PAVOL JOZEF ŠAFÁRIK UNIVERSITY IN KOŠICE
Faculty of Science

Statistical error of FP equation

[1]V. Mykhailenko, [2]P. Bobik

[1]Institute of Physics, Faculty of Science, P. J. Šafárik University, Park Angelinum 9, 041 54 Košice, Slovakia
[2]Institute of Experimental Physics, Slovak Academy of Sciences, Kosice, Slovakia

Abstract

- Cosmic rays propagate the heliosphere from interstellar space till orbit of planets and Sun. The problem of their distribution is solved by a couple of methods, one of the most used is the stochastic method based on Ito lemma.
- Work is focused on the estimation of statistical error of Foker – Plank equation solution of 1D forward stochastic method, for evaluation cosmic rays distribution in the heliosphere. Error dependence on simulation statistics and energy is presented. The 1% precision criterium is defined as a function of solar wind velocity and diffusion coefficient value. The systematic error of the FP equation is also discussed.

Results

For the data processing and obtain the spectra in one case was used so-called binning procedure, and approach well described in [2]. The differential intensity, in this case, was taken to be $J \propto p(m^2c^4 + p^2c^2)^{-1.85}$.

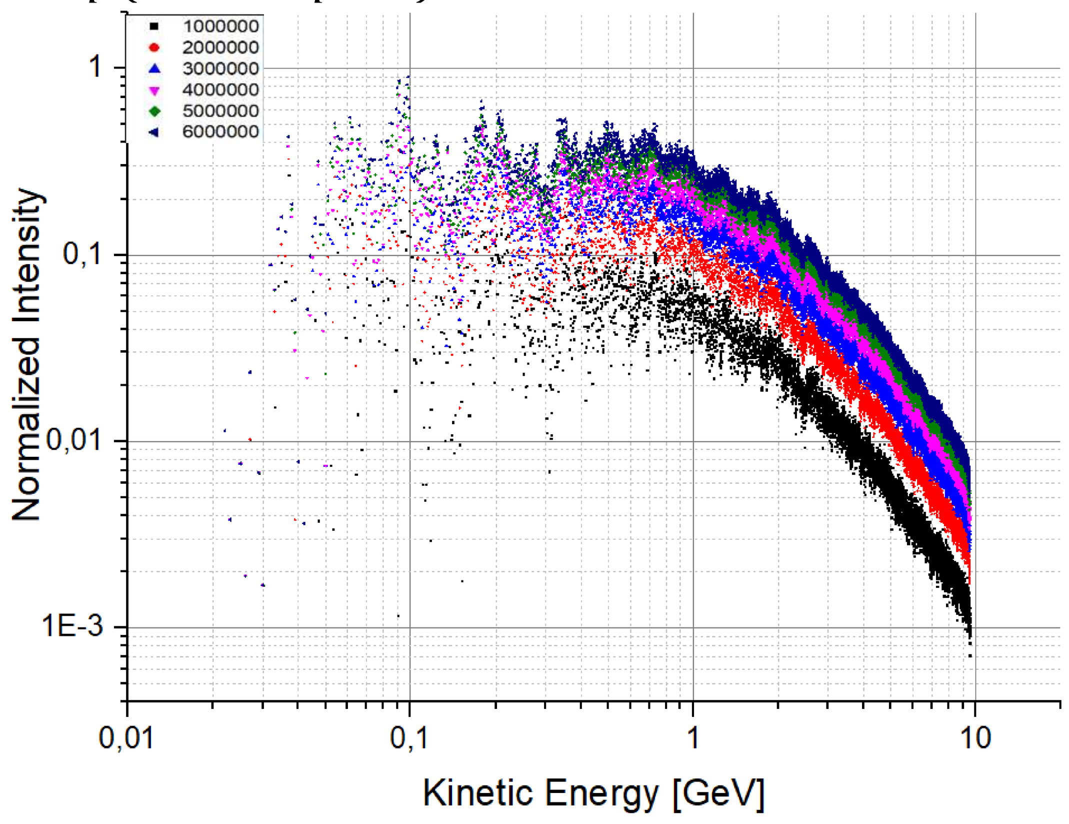

Fig. 1. Normalized intensity with respect to the different number of injected trajectories for each injected energy step [0.001 GeV], from 1000000 injected trajectories to 6000000 respectively.

In the same way integral with respect to number of crosses at 1AU was calculated.

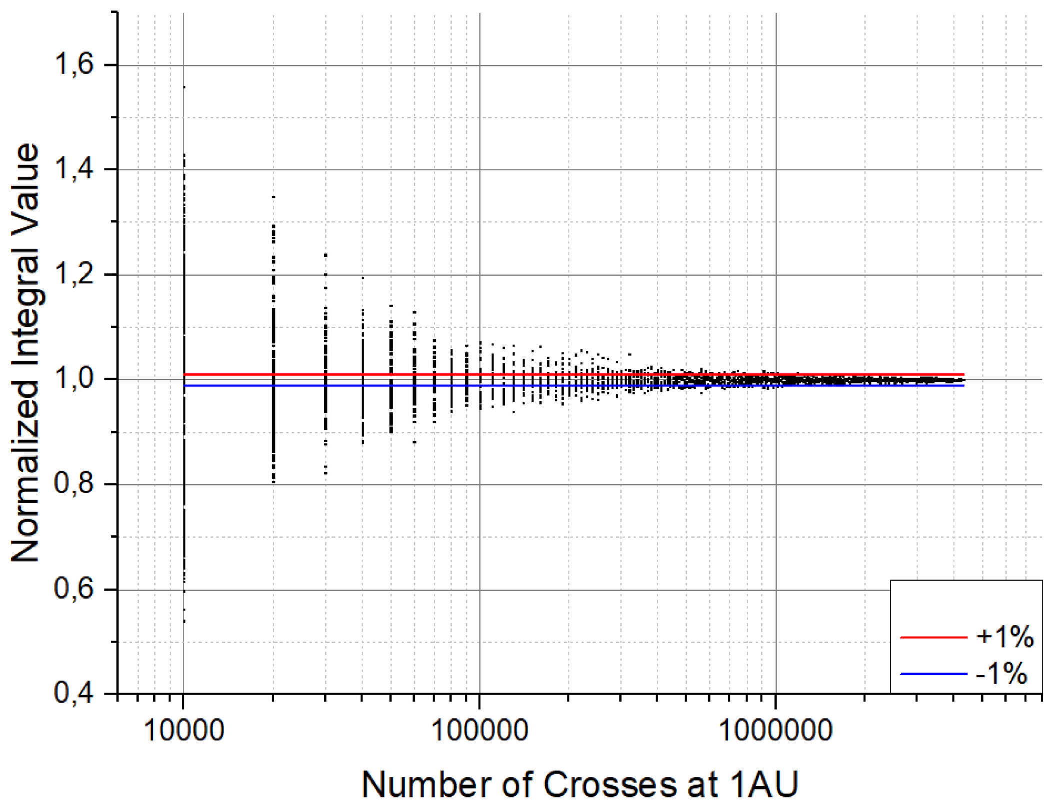

Fig.2. Normalized integral of whole spectrum[0 – 10GeV] value distribution with respect to number of crosses at 1 AU.

Model Description

Forward-in-time see [1]. integration method was used to modulate of the stochastic path of charged particles from the border of the heliosphere to the Earth. The set of SDE for Forward integration with momentum p is:

$$dr = \left(\frac{2K_{diff}}{r} + V_{sw}\right)dt + \sqrt{2K_{diff}}dW$$

$$dp = -\frac{2V_{sw}p}{3r}dt$$

$$L = -\frac{4V_{sw}}{3r}$$

Here $K_{diff} = K_0\beta p$ is diffusion scalar where K_0 is the diffusion parameter in presented simulation is taken to be $K_0 = 5 \times 10^{22}\, cm^2\, s^{-1}\, GV^{-1}$. Solar wind speed V_{sw} in describing simulation is taken to be constant and equal to 400 $km\, s^{-1}$. β, is the particle velocity in speed of light units.

Gaussian distribution of local interstellar spectra was obtained like function with maximum at T_0, $J \propto \exp\left[-300\left(ln\frac{T}{T_0}\right)^2\right]$, for different T_0 values T_0 = 1, 2, 5, 10 GeV. Obtained distribution was compared with spectra evaluated by Crank-Nicolson method.

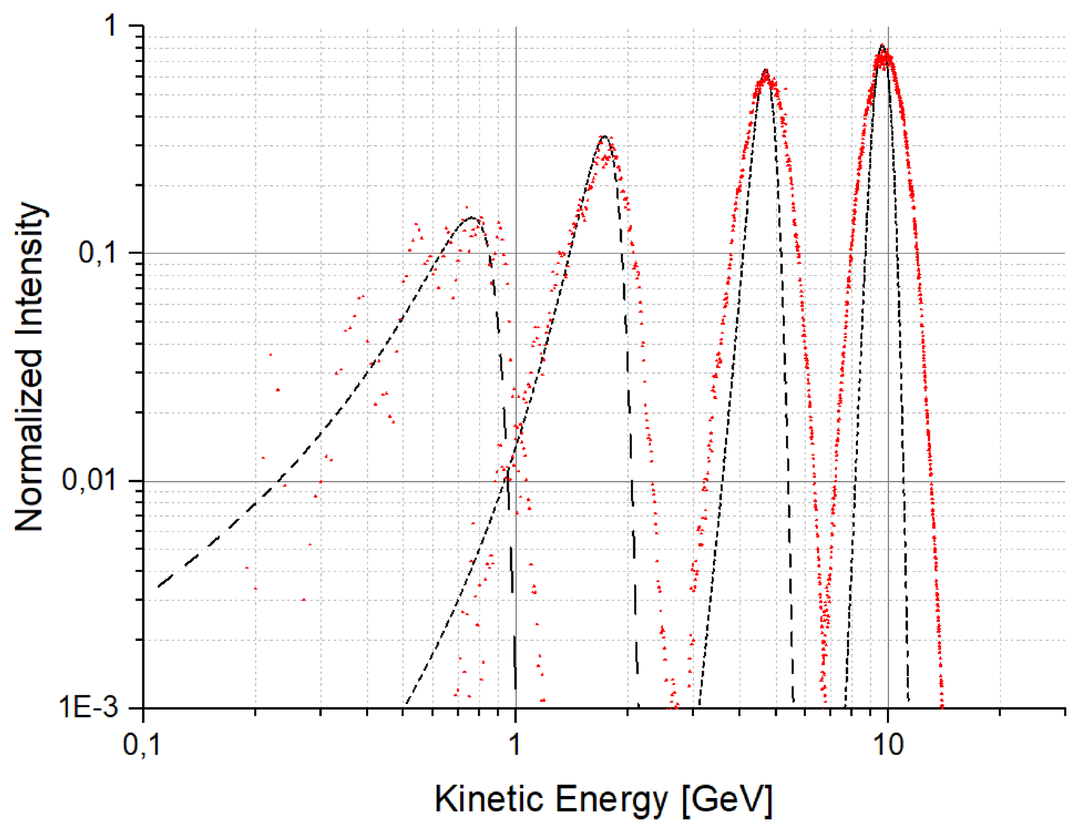

Fig.3. Spectra at 1AU evaluated by forward-in-time integration method(red), and spectra evaluated by Crank-Nicolson method(dashed lines).

Integrals for different T_0 was also evaluated.

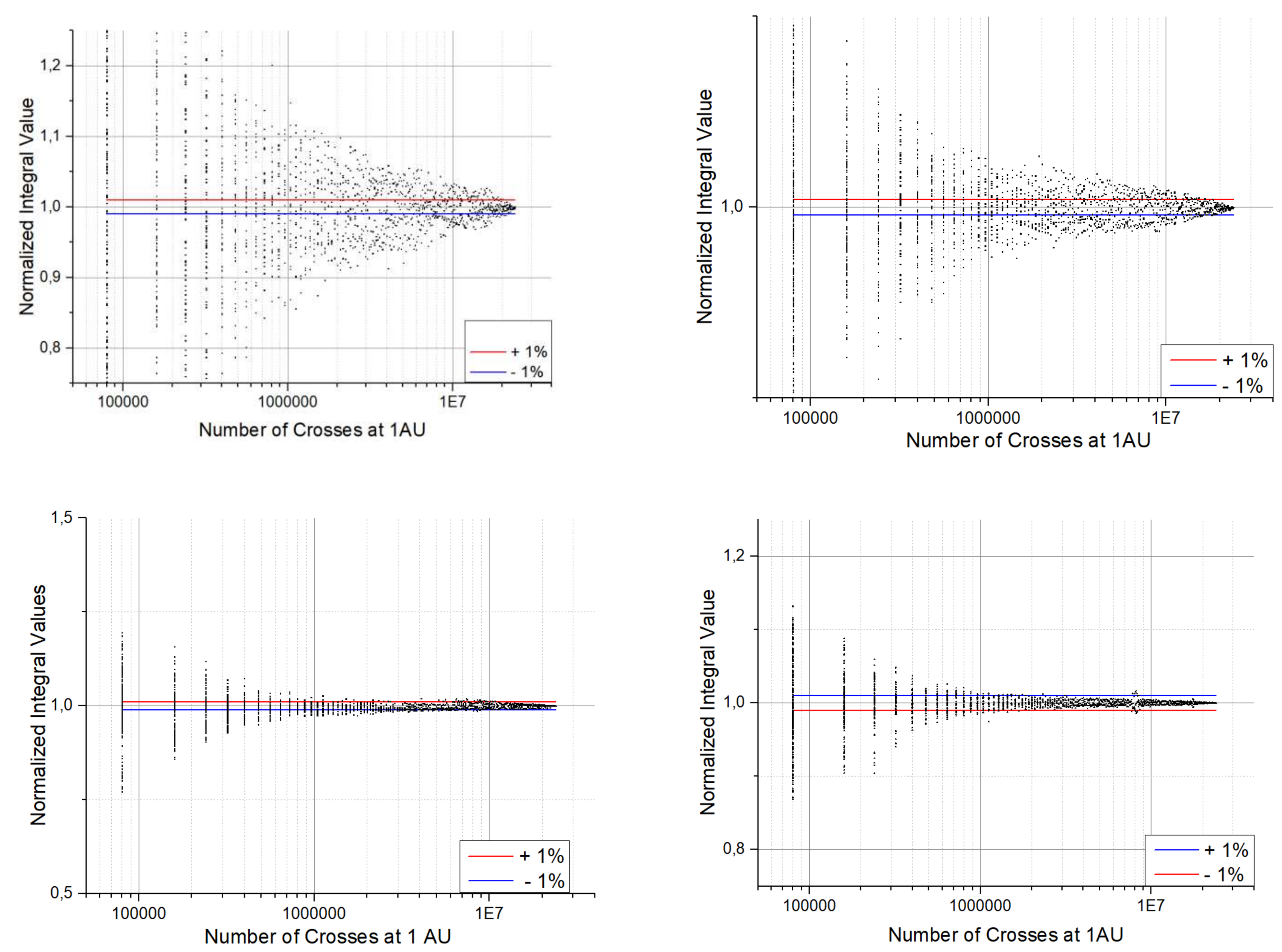

Fig.3. Integrals evaluated from Gaussian spectra for different T_0, T_0 = 1 GeV [top, left], T_0 = 2 GeV [top, right], T_0 = 5 GeV [bottom, left], T_0 = 10 GeV [bottom, right].

Conclusions

- The evaluation of statistical error of cosmic rays modulation in heliosphere at 1AU(input parameters: V_{sw} = 400 $km\, s^{-1}$, $K_0 = 5 \times 10^{22}\, cm^2\, s^{-1}\, GV^{-1}$) was done in F-p method for Gaussian spectra for selected energies and Yamada LIS spectrum;
- For Gaussian spectra needed statistics to reach 1% statistical error was 20 millions for T_0 = 1, and 2 GeV, and 10 millions for T_0 = 5, and 10 GeV;
- For Yamada spectra needed statistics to reach 1% statistical error was approximately 20 millions for 0.001GeV linear registration bins;
- In the future the same methods will be applied for another combination of the input parameters and errors will be evaluated as standard deviation dependency on simulation statistics.

References

[1] Bobik, P., et al. (2016), J. Geophys. Res. Space Physics, 121.
[2] Yamada, Y., S. Yanagita, and T. Yoshida (1998), Geophys. Res. Lett., 25.