Field-space geometry of cosmological attractors

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Lancaster–Manchester–Sheffield Consortium for Fundamental Physics

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November 28, 2019
UK-QFT VIII, King's College London

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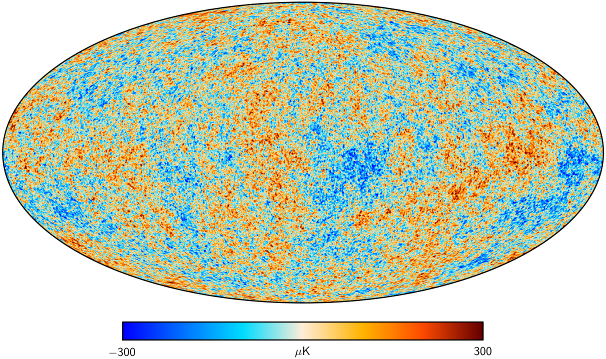
Outline

- Scalar-tensor theories and attractors
- Pole inflation
- Within the poles and beyond
- Initial conditions/vacuum states in attractor models

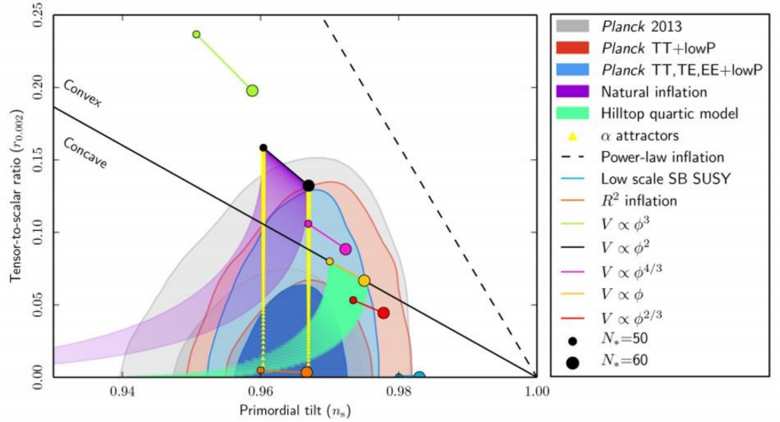
The theory of inflation



- Excellent *generic explanation* for the origin of anisotropies in the CMB from *primordial perturbations*
- “Generic” is a double-edged sword: finding well-motivated models is tough...

The ~~theory~~ framework of inflation

- No definitive driving mechanism for inflation exists; many excluded
- Phenomenology of inflation reflected in CMB spectrum of anisotropies



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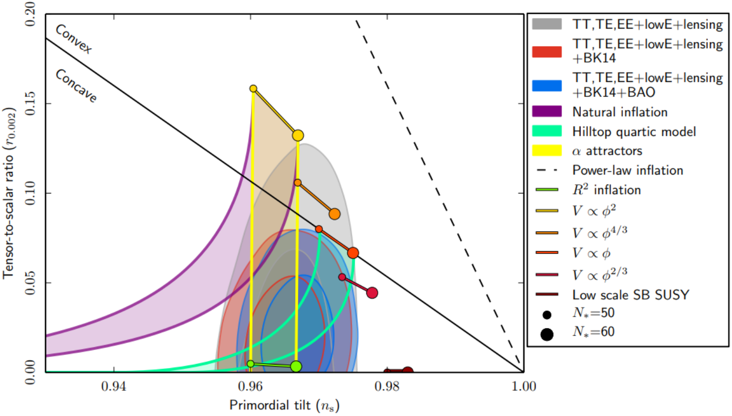


- Observations impose increasingly tighter constraints on inflation

α-attractors

- Originally motivated from spontaneously broken conformally invariant models [Kallosh, Linde, Roest (2013)]

- Consider theory with $\mathrm{SO}(1,1)$ symmetry

$$\mathcal{L} = \sqrt{-g}\left[\frac{\partial_\mu\chi\partial^\mu\chi}{2} - \frac{\partial_\mu\psi\partial^\mu\psi}{2} + \frac{\chi^2-\psi^2}{12}R - \frac{1}{36}F\left(\frac{\psi}{\chi}\right)(\psi^2-\chi^2)^2\right]$$

- Use "rapidity gauge" $\chi^2 - \psi^2 = 6\alpha$, and parametrise remaining degree of freedom as $\tanh(\frac{\phi}{\sqrt{6\alpha}}) = \frac{\psi}{\chi}$

α-attractors

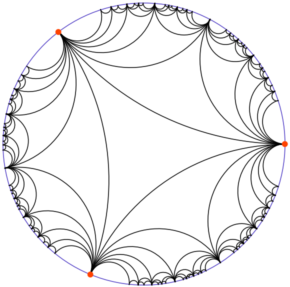
- With some algebra, may be embedded in Poincaré disk (arising e.g. in $\mathcal{N} = 1$ SUGRA)

$$\sqrt{-g}\,\mathcal{L} = -\frac{R}{2} + \frac{1}{2}\frac{(\partial_\mu r)(\partial^\mu r) + r^2(\partial^\mu \theta)(\partial_\mu \theta)}{\left(1 - \frac{r^2}{6\alpha}\right)^2} - V(r)$$

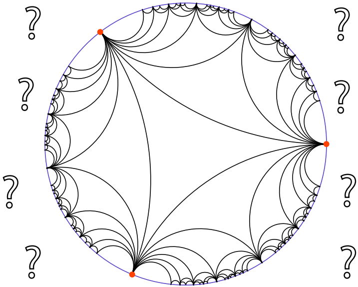
- Regardless of origin, phenomenological effects of such theories are captured by the following Lagrangian

$$\sqrt{-g}\,\mathcal{L} = -\frac{R}{2} + \frac{1}{2}\frac{(\partial_\mu \phi)(\partial^\mu \phi)}{\left(1 - \frac{\phi^2}{6\alpha}\right)^2} - V(\phi)$$

α-attractors *beyond the poles*



- Kinetic term has poles at $\varphi = \pm\sqrt{6\alpha}$
- Significant majority of studies focus on hyperbolic plane, with $|\phi| < \sqrt{6\alpha}$ ($r = \infty$)



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What about $\phi^2 > 6\alpha$?

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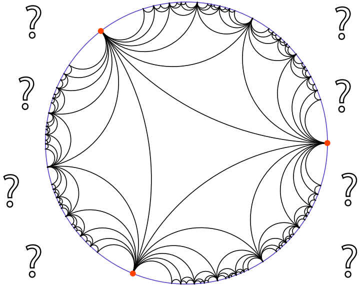

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(beyond the poles)

Why go beyond the poles?

- Within Poincaré disc, must necessarily have $\phi^2 < 6\alpha$, so is there a point?

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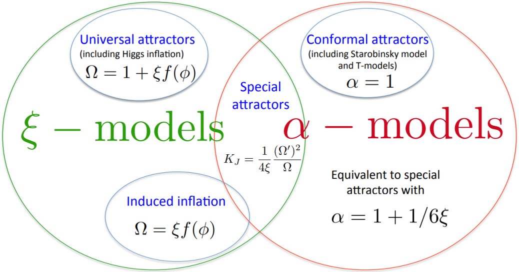
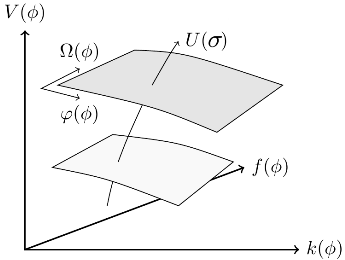
- Within Poincaré disc, must necessarily have $\phi^2 < 6\alpha$, so is there a point?
- Wide class of inflationary models featuring poles in the kinetic term; not all feature fundamentally hyperbolic geometry



[Gallante et al. 2014]

- What does “going beyond the poles” mean in a geometrical context?

The scalar-tensor theory space



Lagrangians linked via frame transformation $\mathcal{L} \sim \widetilde{\mathcal{L}}$ are *physically equivalent*

$$\sqrt{-g}\,\mathcal{L}_{Jordan} = -\frac{f(\phi)}{2}R + \frac{k(\phi)}{2}(\partial\phi)^2 - V(\phi)$$

$$\sqrt{-g}\,\widetilde{\mathcal{L}}_{\text{non-canonical Einstein}} = -\frac{1}{2}R + \frac{1}{2}\left[\frac{k(\phi)}{f(\phi)} + \frac{3}{2}\frac{f'(\phi)^2}{f(\phi)^2}\right](\partial\varphi)^2 - \frac{V(\phi)}{f(\phi)^2}$$

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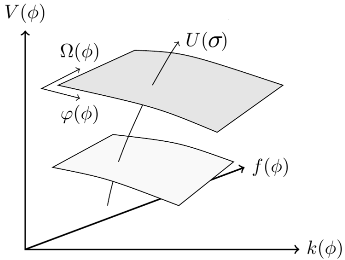

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Charts and atlases

- Why not switch to the canonical Einstein frame and call it a day?

Charts and atlases

- Why not switch to the canonical Einstein frame and call it a day?
- Charts preserve different properties: direction, shape, area, distance, local flatness, bijectivity
- Simultaneous preservation may require multiple charts – an atlas

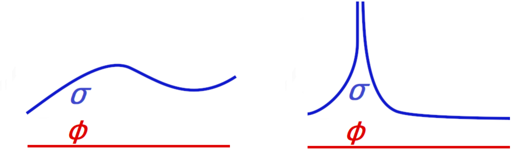
Gnomonic
(nortern hemisphere)

Gnomonic
(southern hemisphere)

Charts and atlases

- Can't have pie (single chart) and eat it too (canonical chart): cannot canonicalise *without specifying interval*

$$\sigma(\phi) = \int_{\phi_0}^{\phi} \frac{d\phi'}{\left|1 - \frac{\phi^2}{6\alpha}\right|}$$



- Can always "untangle" a string into a *single* straight line...
- ...unless the string shoots off to infinity *at a single point*: then, we need *multiple strings*

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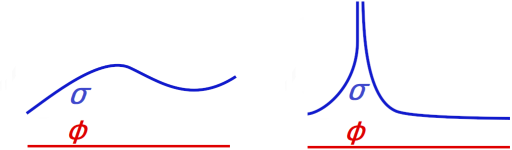

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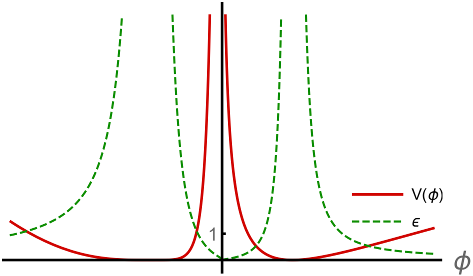
Any theory with poles is a union of multiple canonical theories.

Master models



- Can "glue" multiple canonical models together generating completely phenomenologies depending on side of pole
- For pole of order 2 at $\phi = 0$, gluing potentials $U_-(\phi)$ and $U_+(\phi)$

$$\mathcal{V}(\phi) = \begin{cases} U_+\left(\ln \frac{1}{\phi}\right) & (\phi > 0) \\ U_-\left(\ln \left[-\frac{1}{\phi}\right]\right) & (\phi < 0) \end{cases}$$

Pole inflation

- Close to the pole, observational features are strongly dependent on kinetic term features (order and residue) [Broy 2015 ,Terada 2016]

$$\sqrt{-g}\,\mathcal{L} = -\frac{R}{2} + \frac{\alpha_p}{2}\frac{(\partial_\mu \phi)(\partial^\mu \phi)}{|\phi - \phi_p|^p} - V_0(1 + v_p \phi),$$

- Observables to lowest order are found (for $p > 1$)

$$n_{\mathcal{R}} = 1 - \frac{p}{(p-1)N}, \qquad r = \frac{8v_p^2}{\alpha_p}\left[\frac{\alpha_p}{(p-1)v_p N}\right]^{p/(p-1)}$$

- Observables mostly insensitive to details of well-behaved potential, but side of pole might lead to different phenomenology

Back to α-attractors

- Two poles of order $p = 2$: Lagrangian is

$$\sqrt{-g}\,\mathcal{L} = -\frac{1}{2}R + \frac{1}{2}\frac{(\partial_\mu\phi)(\partial^\mu\phi)}{\left(1 - \frac{\phi^2}{6\alpha}\right)^2} - V(\phi)$$

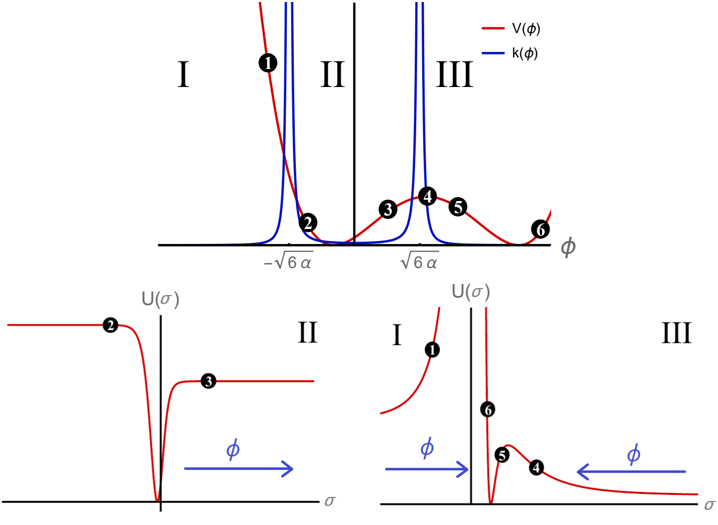
- Usually, solve $\sigma'(\phi) = 1 - \phi^2/6\alpha$ to find

$$\sigma(\phi) = \sqrt{6\alpha}\,\tanh^{-1}\left(\frac{\phi}{\sqrt{6\alpha}}\right)$$

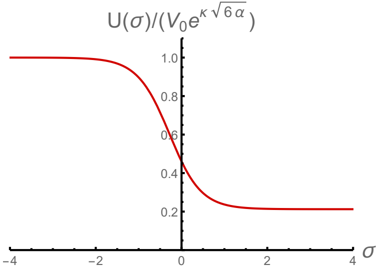
- In general, canonicalised field depends on arbitrary point ϕ_0

$$\sigma = \sqrt{\frac{3\alpha}{2}}\left(\ln\left|\frac{\phi + \sqrt{6\alpha}}{\phi - \sqrt{6\alpha}}\right| - \ln\left|\frac{\phi_0 + \sqrt{6\alpha}}{\phi_0 - \sqrt{6\alpha}}\right|\right)$$

Domains of α-attractors



Arbitrary non-canonical potential $V(\phi)$ and associated canonical potentials $U_{WTP}(\sigma)$ (domain II) and $U_{BTP}(\sigma)$ (I and III)



- Quintessential inflation occurs with $V = V_0 e^{-\kappa\phi}$ within the poles
- Observables for $\alpha \ll 1$:

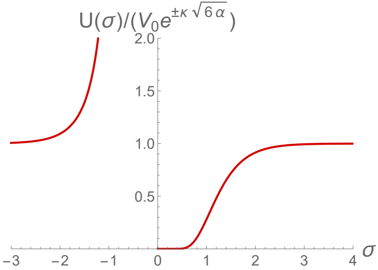
$$n_{\mathcal{R}} \approx 1 - \frac{2}{N} - \frac{\sqrt{3\alpha}}{N^2} - \frac{3\alpha(N-1)}{2N^3}$$

$$r \approx \frac{12\alpha}{N^2} - \frac{12\sqrt{3}\alpha^{3/2}}{N^3}$$

- Inflation ends at $\varphi_{\text{end}} = -\frac{1}{2}\sqrt{\frac{3\alpha}{2}}\ln\left(2\kappa^2\right)$; kination begins as $\epsilon \gg 1$
- Typical distance travelled is O(10), introducing tension with swampland distance conjecture (traversing large field distances implies infinite array of particles becoming exponentially light, invalidating EFT)

$$\Delta\sigma = \frac{1}{\mathcal{O}(1)}\frac{1}{H} \approx 10 \qquad (\textit{Planck}\ 2018)$$

The vanishing inflaton scenario



- Beyond the poles $\phi < 0$ leads to eternal acceleration
- Beyond the poles $\phi > 0$ leads to inflation

$$n_{\mathcal{R}} \approx 1 - \frac{2}{N} + \frac{\sqrt{3\alpha}}{N^2} - \frac{3\alpha(N+1)}{2N^3}$$

$$r \approx \frac{12\alpha}{N^2} - \frac{12\sqrt{3}\alpha^{3/2}}{N^3}$$

- Potential allows ϕ to reach 0 in finite time; corresponds to $\phi = \infty$
- Distance travelled in field space is $\mathcal{O}(1)$; fares better in SDC
- Radiative corrections not expected to prevent vanishing ($\sigma \ll M_P$)

The "edge" of the field space?

- Inflation on projective ray
- Theory is *incomplete*: does not tell us what happens at the edge of the manifold
- Must impose boundary condition at "point at infinity"; choices:
  - if no further evolution, inflaton completely decouples ("vanishes")
  - analytically continue potential
  - compactify field

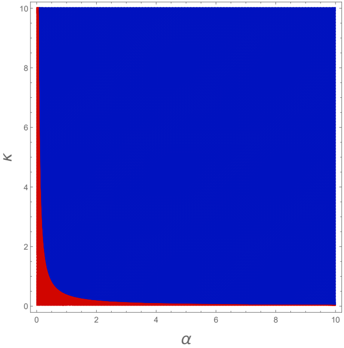
Reheating and quintessence beyond the poles

- Beyond the poles, inflation ends at $\varphi_{\rm end} \approx \frac{1}{2}\sqrt{\frac{3\alpha}{2}}\ln(2\kappa)$; gravitational reheating is generically interrupted:

$$\Delta\sigma \approx \left|\sqrt{\frac{3}{2}}\ln\left(\frac{q g_*^{\rm end}}{1440\pi^2}\frac{U_{\rm out}}{3}\right)\right| \gg \varphi_{\rm end}$$

- Field “freezes” thanks to radiation domination after traversing $\Delta\sigma = \frac{3}{2}\left(1 - \frac{3}{2}\ln\Omega_\gamma^{\rm end}\right)$

- Possible quintessence without introducing potential offset (red area)



- In multifield theories, some singularities are “benign”; some delimit different models, acting as model walls
- Initial conditions/vacuum: which domain to pick?
  - Arguments for favourable field values (e.g. starting on plateau) apply only to canonical models
  - Choosing a domain is equivalent selecting a canonical (pole-free) model
  - Therefore, need entirely new arguments for selecting a domain to initiate evolution

Conclusions

- Single-field models are a collection of canonical models with different predictions
- Require analytic extension to determine the late-time fate of the inflaton beyond the poles depending on potential; not restricted to α-attractors
- Example: quintessense model within poles transforms into “vanishing” (incomplete) model outside the poles
- Must choose a particular “fundamental” representation of theory in order to impose initial conditions/vacuum states