Rencontre de Physique des Particules 2020

Gravitational form factor constraints for states of arbitrary spin

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Outline

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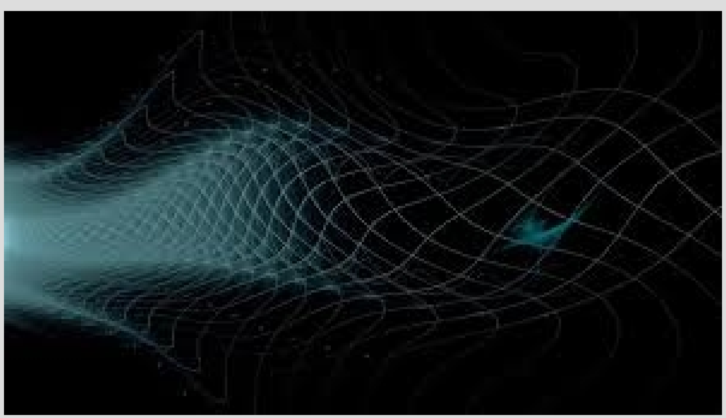
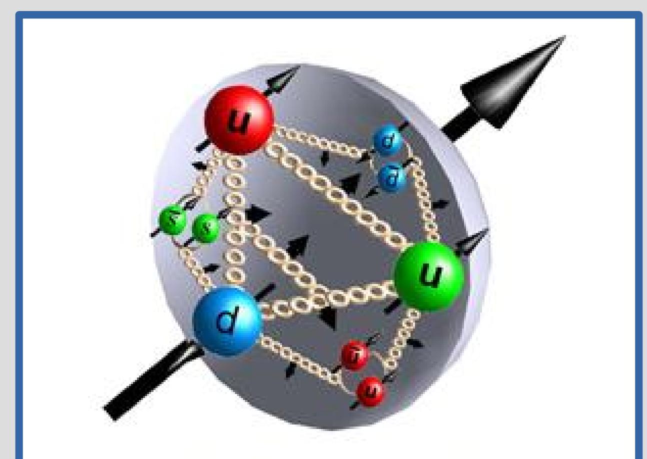




[Iowa State University (2013)]

1. The Gravitational form factors

- The energy-momentum tensor (EMT) matrix elements encode many different dynamical properties, including:
  - *Quantum corrections to the gravitational motion of particles*
  - *Distribution of mass and angular momentum within hadrons*

- The EMT matrix elements can be decomposed into a series of form factors $\rightarrow$ these fully parametrise the **non-perturbative** information

Define on-shell states:

$$\boxed{|p,m;M\rangle = \delta_M^{(+)}(p)|p,m\rangle \equiv 2\pi\,\theta(p^0)\,\delta(p^2-M^2)|p,m\rangle}$$

$$\boxed{\langle p',m';M|T^{\mu\nu}(0)|p,m;M\rangle = \overline{\eta}_{m'}(p')O^{\mu\nu}(p',p)\eta_m(p)\,\delta_M^{(+)}(p')\,\delta_M^{(+)}(p),}$$

"Generalised" polarisation tensor (GPT)

$$\boxed{O^{\mu\nu}(p',p) = \bar{p}^{\{\mu}\bar{p}^{\nu\}}A(q^2) + i\bar{p}^{\{\mu}S^{\nu\}\rho}q_\rho\,G(q^2) + \cdots}$$

$$\boxed{q = p' - p}$$

$$\boxed{\bar{p} = \tfrac{1}{2}(p'+p)}$$

Lorentz generator

1. The Gravitational form factors

→ **What are the constraints on these form factors?**

- Most previous studies chose to focus on massive (canonical spin) states with lower spin, in particular spin 0, ½ or 1
- Analyses often suffered from technical issues, such as the incorrect treatment of non-normalisable states or boundary terms, as detailed in: [Bakker, Leader, Trueman, hep-ph/0406139].
- An approach was developed in [PL, Chiu, Brodsky, 1707.06313] for the **spin-½** case in which the EMT matrix elements were treated rigorously, using their properties as **distributions**

→ Established that the $q \to 0$ limit of $A(q^2)$ and $G(q^2)$ are fixed by the Poincaré transformation properties of the states alone

Central question: *What about states with arbitrary spin?*

2. Constraints for massive states

- Use "distributional matching procedure" [Cotogno, Lorcé, PL, 1905.11969]:

Step 1: Construct rigorous expressions for the Lorentz charge operators in terms of the EMT components

$$J^i = \frac{1}{2}\,\epsilon^{ijk} \lim_{\substack{d\to 0\\ R\to\infty}} \int \mathrm{d}^4x\, f_{d,R}(x)\left[x^j T^{0k}(x) - x^k T^{0j}(x)\right],$$

$$K^i = \lim_{\substack{d\to 0\\ R\to\infty}} \int \mathrm{d}^4x\, f_{d,R}(x)\left[x^0 T^{0i}(x) - x^i T^{00}(x)\right],$$

Need smearing with appropriate test functions

Step 2: Use these definitions, together with the EMT form factor decomposition, to write the rotation and boost generator matrix elements in terms of these form factors

$$\begin{aligned}\langle p', m'; M|J^i|p, m; M\rangle = &-i\epsilon^{ijk}\bar{p}^k(2\pi)^4\delta_M^{(+)}(\bar{p})\,\left[\delta_{m'm}\,\partial^j\delta^4(q) - \partial^j[\overline{\eta}_{m'}(p')\eta_m(p)]\big|_{q=0}\,\delta^4(q)\right]A(q^2)\\ &+\frac{1}{2}\,\epsilon^{ijk}(2\pi)^4\delta_M^{(+)}(\bar{p})\,\left[\overline{\eta}_{m'}(\bar{p})S^{jk}\eta_m(\bar{p})\right]\delta^4(q)\,G(q^2)\end{aligned}$$

$$\begin{aligned}\langle p', m'; M|K^i|p, m; M\rangle = &\,i(2\pi)^4\delta_M^{(+)}(\bar{p})\,\left[\delta_{m'm}\,(\bar{p}^0\partial^i - \bar{p}^i\partial^0)\delta^4(q) - \bar{p}^0\partial^i[\overline{\eta}_{m'}(p')\eta_m(p)]\big|_{q=0}\,\delta^4(q)\right]A(q^2)\\ &+(2\pi)^4\delta_M^{(+)}(\bar{p})\,\left[\overline{\eta}_{m'}(\bar{p})S^{0i}\eta_m(\bar{p})\right]\delta^4(q)\,G(q^2)\end{aligned}$$

2. Constraints for massive states

<u>Step 3</u>: Use the transformation properties of on-shell states under rotations and boosts...

$$U(\alpha)|p,k;M\rangle = \sum_l \mathcal{D}^{(s)}_{lk}(\alpha)|\Lambda(\alpha)p,l;M\rangle,$$

...to write an arbitrary spin representation for the rotation and boost matrix elements in terms of the rest frame spin $\Sigma^i_{m'm}(k) = \overline{\eta}_{m'}(k)\,J^i\,\eta_m(k)$

$$\langle p',m';M|J^i|p,m;M\rangle = (2\pi)^4\delta^{(+)}_M(\bar{p})\left[\Sigma^i_{m'm}(k) - \delta_{m'm}\,i\epsilon^{ijk}\bar{p}^k\frac{\partial}{\partial q_j}\right]\delta^4(q)$$

$$\langle p',m';M|K^i|p,m;M\rangle = (2\pi)^4\delta^{(+)}_M(\bar{p})\left[-\frac{\epsilon^{ijk}\bar{p}^j}{\bar{p}^0+M}\,\Sigma^k_{m'm}(k) + \delta_{m'm}\,i\left(\bar{p}^0\frac{\partial}{\partial q_i} - \bar{p}^i\frac{\partial}{\partial q_0}\right)\right]\delta^4(q)$$

<u>Step 4</u>: Compare the two different representations!

$\rightarrow$ Implies the constraint: $A(q^2)\,\delta^4(q) = G(q^2)\,\delta^4(q) = \delta^4(q)$

...which is simply: $\boldsymbol{A(0)=1}$ and $\boldsymbol{G(0)=1}$

2. Constraints for massive states

- Identical form factor constraints obtained from boost and rotation generators $\rightarrow$ ***not generator specific!***
- In fact, one can instead use the *covariant* operator basis
  - *Pauli-Lubanski,* W^{μ} $\rightarrow$ implies: $\boldsymbol{G(0)=1}$
  - *Covariant boost,* $B^{\mu} = ½(S^{\nu\mu}P_{\nu}+P_{\nu}S^{\nu\mu})$ $\rightarrow$ implies: $\boldsymbol{A(0)=1}$

The constraints are non-perturbative and **independent of both the spin and internal structure of the states** in the matrix elements
$\rightarrow$ *fixed purely from Poincaré covariance of states*

Implications

$\rightarrow$ Spin universality of GPD sum rules

$\rightarrow$ AGM $B(0)=G(0)\text{-}A(0)$ vanishes for particles of **any** spin

3. Arbitrary state generalisation

- Relativistic spin states are **convention dependent**

$$\boxed{|p,\sigma\rangle = U(L(p))|k,\sigma\rangle} \qquad \boxed{\Lambda(L(p))k = p}$$

- Defined by choice of <u>Lorentz transformation</u> and <u>reference vector</u>

  (i) *"Canonical spin state"* $\rightarrow$ k=(m,0,0,0), $L_c(p)$ = pure boost

  (ii) *"Wick helicity state"* $\rightarrow$ k=(κ,0,0,κ), $L_w(p)$ = z-boost & rotation

- Results derived in most of the literature, including [Cotogno, Lorcé, PL, 1905.11969], assumed massive canonical spin states

$\rightarrow$ What happens for <u>arbitrary</u> spin-states?

- It turns out [Lorcé, PL, 1908.02567] that one can apply an analogous matching procedure

$$\boxed{\eta_\sigma(p) = D(L(p))\eta_\sigma(k)}$$

<u>Key:</u> need to take derivative wrt to momentum components of D(L(p))

3. Arbitrary state generalisation

- For the matrix element $\boxed{\langle p',\sigma';M|\widetilde{U}(J^i)|p,\sigma;M\rangle=(2\pi)^4\delta^{(+)}_M(\bar{p})\,\mathcal{J}^i_{\sigma'\sigma}(\bar{p},q),}$

  one can write this in the state-independent form:

$$\boxed{\begin{aligned}\mathcal{J}^i_{\sigma'\sigma}(\bar{p},q)=&-i\epsilon^{ijk}\bar{p}^k\delta_{\sigma'\sigma}\,\partial^j\delta^4(q)A(q^2)+i\epsilon^{ijk}\bar{p}^k\,\overline{\eta}_{\sigma'}(k)\widetilde{D}\left(\frac{\partial L^{-1}(\bar{\boldsymbol{p}})}{\partial\bar{p}_j}L(\bar{\boldsymbol{p}})\right)\eta_\sigma(k)\,\delta^4(q)A(q^2)\\&+\overline{\eta}_{\sigma'}(k)\widetilde{D}\left(L^{-1}(\bar{\boldsymbol{p}})J^iL(\bar{\boldsymbol{p}})\right)\eta_\sigma(k)\,\delta^4(q)\,G(q^2),\end{aligned}}$$

Lie algebra representation of D

- Similarly, the transformation properties of the states under **rotations** implies the general representation:

"Wigner rotation"

$$\boxed{W(\mathcal{R}_i,\bar{p})=L^{-1}\left(\Lambda\left(\mathcal{R}_i\right)\bar{p}\right)e^{-i\beta J^i}L(\bar{p}).}$$

$$\boxed{\mathcal{J}^i_{\sigma'\sigma}(\bar{p},q)=-i\epsilon^{ijk}\bar{p}^k\delta_{\sigma'\sigma}\,\partial^j\delta^4(q)+i\,\delta^4(q)\left.\frac{d}{d\beta}\right|_{\beta=0}\overline{\eta}_{\sigma'}(k)D(W(\mathcal{R}_i,\bar{p}))\eta_\sigma(k).}$$

$$\boxed{\begin{aligned}\mathcal{J}^i_{\sigma'\sigma}(\bar{p},q)=&-i\epsilon^{ijk}\bar{p}^k\delta_{\sigma'\sigma}\,\partial^j\delta^4(q)+i\epsilon^{ijk}\bar{p}^k\,\overline{\eta}_{\sigma'}(k)\widetilde{D}\left(\frac{\partial L^{-1}(\bar{\boldsymbol{p}})}{\partial\bar{p}_j}L(\bar{\boldsymbol{p}})\right)\eta_\sigma(k)\,\delta^4(q)\\&+\overline{\eta}_{\sigma'}(k)\widetilde{D}\left(L^{-1}(\bar{\boldsymbol{p}})J^iL(\bar{\boldsymbol{p}})\right)\eta_\sigma(k)\,\delta^4(q).\end{aligned}}$$

$\rightarrow$ Comparing these expressions implies: ***A*(0)=1** and ***G*(0)=1**

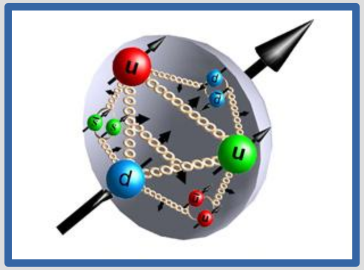
4. Summary & outlook

- By adopting a distributional approach one can prove on a ***non-perturbative level*** that Poincaré symmetry alone is responsible for the $q \to 0$ behaviour of $A(q^2)$ and $G(q^2)$

- These constraints hold ***independently*** of the internal properties of the states (internal structure, spin convention, mass, spin representation)

  $\rightarrow$ GPD spin sum rules are ***state universal***

  $\rightarrow$ AGM vanishes for ***any*** particle



- More generally, these results are relevant for understanding the spin structure of gravitational scattering amplitudes.

4. Summary & outlook

- One can in fact classify ***all*** the possible terms appearing in the form factor decomposition of the EMT matrix elements for (massive) states with arbitrary spin [Cotogno, Lorcé, PL, Morales, 1912.08749]

  $\rightarrow$ Enables one to count the total number of independent gravitational form factors for states of a given spin

- This classification can also be generalised to ***massless*** states (work in progress!)

  $\rightarrow$ Provides an alternative approach for understanding the constraints imposed by conformal symmetry

  $\rightarrow$ New insights in understanding the interplay between gauge symmetry and the structure of massless amplitudes