Gluon TMDs in electroproduction

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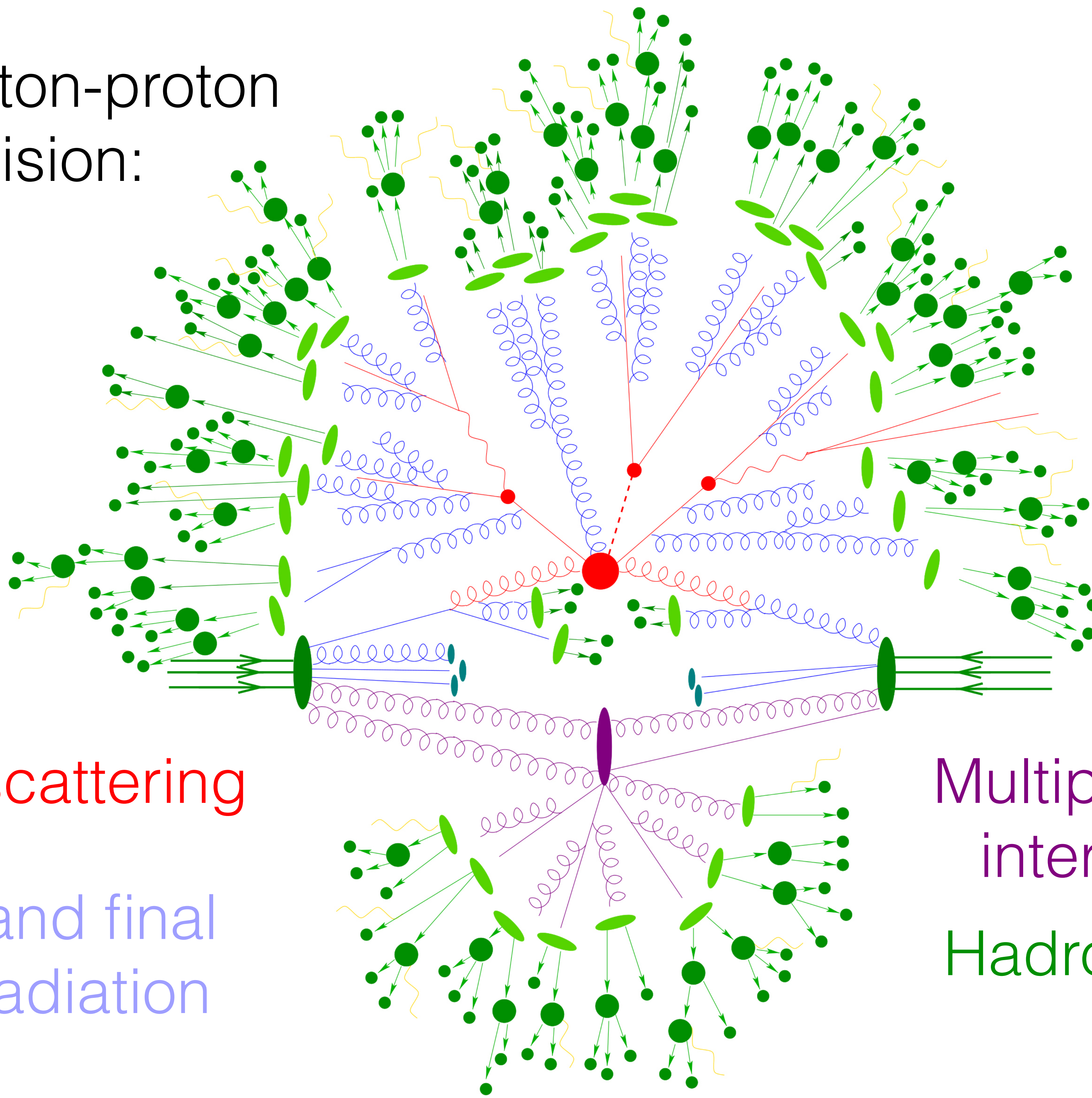
Reminder of deep-inelastic scattering

Hadron collisions

proton-proton collision:



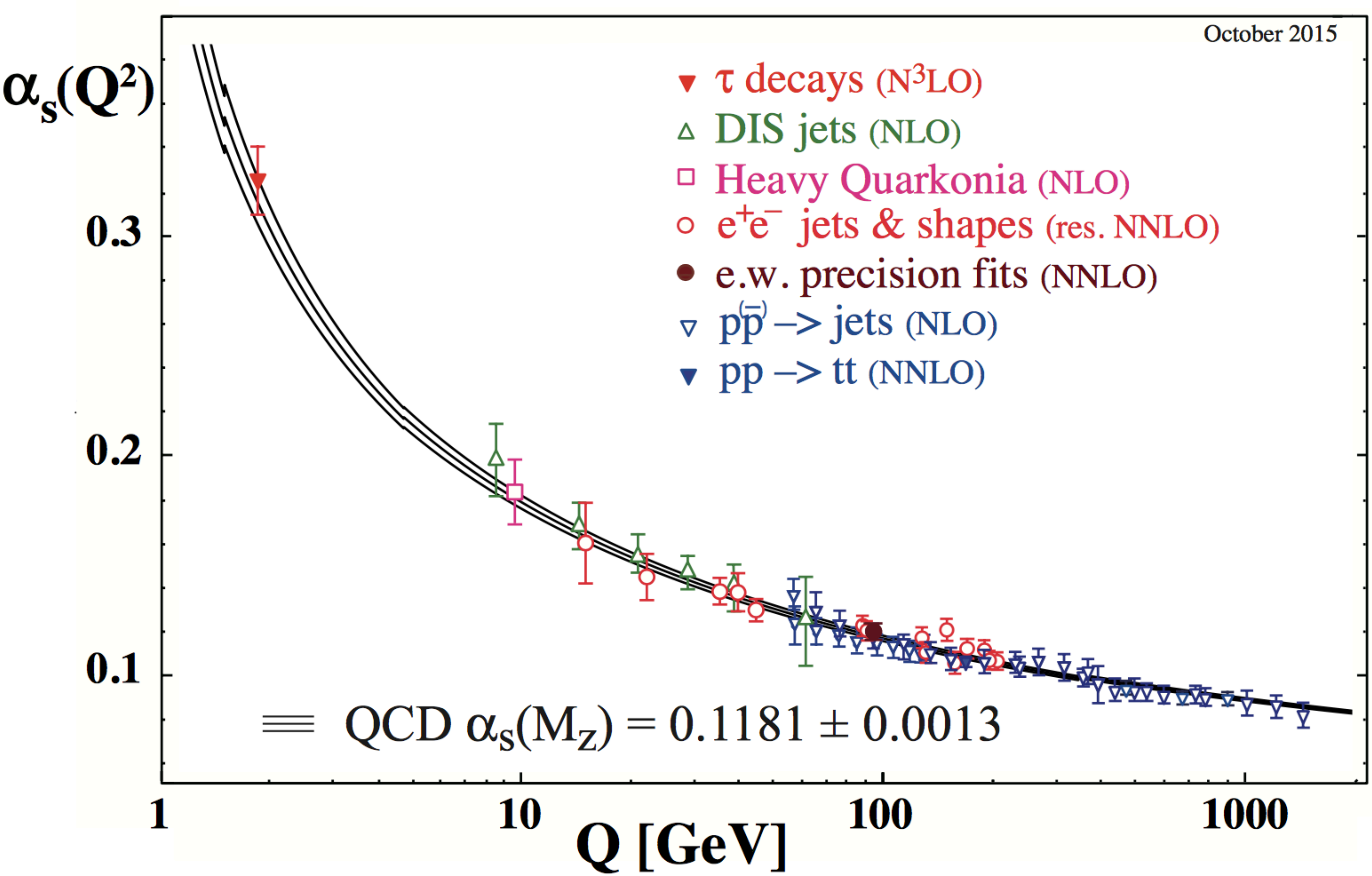
Hard scattering

Initial and final state radiation

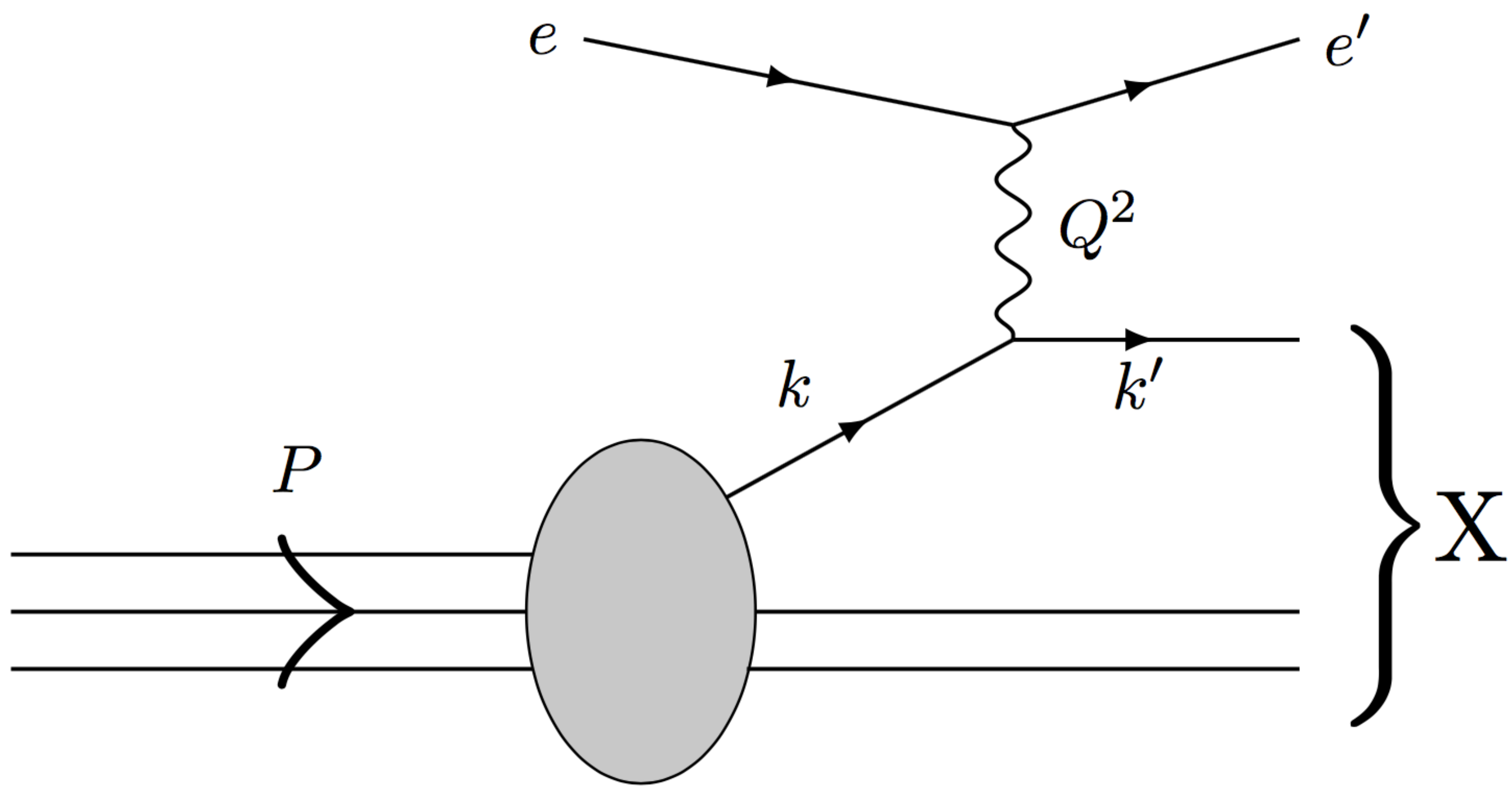
Multiple parton interactions

Hadronization

Running coupling



Deep-inelastic electron-proton scattering (DIS)



Described by two measurable kinematic variables: virtuality Q^2 and Bjorken-x

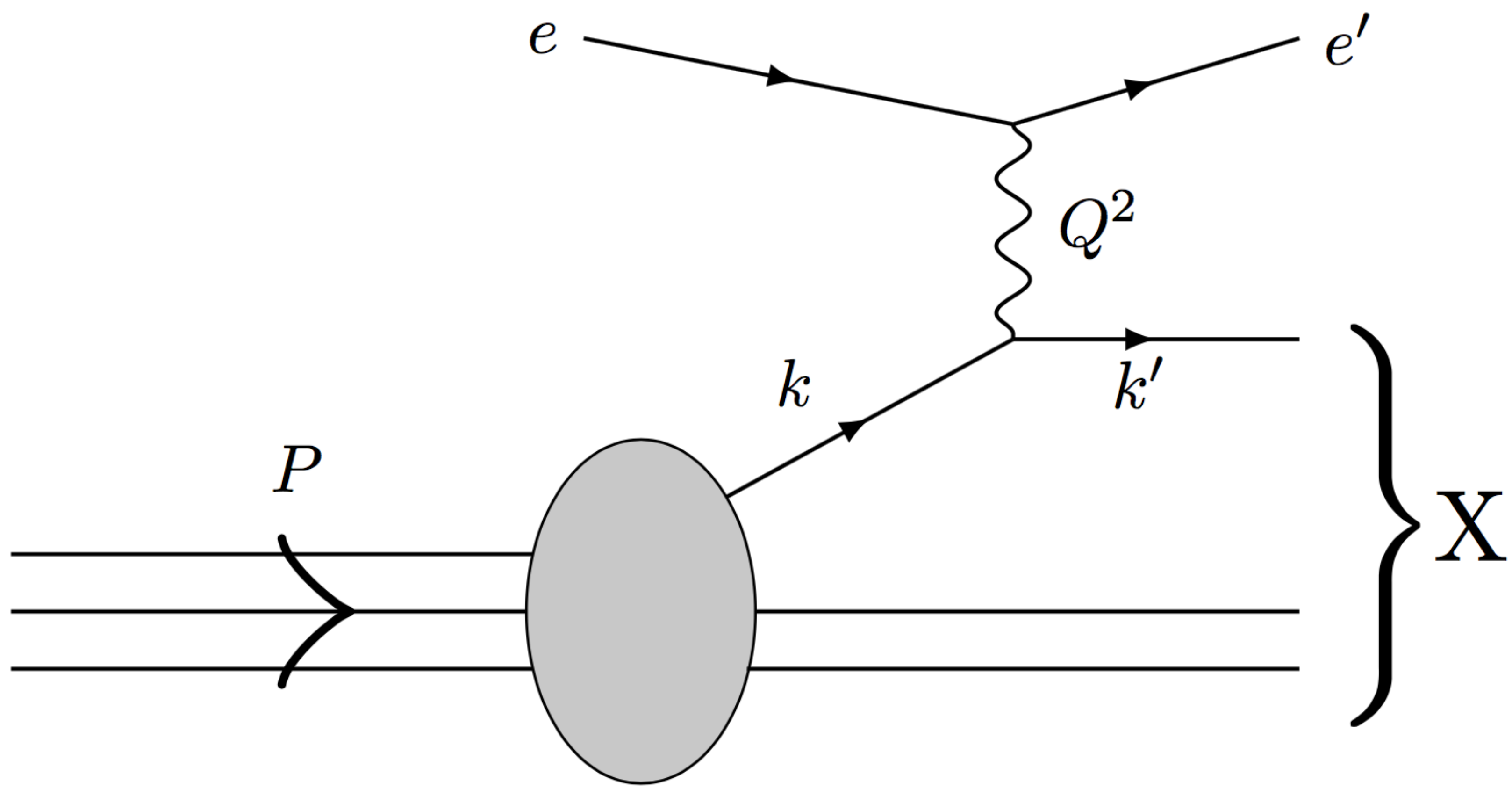
$$Q^2 = -q^2 \equiv -(e - e')^2 \simeq 2EE'(1 - \cos\theta)$$

Hard scale such that $\alpha_s(Q^2) < 1$

measure for transverse resolution: $k_\perp < Q$

$$x = \frac{Q^2}{2P \cdot q} \simeq \frac{k^+}{P^+}$$ energy fraction of proton carried by quark

Deep-inelastic scattering (DIS)



hard part (perturbative)

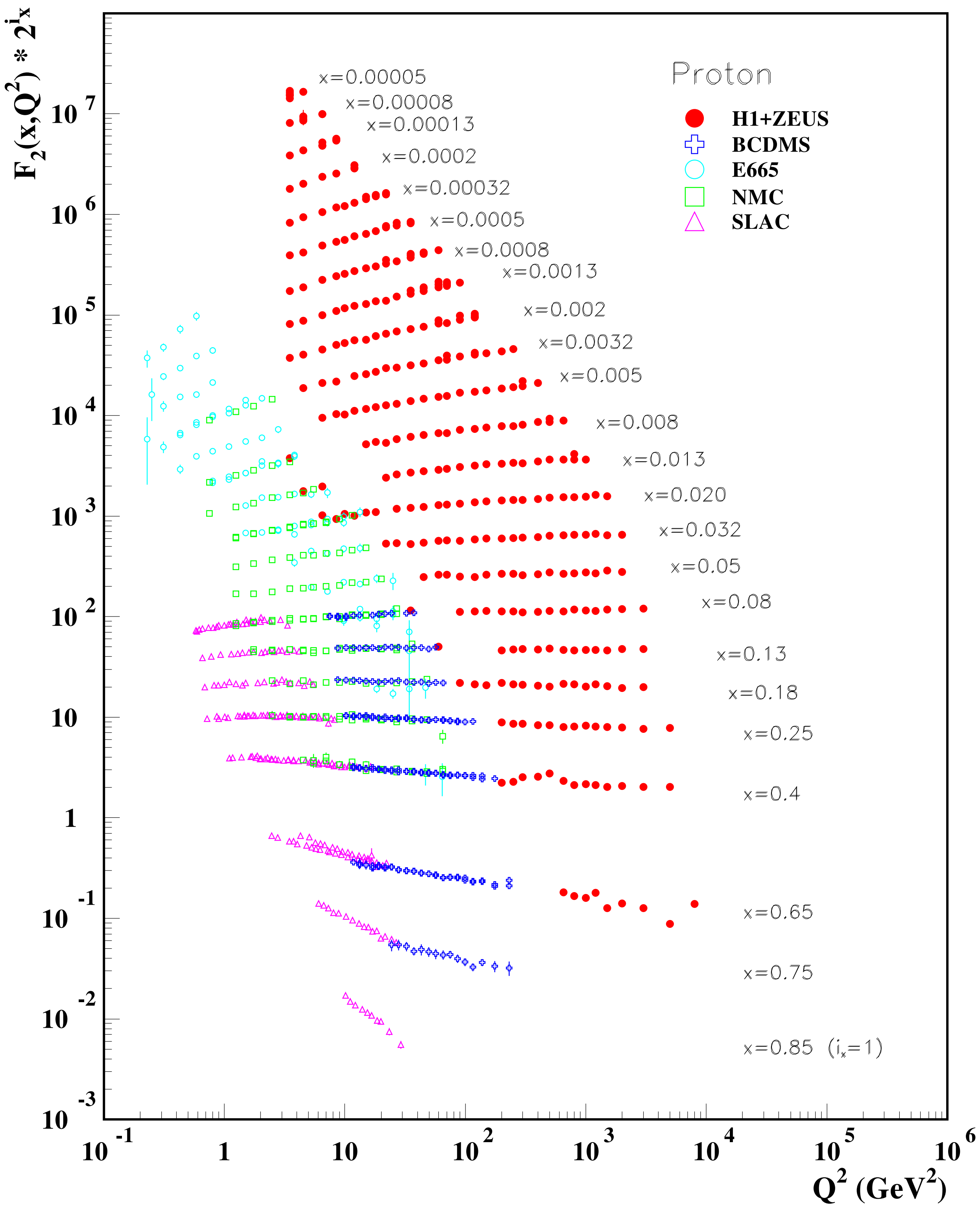
structure function

$$\sigma_{\gamma^* p} = \frac{4\pi^2 \alpha_{\text{em}}}{Q^2} F_2(x, Q^2)$$

$$F_2\left(x, Q^2\right) = \sum_f e_{q_f}^2 \left(x q_f\left(x, Q^2\right) + x \bar{q}_f\left(x, Q^2\right)\right)$$

Parton distribution functions (PDFs): density of quarks/gluons with momentum fraction x at scale Q^2

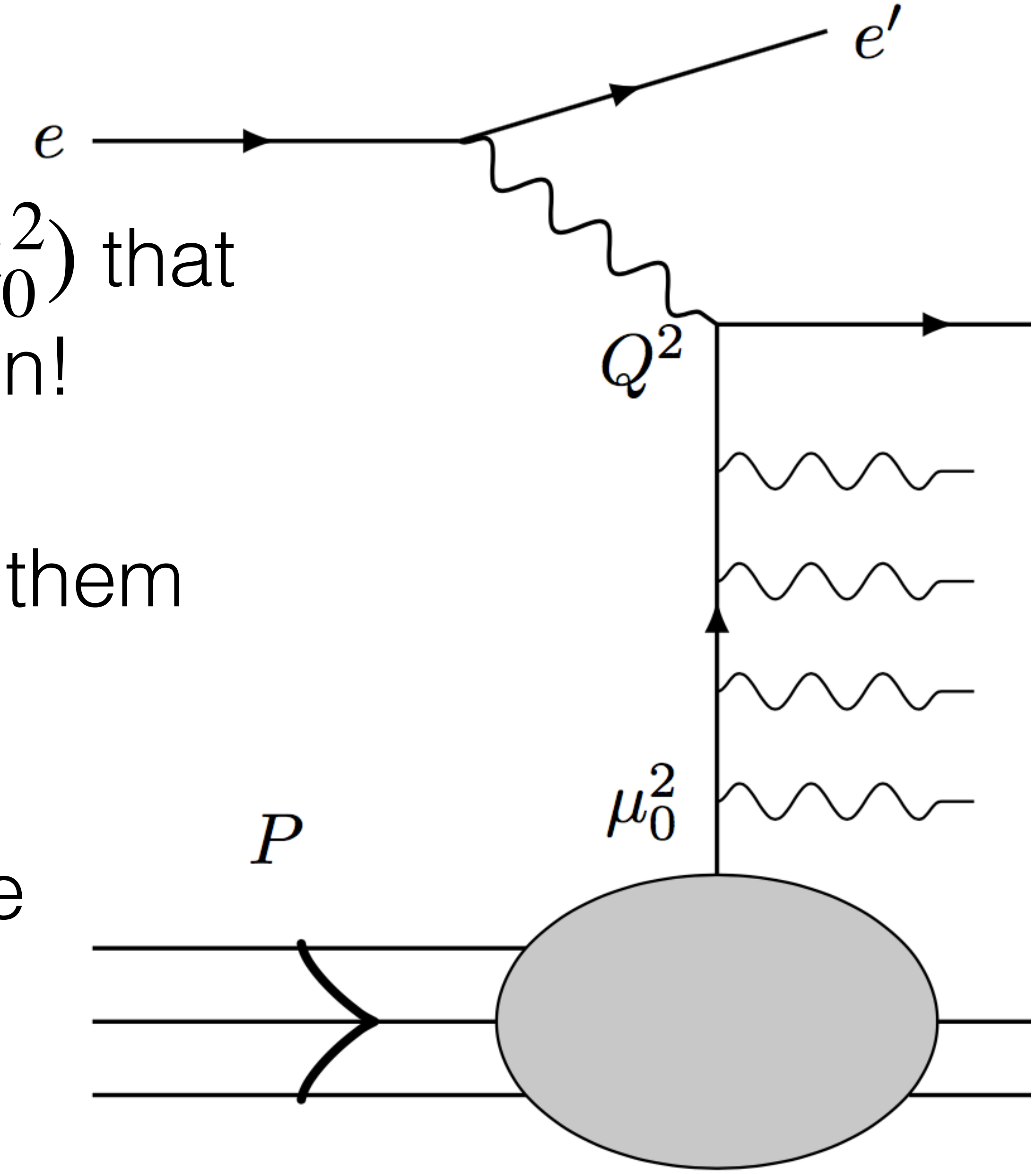
Bjorken scaling violation



$$F_2\left(x, Q^2\right) = \sum_f e_{q_f}^2 \left(x q_f\left(x, Q^2\right) + x \bar{q}_f\left(x, Q^2\right)\right)$$

Structure function is indeed Q^2-dependent

Radiative corrections



Lead to large logarithms $\alpha_s \ln(Q^2/\mu_0^2)$ that can break the perturbative expansion!

Solution is to exponentiate or *resum* them

Perturbatively calculable *evolution equations* govern scale dependence of PDFs

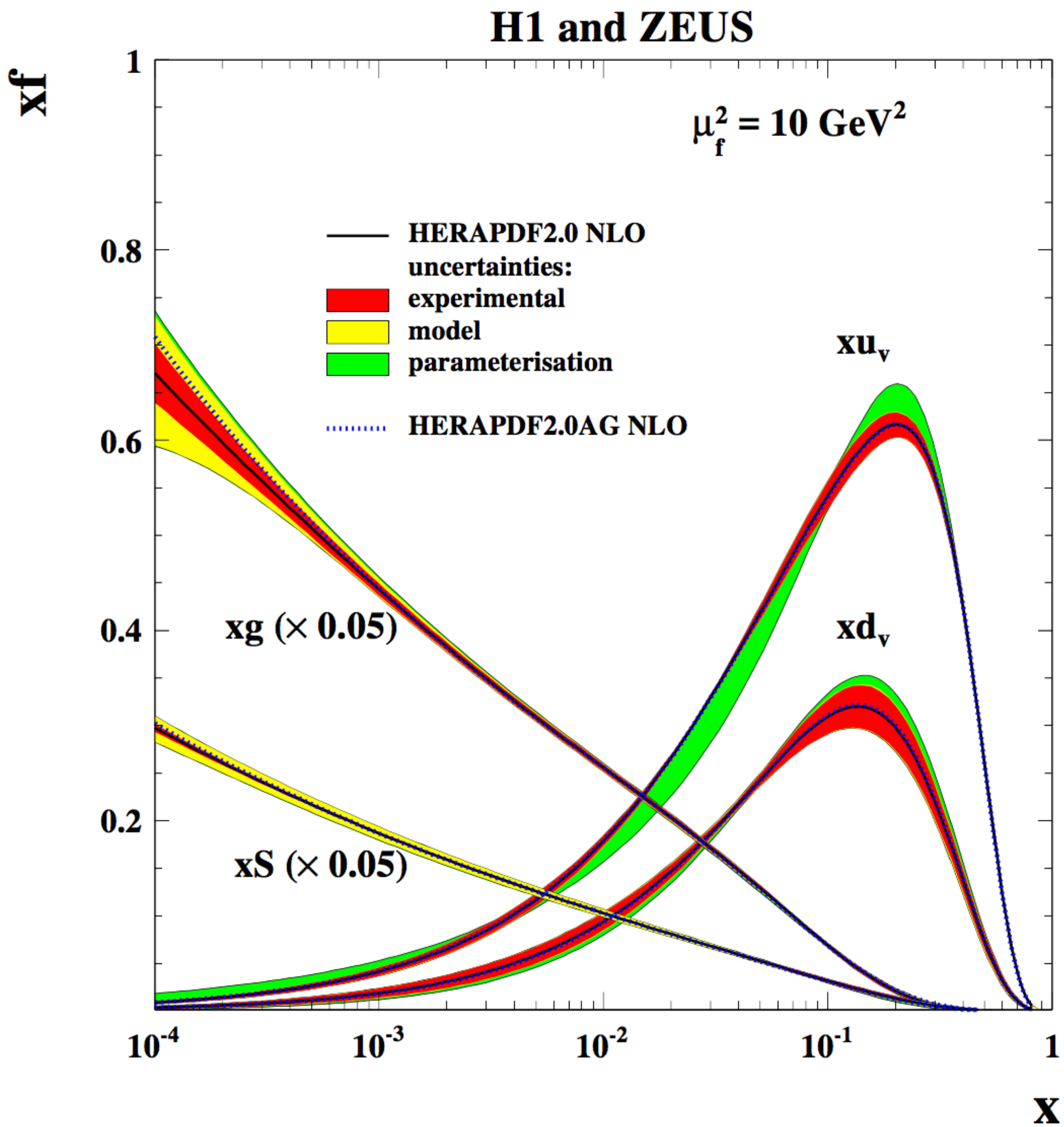
$$\sigma_{\gamma^* p}\left(x, Q^2\right) = \underbrace{\frac{4\pi^2\alpha_{\mathrm{em}}}{Q^2}\sum_f e_{q_f}^2}_{\sigma_{\mathrm{hard}}(x,Q^2)} \underbrace{\left(x q_f\left(x, Q^2\right) + x\bar{q}_f\left(x, Q^2\right)\right)}_{\mathrm{DGLAP}(\mu^2 \to Q^2)\otimes q_f(x,\mu^2)}$$

Dokshitzer, Gribov, Lipatov, Altarelli & Parisi (1972-1977)

Parton distribution functions (PDFs)

Function of:

- Bjorken-x: fraction of energy
- 'Virtuality' Q^2: transverse scale with which hadron is resolved

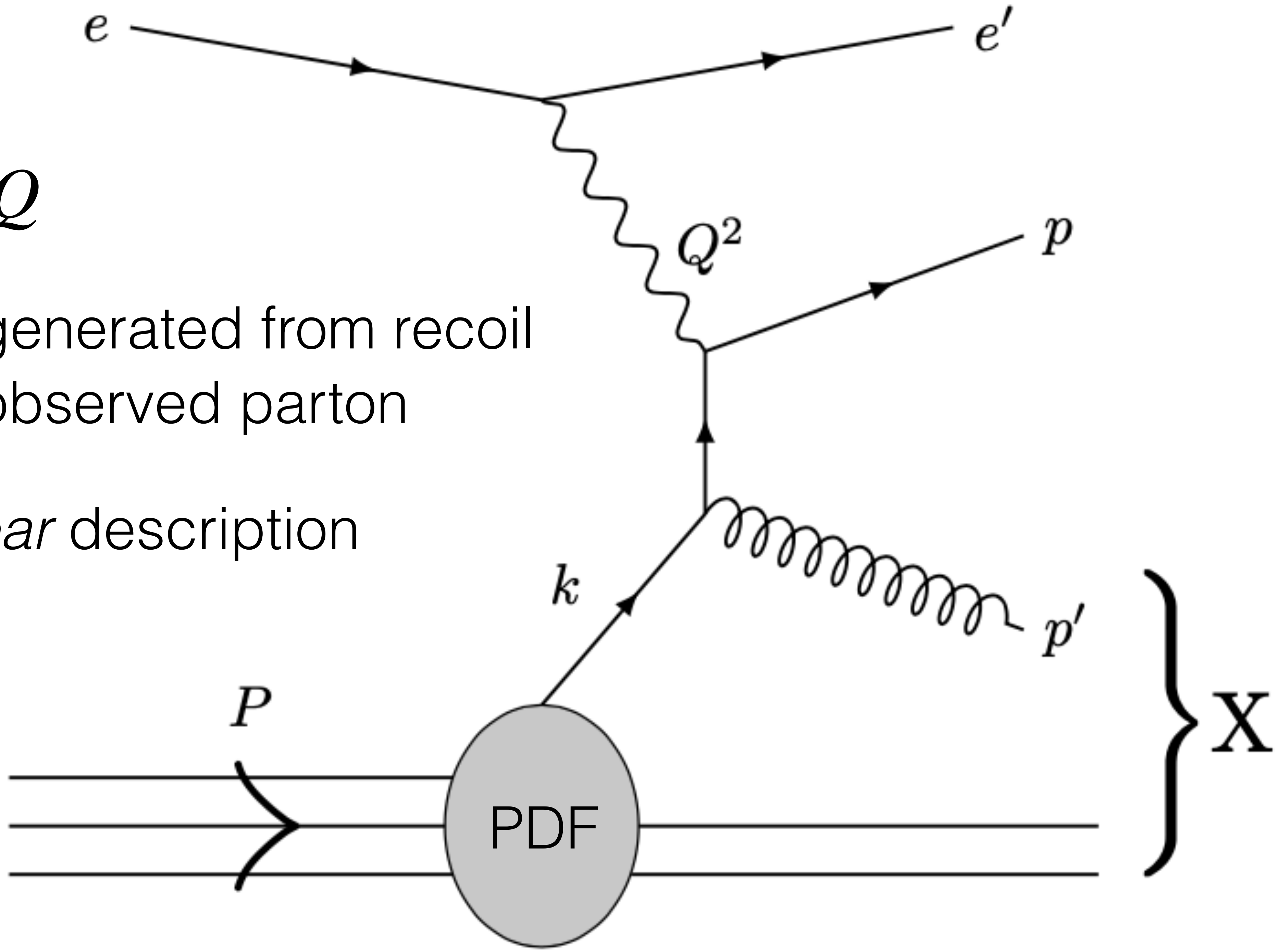


Semi-inclusive DIS (SIDIS)

$p_\perp \sim Q$

$p_\perp$ is generated from recoil off unobserved parton

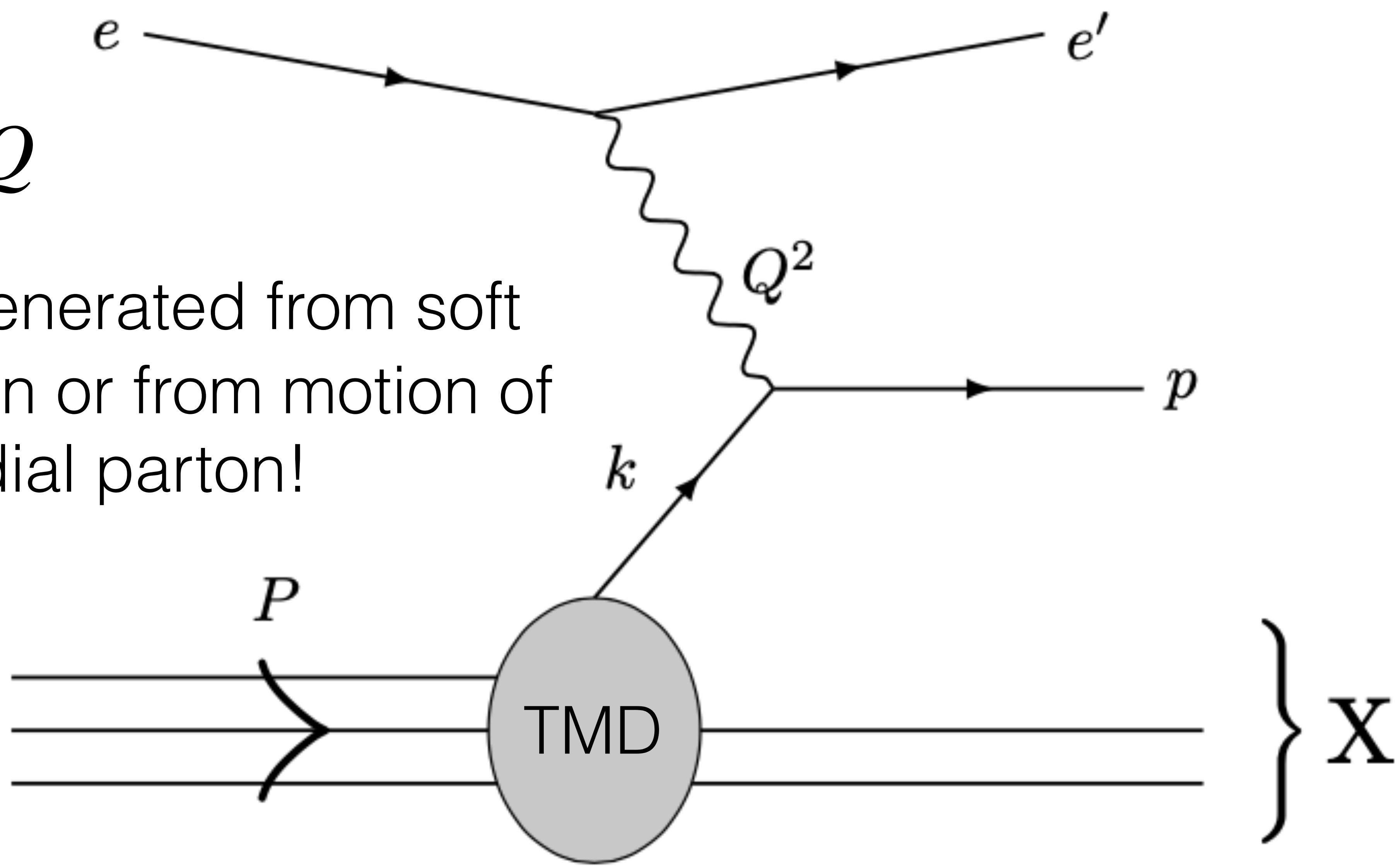
collinear description



Semi-inclusive DIS (SIDIS)

$p_\perp \ll Q$

$p_\perp$ is generated from soft radiation or from motion of primordial parton!



transverse momentum dependent (TMD) description

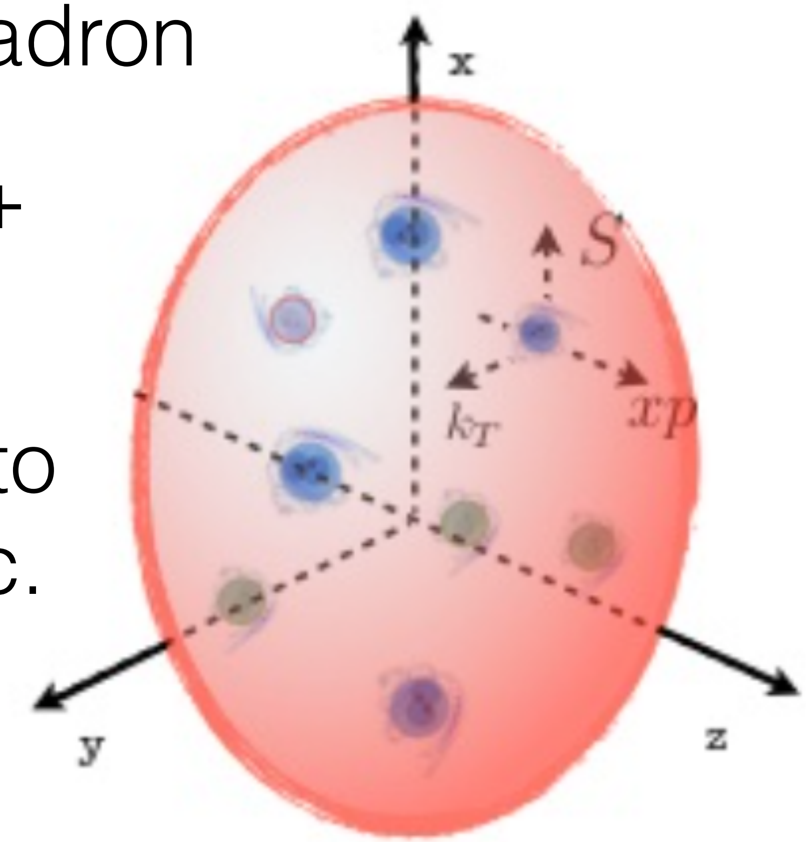
$$f(x, Q^2) \to f(x, k_\perp, Q^2)$$

Transverse momentum dependent PDFs (TMDs)

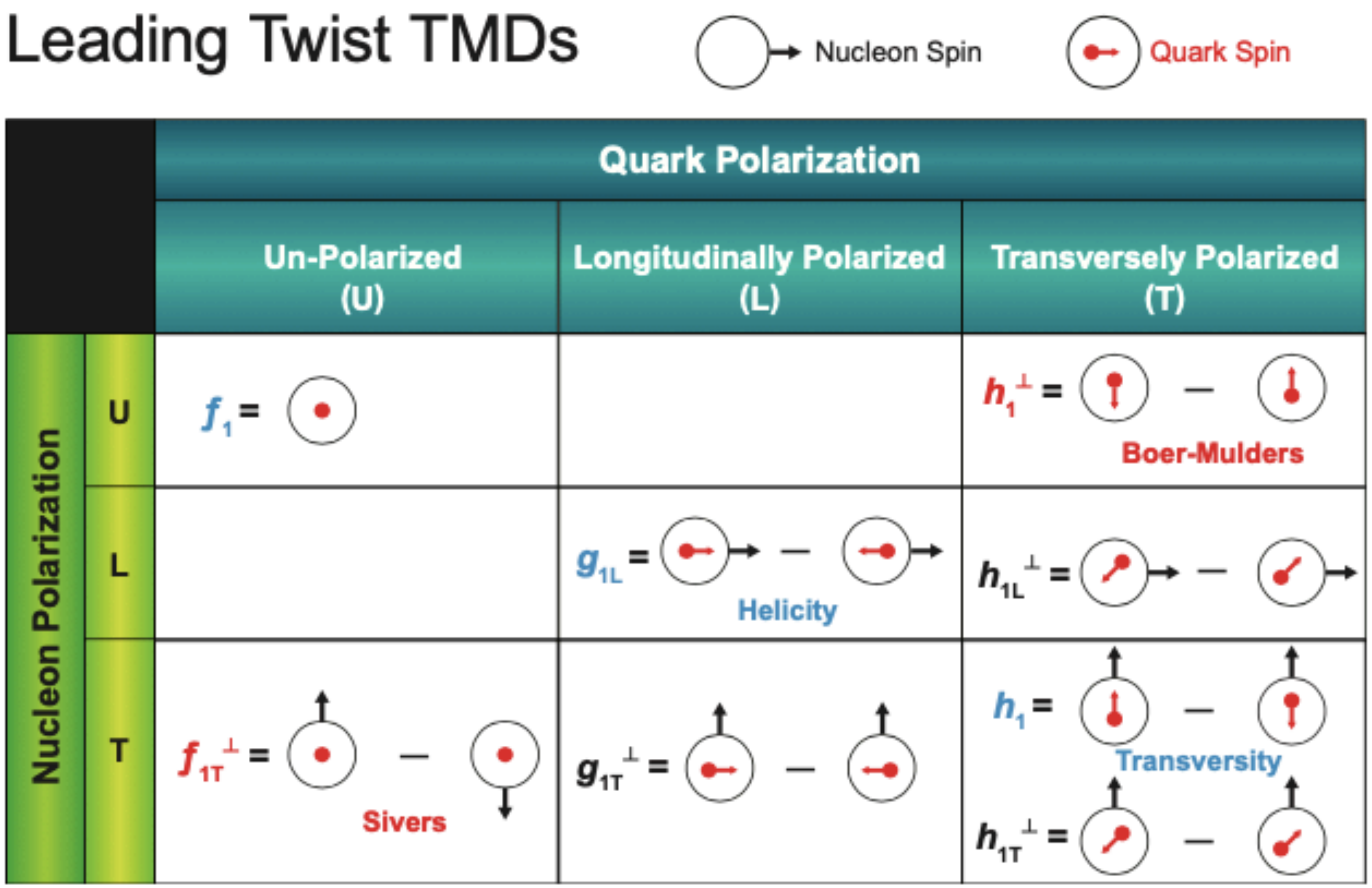
PDFs parametrize longitudinal structure of hadron

TMDs parametrize 3D momentum structure + spin correlations

Many intricacies: process dependence due to gauge invariance, complicated evolution, etc.



Leading Twist TMDs



Collins (2011)

Angelez-Martinez et al. (2015)

Gluon TMDs

Experimentally (almost) completely unknown!

GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g, $h_{1T}^{\perp g}$

Mulders & Rodrigues (2001)

Could influence the low $k_\perp$ spectrum of Higgs production in pp collisions

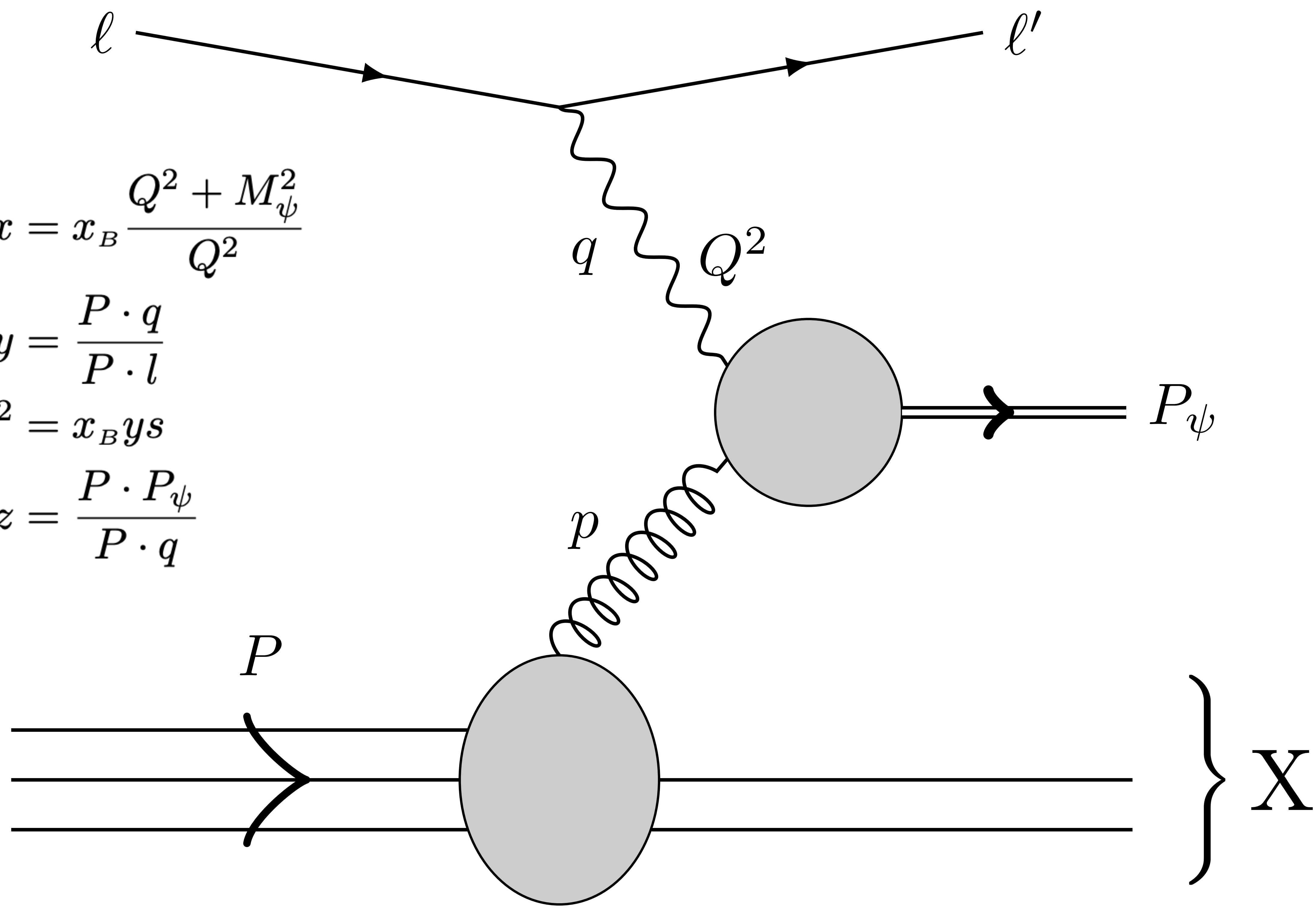
Boer et al. (2012), (2013)

Two important reactions to try to extract them:

1. Quarkonium production
2. Low-x

Gluon TMDs in J/ψ electroproduction

$$\ell + p \to \ell + J/\psi + X$$



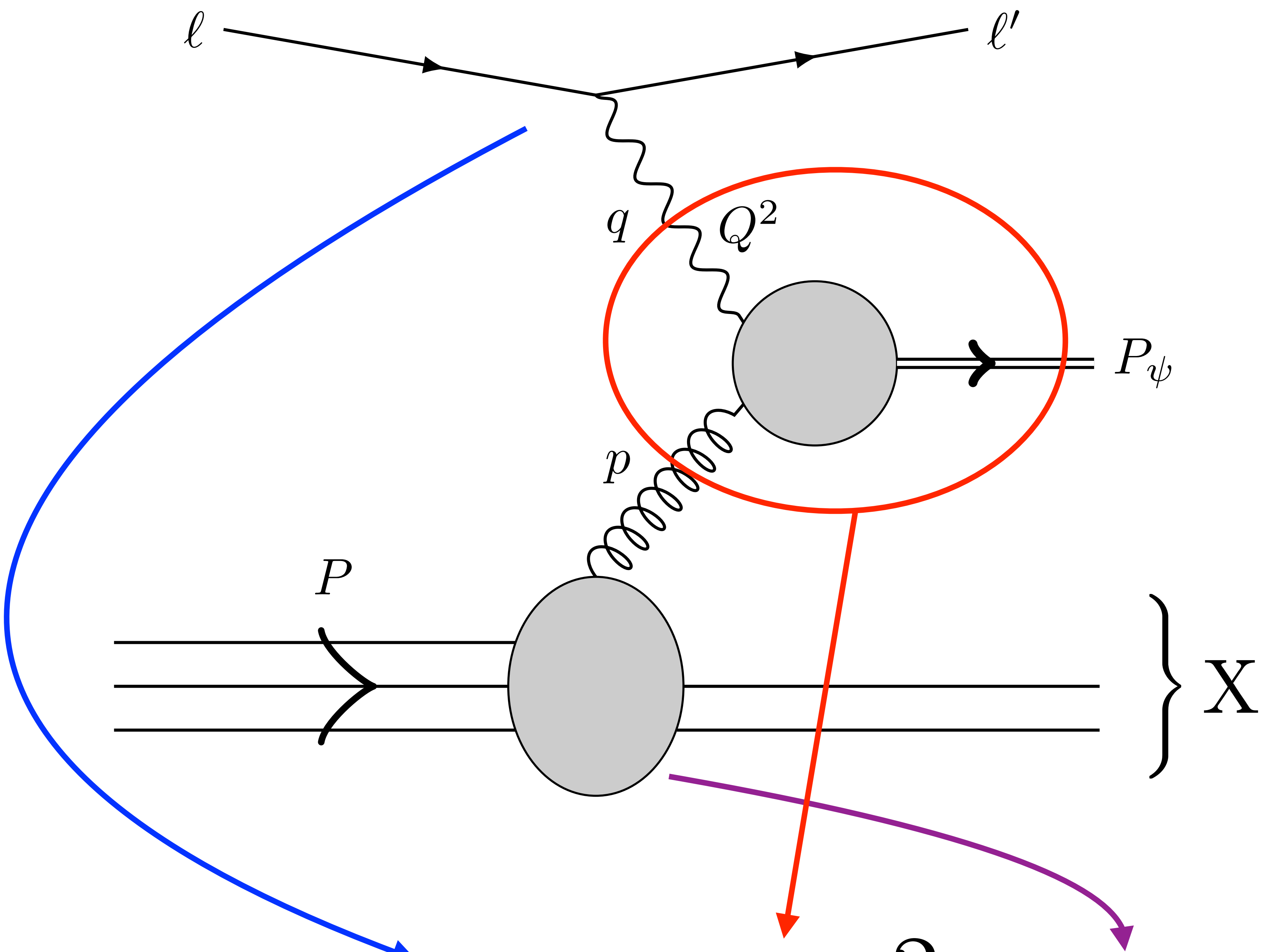
$$x = x_B \frac{Q^2 + M_\psi^2}{Q^2}$$

$$y = \frac{P \cdot q}{P \cdot l}$$

$$Q^2 = x_B y s$$

$$z = \frac{P \cdot P_\psi}{P \cdot q}$$

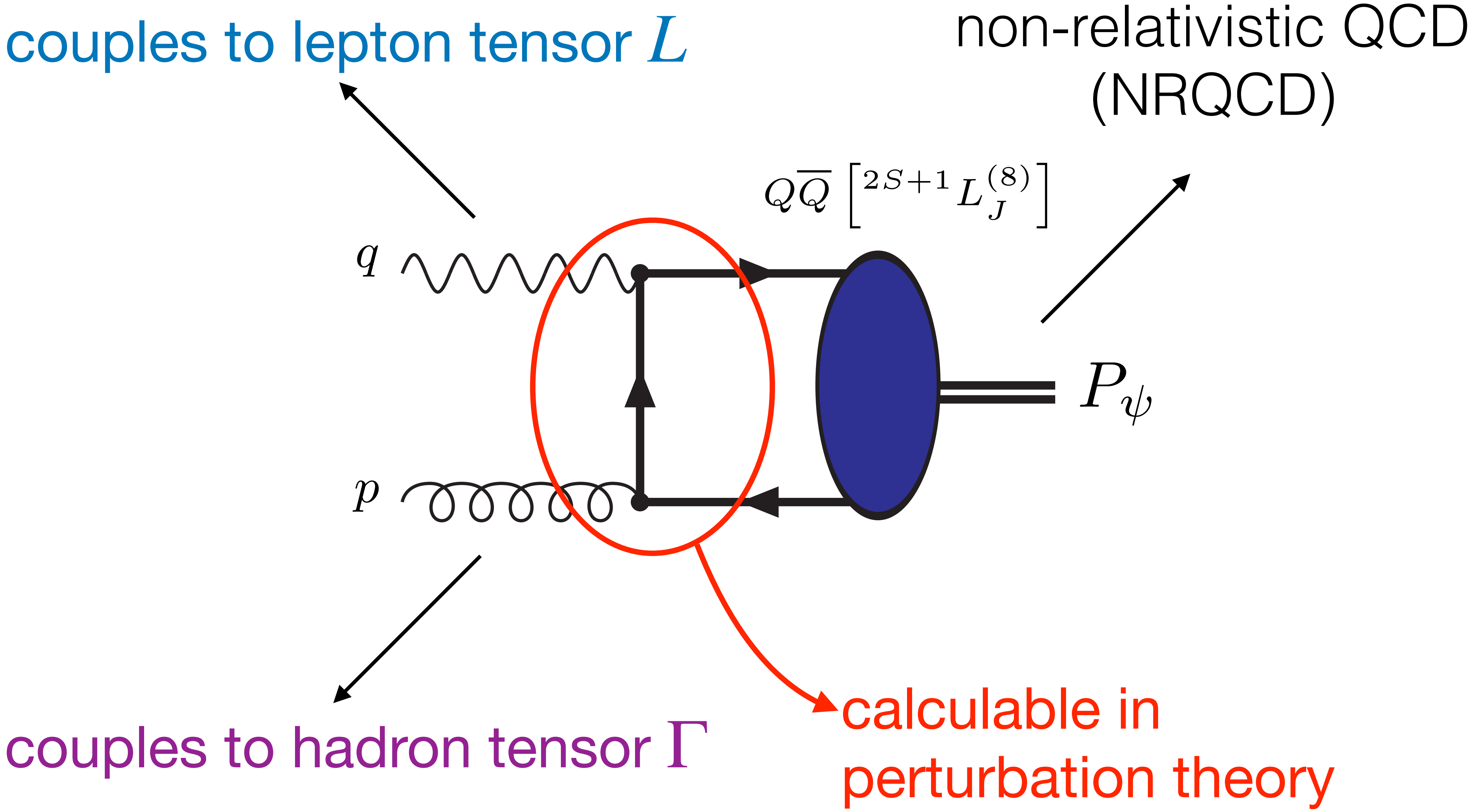
$$\ell + p \to \ell + J/\psi + X$$



$$\mathrm{d}\sigma = L \otimes |M|^2 \otimes \Gamma$$

Lepton tensor **'Hard' part** **Hadron tensor**

‘Hard’ part



couples to lepton tensor L
non-relativistic QCD (NRQCD)
$Q\overline{Q}\left[{}^{2S+1}L_J^{(8)}\right]$
q
p
P_ψ
couples to hadron tensor Γ
calculable in perturbation theory

Hadron tensor

$$\Gamma_U^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{x}{2}\left\{ -g_T^{\mu\nu}\, f_1^g(x,\boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M_p^2}\right) h_1^{\perp\, g}(x,\boldsymbol{p}_T^2)\right\}$$

unpolarized gluons (f_1^g)

linearly polarized gluons ($h_1^{\perp\, g}$)

Sivers function ($f_{1T}^{\perp g}$)

circularly polarized gluons (g_{1T}^g)

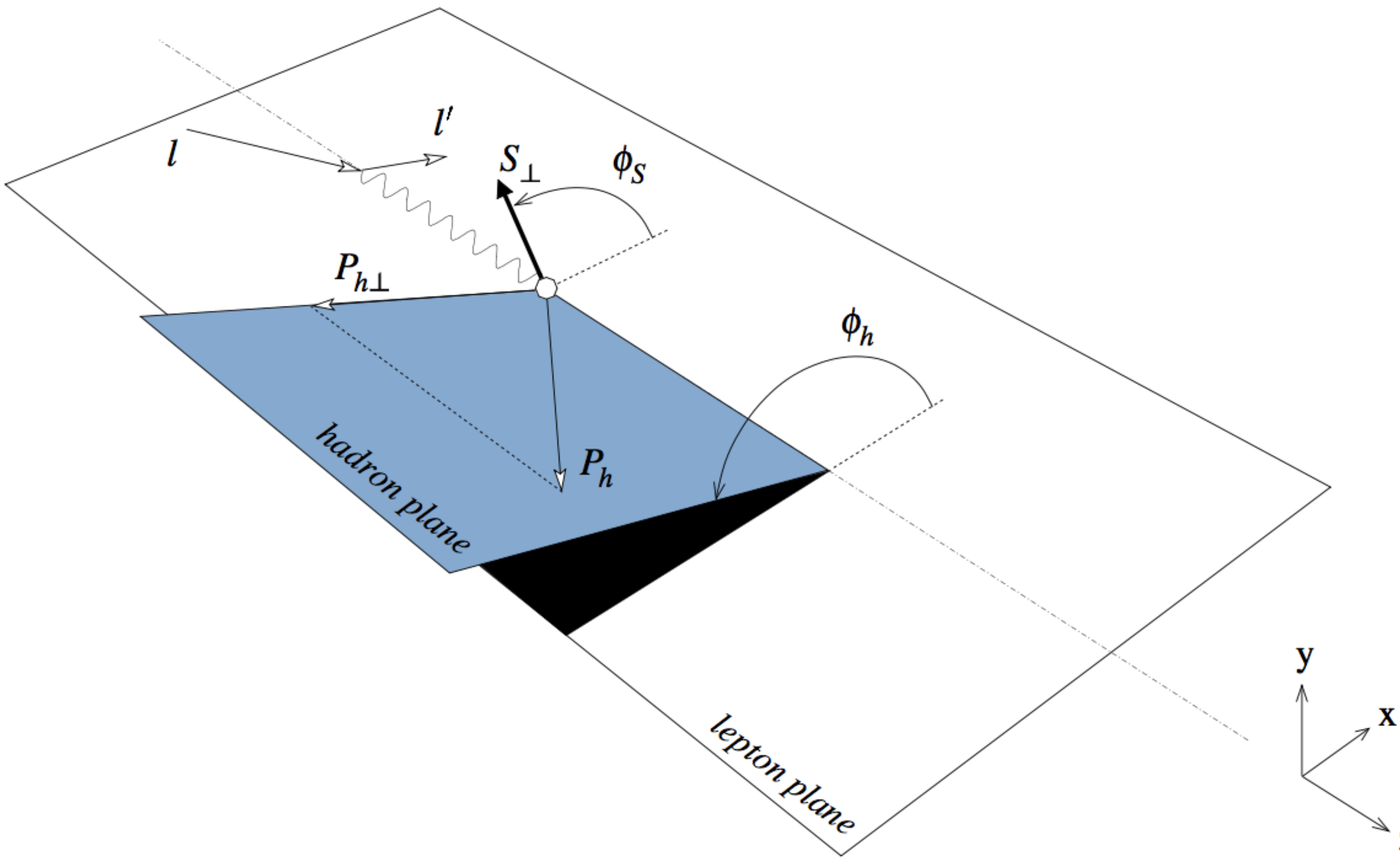
$$\Gamma_T^{\mu\nu}(x,\mathbf{p}_T^2) = \frac{x}{2}\Bigg\{ g_T^{\mu\nu}\frac{\epsilon_T^{\rho\sigma} p_{\perp\rho} S_{\perp\sigma}}{M_p} f_{1T}^{\perp g}(x,\mathbf{p}_T^2) + i\epsilon_T^{\mu\nu}\frac{p_\perp\cdot S_\perp}{M_p} g_{1T}^g(x,\mathbf{p}_T^2)$$

$$+ \frac{p_{\perp\rho}\epsilon_T^{\rho\{\mu} p_\perp^{\nu\}}}{2M_p^2}\frac{p_\perp\cdot S_\perp}{M_p} h_{1T}^{\perp g}(x,\mathbf{p}_T^2) - \frac{p_{\perp\rho}\epsilon_T^{\rho\{\mu} S_\perp^{\nu\}} + S_{\perp\rho}\epsilon_T^{\rho\{\mu} p_\perp^{\nu\}}}{4M_p} h_{1T}^g(x,\mathbf{p}_T^2)\Bigg\}$$

linearly polarized gluons ($h_{1T}^{\perp g}$, h_{1T}^g)

Mulders & Rodrigues (2001); Meissner, Metz & Goeke (2007)

Reference frame



Bacchetta, D'Alesio, Diehl & Miller (2004)

Cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}x_B\,\mathrm{d}^2\mathbf{q}_T} \equiv \mathrm{d}\sigma^U(\phi_T) + \mathrm{d}\sigma^T(\phi_T, \phi_S)$$

$$\mathrm{d}\sigma^U = \mathcal{N}\left[A^U f_1^g(x, \boldsymbol{q}_T^2) + \frac{\boldsymbol{q}_T^2}{M_p^2} B^U h_1^{\perp\, g}(x, \boldsymbol{q}_T^2)\cos 2\phi_T\right]$$

↓ unpolarized ↓ linearly polarized

Sivers function ↑

$$\mathrm{d}\sigma^T = \mathcal{N}|\mathbf{S}_T|\frac{|\mathbf{q}_T|}{M_p}\Bigg\{A^T f_{1T}^{\perp g}(x, \mathbf{q}_T^2)\sin(\phi_S - \phi_T)$$

$$+ B^T\left[h_1^g(x, \mathbf{q}_T^2)\sin(\phi_S + \phi_T) - \frac{\mathbf{q}_T^2}{2M_p^2} h_{1T}^{\perp g}(x, \mathbf{q}_T^2)\sin(\phi_S - 3\phi_T)\right]\Bigg\}$$

↘ linearly polarized ↙

Azimuthal asymmetries

probe *ratios* of gluon TMDs

$$A^{W(\phi_S,\phi_T)} \equiv 2\, \frac{\int \mathrm{d}\phi_S\, \mathrm{d}\phi_T\, W(\phi_S,\phi_T)\, \mathrm{d}\sigma(\phi_S,\, \phi_T)}{\int \mathrm{d}\phi_S\, \mathrm{d}\phi_T\, \mathrm{d}\sigma(\phi_S,\phi_T)}$$

…we have:

$$\int \mathrm{d}\phi_S\, \mathrm{d}\phi_T\, \mathrm{d}\sigma(\phi_S,\phi_T) = (2\pi)^2\, \mathcal{N}\, A^U\, f_1^g(x,\boldsymbol{q}_T^2)$$

$$A^{\cos 2\phi_T} = H(y,M_\psi,Q)\, \frac{\boldsymbol{q}_T^2}{M_p^2}\, \frac{h_1^{\perp\, g}(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

$$A^{\sin(\phi_S-\phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p}\, \frac{f_{1T}^{\perp\, g}(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

$$A^{\sin(\phi_S+\phi_T)} = H(y,M_\psi,Q)\, \frac{|\boldsymbol{q}_T|}{M_p}\, \frac{h_1^g(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

$$A^{\sin(\phi_S-3\phi_T)} = -H(y,M_\psi,Q)\, \frac{|\boldsymbol{q}_T|^3}{2M_p^3}\, \frac{h_{1T}^{\perp\, g}(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

Bachetta, Boer, Pisano, PT (2018)

Upper bounds

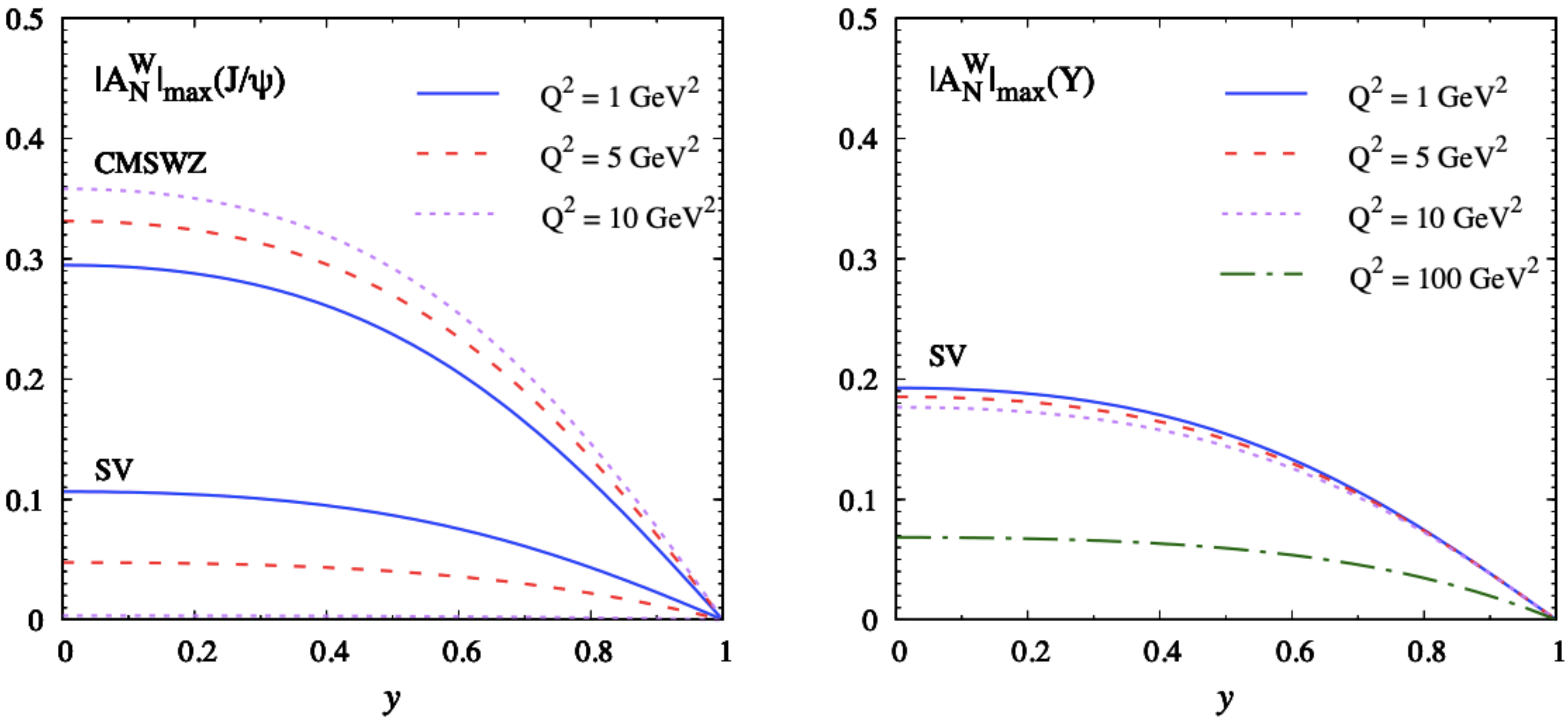
Polarized gluon TMDs satisfy the following positivity bounds:

$$\frac{|\boldsymbol{p}_T|}{M_p}\,|f_{1T}^{\perp\,g}(x,\boldsymbol{p}_T^2)| \le f_1^g(x,\boldsymbol{p}_T^2) \qquad \frac{|\boldsymbol{p}_T|}{M_p}\,|h_1^g(x,\boldsymbol{p}_T^2)| \le f_1^g(x,\boldsymbol{p}_T^2)$$

$$\frac{\boldsymbol{p}_T^2}{2M_p^2}\,|h_1^{\perp\,g}(x,\boldsymbol{p}_T^2)| \le f_1^g(x,\boldsymbol{p}_T^2) \qquad \frac{|\boldsymbol{p}_T|^3}{2M_p^3}\,|h_{1T}^{\perp\,g}(x,\boldsymbol{p}_T^2)| \le f_1^g(x,\boldsymbol{p}_T^2)$$

We can use the maximum allowed values of the gluon TMDs to illustrate the sensitivity of our inclusive charmonium electroproduction to the gluon content of the proton:

Upper bounds

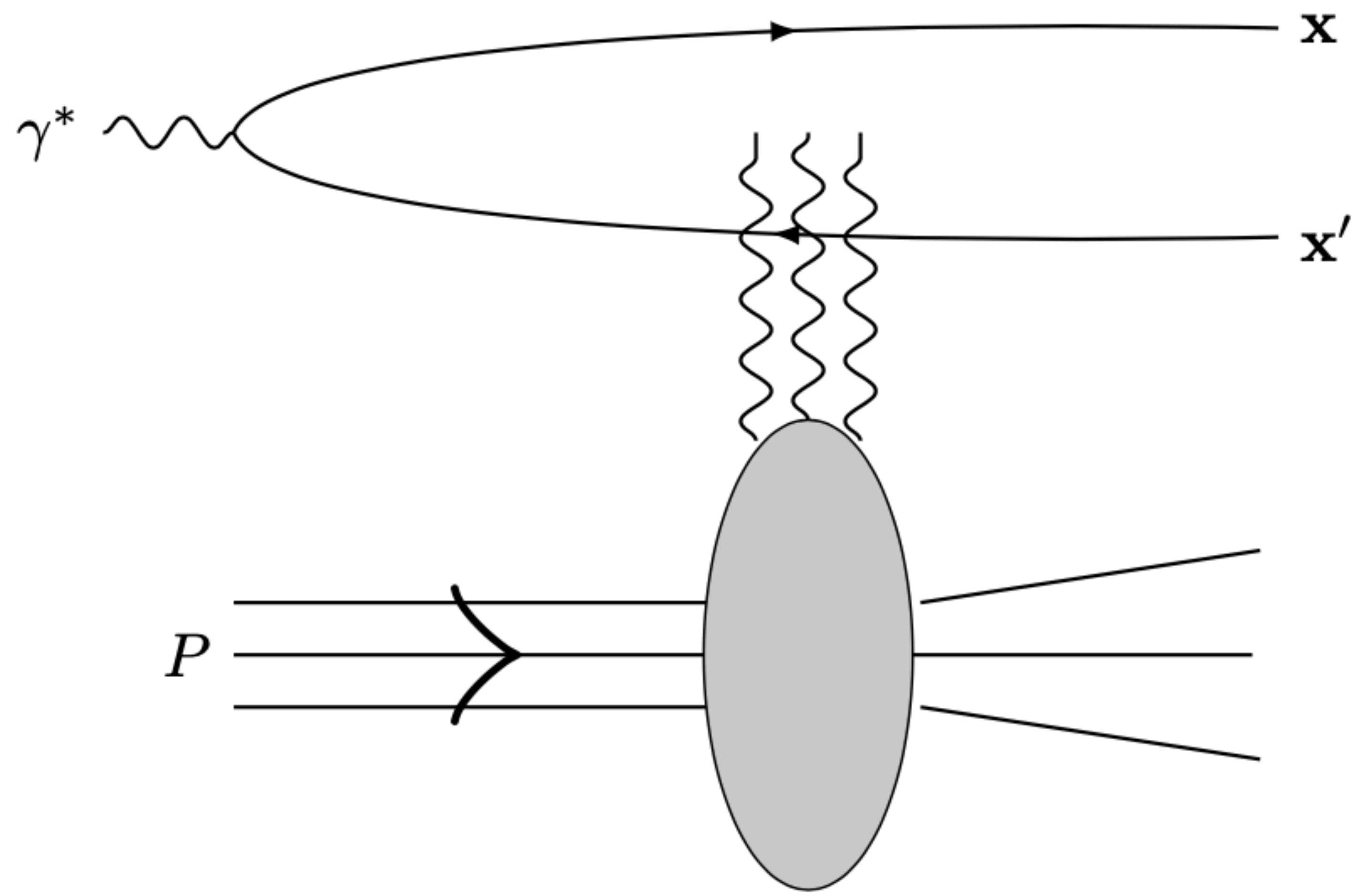


Bachetta, Boer, Pisano, PT (2018)

Large theoretical uncertainty due to nonperturbative parts of the bounding of the heavy-quark pair into charmonium

Gluon TMDs at low x

Dijet production at low x



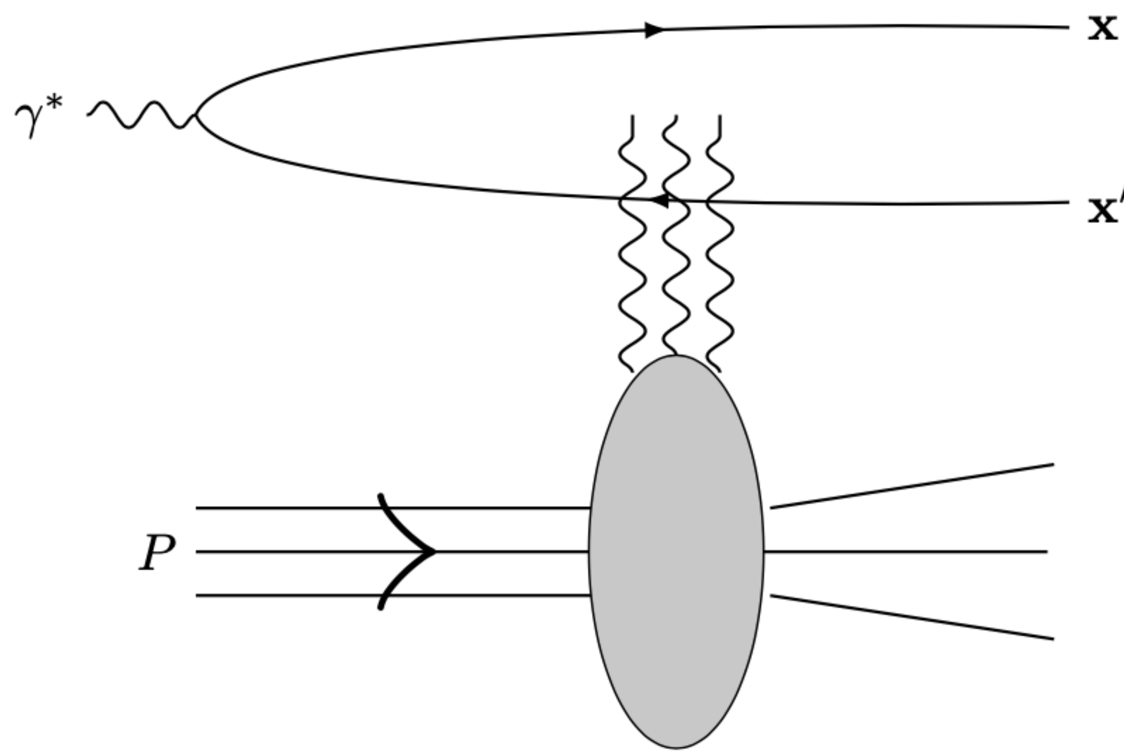
In *dipole frame*, virtual photon dissociates in a quark-antiquark pair long before the scattering off a highly boosted proton

Semiclassical approximation: frozen quark-antiquark dipole interacts with dense classical gluon fields through Wilson lines: $U(\mathbf{x}) = \mathscr{P} e^{i g_s \int \mathrm{d}x^+ A^-(x^+, \mathbf{x})}$

Color Glass Condensate: nonperturbative model + perturbative evolution

McLerran, Venugopalan, Balitsky, Kovchegov, Jalilian-Marian, Kovner, Leonidov, Weigert (1994-2002)

Dijet production at low x



perturbative splitting functions

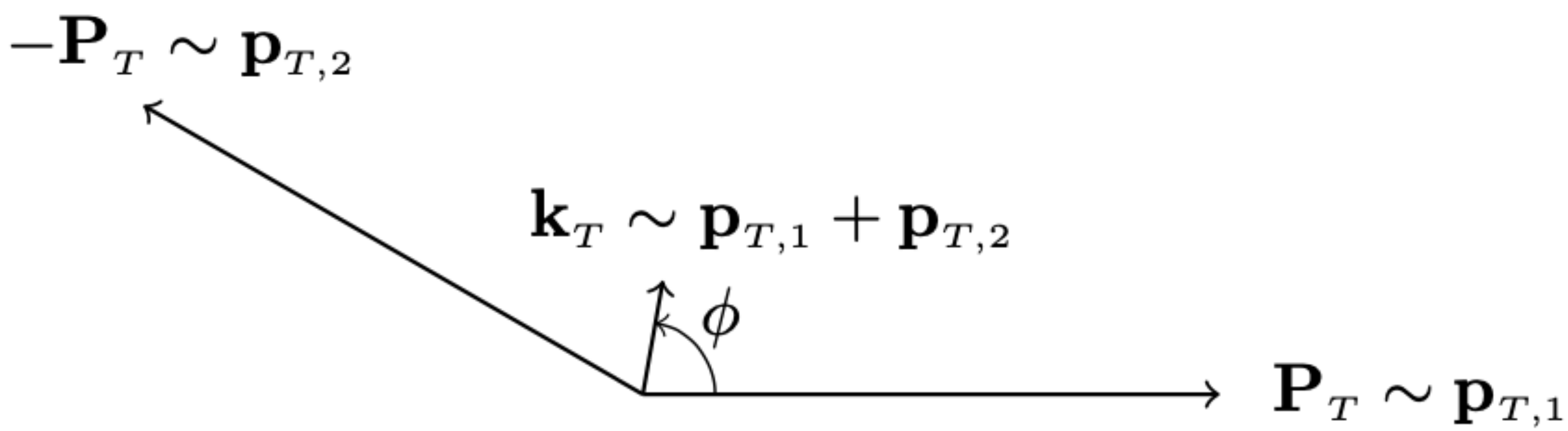
$$\frac{\mathrm{d}\sigma^{\gamma^* p\to q\bar{q}+X}}{\mathrm{d}y_1\mathrm{d}y_2\mathrm{d}^2p_{1T}\mathrm{d}^2p_{2T}} = N_c\alpha_{em}e_q^2 z(1-z)\,\delta\left(1-\frac{p_1^+ + p_2^+}{p^+}\right)\int\frac{\mathrm{d}^2\mathbf{u}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{u}'}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{v}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{v}'}{(2\pi)^2}$$

$$\times\, e^{-i\mathbf{k}_T\cdot(\mathbf{v}-\mathbf{v}')}e^{-i\mathbf{P}_T\cdot(\mathbf{u}-\mathbf{u}')}p^+\sum_{\lambda\alpha\beta}\psi_{\alpha\beta}^{L,T\lambda}(\mathbf{u})\,\psi_{\alpha\beta}^{L,T\lambda*}(\mathbf{u}')$$

$$\times\left[1+\langle Q(\mathbf{x},\mathbf{x}',\mathbf{b}',\mathbf{b})\rangle_x - \langle D(\mathbf{x},\mathbf{b})\rangle_x - \langle D(\mathbf{x}',\mathbf{b}')\rangle_x\right]$$

eikonal interaction of dipole with semiclassical gluon fields through Wilson lines

Dijet production at low x: correlation limit



In the transverse back-to-back regime, one recovers two ordered scales: $k_T \ll P_T \sim Q$

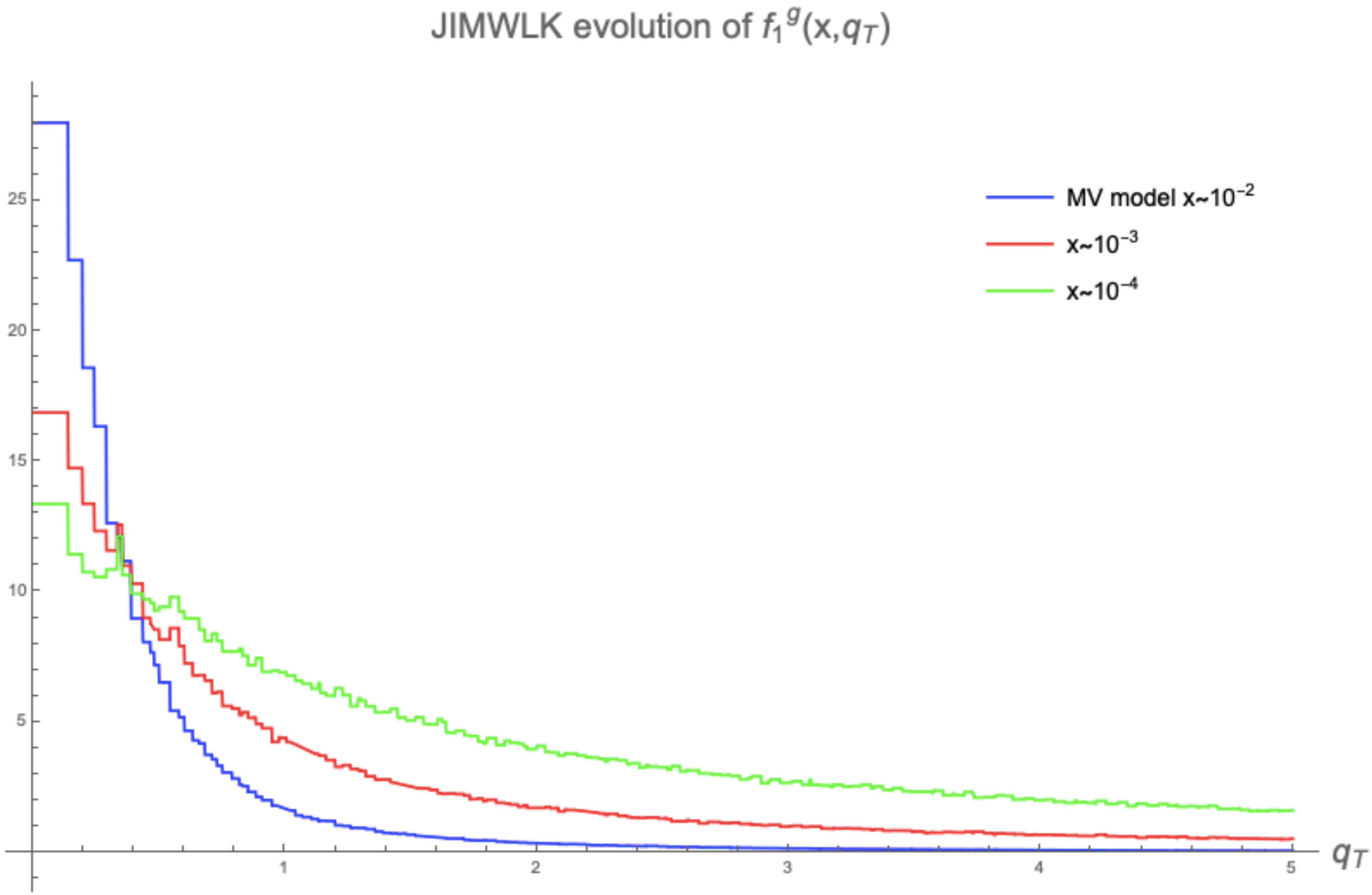
$$(2\pi)^6 \frac{\mathrm{d}\sigma^{\gamma^* p \to q\bar{q}+X}}{\mathrm{d}^3\vec{p}_1 \mathrm{d}^3\vec{p}_2}\bigg|_{\text{corr. limit}} = 2\pi\delta\left(p^+ - p_1^+ - p_2^+\right)\, \left[\mathrm{H}\right]_{ij}$$

$$\times\left[\frac{1}{2}\delta^{ij} f_{1,WW}^{g}(x,\mathbf{k}_T) + \frac{1}{2}\left(2\frac{k_T^i k_T^j}{\mathbf{k}_T^2} - \delta^{ij}\right)\frac{\mathbf{k}_T^2}{2M_p^2} h_{1,WW}^{g\perp}(x,\mathbf{k}_T)\right]$$

TMD-factorized expression is recovered!

Dominguez, Marquet, Xiao, Yuan (2011)

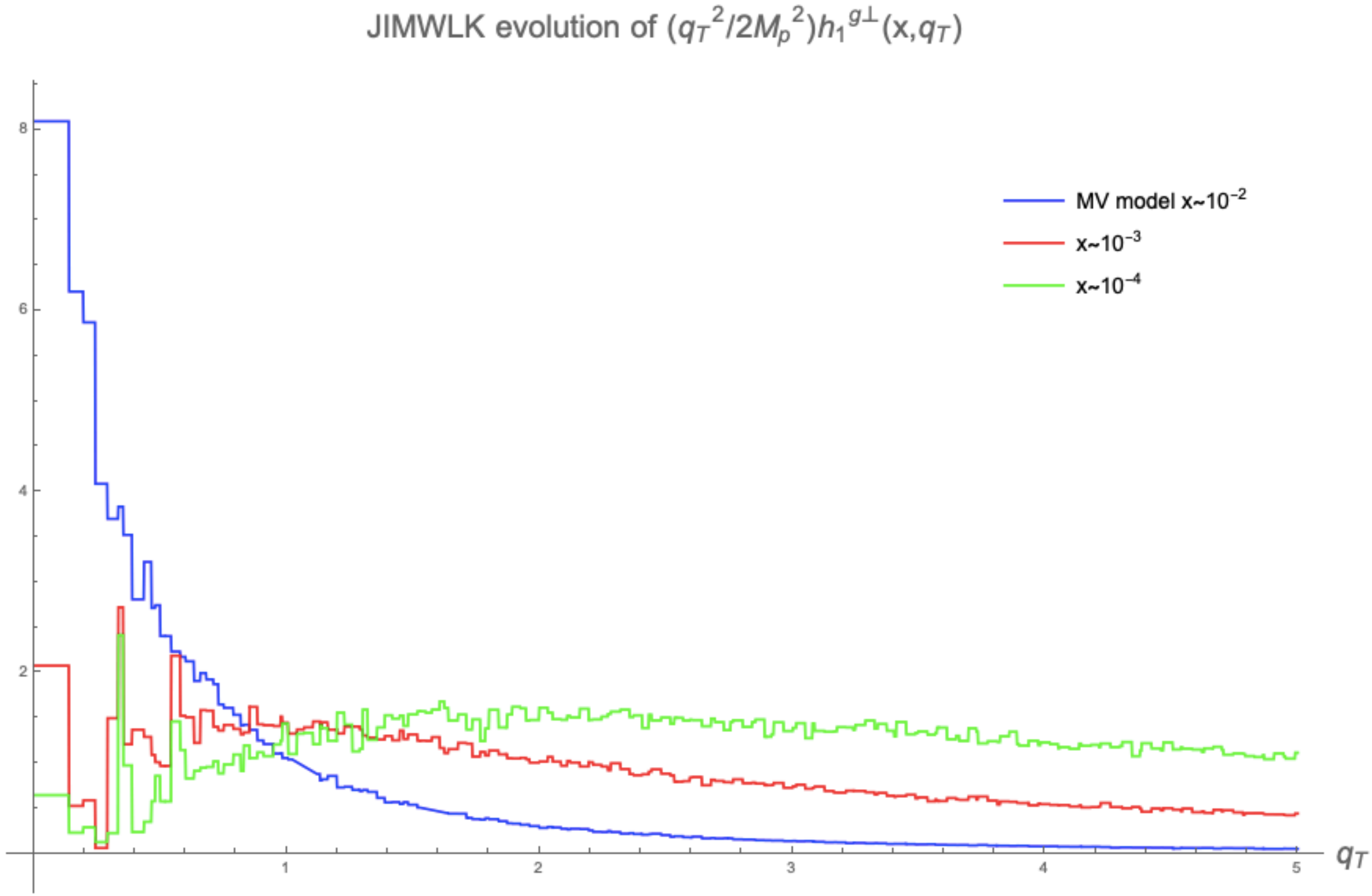
JIMWLK evolution of f_1^g and $h_1^{g\perp}$



Langevin formulation of JIMWLK: Weigert (2002), Rummukainen; Weigert (2004); Lappi (2008)

Marquet, Petreska, Roiesnel (2016); Marquet, Roiesnel, PT (2018)

JIMWLK evolution of f_1^g and $h_1^{g\perp}$



Conclusions & outlook

Conclusions & outlook

Gluon TMDs provide invaluable insight in three-dimensional proton structure

Leptoproduction of J/ψ (+jet) at the Electron-Ion Collider seems a very promising process to probe gluon TMDs

Largest source of theoretical uncertainty are the nonperturbative parameters of NRQCD binding mechanism (Large-Distance Matrix Elements)

For a real accurate extraction, we need a dedicated fit at low k_T of the shape functions (= TMD generalization of LDMEs)

Conclusions & outlook

At low x, CGC-TMD correspondence is pushed further, NLO calculation in progress

If Electron-Ion Collider doesn't reach small enough x, we could consider ultra peripheral collisions at the LHC

Many interesting pp and pA studies, experimental feasibility remains to be seen

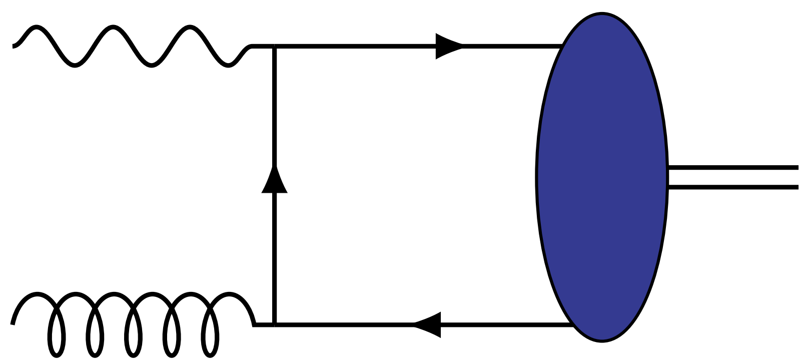
Thanks to the organizers,
thanks for your attention!

Backup slides

Non-relativistic QCD (NRQCD)

Color octet (CO) mechanism: heavy-quark pair is produced with *all allowed quantum numbers* and then hadronizes as encoded in the non-perturbative long-distance matrix elements (LDMEs)

$$\langle|\mathcal{O}_8^{J/\psi}(^3P_J)|0\rangle \qquad \langle|\mathcal{O}_8^{J/\psi}(^1S_0)|0\rangle$$



$Q\bar{Q}[^{2S+1}L_J^{(8)}]$ $\quad \mathcal{O}(\alpha_{\rm em}\alpha_{\rm s})$

NRQCD also encompasses color singlet (CS) mechanism where J/ψ is directly produced with correct quantum numbers

At least for leptoproduction, CO mechanism is the dominant one in the low-$P_{\psi\perp}$ regime Fleming and Mehen (1998)