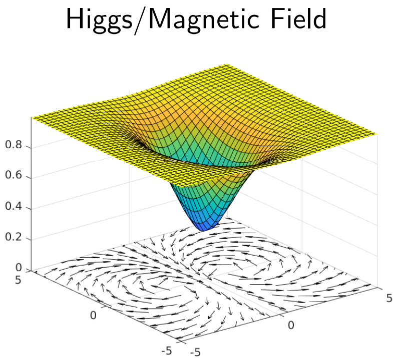



- Clear separation of magnetic charges
- Sphaleron energy well-approximated by pointlike energy

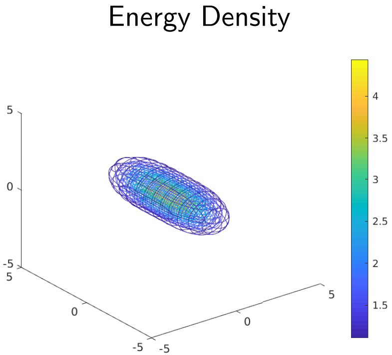
# Results: intermediate magnetic fields

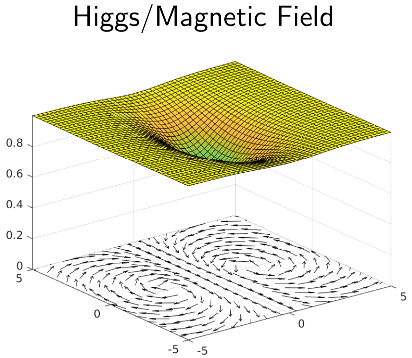



- Magnetic charges “annihilate”
- Dipole moment still present due to ring of electric current density
- Sphaleron energy is lower than pointlike case

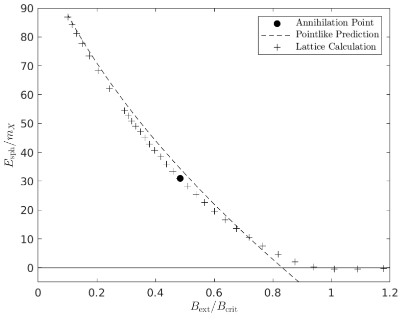
# Results: strong magnetic fields




- Sphaleron energy approaches zero
- Field configuration approaches vacuum

# Sphaleron energy as a function of external field strength




- Sphaleron energy vanishes above critical point coinciding with the Ambjørn-Olesen critical field

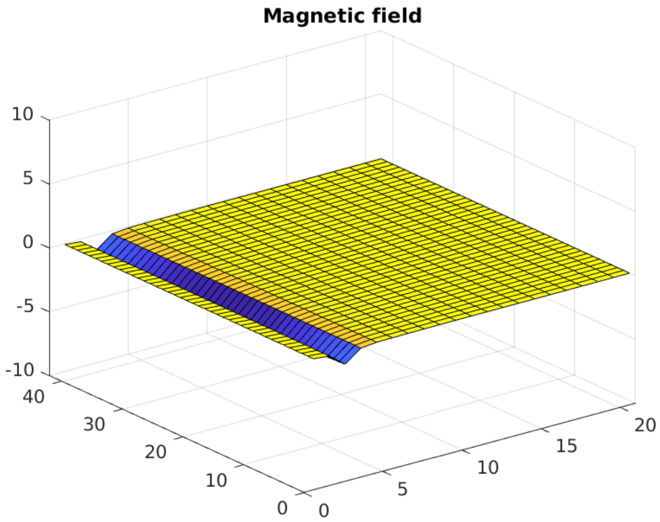
$$B_{\text{crit}} = \frac{m_{\text{v}}^2}{g}.$$

- No energy barrier → monopole production via **classical** process!

# Implications

- Below critical field, solitonic sphaleron energy is lower than pointlike: mass bounds stronger for solitonic monopoles.
- Lower bound on critical field strength for monopole production using bounds on BSM charged boson masses: $B_{\mathrm{crit}} > 10^{24}\,\mathrm{T}$.
- $O(10^7)$ times stronger than LHC fields, but possibility of generation in the early universe.
- Supercritical fields cannot have been present post-inflation, or monopoles would still be around today.

# Watch monopoles forming!



# Conclusions and further work

- Numerically investigated the energy barrier to production of solitonic monopoles
- We have shown this barrier vanishes at Ambjørn-Olesen critical field strength
$$B_{\rm crit} = \frac{m_{\rm v}^2}{g}$$
- Above this field strength, if they exist, monopoles are produced classically even at zero temperature

Unanswered questions:

- Effect of spacetime dependence on solitonic monopole production (cf. Gould, Ho & Rajantie 2019)
- Production of monopoles with mass not set by vector boson mass scale, e.g. Cho-Maison monopole