INTRODUCTION AND MODEL

Bipartite quantum entanglement belongs to hot topics of modern physics due to its significant role in quantum information theory [1]. However, in two-dimensional models, it is examined mainly in the ground state or at very low temperatures [2].

Inspired by this fact, in this work we will examine the bipartite quantum entanglement in the spin-1/2 Ising-Heisenberg model on the infinite planar lattice consisting of trigonal bipyramids schematically depicted in figure 1, which is exactly solvable by means of the generalized decoration-iteration transformation method (for more computational details see our recent work [3]). The total Hamiltonian of the model can be written as a sum of the identical plaquette Hamiltonians $H_j$:

$$H = \sum_{j} H_j, \quad H_j = -J_H \sum_{k=1}^{3} \left[ \Delta \left( \frac{S^{x}_{j,k} S^{x}_{j,k+1} + S^{y}_{j,k} S^{y}_{j,k+1} + S^{z}_{j,k} S^{z}_{j,k+1}}{3} \right) + \frac{1}{\sqrt{3}} \sum_{k=1}^{3} \frac{S^{\alpha}_{j,k} (\sigma^{\alpha}_{j} + \sigma^{\alpha}_{j+1})}{2} \right],$$

where $\Delta$, $J_H$, $J_I$, and $J_J$ are the parameters of the model.

CONCURRENCE AS A TOOL FOR STUDY OF THE BIPARTITE QUANTUM ENTANGLEMENT

The spins may be entangled only within the Heisenberg triangular clusters (blue triangles in figure 1). The spins of different Heisenberg triangles can never be entangled, because the Ising spins at common vertices of the neighbouring trigonal bipyramids make a barrier for development of any quantum correlation between these spins. For investigation of the quantum entanglement between two Heisenberg spins of the $j$-th triangular cluster we use the quantity called concurrence [4]:

$$\mathcal{C} = 2 \max \left( 0, \frac{1}{2} |C_{\Delta}^{\pm}| - \left( \frac{1}{\Delta} + \frac{1}{J_H} + \frac{1}{J_I} \right) \right),$$

where $C_{\Delta}^{\pm}$ are the correlation functions and the spontaneous magnetization of the $j$-th Heisenberg triangle, respectively.

NUMERICAL RESULTS AND CONCLUSIONS

The ground state of the model is spontaneously ordered:

- for $J_H > \frac{2}{3} J_I$ the classical ferromagnetic (CF) phase
  $$|CF\rangle = \prod_{j} |\downarrow\rangle_j \otimes |\downarrow\rangle_j,$$
  with no entangled spin states ($\mathcal{C} = 0$) is stable;
- for $J_H > \frac{2}{3} J_I$ the quantum ferromagnetic (QF) phase
  $$|QF\rangle = \prod_{j} |\downarrow\rangle_j \otimes |\uparrow\rangle_j \otimes |\uparrow\rangle_j \otimes |\uparrow\rangle_j,$$
  where the unsaturated bipartite quantum entanglement ($\mathcal{C} = 2/3$) between the Heisenberg spins can be observed.

The quantum entanglement of the Heisenberg spins existing in the QF phase is much more resistant to temperature than the respective spontaneous long-range spin order. See figure 2.

The weak quantum entanglement of the Heisenberg spin pairs can be invoked by temperature above the CF phase in a vicinity of the ground-state boundary CF-QF due to thermal fluctuations to the energetically close Heisenberg spin states peculiar to the neighbouring QF phase. See figure 2 and also the thermal variations of the concurrence $\mathcal{C}$ plotted for $\Delta = 1.8$ in figure 3.

Very weak (almost unobservable) re-entrance of the bipartite entanglement between the Heisenberg spins (concurrence $\mathcal{C} \geq 0$) can also be found slightly above the critical temperature of the QF phase due to thermal activation of quantum pair correlations in some Heisenberg triangles. See insets in figures 2 and 3.

References


Acknowledgement

This work was financially supported by the grants APVV-16-0186, VEGA 1/0301/20.