THERMAL TREND OF THE BIPARTITE ENTANGLEMENT IN THE SPIN-1/2 ISING-HEISENBERG PLANAR LATTICE OF INTER-CONNECTED TRIGONAL BIPYRAMIDS

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INTRODUCTION AND MODEL

Bipartite quantum entanglement belongs to hot topics of modern physics due to its significant role in quantum information theory [1]. However, in two dimensional models, it is examined mainly in the ground state or at very low temperatures [2].

Inspired by this fact, in this work we will examine the bipartite quantum entanglement in **the spin-1/2 Ising-Heisenberg model on the infinite planar lattice consisting of trigonal bipyramids** schematically depicted in figure 1, which is exactly solvable by means of the generalized decoration-iteration transformation method (for more computational details see our recent work [3]). The total Hamiltonian of the model can be written as a sum of the identical plaquette Hamiltonians $\widehat{H}_j$:

$$\widehat{H} = \sum_{j=1} \widehat{H}_j \,,$$

$$\widehat{H}_j = -J_H \sum_{k=1}^{3} \left[\Delta\left(\hat{S}^x_{j,k}\hat{S}^x_{j,k+1} + \hat{S}^y_{j,k}\hat{S}^y_{j,k+1} \right) + \hat{S}^z_{j,k}\hat{S}^z_{j,k+1} \right] - J_I \sum_{k=1}^{3} \hat{S}^z_{j,k}\left(\hat{\sigma}^z_j + \hat{\sigma}^z_{j+1}\right), \ \left(\hat{S}^\alpha_{j,4} \equiv \hat{S}^\alpha_{j,1}\,, \alpha = x, y, z\right).$$

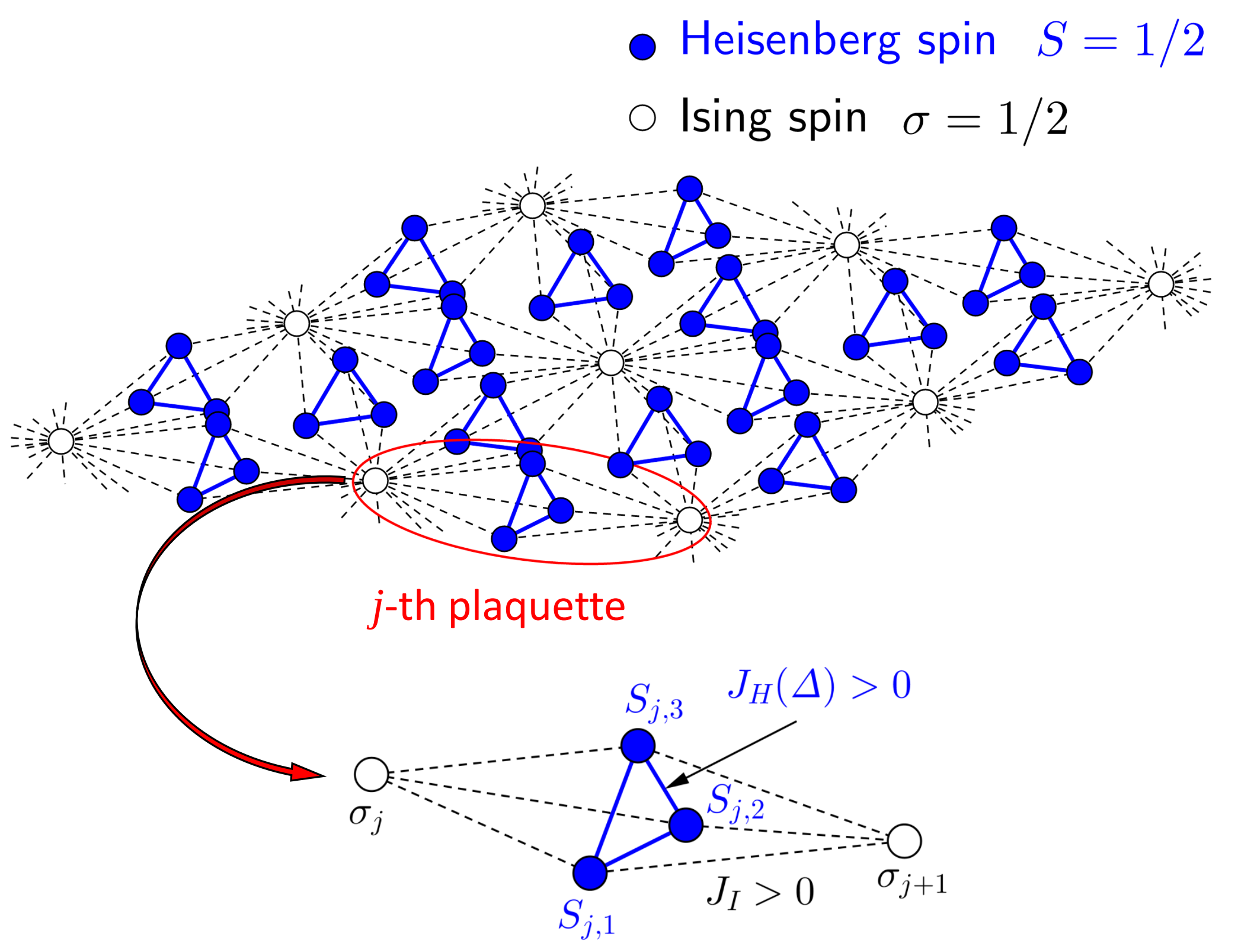

Figure 1. The part of the studied spin-1/2 Ising-Heisenberg lattice and the j-th trigonal bipyramidal plaquette .

CONCURRENCE AS A TOOL FOR STUDY OF THE BIPARTITE QUANTUM ENTANGLEMENT

The spins may be entangled only within the Heisenberg triangular clusters (blue triangles in figure 1). The spins of different Heisenberg triangles can never be entangled, because the Ising spins at common vertices of the neighbouring trigonal bipyramids make a barrier for development of any quantum correlation between these spins. For investigation of the quantum entanglement between two Heisenberg spins of the j-th triangular cluster we use the quantity called **concurrence** [4]:

$$\mathscr{C} = 2\max\left\{0, \frac{2}{3}|C^{xx}_\Delta| - \sqrt{\left(\frac{1}{4} + \frac{C^{zz}_\Delta}{3}\right)^2 - \frac{M^2_\Delta}{9}}\right\},$$

where $C^{xx}_\Delta = \left\langle \sum_{j=1}^{3} \hat{S}^{x(y)}_{j,k}\hat{S}^{x(y)}_{j,k+1} \right\rangle$, $C^{zz}_\Delta = \langle \sum_{j=1}^{3} \hat{S}^z_{j,k}\hat{S}^z_{j,k+1} \rangle$ and $M_\Delta = \langle \sum_{j=1}^{3} \hat{S}^z_{j,k} \rangle$ are the pair correlation function and the spontaneous magnetization of the j-th Heisenberg triangle, respectively [3].

Convention:

- $\mathscr{C} > 0$ … the spins forming the Heisenberg triangular clusters are **entangled**,
- $\mathscr{C} = 0$ … the spins forming in the Heisenberg triangular clusters are **non-entangled**.

NUMERICAL RESULTS AND CONCLUSIONS

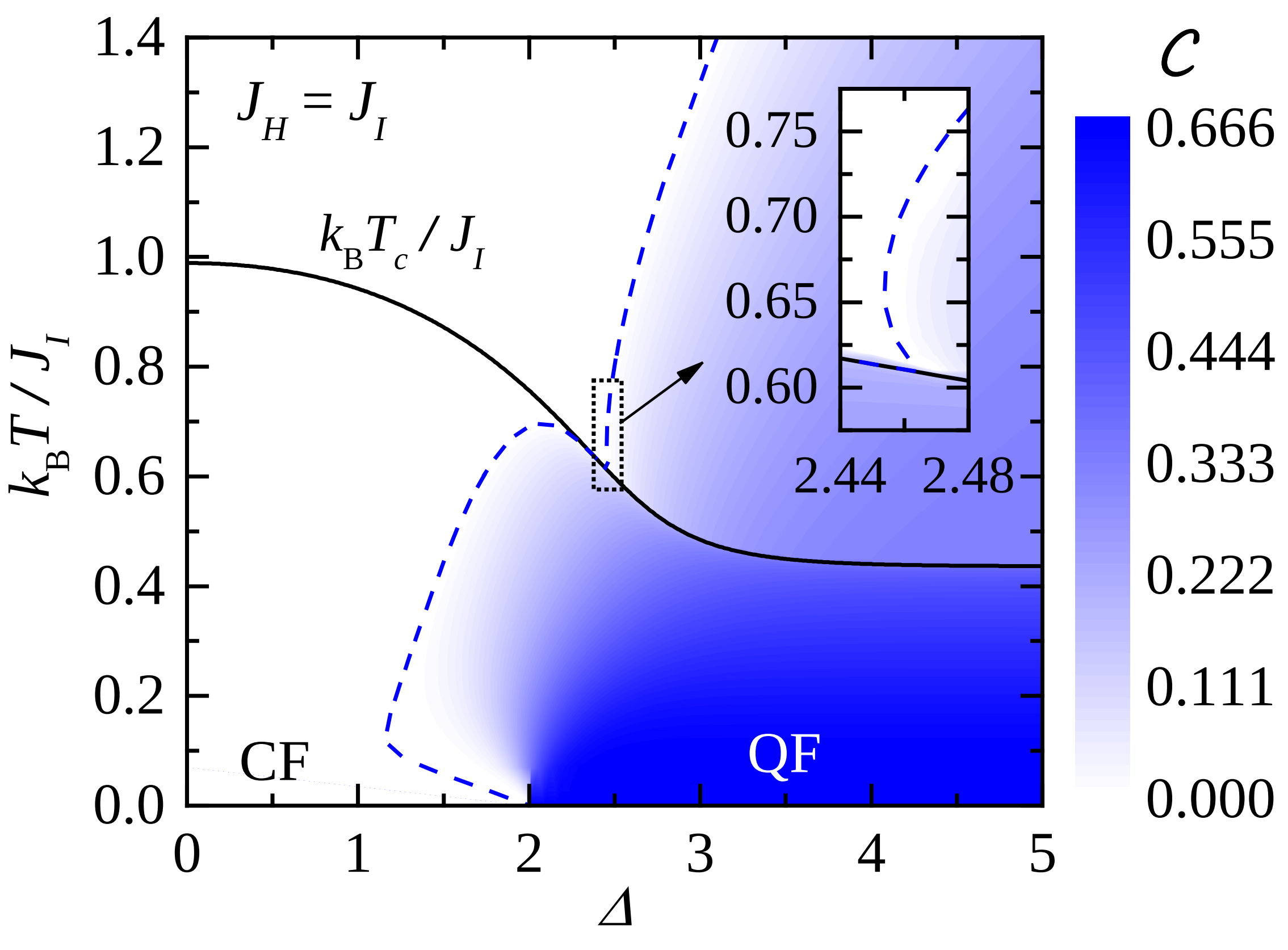

Figure 2. The density plot of the concurrence $\mathscr{C}$ in the $\Delta - k_B T/J_I$ plane. The blue dashed curve is the threshold between the entangled ($\mathscr{C} > 0$) and non-entangled ($\mathscr{C} = 0$) regions and the black solid curve shows the critical temperature $k_B T_c/J_I$ of the model.

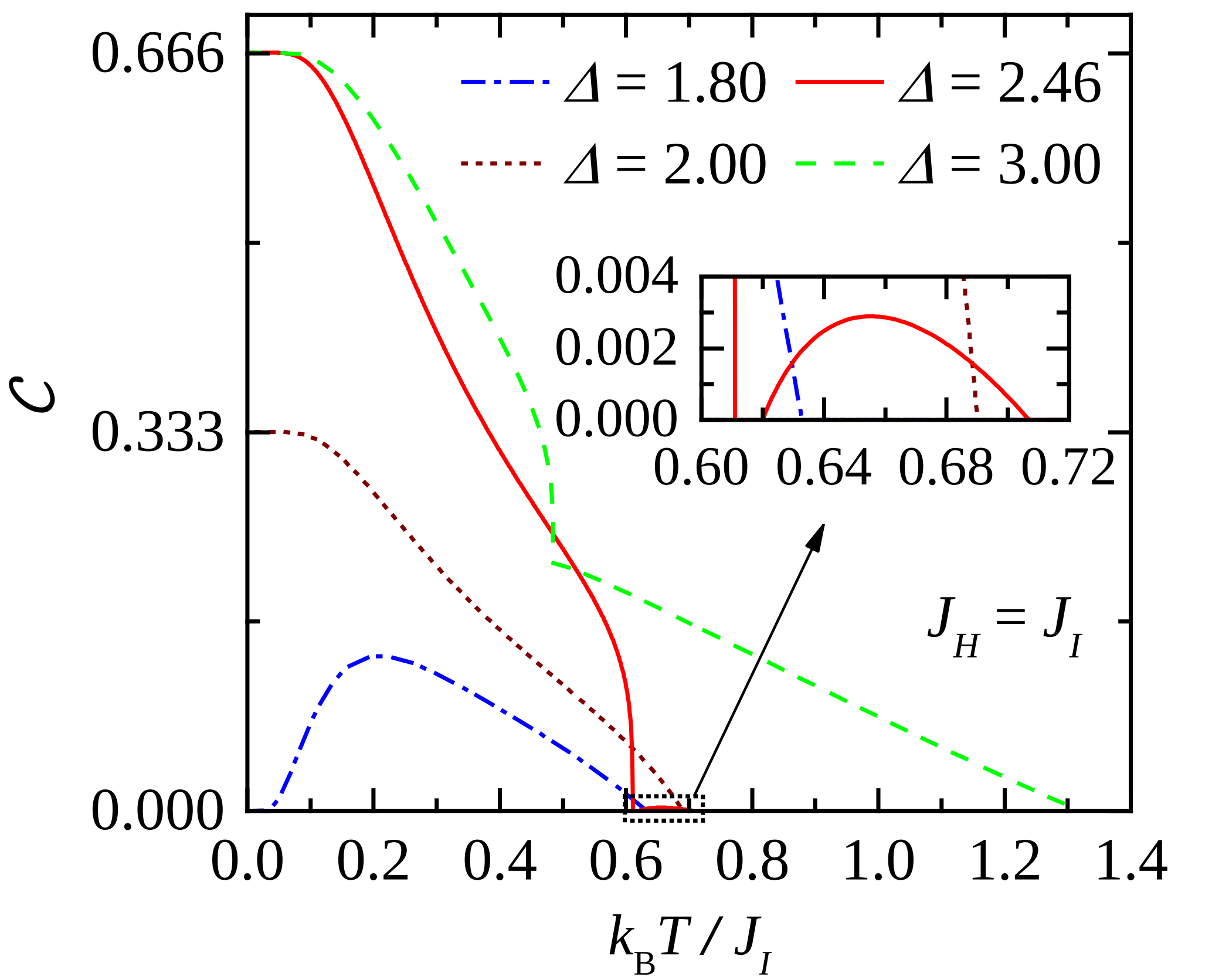

Figure 3. Typical thermal variations of the concurrence $\mathscr{C}$ for four representative values of the exchange anisotropy parameter Δ.

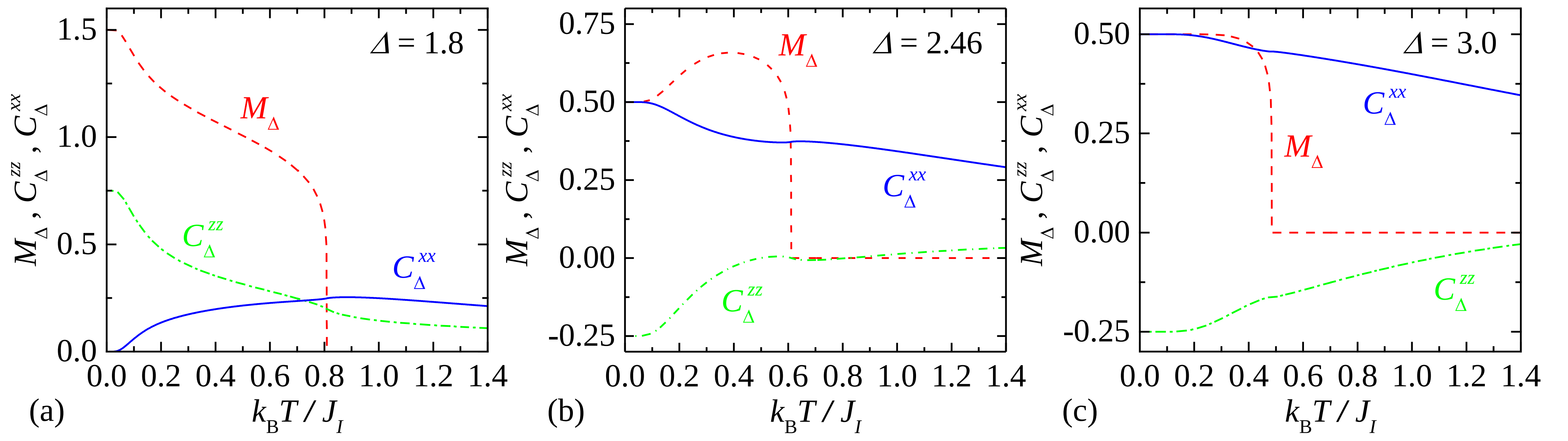

Figure 4. Thermal variations of the spontaneous magnetization M_Δ and pair correlation functions C^{xx}_Δ, C^{zz}_Δ of the j-th Heisenberg triangle that correspond to the thermal variations of the concurrence $\mathscr{C}$ plotted for the exchange anisotropies $\Delta = 1.8, 2.46, 3.0$ in figure 3.

- The ground state of the model is spontaneously ordered:
  - for $\Delta < \frac{J_I}{J_H} + 1$ the classical ferromagnetic (CF) phase
    $$|\mathrm{CF}\rangle = \prod_j |\uparrow\rangle_{\sigma_j} \otimes |\uparrow\uparrow\uparrow\rangle_j$$
    with **no entangled** spin states ($\mathscr{C} = 0$) is stable;
  - for $\Delta > \frac{J_I}{J_H} + 1$ the quantum ferromagnetic (QF) phase
    $$|\mathrm{QF}\rangle = \prod_j |\uparrow\rangle_{\sigma_j} \otimes \frac{1}{\sqrt{3}}\left(|\uparrow\uparrow\downarrow\rangle_j + |\uparrow\downarrow\uparrow\rangle_j + |\downarrow\uparrow\uparrow\rangle_j\right),$$
    where **the unsaturated bipartite quatum entanglement** ($\mathscr{C} = 2/3$) between the Heisenberg spins can be observed.

  See the horizontal axis corresponding to zero temperature in figure 2.
- **The quantum entanglement** of the Heisenberg spins existing in the QF phase is much more resistant to temperature than the respective spontaneous long-range spin order. See figure 2.
- **The weak quantum entanglement** of the Heisenberg spin pairs can be invoked by temperature above the CF phase in a vicinity of the ground-state boundary CF-QF due to thermal fluctuations to the energetically close Heisenberg spin states peculiar to the neighbouring QF phase. See figure 2 and also the thermal variations of the concurrence $\mathscr{C}$ plotted for $\Delta = 1.8$ in figure 3.
- **Very weak** (almost unobservable) **re-entrance of the bipartite entanglement** between the Heisenberg spins (**concurrence** $\mathscr{C} \gtrsim 0$) can also be found slightly above the critical temperature of the QF phase due to thermal activation of quantum pair correlations in some Heisenberg triangles. See insets in figures 2 and 3.

References

[1] M. A. Nielsen and I. L. Chuang: *Quantum Compu-tation and Quantum Information* (Cambridge University Press, Cambridge - 2000). [2] T. Roscilde, *et al.*, Phys. Rev. Lett., 147208 (2005). [3] L. Gálisová, J. Phys.: Condens. Matter **31**, 465801 (2019). [4] L. Amico, *et al.,* Phys. Rev. A **69** 022304 (2004).

Acknowledgement

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