

# YFS Photon Resummation

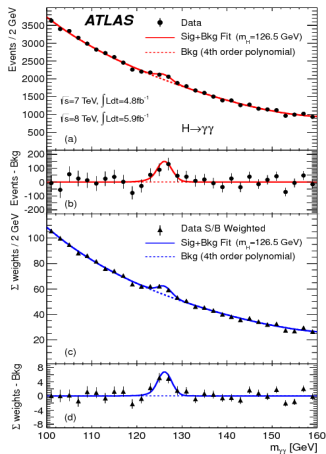
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1. Motivation
2. YFS Resummation
3. Validation and Results
4. Conclusion



- For precision physics a lepton-lepton collider is desirable
- The discovery of Higgs Boson tells us at what energy to run
- High precision understanding of the Higgs Boson requires below 1% uncertainties in the couplings
- $e^+e^- \rightarrow ZH$  optimal at 240GeV. Uncertainties mainly due to loops
- @ two-loop intrinsic uncertainty  $\mathcal{O}(1\%)$ , compared to expected experimental error  $\mathcal{O}(0.4\%)$  (FCC-ee)

We must not forget that an  $e^+e^-$  will precisely measure EW observables

Observable	Where from	Present (LEP)	FCC stat.	FCC syst	$\frac{\text{Now}}{\text{FCC}}$	Challenge
$M_Z$ [MeV]	Z linesh.	$91187.5 \pm 2.1\{0.3\}$	0.005	0.1	3	QED Corrections
$\Gamma_Z$ [MeV]	Z linesh.	$2495.2 \pm 2.1\{0.2\}$	0.008	0.1	2	QED Corrections
$R_f^Z = \Gamma_h/\Gamma_f$	$\sigma(M_Z)$	$20.767 \pm 0.025\{0.012\}$	$6 \cdot 10^{-5}$	$1 \cdot 10^{-3}$	12	QED Corrections
$N_\nu$	$\sigma(M_Z)$	$2.984 \pm 0.008\{0.006\}$	$5 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	6	Bhabha scattering (QED)
$M_W$ [MeV]	ADLO	$80376 \pm 33\{6\}$	0.5	0.3	12	QED Corrections
$A_{FB,\mu}^{M_Z \pm 3.5\text{GeV}}$	$\frac{d\sigma}{d\cos\theta}$	$\pm 0.020\{0.001\}$	$1.0 \cdot 10^{-5}$	$0.3 \cdot 10^{-5}$	100	

**Table:** Adapted from Arxiv:1903.09895

Consider  $e^+e^- \rightarrow X + n\gamma$

QED corrections (ISR) can be parametrised as,

$$\gamma_{n,m} = \left(\frac{\alpha}{\pi}\right)^n \left(2 \ln\left(\frac{m_Z^2}{m_e^2}\right)\right)^m \quad 0 \leq m \leq n$$

- For future lepton collider  $\gamma \leq 10^{-5} \rightarrow$  need to include corrections up to  $\mathcal{O}(\alpha^3, L_e^3)$ ,  $L_e = \ln\left(\frac{s}{m_e}\right)$
- Soft and Collinear photons need to be resummed

# CROSS SECTION CALCULATION

- The Problem: Emission of soft and/or Collinear photons
- Solution: Calculate the Cross Section for  $e^+e^- \rightarrow X + n_\gamma$

$$\sigma = \frac{1}{\text{flux}} \int \sum_{i=0}^{n_R} d\Phi_f d\Phi_{k_i} (2\pi)^4 \delta\left(P - \sum q_f - \sum k_i\right) \left| \sum_{n_V=0}^{\infty} \mathcal{M}_{n_R}^{n_V + \frac{1}{2}n_R} \right|^2$$

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- Lets first consider virtual photon corrections in the soft limit

$$\mathcal{M}_0^0 = \underbrace{M_0^0}_{\text{Born ME}}$$

$$\mathcal{M}_0^1 = \alpha B M_0^0 + M_0^1$$

$$\mathcal{M}_0^2 = \frac{(\alpha B)^2}{2} M_0^0 + \alpha B M_0^1 + M_0^2$$

...

$$\mathcal{M}_0^{n_V} = \sum_{r=0}^{n_V} M_0^{n_V} \frac{(\alpha B)^r}{r!}$$

where B is the virtual infrared factor.

$$B = 2\alpha\Re \int \frac{d^4 k}{k^2} \frac{i}{(2\pi)^2} \left( \frac{2p_1 - k}{2kp_1 - k^2} - \frac{2p_2 - k}{2kp_2 - k^2} \right)^2$$



Summing to infinity yields,

$$\sum_{n_V=0}^{\infty} \mathcal{M}_0^{n_V} = e^{\alpha B} \sum_{n_V=0}^{\infty} M_0^{n_V}$$

Due to the Abelian nature of QED this can be generalised to  $n_R$  real photons,

$$\sum_{n_V=0}^{\infty} \left| \mathcal{M}_{n_R}^{n_V + \frac{1}{2} n_R} \right|^2 = e^{2\alpha B} \sum_{n_V=0}^{\infty} \left| M_{n_R}^{n_V + \frac{1}{2} n_R} \right|^2$$

For a single photon emission we have,

$$\frac{1}{2(2\pi)^3} \sum_{n_V=0}^{\infty} \left| M_1^{n_V+\frac{1}{2}} \right|^2 = \tilde{S}(k) \left| M_0^{n_V+\frac{1}{2}} \right|^2 + \sum_{n_V=0}^{\infty} \tilde{\beta}_1^{n_V+1}(k)$$

- Factorisation of real emissions occurs at the amplitude squared level
- Eikonal term  $\tilde{S}(k) = -\frac{\alpha}{4\pi^2} \left( \frac{p_1}{p_1 k} - \frac{p_2}{p_2 k} \right)^2$
- $\tilde{\beta}_{n_R}^{n_V+n_R}$  complete IR finite squared matrix element for born process plus  $n_V$  virtual and  $n_R$  photons
- Note abbreviation used in this talk

$$\tilde{\beta}_{n_R} = \sum_{n_V=0}^{\infty} \tilde{\beta}_{n_R}^{n_V+\frac{1}{2}n_R}$$

For  $n_R$  photons summed over all virtual contributions,

$$\begin{aligned}
 & \left( \frac{1}{2(2\pi)^3} \right)^{n_R} \left| \sum_{n_V=0}^{\infty} M_{n_R}^{n_V + \frac{1}{2}n_R} \right|^2 \\
 &= \tilde{\beta}_0 \prod_{i=1}^{n_R} [\tilde{S}(k_i)] + \sum_{i=1}^{n_R} \left[ \frac{\tilde{\beta}_1(k_i)}{\tilde{S}(k_i)} \right] \prod_{j=1}^{n_R} [\tilde{S}(k_j)] \\
 &+ \sum_{\substack{i,j=1 \\ i \neq j}}^{n_R} \left[ \frac{\tilde{\beta}_2(k_i, k_j)}{\tilde{S}(k_i)\tilde{S}(k_j)} \right] \prod_{l=1}^{n_R} [\tilde{S}(k_l)] + \dots \\
 &+ \sum_{i=1}^{n_R} \left[ \tilde{\beta}_{n_R-1}(k_1, \dots, k_{i-1}, k_{i+1}, \dots, k_{n_R}) \tilde{S}(k_i) \right] + \tilde{\beta}_{n_R}(k_1, \dots, k_{n_R})
 \end{aligned}$$

# MASTER EQUATION

This gives us our cross section

$$\sigma = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\Phi_q e^{2\alpha B + 2\alpha \tilde{B}} \prod_{j=1}^n \tilde{S}(k_j) \theta(\Omega; k_j) \left[ \tilde{\beta}_0(p_1, p_2; q_1, \dots, q_{n'}) \right. \\ \left. + \sum_{j=1}^n \frac{\tilde{\beta}_1(p_1, p_2; q_1, \dots, q_{n'}, k_j)}{S(k_j)} \right. \\ \left. + \sum_{\substack{j,l=1 \\ j \neq l}}^n \frac{\tilde{\beta}_2(p_1, p_2; q_1, \dots, q_{n'}, k_j, k_l)}{S(k_j)S(k_l)} + \dots \right]$$

The exponentiation of real emissions gives a factor

$$\tilde{B} = -\frac{1}{8\pi^2} \int \frac{d^3k}{k^0} \Theta(\Omega, k) \left( \frac{p_1}{p_1 k} - \frac{p_2}{p_2 k} \right)^2$$

$$\tilde{\beta}_0^0 = M_0^0 M_0^{0*}$$

$$\tilde{\beta}_0^1 = M_0^1 M_0^{0*} + M_0^0 M_1^{0*}$$

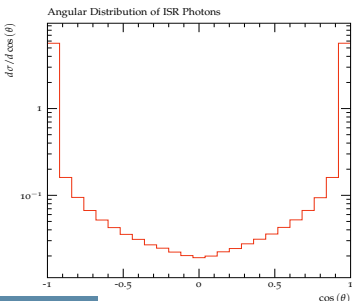
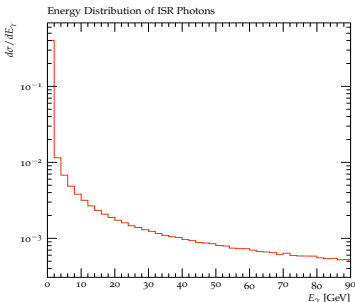
$$\tilde{\beta}_1^1 = \frac{1}{2(2\pi)^3} M_0^{\frac{1}{2}} M_0^{\frac{1}{2}*} - \tilde{S}(k) M_0^0 M_0^{0*} = \frac{1}{2(2\pi)^3} M_0^{\frac{1}{2}} M_0^{\frac{1}{2}*} - \tilde{S}(k) \tilde{\beta}_0^0$$

- These are the infrared subtracted squared matrix elements read, up to  $\mathcal{O}(\alpha)$
- They have been implemented in SHERPA up to  $\mathcal{O}(\alpha^2)$  for ISR

1. First the phase space is rearranged by a change of variable  $v = 1 - \frac{s'}{s}$
2.  $v$  is then generated by standard Monte-Carlo Methods e.g. Importance Sampling
3. If  $v < \epsilon$  no photons are generated. For  $v \geq \epsilon$  the multiplicity is generated according to a Poissonian distribution with average 
$$\mu = \frac{2\alpha}{\pi} \log\left(\frac{s}{m_e^2}\right) \log\left(\frac{v}{\epsilon}\right)$$
4. The  $n$  photon momenta are distributed according to the Eikonal  $\tilde{S}(k)$  and then rescaled to ensure 4-momentum conservation
5. The perturbative parts can be calculated separately using ME generator
6. Give everything to the master equation to calculate the cross section

# THE POWER OF YFS

- $\tilde{\beta}$  are infrared finite and are calculated perturbatively *order by order*
  - ▶ Non trivial for other methods e.g Structure function see Talk by Frixione
- Photons are explicitly created
  - ▶ Allows us to calculate properties of soft photons e.g Energy, Angular distribution
  - ▶ Allows for decays e.g  $\gamma \rightarrow e^+e^-$



- For LEP there were dedicated MC codes that implemented YFS for specific processes
  - KKMC [Jadach et.al, ARXIV:9912214](#)
  - KoralWW [Jadach et.al, ARXIV:0012094](#)
  - YFSWW [Jadach et.al, ARXIV:0012094](#)



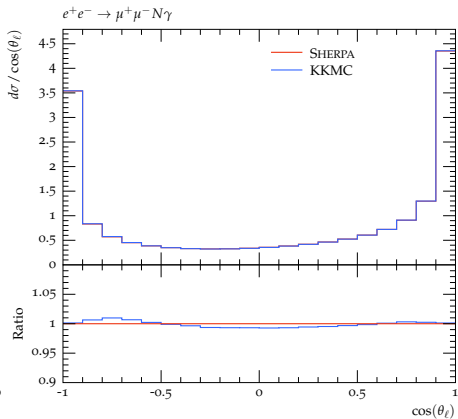
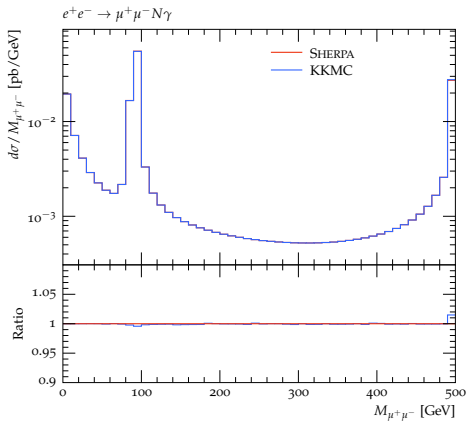
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YFS $\mathcal{O}(\alpha^0)$	KKMC 500 GeV	SHERPA 500 GeV
$e^+e^- \rightarrow \mu^+\mu^-$	1.748(3) pb	1.746(3) pb
$e^+e^- \rightarrow \nu_\mu\bar{\nu}_\mu$	1.6235(8) pb	1.626(3) pb
$e^+e^- \rightarrow \nu_\tau\bar{\nu}_\tau$	1.6235(8) pb	1.626(3) pb
$e^+e^- \rightarrow jj$	19.94(2) pb	19.98(8) pb

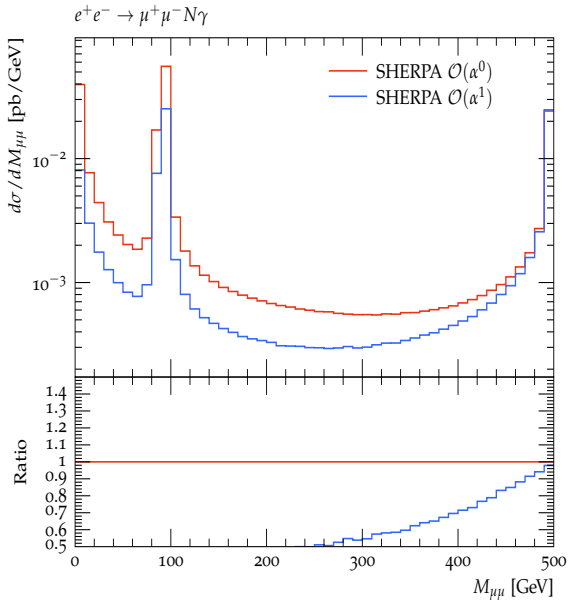
YFS $\mathcal{O}(\alpha^1)$	KKMC 500 GeV	SHERPA 500 GeV
$e^+e^- \rightarrow \mu^+\mu^-$	1.024(3) pb	1.020(3) pb
$e^+e^- \rightarrow \nu_\mu\bar{\nu}_\mu$	0.8072(7) pb	0.808(2) pb
$e^+e^- \rightarrow \nu_\tau\bar{\nu}_\tau$	0.8072(7) pb	0.808(2) pb
$e^+e^- \rightarrow jj$	10.54(1) pb	10.21(4) pb

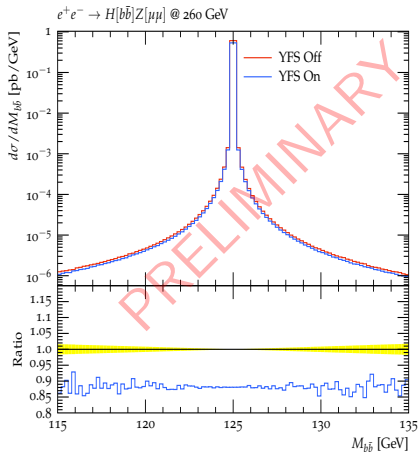
YFS $\mathcal{O}(\alpha^2)$	KKMC 500 GeV	SHERPA 500 GeV
$e^+e^- \rightarrow \mu^+\mu^-$	1.080(3) pb	1.082(6) pb
$e^+e^- \rightarrow \nu_\mu\bar{\nu}_\mu$	0.8604(7) pb	0.860(3) pb
$e^+e^- \rightarrow \nu_\tau\bar{\nu}_\tau$	0.8604(7) pb	0.860(3) pb
$e^+e^- \rightarrow jj$	11.18(2) pb	11.10(8) pb

# DIFFERENTIAL DISTRIBUTIONS



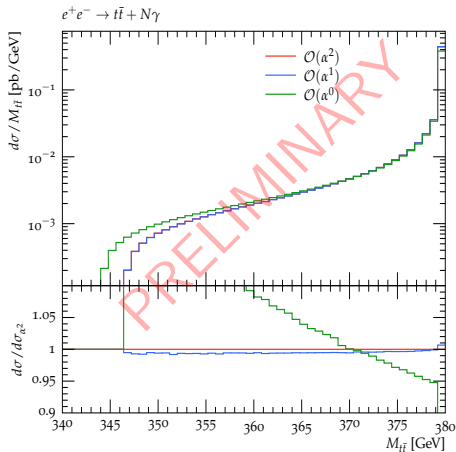
# DIFFERENTIAL DISTRIBUTIONS





- At the Higgs pole ISR contributes a  $\approx 10\%$  effect

# $t\bar{t}$ PRODUCTION



- The impact of YFS  
Matching on  $t\bar{t}$  threshold

- Interface with other SHERPA modules e.g ME Generator, PS, Hadronization, UFO...
- Fully automated for arbitrary processes
- Higher order corrections have been hard coded for ISR
- YFS has been fully implemented for FSR up to NNLO QCD and  $NLO_{EW}$  [Krauss et.al, ARXIV:1809.10650](#)
- Matching of YFS resummation to  $NLO_{EW}$  is under-way
  - EW Loops are now provided in full by OPENLOOPS [Buccioni et.al, ARXIV:1907.13071](#)
  - Real emission ME provided by COMIX/AMEGIC

- SHERPA is dedicated to becoming a state of the art event generator for lepton-lepton colliders
- Soft and Collinear photons are resummed in SHERPA via YFS
  - This will be soon matched to fixed order calculations i.e  $\text{NLO}_{\text{EW}}$
- Beam-Strahlung is also being implemented
- Underlying events such as  $\gamma\gamma$  will be modelled in the future



# The SHERPA 2.2 event generator framework

## User Inputs

### Initial Beams

- collider setup
- PDFs (built-in, LHAPDF)
- beam spectra

### Parameters/Models

- FeynRules/UFO
- couplings
- masses
- variations
- shower settings
- non-perturbative parameters

### Physics Process

- parton level
- perturbative order (QCD/EW)
- selectors
- matching/merging
- partonic decays



## Matrix Elements

### Matrix Element Generators

- AMEGIC
- COMIX
- CS subtraction

### 1-loop Amplitudes

- OpenLoops
- Recola
- GoSam
- BLHA



## Parton Showers

### CS-Shower (default)

- dipole shower
- fully massive
- QED splittings

### DIRE

- hybrid dipole-parton shower algorithm
- fully massive



## Soft Physics

### Hadronisation

- AHADIC: a cluster fragmentation model
- interface to Pythia string fragmentation



### Hadron Decays

- decay tables for hadronic resonances
- dedicated form-factor models, e.g.  $\tau$ , B,  $\Lambda$
- spin correlations
- YFS QED corrections
- partonic channels



### Underlying Event

- multiple parton interactions
- beam-remnant colours
- intrinsic transverse momentum

## Interfaces/Outputs

### Output Formats

- HepMC
- LHEF
- Root Ntuple



### Interfaces

- RIVET analyses
- C++/Python ME access
- MCgrid
- integration into ATLAS/CMS



### Code/Docu

- HepForge
- GitLab
- online documentation

sherpa.hepforge.org

gitlab.com/sherpa-team/sherpa

## Matching and Merging

### Automated MC@NLO style matching

### Multijet-merging algorithms

- based on truncated showers
- tree-level and one-loop matrix elements: MEPS@LO and MEPS@NLO
- approximate electroweak corrections

### NNLO QCD with parton showers

- selected processes only