Super-Klein tunneling of Dirac fermions through electrostatic gratings in graphene

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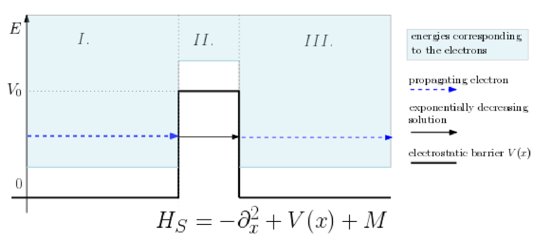
Abstract

We use the Wick-rotated time-dependent supersymmetry to construct models of two- dimensional Dirac fermions in presence of an electrostatic grating. We show that there appears omnidirectional perfect transmission through the grating at specific energy. Additionally to being transparent for incoming fermions, the grating hosts strongly localized states, see [7] for more details.

Introduction

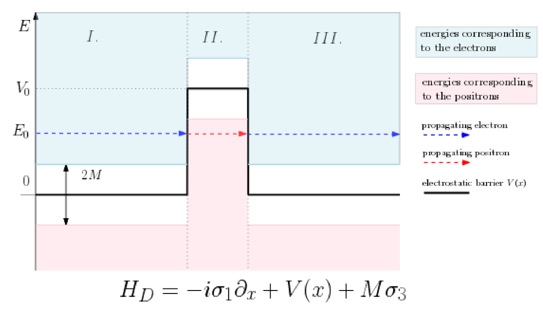
Klein tunneling is a phenomenon in relativistic quantum mechanics where the particle can tunnel through an electrostatic barrier without being reflected, provided that the barrier is sufficiently high. This counter-intuitive behavior stems from the fact that the electron converts into the positron inside the barrier, which is not allowed in non-relativistic case.

Nonrelativistic system described by one-dimensional Schrödinger Hamiltonian H_S:



In the nonrelativistic systems described by Schrödinger equation, the energy spectrum is bounded from below. There are only exponentially decreasing solutions available in the barrier, which implies that the wave function of the particle gets dampened and partially reflected.

Relativistic system described by one-dimensional stationary Dirac Hamiltonian H_D:



In the relativistic wave equations (e.g. Dirac equation), there are both positive and negative energy solutions. They correspond to particles and antiparticles, respectively. When the electrostatic barrier is strong enough ($V_0 > 2M$), then an incoming particle (electron) of energy $E_0 < (V_0 - M)$ can propagate as an antiparticle (positron) inside the barrier and perfect transmission can occur.

Klein tunneling was not observed for elementary particles so far as the strength of the electrostatic field necessary to observe the phenomenon surpasses possibilities of the current experiments. However, it was observed in condensed matter systems.

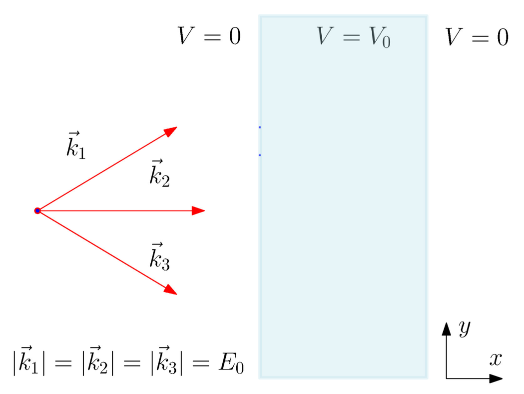
In graphene, the quasi-particles behave as mass-less, two-dimensional Dirac fermions. The effective Hamiltonian reads

$$H = -i\sigma_1\partial_x - i\sigma_2\partial_y + V(x,y) \tag{1}$$

where the (matrix) potential term $V(x,y)$ represents the external fields.

The transport properties of the system described by (1), where $V = V(x)$ is a rectangular potential barrier as depicted above, were studied in [1]. The transmission T is strongly angle-dependent. The perfect tunneling $T = 1$ occurs for a discrete set of incidence angles only.

Scheme of the scattering experiment



Dirac fermion described by (1) bouncing on the rectangular electrostatic barrier $V = V(x)$ under different angles.

Dependence of the transmission amplitude on the incidence angle

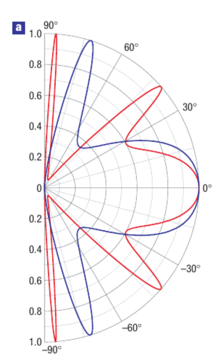


Figure from [1]: *Transmission probability T for Dirac Fermions in graphene through 100nm-wide barrier, $V_0 = 200meV$ (red), $V_0 = 285meV$ (blue), energy of incident particle $E_0 = 80meV$.*

Super-Klein tunneling (also called all-angle or omnidirectional Klein tunneling) is a phenomenon where relativistic particles can go through an electrostatic barrier **at any angle** without being reflected. It was revealed recently in the condensed-matter systems where the quasi-particles behave as relativistic ones with spin one [2], [3] one-half [4] and zero spin [5].

Omnidirectional Klein tunneling occurs exclusively for a specific energy. When the energy of the incoming particle is mismatched with this critical value, transmission amplitude gets strongly angle-dependent. **In all the studied cases, the system possessed translational invariance in one direction, i.e. $V = V(x)$.**

Q: *Does the super-Klein tunneling exist in the systems without translational symmetry?*
A: *Yes, it does. We found an explicit example.*

Exactly solvable 1+1D Dirac equation via time-dependent supersymmetry

Supersymmetric transformations allow to construct new solvable systems from the known ones. The Hamiltonians H_0 of the original and H_1 of the new system are intertwined by operator L that represents the supersymmetric (Darboux) transformation,

$$LH_0 = H_1 L, \quad L^\dagger H_1 = H_0 L^\dagger. \tag{2}$$

The intertwining relations are powerful as they allow to find solutions of $(H_1 - \lambda)\xi = 0$ from $(H_0 - \lambda)\psi = 0$ where $\xi \equiv L\psi$.

We utilize the results of [6] where H_0 was fixed as the 1+1D Dirac operator of free particle,

$$H_0\psi = (i\partial_t - i\sigma_2\partial_z - m\sigma_3)\psi = 0, \tag{3}$$

and the operator L and H_1 were found in the following form,

$$L = \partial_z - \frac{1}{2D_1}\left(\frac{\omega^2\sinh(2kz)}{k}\sigma_0 - 2m\cosh^2(\omega t)\,\sigma_1 + \omega\sinh(2\omega t)\,\sigma_2\right), \tag{4}$$

$$H_1 = i\partial_t - i\sigma_2\partial_z + \left(-m + \frac{4mk^2\cosh^2(\omega t)}{m^2 + k^2\cosh(2\omega t) + \omega^2\cosh(2kz)}\right)\sigma_3. \tag{5}$$

Here $D_1(z,t) = (m^2 + k^2\cosh(2\omega t) + \omega^2\cosh(2kz))/2k^2$. As a by-product of the supersymmetric transformation, one gets two "extra" solutions v_1 and v_2 that have no analog in the original system,

$$v_1(z,t) = \frac{1}{D_1}\left([m\cosh(\omega t) + i\omega\sinh(\omega t)]\sinh kz, k\cosh(\omega t)\cosh(zk)\right)^T, \quad v_2(z,t) = \sigma_1 v_1(z,t)^*.$$

They satisfy $L^\dagger v_a = 0$, $H_1 v_a = 0$.

From 1+1D to 2+0D: the Wick rotation

The Wick rotation was used originally to get solutions of Bethe-Salpeter equation in Minkowski space from those defined in the Euclidean space. Let us go in the opposite direction and define a system living in two-dimensional Euclidean space from a related 1+1 dimensional relativistic model and get a relevant information on its physical properties.

We make the following change of coordinates, $z = ix, \partial_z = -i\partial_x, t = y$, multiply the equation by σ_3 and make an additional gauge-transformation of the Hamiltonian,

$$\begin{aligned}\widetilde{H}_1(x,y)\widetilde{\psi}(x,y) &= -\mathbf{U}\sigma_3 H_1(ix,y)\mathbf{U}^{-1}\widetilde{\psi}(x,y)\\ &= \left(-i\sigma_1\partial_x - i\sigma_2\partial_y + m - \frac{4mk^2\cosh^2(\omega y)}{m^2 + k^2\cosh(2\omega y) + \omega^2\cos(2kx)}\sigma_0\right)\mathbf{U}\psi(ix,y) = 0,\end{aligned}$$

where $\mathbf{U} = e^{i\frac{\pi}{4}\sigma_1}$ and $k = \sqrt{m^2 + \omega^2}$. We get a stationary Dirac equation for zero energy in two dimensions. For large $|y|$, the potential term tends to $-m$. We substract this constant value and attribute it to the energy. We get stationary equation for energy $E = m$,

$$\left(-i\sigma_1\partial_x - i\sigma_2\partial_y + \mathcal{V}_A(x,y)\sigma_0\right)\widetilde{\psi}(x,y) = m\widetilde{\psi}(x,y), \tag{6}$$

where $\mathcal{V}_A(x,y) = -\frac{4m\omega^2\sin^2(kx)}{m^2+k^2\cosh(2\omega y)+\omega^2\cos(2kx)}$. The potential term vanishes for large $|y|$ and is periodic in x, $\lim_{|y|\to\infty}\mathcal{V}_A(x,y) = 0$, $\mathcal{V}_A(x+\pi/k,y) = \mathcal{V}_A(x,y)$, $\mathcal{V}_A(x,y) = \mathcal{V}_A(-x,-y)$. It forms an electrostatic grating in the graphene sheet.

The transformed extra solutions represent states that are strongly localized at the grating

$$\widetilde{v}_1 = \mathbf{U}v_1(ix,y), \quad \widetilde{v}_2 = \mathbf{U}v_2(ix,y). \tag{7}$$

Scattering

The intertwining relation (2) reduces to

$$(-i\sigma_1\partial_x - i\sigma_2\partial_y + m\sigma_0)\widetilde{\psi} = 0 \Rightarrow \left(-i\sigma_1\partial_x - i\sigma_2\partial_y + \mathcal{V}_A(x,y)\sigma_0 - m\right)\widetilde{L}\widetilde{\psi} = 0. \tag{8}$$

It allows us to get scattering solutions of (6):

The free-particle solutions of $\widetilde{H}_0\widetilde{\psi}_0(x,y,\phi) = 0$ can be written as

$$\widetilde{\psi}_0(x,y,\phi) = e^{im(\sin\phi\, x + \cos\phi\, y)}\begin{pmatrix}1\\ -ie^{-i\phi}\end{pmatrix}, \quad k_x = m\sin\phi, \quad k_y = m\cos\phi, \quad \phi = (-\pi/2,\pi/2),$$

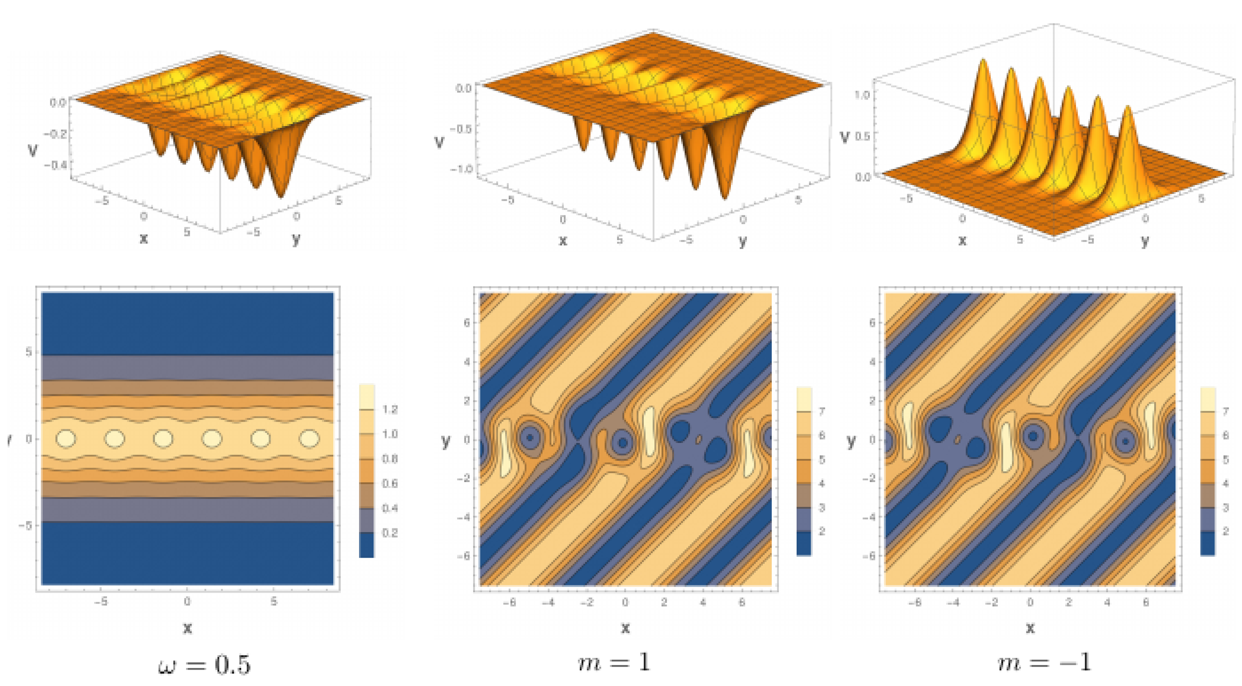
where ϕ corresponds to the incidence angle of the plane wave. Let us investigate asymptotic behavior of $\widetilde{L}\widetilde{\psi}_0$ for large $|y|$. The operator $\widetilde{L}$ has the following asymptotic form

$$\widetilde{L} \sim -i\partial_x + m\sigma_1 \mp \omega\sigma_3, \quad y \to \mp\infty,$$

Comparing the behavior of $\widetilde{L}\widetilde{\psi}_0$ at $y \to -\infty$ and $y \to +\infty$, there is just a phase shift,

$$\lim_{y\to+\infty}\widetilde{L}\widetilde{\psi}_0 = \frac{\omega - im\cos\phi}{-\omega - im\cos\phi}\left(\lim_{y\to-\infty}\widetilde{L}\widetilde{\psi}_0\right).$$

Hence, the function $\widetilde{L}\widetilde{\psi}_0$ acquires just a phase shift when passing through the barrier, independently on the incidence angle. Therefore, there is super-Klein tunneling in the system for energy $E = m$.



The potential term $\mathcal{V}_A(x,y)$ (upper row) with density of probability of the confined state $\tilde{v}_1$ (lower left). The columns differ by the choice of ω and m. In the first column, we used $m = 1, \omega = 0.5$. In the lower middle and right, we show density of probability (interference pattern) of a linear combination $F_A(x,y,\phi_1,\phi_2) = \widetilde{L}\widetilde{\psi}_0(x,y,\phi_1) + \widetilde{L}\widetilde{\psi}_0(x,y,\phi_2)$ of a free particle (left) and the asymptotically plane-wave solutions fro the upper row potential for $m = 1$ (center) and $m = -1$ (right). In these plots, we used $\omega = 0.75$, $\phi_1 = 0$, $\phi_2 = \pi/2$.

Conclusions

- To our best knowledge, we found the first genuinely two-dimensional configuration of the electrostatic field that permits super-Klein tunneling for the two-dimensional Dirac fermions.
- The model presented here shows that the omnidirectional Klein tunneling is not specific only for the systems with translational symmetry.

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