Probing the flavor of New Physics with dipoles

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Rencontres de Physique Théorique

Outline

1. Introduction
2. Dim.-6 corrections to Yukawa couplings
3. Four-fermion operators
4. Conclusions

Outline

Radiative processes

$\rightarrow$ Low-energy dipole ops.: $\mathcal{L}_{dipole} = e\frac{v_{\rm EW}}{\sqrt{2}}\mathcal{C}^{\beta\alpha}_{\psi\gamma}\bar{\psi}_\beta\sigma^{\mu\nu}P_R\psi_\alpha F_{\mu\nu} + \text{h.c.}$

E.M. form factors: *Magnetic Dipole Moment* (MDM), *Electric Dipole Moment* (EDM), etc.

Flavor transitions: $\mu \to e\gamma$, $\tau \to (e,\mu)\gamma$, $\nu' \to \nu\gamma$, $s \to d\gamma$, $b \to (s,d)\gamma$, etc.

$\rightarrow$ Multitask tool: **structure of flavor, sources of CP violation** of the SM and beyond, in both quark and lepton sectors

$\rightarrow$ NP flavour structure:

eEDM: $|\mathrm{Im}[\mathcal{C}^{ee}_{e\gamma}]| \lesssim (8 \times 10^5\ \mathrm{TeV})^{-2}$ [ACME]

$\mu \to e\gamma$: $\sqrt{|\mathcal{C}^{e\mu}_{e\gamma}|^2 + |\mathcal{C}^{\mu e}_{e\gamma}|^2} \lesssim (4 \times 10^4\ \mathrm{TeV})^{-2}$ [MEG]

nEDM: $\left|\mathrm{Im}[\mathcal{C}^{dd}_{d\gamma}]\right|, \left|\mathrm{Im}[\mathcal{C}^{uu}_{u\gamma}]\right| \lesssim (2 \times 10^4\ \mathrm{TeV})^{-2}$ [PRD92, 092003 (2015)]

SMEFT way: NP sector much above EW scale

→ Persistent absence of experimental evidence for non-SM particles below the EW scale

→ Generic NP involving new heavy d.o.f. $\sim \Lambda \gg v_{\rm EW}$ lead to higher dimensional operators

$$\frac{C^{(5)} \times Q^{(5)}}{\Lambda}, \; \frac{C^{(6)} \times Q^{(6)}}{\Lambda^2}, \; \frac{C^{(7)} \times Q^{(7)}}{\Lambda^3}, \; \frac{C^{(8)} \times Q^{(8)}}{\Lambda^4}, \text{ etc.}$$

→ Consider operators $Q^{(n)}$ respecting SM local symmetries and containing SM d.o.f. only

→ Non-SM interactions $C^{(n)}$ among the d.o.f. that we know

→ New weak sector: typically effects from lower-dimensionality operators are more important for low-energy observables

Basis of dimension-six operators

$\rightarrow$ Focus on operators of **dimension-six**

$\rightarrow$ Equations Of Motion (EOMs) eliminate redundant cases:
59 linearly independent operators,
with 1350 CP-even + 1149 CP-odd couplings,
assuming SM global symmetries, $B_{\rm tot}$ and $L_{\rm tot}$

Warsaw: X^3, H^6, H^4D^2, ψ^2H^3, X^2H^2, ψ^2XH, ψ^2H^2D, ψ^4

[ψ fermions; D cov. derivative; X field strengths]

[Buchmüller, Wyler '86; Grzadkowski, Iskrzyński, Misiak, Rosiek '10]

$\underbrace{\psi^2XH \text{ class}}_{\mathcal{L}_{dipole} \text{ @ tree}}$: $(\bar{q}\sigma^{\mu\nu}d)HB_{\mu\nu}$, $(\bar{q}\sigma^{\mu\nu}d)\tau^IHW^I_{\mu\nu}$, $(\bar{q}\sigma^{\mu\nu}T^Ad)HG^A_{\mu\nu}$, etc.

[q (d) $SU(2)$ doublet (singlet)]

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[Buchmüller, Wyler '86; Grzadkowski, Iskrzyński, Misiak, Rosiek '10]

ψ^2H^3 class: $(H^\dagger H)(\bar{q}dH)$, $(H^\dagger H)(\bar{q}u\tilde{H})$, $(H^\dagger H)(\bar{\ell}eH)$

[q, ℓ (d, u, e) $SU(2)$ doublets (singlets)]

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[ψ fermions; D cov. derivative; X field strengths]

[Buchmüller, Wyler '86; Grzadkowski, Iskrzyński, Misiak, Rosiek '10]

ψ^4 class: $(\bar{q}\gamma_\mu q)(\bar{q}\gamma^\mu q)$, $(\bar{d}\gamma_\mu d)(\bar{d}\gamma^\mu d)$, $(\bar{q}\gamma_\mu T^A q)(\bar{d}\gamma^\mu T^A d)$, etc.

[q (d) $SU(2)$ doublet (singlet)]

Probing **non-dipole** operators

Here, $\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_i C_i Q_i$, C_i scales as Λ^{-2}

Mixing with dipole:

$$16\pi^2 \frac{d}{d\ell n(\mu)} C_{\psi^2 XH}(\mu) = \Sigma_i (C_{\psi^2 XH}, C_{\psi^4}, C_{X^3}, C_{X^2H^2})_i(\mu) \gamma^{(1\text{-loop})}_{i,\psi^2 XH}$$

$$\{\psi^2 XH, \psi^4, X^3, X^2H^2\} \xrightarrow[1Loop]{RGE} \psi^2 XH$$

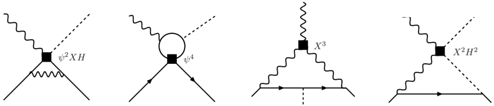
[1-loop: Alonso, Jenkins, Manohar, Trott '13]

[Pruna, Signer '14; Davidson '16; Crivellin, Davidson, Pruna, Signer '17]

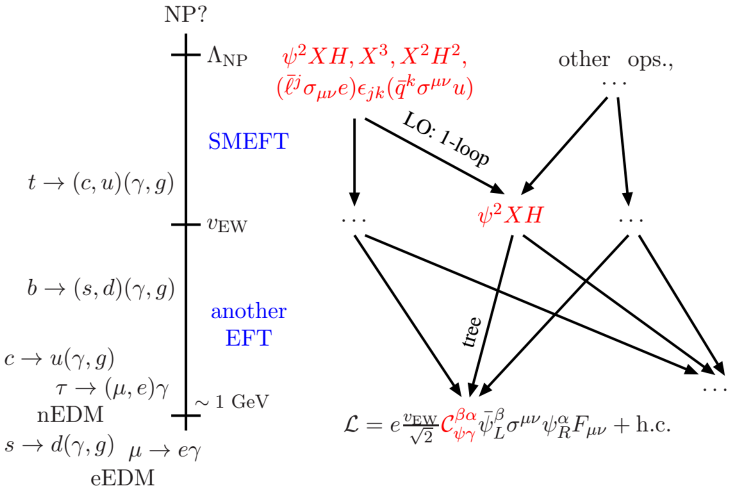
Ex. of bound: [ACME]

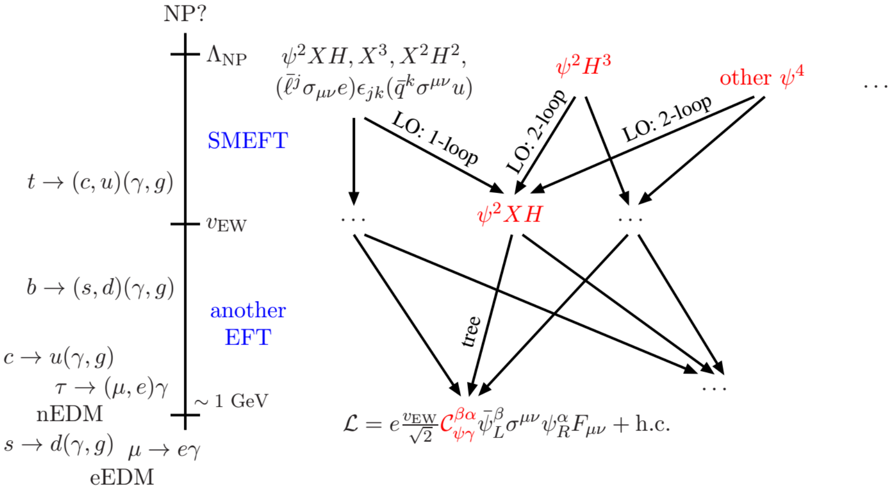
$$\left|{\rm Im}\, C^{(3),eett}_{\ell equ}\right| \lesssim (3 \times 10^5\ {\rm TeV})^{-2}$$

[■: possible vertices]

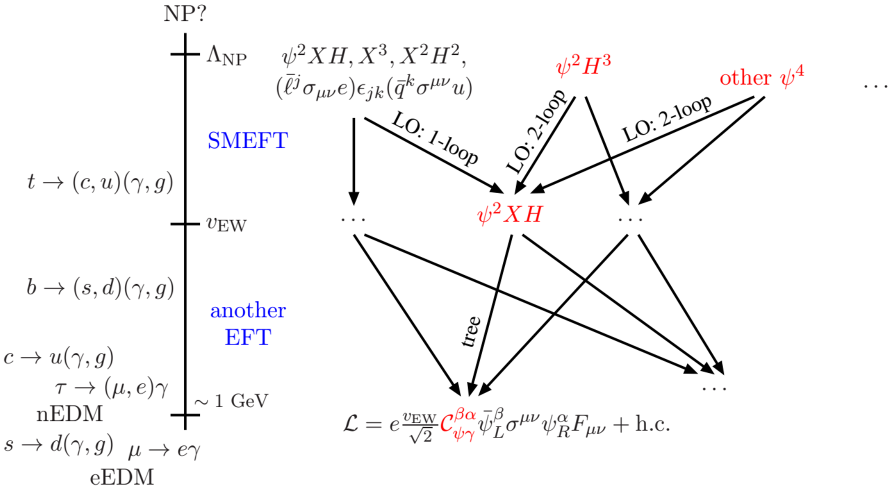


Roadmap to phenomenology





NP?
$\Lambda_{\rm NP}$
$\psi^2 XH, X^3, X^2H^2,$
$(\bar{\ell}^j\sigma_{\mu\nu}e)\epsilon_{jk}(\bar{q}^k\sigma^{\mu\nu}u)$
$\psi^2 H^3$
other ψ^4
SMEFT
LO: 1-loop
LO: 2-loop
LO: 2-loop
$t \to (c,u)(\gamma,g)$
$v_{\rm EW}$
$\psi^2 XH$
$b \to (s,d)(\gamma,g)$
another EFT
tree
$c \to u(\gamma,g)$
$\tau \to (\mu,e)\gamma$
nEDM
~ 1 GeV
$\mathcal{L} = e\frac{v_{\rm EW}}{\sqrt{2}} C^{\beta\alpha}_{\psi\gamma} \bar{\psi}^\beta_L \sigma^{\mu\nu} \psi^\alpha_R F_{\mu\nu} + \text{h.c.}$
$s \to d(\gamma,g)$
$\mu \to e\gamma$
eEDM



HERE: Operators for which $\gamma^{(\text{1-loop})}_{i,\psi^2XH}=0$ (i.e., no mixing at 1-loop)

$\rightarrow$ Leading Order mixing with the dipole arriving at 2-loops

$\rightarrow$ How flavor in other operators feed into dipole operators

Outline

Renormalization of $\psi^2 H^3$ operators

→ 1-loop: $\psi^2 H^3$ mix only into $\psi^2 H$ (dim.=4), & $H^6, \psi^2 H^3$ (dim.=6)

→ Mixing with the dipole: LO at 2-loops

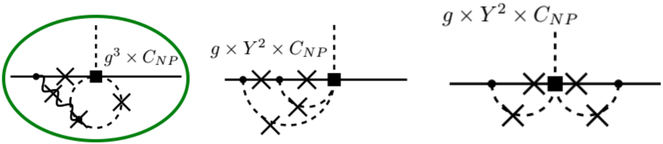
$$\psi^2 H^3 \xrightarrow[1Loop]{RGE} \{\psi^2 H; H^6, \psi^2 H^3\}$$

Renormalization of $\psi^2 H^3$ operators

$\rightarrow$ 1-loop: $\psi^2 H^3$ mix only into $\psi^2 H$ (dim.=4), & $H^6, \psi^2 H^3$ (dim.=6)

$\rightarrow$ Mixing with the dipole: LO at 2-loops $\quad \psi^2 H^3 \xrightarrow[1Loop]{RGE} \{\psi^2 H; H^6, \psi^2 H^3\}$

Ex. of diagrams necessary to determine the mixing with dipoles:



[×: gauge boson]

Neglecting Yukawa couplings: $Z_{\psi^2 H^3, \psi^2 XH} \overset{\overset{(-)}{MS}}{=} \underset{1/\varepsilon}{\text{Coef.}}[\mathcal{G}_{\psi^2 H^3}(\psi^2 AH), \not{k}\,\not{\epsilon}(k)]$

[Kinetic basis: $p \cdot \epsilon(k)$, $p' \cdot \epsilon(k)$, $k \cdot \epsilon(k)$, $\not{p}\,\not{\epsilon}(k)$, $\not{p}'\,\not{\epsilon}(k)$, $\not{k}\,\not{\epsilon}(k)$]

Anomalous Dimension Matrix elements

Dipole: $g_X(\bar{\psi}_L \sigma^{\mu\nu} t^I \xi_R \varphi) X^I_{\mu\nu}$, $(q_L, u_R), (q_L, d_R), (\ell_L, e_R), (\ell_L, \nu_R)$

$$(16\pi^2)^2 \frac{d}{d\ell n(\mu)} C_{\psi^2 XH}(\mu) = \left(g_Y^2 \gamma_Y^X + g_L^2 \gamma_L^X + g_C^2 \gamma_c^X + Y^2 \gamma^X \right) C_{\psi^2 H^3}(\mu)$$

	$X^{\mu\nu} = B^{\mu\nu}$	$X^{\mu\nu} = W^{\mu\nu}$	$X^{\mu\nu} = G^{\mu\nu}$
γ_Y^X	$3 Q_\varphi^Y Q_\varphi^Y (Q_L^Y + Q_R^Y)$	$\frac{1}{2} Q_\varphi^Y (Q_L^Y + Q_R^Y)$	0
γ_L^X	$\frac{3}{4} Q_\varphi^Y$	$\frac{3}{8}$	0
γ_c^X	0	0	0

$Q_\phi^Y = -Q_{\tilde{\phi}}^Y = 1/2$; $Q_{\ell_L}^Y = -1/2$, $Q_{e_R}^Y = -1$, $Q_{\nu_R}^Y = 0$; $Q_{q_L}^Y = 1/6$, $Q_{u_R}^Y = +2/3$, $Q_{d_R}^Y = -1/3$

γ^X: cases involving dim.=4 Yukawas Y given in the spare slides

Checks: arbitrary Feynman gauge $\surd$

relations among Green's functions $\surd$

Also, results agree with γ_Y^X, γ_L^X discussed by [Panico, Pomarol, Riembau '18]

Running of dipole Wilson coef.

$$\mathcal{L}_{dipole} = e\frac{v_{\rm EW}}{\sqrt{2}}\mathcal{C}^{\beta\alpha}_{\psi\gamma}\bar{\psi}_\beta\sigma^{\mu\nu}P_R\psi_\alpha F_{\mu\nu} + {\rm h.c.}$$

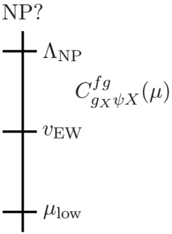
$$\mathcal{C}_{e\gamma} = C_{g_Y eB} - C_{g_L eW}\,, \qquad \mathcal{C}_{d\gamma} = C_{g_Y dB} - C_{g_L dW}$$
$$\mathcal{C}_{\nu\gamma} = C_{g_Y \nu B} + C_{g_L \nu W}\,, \qquad \mathcal{C}_{u\gamma} = C_{g_Y uB} + C_{g_L uW}$$

[ex. pheno: Crivellin, Najjari, Rosiek '13; Panico, Pomarol, Riembau '18]

→ Consider the pheno. of $C^{fg}_{\psi H}(\Lambda) \neq 0$

→ **Solution of the RGE**:

$$C^{fg}_{g_X\psi X}(\mu) = C^{fg}_{g_X\psi X}(\Lambda)$$
$$-\ell n\left(\frac{\Lambda}{\mu}\right) \times C^{fg}_{\psi H}(\Lambda) \times \left(\frac{g_Y^2}{(4\pi)^4}\gamma_Y^X + \frac{g_L^2}{(4\pi)^4}\gamma_L^X\right)$$
$$+(\text{sub-leading terms})$$



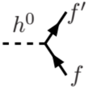
Tree-level constraints on $\psi^2 H^3$

$\psi^2 H^3$ changes couplings of the physical Higgs h:

$$\mathcal{L} = -\bar{u}' M_u u' - h \bar{u}' \mathcal{Y}_u u' + \ldots$$

$$[M_\psi]_{ij} \simeq \frac{v_{\rm EW}}{\sqrt{2}} \left([Y_\psi]_{ij} - \frac{1}{2} v_{\rm EW}^2 [C^\dagger_{\psi H}]_{ij} \right) ,$$

$$[\mathcal{Y}_\psi]_{ij} \simeq \frac{1}{v_{\rm EW}} [M_\psi]_{ij} - \frac{v_{\rm EW}^2}{\sqrt{2}} [C^\dagger_{\psi H}]_{ij} , \quad \psi = u, d, e$$



$\rightarrow$ Possible tuning if $\tilde{Y}_\psi \sim \frac{1}{2} v_{\rm EW}^2 \tilde{C}^\dagger_{\psi H} \gg \tilde{M}_\psi$

$$\mathcal{B}(h \to e\mu) \Rightarrow \sqrt{|\tilde{C}^{e\mu}_{eH}|^2 + |\tilde{C}^{\mu e}_{eH}|^2} < 5.2 \times 10^{-3} \ {\rm TeV}^{-2}$$

$$\mathcal{B}(h \to e\tau) \Rightarrow \sqrt{|\tilde{C}^{e\tau}_{eH}|^2 + |\tilde{C}^{\tau e}_{eH}|^2} < 4.6 \times 10^{-2} \ {\rm TeV}^{-2}$$

$$\mathcal{B}(h \to \mu\tau) \Rightarrow \sqrt{|\tilde{C}^{\mu\tau}_{eH}|^2 + |\tilde{C}^{\tau\mu}_{eH}|^2} < 3.6 \times 10^{-2} \ {\rm TeV}^{-2}$$

$$\mathcal{B}(h \to \ell\ell) \Rightarrow |\tilde{C}^{ee}_{eH}| < 2.9 \times 10^{-2} \ {\rm TeV}^{-2} , \quad |\tilde{C}^{\mu\mu}_{eH}| < 1.7 \times 10^{-2} \ {\rm TeV}^{-2}$$

$\rightarrow$ **Meson-mixing** (K^0, D^0, B^0_d, B^0_s systems) dominates constraints on s, c, b, t flavor-changing currents w.r.t. $s \to d\gamma$, $b \to (s,d)\gamma$, etc.

[cf., e.g., Harnik, Kopp, Zupan '12]

Two-loop constraints on $\psi^2 H^3$

Radiative decays:

$$\mathcal{B}(\mu \to e\gamma) \Rightarrow \sqrt{|\tilde{C}_{eH}^{e\mu}(\Lambda)|^2 + |\tilde{C}_{eH}^{\mu e}(\Lambda)|^2} \lesssim 9 \times 10^{-5}\ \mathrm{TeV}^{-2} = 0.1 \times \frac{\sqrt{2 m_e m_\mu}}{v_{\mathrm{EW}}^3}$$

$$\mathcal{B}(\tau \to e\gamma) \Rightarrow \sqrt{|\tilde{C}_{eH}^{e\tau}(\Lambda)|^2 + |\tilde{C}_{eH}^{\tau e}(\Lambda)|^2} \lesssim 1\ \mathrm{TeV}^{-2} = 400 \times \frac{\sqrt{2 m_e m_\tau}}{v_{\mathrm{EW}}^3}$$

$$\mathcal{B}(\tau \to \mu\gamma) \Rightarrow \sqrt{|\tilde{C}_{eH}^{\mu\tau}(\Lambda)|^2 + |\tilde{C}_{eH}^{\tau\mu}(\Lambda)|^2} \lesssim 1\ \mathrm{TeV}^{-2} = 30 \times \frac{\sqrt{2 m_\mu m_\tau}}{v_{\mathrm{EW}}^3}$$

Anomalous Magnetic Moments (AMMs):

$$\Delta a_e = a_e^{\mathrm{exp}} - a_e^{\mathrm{SM}} \Rightarrow 0.06\ \mathrm{TeV}^{-2} \lesssim \mathrm{Re}[\tilde{C}_{eH}^{ee}(\Lambda)] \lesssim 0.6\ \mathrm{TeV}^{-2}\ @\ 2\sigma$$

$$\Delta a_\mu = a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} \Rightarrow -7\ \mathrm{TeV}^{-2} \lesssim \mathrm{Re}[\tilde{C}_{eH}^{\mu\mu}(\Lambda)] \lesssim -2\ \mathrm{TeV}^{-2}\ @\ 2\sigma$$

However, $|\mathrm{Re}[\tilde{C}_{eH}^{\ell\ell}(\Lambda)]| \gg m_\ell / v_{\mathrm{EW}}^3$

EDMs:

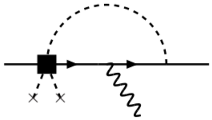
$$|d_e| \Rightarrow |\mathrm{Im}[\tilde{C}_{eH}^{ee}(\Lambda)]| \lesssim 2 \times 10^{-7}\ \mathrm{TeV}^{-2} = 0.004 \times \frac{\sqrt{2} m_e}{v_{\mathrm{EW}}^3}$$

$$|d_N| \Rightarrow \left|\mathrm{Im}[\tilde{C}_{dH}^{dd}(\Lambda)] + 1.3 \times \mathrm{Im}[\tilde{C}_{uH}^{uu}(\Lambda)]\right| \lesssim 4 \times 10^{-3}\ \mathrm{TeV}^{-2} \sim 9 \times \frac{\sqrt{2} m_d}{v_{\mathrm{EW}}^3}$$

Finite contributions

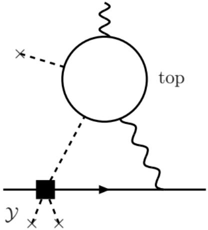
→ Loop-induced **finite** effects from flavor violating Higgs couplings:

Ex. of 1-loop diagram:



e.g., $\mu \to e\gamma$, EDMs

Ex. of Barr-Zee diagram:



[cf. Barr, Zee '90; Harnik, Kopp, Zupan '12; Brod, Haisch, Zupan '13; Davidson '16]

→ By avoiding a small Yukawa coupling, 2-loop diagrams may (over) compensate for the loop-suppression

→ Analogously, the 2-loop **mixing**-induced effect will also be enhanced compared to 1-loop finite terms

Bounds from effects induced at 2-loop

$\rightarrow$ Improvement by Barr-Zee diagrams:

$$\textbf{eEDM}: |\mathrm{Im}[\tilde{C}^{ee}_{eH}(\Lambda)]| \lesssim 0.004 \times \frac{\sqrt{2}m_e}{v^3_{\mathrm{EW}}}$$

[ACME]

$$\mu \to e\gamma: \; (|\tilde{C}^{e\mu}_{eH}(\Lambda)|^2 + (\mu \leftrightarrow e))^{1/2} \lesssim 0.1 \times \frac{\sqrt{2m_e m_\mu}}{v^3_{\mathrm{EW}}}$$

[MEG]

$$\textbf{nEDM}: \left|\mathrm{Im}[\tilde{C}^{\psi\psi}_{\psi H}(\Lambda)]\right|_{(\psi=u,d)} \lesssim 10 \times \frac{\sqrt{2}m_d}{v^3_{\mathrm{EW}}}$$

[PRD92, 092003 (2015); theory: e.g., Pospelov, Ritz '05]

[Values for $\Lambda = 1$ TeV, and $\mu = M_H$: further RGE corrections are omitted]

Bounds from effects induced at 2-loop

$\rightarrow$ Improvement by Barr-Zee diagrams:

$$\textbf{eEDM}:\ |\mathrm{Im}[\tilde{C}_{eH}^{ee}(\Lambda)]| \lesssim 0.004 \times \frac{\sqrt{2}m_e}{v_{\rm EW}^3} \overset{\text{Barr-Zee}}{\rightarrow} 0.002 \times \frac{\sqrt{2}m_e}{v_{\rm EW}^3}$$

[ACME]

$$\mu \to e\gamma:\ (|\tilde{C}_{eH}^{e\mu}(\Lambda)|^2 + (\mu \leftrightarrow e))^{1/2} \lesssim 0.1 \times \frac{\sqrt{2m_e m_\mu}}{v_{\rm EW}^3} \overset{\text{Barr-Zee}}{\rightarrow} 0.02 \times \frac{\sqrt{2m_e m_\mu}}{v_{\rm EW}^3}$$

[MEG]

$$\textbf{nEDM}:\ \left|\mathrm{Im}[\tilde{C}_{\psi H}^{\psi\psi}(\Lambda)]\right|_{(\psi=u,d)} \lesssim 10 \times \frac{\sqrt{2}m_d}{v_{\rm EW}^3} \overset{\text{Barr-Zee}}{\rightarrow} 3 \times \frac{\sqrt{2}m_d}{v_{\rm EW}^3}$$

[PRD92, 092003 (2015); theory: e.g., Pospelov, Ritz '05]

[Values for $\Lambda = 1$ TeV, and $\mu = M_H$: further RGE corrections are omitted]

$\rightarrow$ Enhancements in the **finite** corrections (such as $N_c = 3$) compared to **mixing**-induced effects (for which $\gamma^X \sim 1/4$)

Summary

$$\psi^2 H^3 \xrightarrow[2Loop]{RGE} \psi^2 XH$$

Channel	Coupling	Bound
$\mu \to e\gamma$	$\sqrt{\lvert\tilde{C}_{eH}^{e\mu}(\Lambda)\rvert^2 + \lvert\tilde{C}_{eH}^{\mu e}(\Lambda)\rvert^2}$	$\lesssim 0.02 \times \frac{\sqrt{2 m_e m_\mu}}{v_{\rm EW}^3}$
eEDM	$\lvert \mathrm{Im}[\tilde{C}_{eH}^{ee}(\Lambda)]\rvert$	$\lesssim 0.002 \times \frac{\sqrt{2}m_e}{v_{\rm EW}^3}$
$h \to e\tau$	$\sqrt{\lvert\tilde{C}_{eH}^{e\tau}\rvert^2 + \lvert\tilde{C}_{eH}^{\tau e}\rvert^2}$	(tree)
$h \to \mu\tau$	$\sqrt{\lvert\tilde{C}_{eH}^{\mu\tau}\rvert^2 + \lvert\tilde{C}_{eH}^{\tau\mu}\rvert^2}$	(tree)
$h \to ee$	$\lvert\tilde{C}_{eH}^{ee}\rvert$	(tree)
$h \to \mu\mu$	$\lvert\tilde{C}_{eH}^{\mu\mu}\rvert$	(tree)
nEDM	$\left\lvert \mathrm{Im}[\tilde{C}_{\psi H}^{\psi\psi}(\Lambda)]\right\rvert_{(\psi=u,d)}$	$\lesssim 3 \times \frac{\sqrt{2}m_d}{v_{\rm EW}^3}$
$\lvert\Delta q'\rvert, \lvert\Delta q\rvert = 2$ $(q, q' = u, d, s, c, b)$	$\lvert\tilde{C}_{\psi H}^{qq'}(\Lambda)\rvert^2 + \lvert\tilde{C}_{\psi H}^{q'q}(\Lambda)\rvert^2$	(tree)

2-Loop effects set most important bounds in many cases

Outline

→ **Four-fermions**: only $Q_{\ell equ}^{(3)}$ mixes directly w/ the dipole at 1-loop

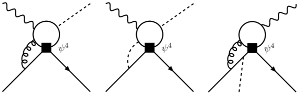
$$Q_{\ell equ}^{(1)} \xrightarrow[1Loop]{RGE} Q_{\ell equ}^{(3)} \xrightarrow[1Loop]{RGE} \psi^2 XH$$

LRLR operators

$$\begin{aligned}
Q_{\ell equ}^{(1)}(prst) &= (\bar{\ell}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)\\
Q_{\ell equ}^{(3)}(prst) &= (\bar{\ell}_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)\\
Q_{\ell\nu qd}^{(1)}(prst) &= (\bar{\ell}_p^j \nu_r)\epsilon_{jk}(\bar{q}_s^k d_t)\\
Q_{\ell\nu qd}^{(3)}(prst) &= (\bar{\ell}_p^j \sigma_{\mu\nu} \nu_r)\epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} d_t)
\end{aligned}$$

LRLR operators

$$\begin{aligned}
Q_{\ell\nu\ell e}(prst) &= (\bar{\ell}_p^j \nu_r)\epsilon_{jk}(\bar{\ell}_s^k e_t)\\
Q_{quqd}^{(1)}(prst) &= (\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)\\
Q_{quqd}^{(8)}(prst) &= (\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)
\end{aligned}$$



LRRL operators

$$\begin{aligned}
Q_{\ell edq}(prst) &= (\bar{\ell}_p e_t)(\bar{d}_s q_r)\\
Q_{\ell\nu uq}(prst) &= (\bar{\ell}_p \nu_t)(\bar{u}_s q_r)
\end{aligned}$$

[Fierzed] LLRR operators

$$\begin{aligned}
Q_{\ell e}(prst) &= (\bar{\ell}_p e_t)(\bar{e}_s \ell_r)\\
Q_{\ell\nu}(prst) &= (\bar{\ell}_p \nu_t)(\bar{\nu}_s \ell_r)\\
Q_{qu}^{(1)}(prst) &= (\bar{q}_p^\alpha u_t^\beta)(\bar{u}_s^\beta q_r^\alpha)\\
Q_{qu}^{(8)}(prst) &= (\bar{q}_p^\alpha T^A_{\alpha\tilde{\alpha}} u_t^{\tilde{\beta}})(\bar{u}_s^\beta T^A_{\beta\tilde{\beta}} q_r^{\tilde{\alpha}})\\
Q_{qd}^{(1)}(prst) &= (\bar{q}_p^\alpha d_t^\beta)(\bar{d}_s^\beta q_r^\alpha)\\
Q_{qd}^{(8)}(prst) &= (\bar{q}_p^\alpha T^A_{\alpha\tilde{\alpha}} d_t^{\tilde{\beta}})(\bar{d}_s^\beta T^A_{\beta\tilde{\beta}} q_r^{\tilde{\alpha}})
\end{aligned}$$

[Fierzed] LLRR operators

$$\begin{aligned}
Q_{\ell u}(prst) &= (\bar{\ell}_p u_t)(\bar{u}_s \ell_r)\\
Q_{\ell d}(prst) &= (\bar{\ell}_p d_t)(\bar{d}_s \ell_r)\\
Q_{qe}(prst) &= (\bar{q}_p e_t)(\bar{e}_s q_r)\\
Q_{q\nu}(prst) &= (\bar{q}_p \nu_t)(\bar{\nu}_s q_r)
\end{aligned}$$

LLLL operators

$$\begin{aligned}
Q_{\ell\ell}(prst) &= (\bar{\ell}_p \gamma_\mu \ell_r)(\bar{\ell}_s \gamma^\mu \ell_t)\\
Q_{qq}^{(1)}(prst) &= (\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)\\
Q_{qq}^{(3)}(prst) &= (\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)\\
Q_{\ell q}^{(1)}(prst) &= (\bar{\ell}_p \gamma_\mu \ell_r)(\bar{q}_s \gamma^\mu q_t)\\
Q_{\ell q}^{(3)}(prst) &= (\bar{\ell}_p \gamma_\mu \tau^I \ell_r)(\bar{q}_s \gamma^\mu \tau^I q_t)
\end{aligned}$$

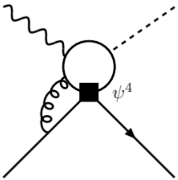
RRRR operators

$$\begin{aligned}
Q_{ee}(prst) &= (\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)\\
Q_{\nu\nu}(prst) &= (\bar{\nu}_p \gamma_\mu \nu_r)(\bar{\nu}_s \gamma^\mu \nu_t)\\
Q_{uu}(prst) &= (\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)\\
Q_{dd}(prst) &= (\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)\\
Q_{eu}(prst) &= (\bar{u}_p \gamma^\mu u_r)(\bar{e}_s \gamma_\mu e_t)\\
Q_{ed}(prst) &= (\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)\\
Q_{\nu u}(prst) &= (\bar{\nu}_p \gamma_\mu \nu_r)(\bar{u}_s \gamma^\mu u_t)\\
Q_{\nu d}(prst) &= (\bar{\nu}_p \gamma_\mu \nu_r)(\bar{d}_s \gamma^\mu d_t)\\
Q_{e\nu}(prst) &= (\bar{\nu}_p \gamma_\mu \nu_r)(\bar{e}_s \gamma^\mu e_t)\\
Q_{ud}^{(1)}(prst) &= (\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)\\
Q_{ud}^{(8)}(prst) &= (\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)\\
Q_{du\nu e}(prst) &= (\bar{d}_p \gamma_\mu u_r)(\bar{\nu}_s \gamma^\mu e_t)
\end{aligned}$$

→ □ 1-loop, □ main interest (preliminary), □ ongoing calculation

→ Focus on light external fermions: other cases ∝ external masses

Mixing of four-fermion ops. into dipoles



Possible enhancements: large Yukawa, strong coupling, color factor

→ Analogous to Barr-Zee type of contributions

→ In the following: preliminary bounds from $\mu \to e\gamma$, EDMs

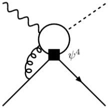
→ γ_5: Breitenlohner-Maison-'t Hooft-Veltman scheme

CP violation in quark dipoles

→ One-loop: $Q^{(1)}_{\ell equ}$, $Q^{(3)}_{\ell equ}$

→ Two-loop, y_t-enhancement: $Q^{(1)}_{qu}$, $Q^{(8)}_{qu}$, $Q^{(1)}_{quqd}$, $Q^{(8)}_{quqd}$

→ Two-loop: $Q^{(1)}_{qd}$, $Q^{(8)}_{qd}$, $Q_{\ell edq}$



$$(16\pi^2)^2 \frac{d}{d\ell n(\mu)} C_{\psi^2 XH}(\mu) = \left(g_Y^2 \gamma_Y^X + g_L^2 \gamma_L^X + g_C^2 \gamma_c^X + Y^2 \gamma^X \right) C_{\psi^2 H^3}(\mu)$$

$$Q^{(1)}_{qu} = (\bar{q}^{\alpha}_p u^{\beta}_t)(\bar{u}^{\beta}_s q^{\alpha}_r) \qquad Q^{(8)}_{qu} = (\bar{q}^{\alpha}_p T^A_{\alpha\tilde{\alpha}} u^{\tilde{\beta}}_t)(\bar{u}^{\beta}_s T^A_{\beta\tilde{\beta}} q^{\tilde{\alpha}}_r)$$

	$X = B$	$X = W$	$X = G$
γ_Y^X	$-\frac{835}{3456}$	$+\frac{137}{384}$	$+\frac{7}{72}$
γ_L^X	$+\frac{509}{768}$	$-\frac{923}{768}$	$+\frac{37}{64}$
γ_c^X	$-\frac{20}{9}$	$-\frac{4}{3}$	$+\frac{11}{3}$

	$X = B$	$X = W$	$X = G$
γ_Y^X	$-\frac{835}{648}$	$+\frac{137}{72}$	$-\frac{25}{16}$
γ_L^X	$+\frac{509}{144}$	$-\frac{923}{144}$	$-\frac{983}{192}$
γ_c^X	$+\frac{760}{27}$	$+\frac{152}{9}$	$+\frac{446}{9}$

→ $X = G$: Chromo-Magnetic Dipole Moment

Quark EDMs, pheno

→ Electric Dipole Moment:

$$\mathcal{C}_{u\gamma}(\mu) \simeq \frac{1}{(16\pi^2)^2} \times \ell n\left(\frac{\Lambda^2}{\mu^2}\right) \times \mathbf{y_{top}} \times \left\{ C_{qu}^{(1)}(\Lambda)\left(-0.8 \times g_L^2 + 0.4 \times g_c^2\right) + C_{qu}^{(8)}(\Lambda)\left(-4.4 \times g_L^2 - 5.6 \times g_c^2\right) \right\}$$

→ Chromo-MDM generates a CPV πNN coupling [see, e.g., Pospelov, Ritz '05]

$$\mathcal{C}_{uG}(\mu) \simeq \frac{1}{(16\pi^2)^2} \times \ell n\left(\frac{\Lambda^2}{\mu^2}\right) \times \mathbf{y_{top}} \times \left\{ C_{qu}^{(1)}(\Lambda)\left(-0.3 \times g_L^2 \mathbf{-1.8} \times g_c^2\right) + C_{qu}^{(8)}(\Lambda)\left(2.8 \times g_L^2 \mathbf{-24.8} \times g_c^2\right) \right\}$$

	$\lvert\mathrm{Im}\{\tilde{C}_{qu}^{(1)}(\Lambda)\}\rvert \times y_{top}$	$\lvert\mathrm{Im}\{\tilde{C}_{qu}^{(8)}(\Lambda)\}\rvert \times y_{top}$
$\lvert d_N\rvert$	$\mathcal{O}(10^{-4})\,\mathrm{TeV}^{-2} \sim \mathcal{O}(1)\frac{\sqrt{2}m_u}{v^3}$	$\mathcal{O}(10^{-6})\,\mathrm{TeV}^{-2} \sim \mathcal{O}(10^{-2})\frac{\sqrt{2}m_u}{v^3}$
$\lvert d_{Hg}\rvert$	$\mathcal{O}(10^{-6})\,\mathrm{TeV}^{-2} \sim \mathcal{O}(10^{-2})\frac{\sqrt{2}m_u}{v^3}$	$\mathcal{O}(10^{-7})\,\mathrm{TeV}^{-2} \sim \mathcal{O}(10^{-3})\frac{\sqrt{2}m_u}{v^3}$

→ No tops below EW scale: effects from SMEFT mixing only

→ Wilson coefficients $\lesssim \mathcal{O}(10^{-6}) - \mathcal{O}(10^{-7})\,\mathrm{TeV}^{-2}$

Summary

$\rightarrow$ Typically, Wilson coefficients $< \mathcal{O}(0.1)\,\mathrm{TeV}^{-2}$ **w/o** considering **mixing w/ dipoles** [e.g., (semi-)leptonic, low-energy + LHC: Falkowski, González-Alonso, Mimouni '17]

$\psi^4 \xrightarrow[2Loop]{RGE} \psi^2 XH$, preliminary

	Channel	Coupling	Bound
$Q_{qu}^{(1)}$	Hg-EDM	$y_{top} \times \|\mathrm{Im}[\tilde{C}_{qu}^{(1),uutt}(\Lambda)]\|$	$\lesssim \mathcal{O}(10^{-6})\,\mathrm{TeV}^{-2}$
$Q_{qu}^{(8)}$		$y_{top} \times \|\mathrm{Im}[\tilde{C}_{qu}^{(8),uutt}(\Lambda)]\|$	$\lesssim \mathcal{O}(10^{-7})\,\mathrm{TeV}^{-2}$
$Q_{\ell e}$	$\mu \to e\gamma$	$y_\tau \times \sqrt{\|\tilde{C}_{\ell e}^{e\mu\tau\tau}(\Lambda)\|^2 + \|\tilde{C}_{\ell e}^{\mu e\tau\tau}(\Lambda)\|}$	$\lesssim \mathcal{O}(10^{-5})\,\mathrm{TeV}^{-2}$
	eEDM	$y_\tau \times \|\mathrm{Im}[\tilde{C}_{\ell e}^{ee\tau\tau}(\Lambda)]\|$	$\lesssim \mathcal{O}(10^{-7})\,\mathrm{TeV}^{-2}$
$Q_{\ell edq}$	eEDM	$y_b \times \|\mathrm{Im}[\tilde{C}_{\ell edq}^{eebb}(\Lambda)]\|$	$\lesssim \mathcal{O}(10^{-7})\,\mathrm{TeV}^{-2}$
	$\mu \to e$ conv.	$y_b \times \sqrt{\|\tilde{C}_{\ell edq}^{e\mu bb}(\Lambda)\|^2 + \|\tilde{C}_{\ell edq}^{\mu ebb}(\Lambda)\|}$	(*1Loop*)

(ongoing analysis for further operators, channels, and couplings)

2-Loop effects set most important bounds in many cases

Outline

Conclusions

$\rightarrow$ Dipoles: probe very high energy scales, e.g., EDMs, $\mu \rightarrow e\gamma$, leading to a broad physics program

$\rightarrow$ **Generic tool** for improving our understanding of flavor and CPV

$\rightarrow$ SMEFT: systematic approach in the absence of new d.o.f. (so far)

$\rightarrow$ Important 2-loop effects generated by operator mixing

Conclusions

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Merci !

Backup

Charged lepton dipoles

eEDM, $\mu \to e\gamma$:

$\to$ One-loop: $Q_{\ell equ}^{(1)}$, $Q_{\ell equ}^{(3)}$

$\to$ Two-loop, y_τ, y_b-enhanced: $Q_{\ell e}$, $Q_{\ell edq}$

$$(16\pi^2)^2 \frac{d}{d\ell n(\mu)} C_{\psi^2 XH}(\mu) = \left(g_Y^2 \gamma_Y^X + g_L^2 \gamma_L^X + g_C^2 \gamma_c^X + Y^2 \gamma^X \right) C_{\psi^2 H^3}(\mu)$$

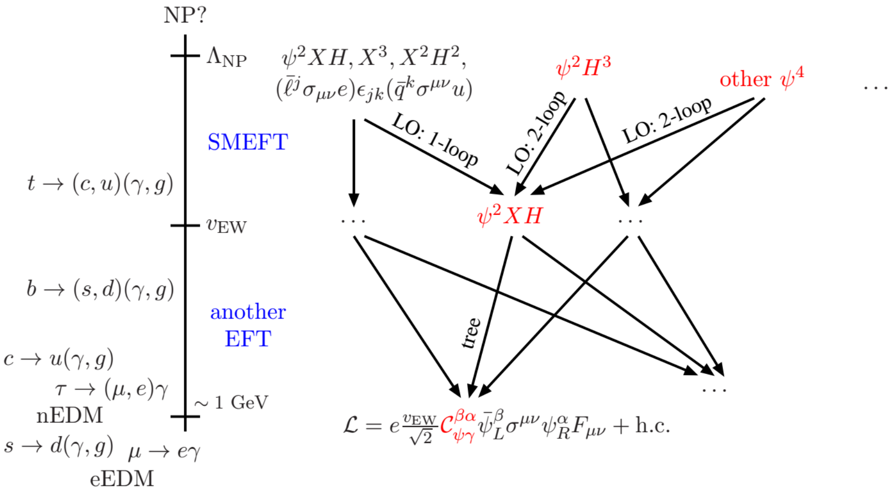
$$Q_{\ell e} = (\bar{\ell}_p e_t)(\bar{e}_s \ell_r)$$

	$B^{\mu\nu}$	$W^{\mu\nu}$	$G^{\mu\nu}$
γ_Y^X	$+\frac{93}{128}$	$+\frac{67}{128}$	0
γ_L^X	$-\frac{219}{256}$	$-\frac{923}{768}$	0
γ_c^X	0	0	0

$$Q_{\ell edq} = (\bar{\ell}_p e_t)(\bar{d}_s q_r)$$

	$X = B$	$X = W$	$X = G$
γ_Y^X	$-\frac{67}{128}$	$+\frac{115}{128}$	0
γ_L^X	$-\frac{315}{256}$	$-\frac{923}{768}$	0
γ_c^X	0	0	0

$$\mathcal{C}_{e\gamma}(\mu) \simeq \frac{1}{(16\pi^2)^2} \times \ell n \left(\frac{\Lambda^2}{\mu^2} \right) \times \{ -0.2 \times C_{\ell e}(\Lambda) \times y_\tau + 0.3 \times C_{\ell edq}(\Lambda) \times y_b \} \times g_L^2$$



Mixing below EW scale, e.g., $(\bar\ell P_L \ell')(\bar f P_R f)$, $\ell, \ell' = \mu, e$, $f = b, \tau$

[RGE below EW scale: Ciuchini, Franco, Reina, Silvestrini '93]

Charged lepton dipoles, pheno

eEDM: $Q_{\ell edq}$, $Q_{\ell e}$

$|\mathrm{Im}\{\tilde{C}^{eebb(,ee\tau\tau)}_{\ell edq(,\ell e)}(\Lambda)\}| \times y_{b(,\tau)} \lesssim \mathcal{O}(10^{-7})\,\mathrm{TeV}^{-2} \sim \mathcal{O}(10^{-3})\frac{\sqrt{2}m_e}{v^3}$

$\rightarrow$ Running below EW scale: improves bound by a factor ~ 1

[Similar bounds found by Panico, Pomarol, Riembau '18]

$\mu \rightarrow e\gamma$**:** $Q_{\ell e}$

$|\tilde{C}^{\mu e\tau\tau}_{\ell e}(\Lambda)| \times y_\tau \lesssim \mathcal{O}(10^{-5})\,\mathrm{TeV}^{-2} \sim \mathcal{O}(0.01) \times \frac{\sqrt{2m_e m_\mu}}{v^3}$

$\rightarrow$ Running below EW scale: improves bound by a factor $\mathcal{O}(\mathrm{few})$

$\mu \rightarrow e$ **conversion in nuclei**: $Q_{\ell edq}$

bound on $\tilde{C}^{\mu ebb}_{\ell edq}(\Lambda)$ stronger by a factor ~ 20 [Crivellin, Davidson, Pruna, Signer '17]

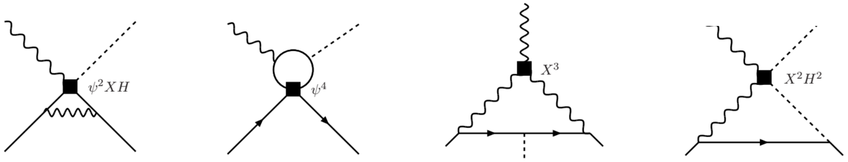
$\rightarrow$ Wilson coefficients $\lesssim \mathcal{O}(10^{-3}) - \mathcal{O}(10^{-5})\,\mathrm{TeV}^{-2}$

Off-shell renormalization of SM + dim.=6 ops.

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_i C_i Q_i, \qquad C_i \text{ scales as } \Lambda^{-2}$$

$$\mathcal{L}_{\rm SM} = -\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^I_{\mu\nu} W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D_\mu H^\dagger)(D^\mu H) + \sum_{\psi=q,u,d,\ell,e} \bar{\psi} i \not{D} \psi$$
$$-\lambda \left(H^\dagger H - \frac{1}{2} v^2_{\rm EW} \right)^2 - \left[H^{\dagger j} \bar{d} Y_d q_j + \tilde{H}^{\dagger j} \bar{u} Y_u q_j + H^{\dagger j} \bar{e} Y_e \ell_j + \text{h.c.} \right]$$

[plus gauge fixing and ghost terms]



Insertions of dim.-6 ops. $\mathcal{O}$ that require dipoles as counter-terms:
$\mathcal{O} = \psi^2 XH, \psi^4, X^3, X^2H^2$, $[\mathcal{O}]^{\rm ren} \supset Z_{\mathcal{O},\psi^2 XH} \times \psi^2 XH$

Full set of local operators required in the renormalization program?

Full basis of operators

class $\mathcal{O}, \mathcal{Q}$: gauge-invariant operators, *e.g.*, *Warsaw basis*
class **A**: BRST-exact operators, i.e., $A = \delta_{BRST} A'$
class **B**: vanish via the equations of motion

$$\begin{pmatrix} [\mathcal{O}]^{\rm ren} \\ [\mathbf{A}]^{\rm ren} \\ [\mathbf{B}]^{\rm ren} \end{pmatrix} = \begin{pmatrix} Z_{\mathcal{O}\mathcal{Q}} & Z_{\mathcal{O}A} & Z_{\mathcal{O}B} \\ 0 & Z_{AA} & Z_{AB} \\ 0 & 0 & Z_{BB} \end{pmatrix} \begin{pmatrix} \mathcal{Q} \\ \mathbf{A} \\ \mathbf{B} \end{pmatrix}$$

$$\langle 0| T\{A\Phi\}|0\rangle_{\text{on-shell}} = 0$$
$$\langle 0| T\{B\Phi\}|0\rangle_{\text{on-shell}} = 0$$
Φ: set of local fields

[Dixon, Taylor '74; Kluberg-Stern, Zuber '75; Joglekar, Lee '76; Joglekar '77; Collins '84]

[cf. Herrlich, Nierste '96 for double insertions]

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$$\langle 0 | T\{A\Phi\} | 0 \rangle_{\text{on-shell}} = 0$$
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[cf. Herrlich, Nierste '96 for double insertions]

Non-phys. ops. are enumerated systematically by extending BRST

[Henneaux '93; Barnich, Brandt, Henneaux '00]

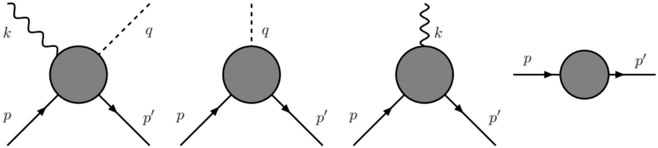
Ex. of gauge non-inv. structure from non-physical operators:

$$g_A[(\partial_\nu \bar{\psi}_L)\Gamma^{\mu\nu} t^I \xi_R \varphi] A^I_\mu, \ \ldots$$

Mixing into dipole operators

Consider all operators Q that contribute to $\Sigma_Q Z_{OQ} \mathcal{G}_Q^{\rm tree}(\cdot) = \mathcal{G}_O(\cdot)$

Need to look at many Green's functions to pin down Z_{OQ}



$\Rightarrow Z_{OQ} =$ linear combination of $\mathcal{G}_O(\psi^2 A^\mu H), \mathcal{G}_O(\psi^2 H), \mathcal{G}_O(\psi^2 A^\mu), \mathcal{G}_O(\psi^2)$

[$\mathcal{G}_O$: Green's functions of single insertions of (bare) operator O]

[cf., e.g., Grinstein, Springer, Wise '88 on $b \to s\gamma$]

Again, we want to determine $Z_{O,\psi^2 XH}$ for O such that $\gamma^{\text{(1-loop)}}_{O,\psi^2 XH} = 0$

Extended BRST-variation

$\rightarrow$ Extend BRST-variation to "anti-fields"; $\delta_{BRST} = \delta + \gamma$ increases the mass power counting and the ghost number by one unit

$\rightarrow$ Consider all polynomials of dim. ≤ 5, and ghost number -1

Z	δZ	γZ
A^I_μ	0	$D_\mu C^I$
ξ^i	0	$-eC^I T^i_{Ij}\xi^j$
C^I	0	$\frac{1}{2} ef^I_{KJ} C^J C^K$
$C^\ddagger_I$	$-D_\mu A^{\ddagger\mu}_I - e\xi^\ddagger_i T^i_{Ij}\xi^j$	$ef^K_{JI} C^J C^\ddagger_K$
$A^{\ddagger\mu}_I$	L^μ_I	$ef^K_{JI} C^J A^{\ddagger\mu}_K$
$\xi^\ddagger_i$	L_i	$eC^I \xi^\ddagger_j T^j_{Ii}$

$D_\mu C^I = \partial_\mu C^I + ef^I_{JK} A^J_\mu C^K$, $D_\mu A^{\ddagger\mu}_I = \partial_\mu A^{\ddagger\mu}_I - ef^K_{JI} A^J_\mu A^{\ddagger\mu}_K$, and $L^\mu_I = \frac{\delta L}{\delta A^I_\mu}$, $L_i = (-1)^{\epsilon_i} \frac{\delta L}{\delta \xi^i}$, where L is the action.

The field ξ designates a fermion or a scalar.

[Batalin-Vilkovisky; Henneaux '93; Collins, Scalise '94; Barnich, Brandt, Henneaux '00]

Some calculation aspects

$$\int f\,[\text{internal momenta } q\text{, external momenta } p\text{, masses } M]$$

Expansion in external momenta for simplifying integrals:

$$\underbrace{\frac{1}{(q+p)^2 - M^2}}_{\text{sup. deg. of div. }+2} \overset{exact}{=} \underbrace{\frac{1}{q^2 - m_R^2}}_{\text{sup. deg. of div. }+2} + \underbrace{\frac{M^2 - p^2 - 2q\cdot p - m_R^2}{q^2 - m_R^2} \frac{1}{(q+p)^2 - M^2}}_{\text{sup. deg. of div. }+3}$$

[Chetyrkin, Misiak, Münz '97; Gambino, Gorbahn, Haisch '03; Zoller '14]

Basic formulas

$$\frac{dC^T}{d\ell n(\mu)} = -C^T \left(\frac{dZ}{d\ell n(\mu)} Z^{-1} - Z(\epsilon\Delta + \gamma_M N) Z^{-1} \right) \equiv C^T \gamma$$

for $\psi^2 H^3$: $\Delta = -3, n = 2$;
for ψ^4: $\Delta = -2, n = 2$;
for $g\psi^2 XH$: $\Delta = -1, n = 2$.

$$\mathcal{L}^{(6)}(\beta\alpha) = \Sigma_i M^{-2} \mu^{-\Delta_i \epsilon} [C_i(\mu)]^{\beta\alpha} [Q_i^{bare}]^{\beta\alpha} + \Sigma_{i,j,f,g} M^{-2} \mu^{-\Delta_j \epsilon} [C_i(\mu)]^{\beta\alpha} [(Z_{ij}^X - \delta_{ij})]^{\beta\alpha fg} [Q_j^{bare}]^{fg} + \ldots + \mathrm{h.c.}$$

$$Z^{X,\ \beta\alpha fg}_{\psi^2 H^3, g\psi^2 XH} = \Bigg[\left(\frac{g_Y^2}{(4\pi)^4} (Z_Y^X)_1^{(1)} + \frac{g_L^2}{(4\pi)^4} (Z_L^X)_1^{(1)} + \frac{g_c^2}{(4\pi)^4} (Z_c^X)_1^{(1)} + \frac{\lambda}{(4\pi)^4} (Z_\lambda^X)_1^{(1)} + \frac{\Sigma_{k,l} Y_{kl}^* \times Y_{lk}}{(4\pi)^4} (Z_{\mathrm{det}^2}^X)_1^{(1)} \right) \delta_{f\beta} \delta_{g\alpha} + \frac{\Sigma_l (Y^\dagger)_{fl} \times Y_{l\beta}}{(4\pi)^4} \delta_{g\alpha} (Z_{y,y}^X)_1^{(1)} + \frac{(Y^\dagger)_{g\beta} \times (Y^\dagger)_{\alpha f}}{(4\pi)^4} (Z_{Y,y}^X)_1^{(1)} + \frac{\Sigma_k Y_{\alpha k} \times (Y^\dagger)_{kg}}{(4\pi)^4} \delta_{f\beta} (Z_{Y,Y}^X)_1^{(1)} \Bigg] \frac{1}{\epsilon} + \ldots$$

Dipole: $g_X(\bar{\psi}_L \sigma^{\mu\nu} t^I \xi_R \varphi) X^I_{\mu\nu}$, $(q_L, u_R), (q_L, d_R), (\ell_L, e_R), (\ell_L, \nu_R)$

	$X = B$	$X = W$	$X = G$
$(Z^X_Y)^{(1)}_1$	$\frac{3}{4} Q^Y_\varphi Q^Y_\varphi (Q^Y_L + Q^Y_R)$	$\frac{1}{8} Q^Y_\varphi (Q^Y_L + Q^Y_R)$	0
$(Z^X_L)^{(1)}_1$	$\frac{3}{16} Q^Y_\varphi$	$\frac{3}{32}$	0
$(Z^X_c)^{(1)}_1$	0	0	0
$(Z^X_\lambda)^{(1)}_1$	0	0	0
$(Z^X_{y,y})^{(1)}_1$	$\frac{1}{16}(5\, Q^Y_L + Q^Y_R)$	$\frac{1}{32}$	$\frac{3}{8}$
$(Z^X_{Y,y})^{(1)}_1$	0	0	0
$(Z^X_{Y,Y})^{(1)}_1$	$\frac{1}{16}(Q^Y_L + 5\, Q^Y_R)$	$\frac{1}{96}$	$\frac{3}{8}$
$(Z^X_{\mathrm{det}^2})^{(1)}_1$	0	0	0

$Q^Y_\phi = -Q^Y_{\tilde{\phi}} = 1/2$; $Q^Y_{\ell_L} = -1/2$, $Q^Y_{e_R} = -1$, $Q^Y_{\nu_R} = 0$; $Q^Y_{q_L} = 1/6$, $Q^Y_{u_R} = +2/3$, $Q^Y_{d_R} = -1/3$

Channel	Coupling	Bound
$\mu \to e\gamma$	$\sqrt{\lvert Y_{\mu e}\rvert^2 + \lvert Y_{e\mu}\rvert^2}$	$< 0.63 \times 10^{-6}$
$\mu \to 3e$	$\sqrt{\lvert Y_{\mu e}\rvert^2 + \lvert Y_{e\mu}\rvert^2}$	$\lesssim 3.1 \times 10^{-5}$
electron $g-2$	$\mathrm{Re}(Y_{e\mu}Y_{\mu e})$	$-0.0022 \ldots -0.0009$
electron EDM	$\lvert\mathrm{Im}(Y_{e\mu}Y_{\mu e})\rvert$	$< 0.10 \times 10^{-8}$
$\mu \to e$ conversion	$\sqrt{\lvert Y_{\mu e}\rvert^2 + \lvert Y_{e\mu}\rvert^2}$	$< 1.2 \times 10^{-5}$
M-$\bar{M}$ oscillations	$\lvert Y_{\mu e} + Y^*_{e\mu}\rvert$	< 0.079
$\tau \to e\gamma$	$\sqrt{\lvert Y_{\tau e}\rvert^2 + \lvert Y_{e\tau}\rvert^2}$	< 0.014
$\tau \to 3e$	$\sqrt{\lvert Y_{\tau e}\rvert^2 + \lvert Y_{e\tau}\rvert^2}$	$\lesssim 0.12$
electron $g-2$	$\mathrm{Re}(Y_{e\tau}Y_{\tau e})$	$[-0.24 \ldots -0.10] \times 10^{-3}$
electron EDM	$\lvert\mathrm{Im}(Y_{e\tau}Y_{\tau e})\rvert$	$< 0.01 \times 10^{-8}$
$\tau \to \mu\gamma$	$\sqrt{\lvert Y_{\tau\mu}\rvert^2 + \lvert Y_{\mu\tau}\rvert^2}$	0.016
$\tau \to 3\mu$	$\sqrt{\lvert Y^2_{\tau\mu} + \lvert Y_{\mu\tau}\rvert^2}$	$\lesssim 0.25$
muon $g-2$	$\mathrm{Re}(Y_{\mu\tau}Y_{\tau\mu})$	$(2.5 \pm 0.71) \times 10^{-3}$
muon EDM	$\mathrm{Im}(Y_{\mu\tau}Y_{\tau\mu})$	$-0.8 \ldots 1.0$
$\mu \to e\gamma$	$(\lvert Y_{\tau\mu}Y_{e\tau}\rvert^2 + \lvert Y_{\mu\tau}Y_{\tau e}\rvert^2)^{1/4}$	$< 0.60 \times 10^{-4}$
neutron EDM [37, 52]	$\lvert\mathrm{Im}(Y_{ut}Y_{tu})\rvert$	$< 4.4 \times 10^{-7}$
	$\lvert\mathrm{Im}(Y_{ct}Y_{tc})\rvert$	$< 5.2 \times 10^{-4}$

[Table adapted from Harnik, Kopp, Zupan '12]

$\mathcal{B}(h \to \ell\ell')$ @ 95% CL:

$$\sqrt{|\tilde{C}_{eH}^{e\mu}|^2 + |\tilde{C}_{eH}^{\mu e}|^2} < 5.2 \times 10^{-3}\ \mathrm{TeV}^{-2}$$

$$\sqrt{|\tilde{C}_{eH}^{e\tau}|^2 + |\tilde{C}_{eH}^{\tau e}|^2} < 4.6 \times 10^{-2}\ \mathrm{TeV}^{-2}$$

$$\sqrt{|\tilde{C}_{eH}^{\mu\tau}|^2 + |\tilde{C}_{eH}^{\tau\mu}|^2} < 3.6 \times 10^{-2}\ \mathrm{TeV}^{-2}$$

$$Y_{ij}^{HKZ} \to -\frac{v_{\mathrm{EW}}^2}{\sqrt{2}}[\tilde{C}^\dagger]_{ij}$$

$$\mathcal{B}(h \to e\mu) < 6.1 \times 10^{-5} \ (95\% \ \mathrm{CL})$$ [Aad:2019ojw]

$$\mathcal{B}(h \to e\tau) < 4.7 \times 10^{-3} \ (95\% \ \mathrm{CL})$$ [Aad:2019ugc]

$$\mathcal{B}(h \to \mu\tau) < 2.8 \times 10^{-3} \ (95\% \ \mathrm{CL})$$ [Aad:2019ugc]

$$\mathcal{B}(\mu \to e\gamma) < 4.2 \times 10^{-13} \ (90\% \ \mathrm{CL})$$ [TheMEG:2016wtm]

$$\mathcal{B}(\tau \to e\gamma) < 3.3 \times 10^{-8} \ (90\% \ \mathrm{CL})$$ [Aubert:2009ag]

$$\mathcal{B}(\tau \to \mu\gamma) < 4.4 \times 10^{-8} \ (90\% \ \mathrm{CL})$$ [Aubert:2009ag]

$$\Delta a_e = a_e^{\rm exp} - a_e^{\rm SM} = -0.88(0.36) \times 10^{-12} \ @ \ 1\sigma$$ [Parker:2018]

$$\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = 268(63)(43) \times 10^{-11} \ @ \ 1\sigma$$ [Tanabashi:2018oca]

$$|d_e|/e < 1.1 \times 10^{-29} \ \mathrm{cm} \ (90\% \ \mathrm{CL})$$ [Andreev:2018ayy]

$$|d_\mu|/e < 1.9 \times 10^{-19} \ \mathrm{cm} \ (95\% \ \mathrm{CL})$$ [Bennett:2008dy]

$$d_\tau/e \in [-2.2, 4.5] \times 10^{-17} \ \mathrm{cm} \ (95\% \ \mathrm{CL})$$ [Inami:2002ah]

$$|d_N|/e < 3.0 \times 10^{-26} \ \mathrm{cm} \ (90\% \ \mathrm{CL})$$ [Afach:2015sja]

$$|d_{\rm Hg}| < 7.4 \times 10^{-30} \ e\,\mathrm{cm} \ (95\% \ \mathrm{CL})$$ [Graner:2016ses]