Thermodynamics and transport properties of the quark-gluon plasma: recent advancements

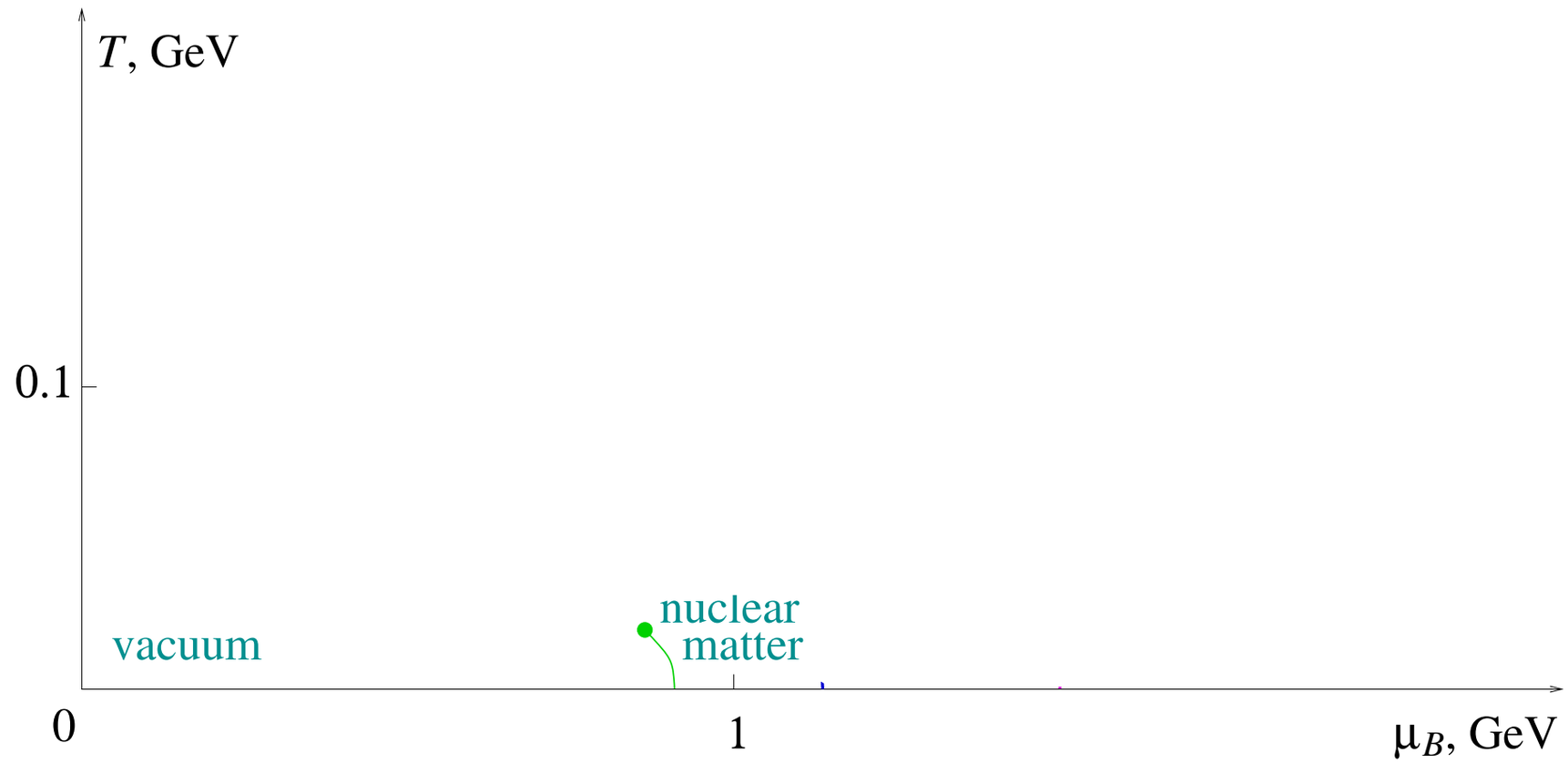
Jacopo Ghiglieri, SUBATECH, Nantes

RPP, École polytechnique, January 29 2020

Overview

The phase diagram of QCD

- In the temperature/baryon chemical potential plane:

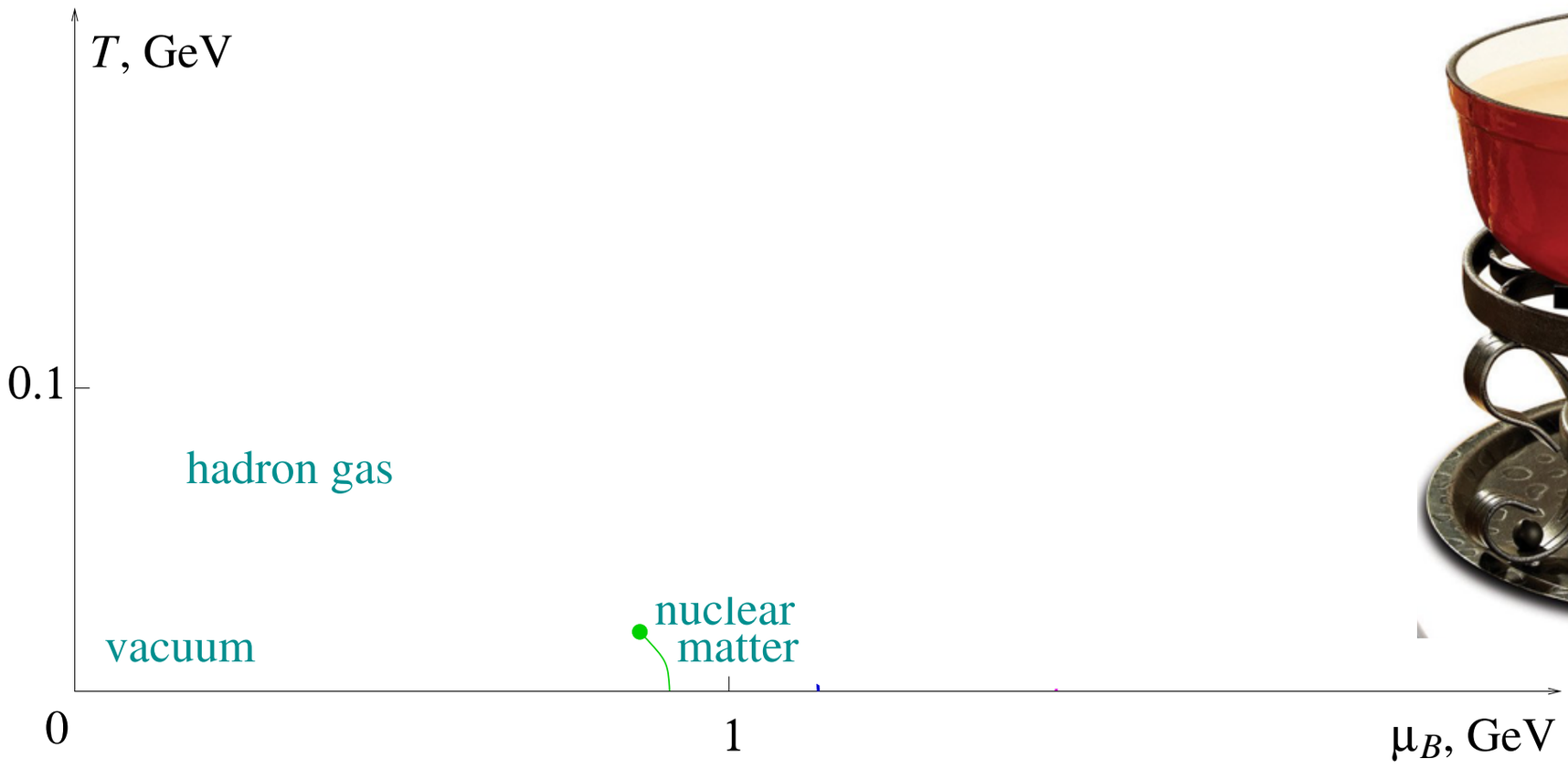


- At low temperature and moderate densities: ordinary hadrons and nuclear matter.

- Colour confinement

The quark-gluon plasma

- As the temperature is increased:



The quark-gluon plasma

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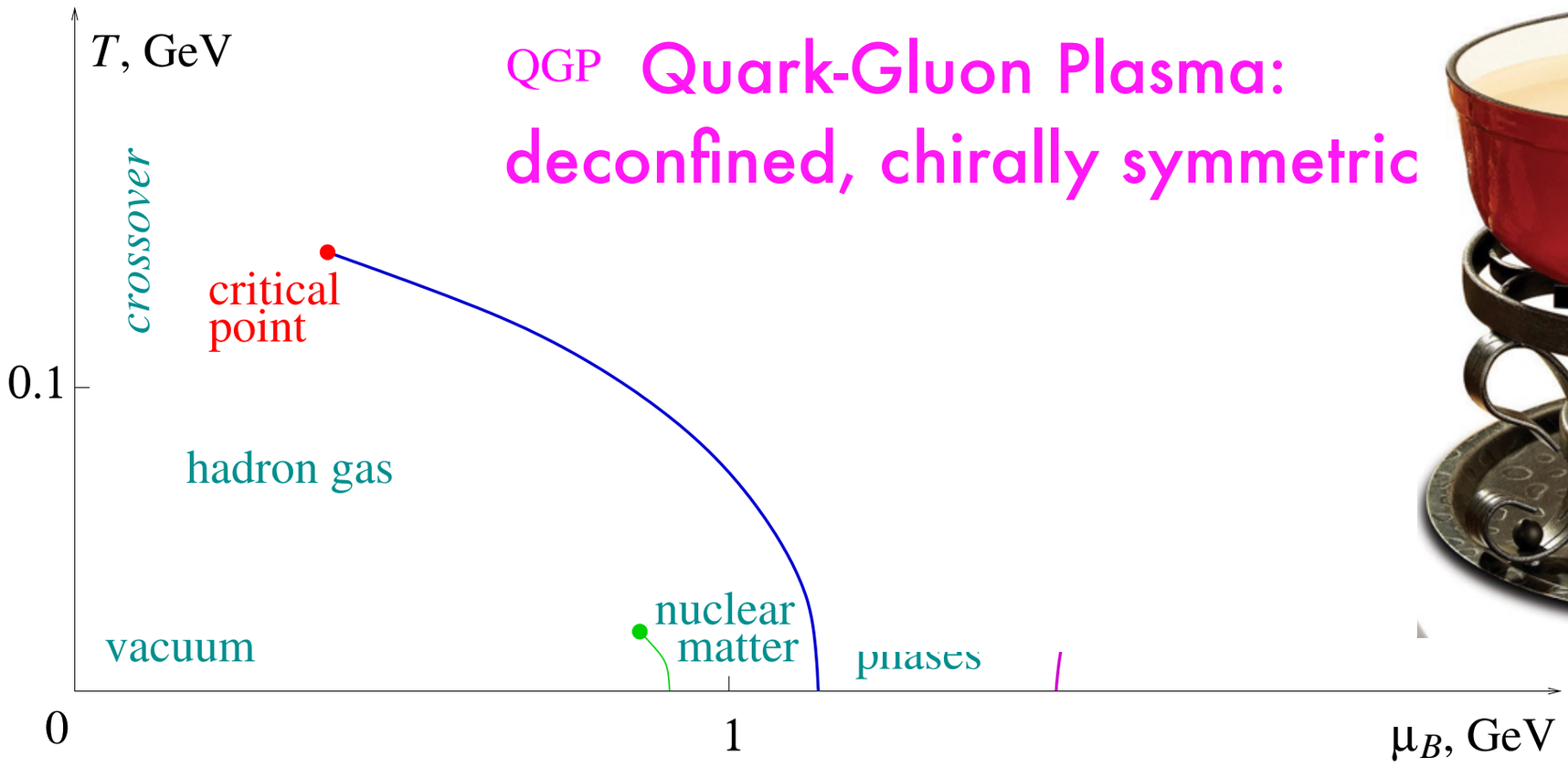

QGP Quark-Gluon Plasma: deconfined, chirally symmetric

- In the upper-left region, lattice QCD indicates a (pseudo)critical temperature T_c~150 MeV ~2x10^{12} K
- For comparison, sun's core: T~1.5x10^{7} K

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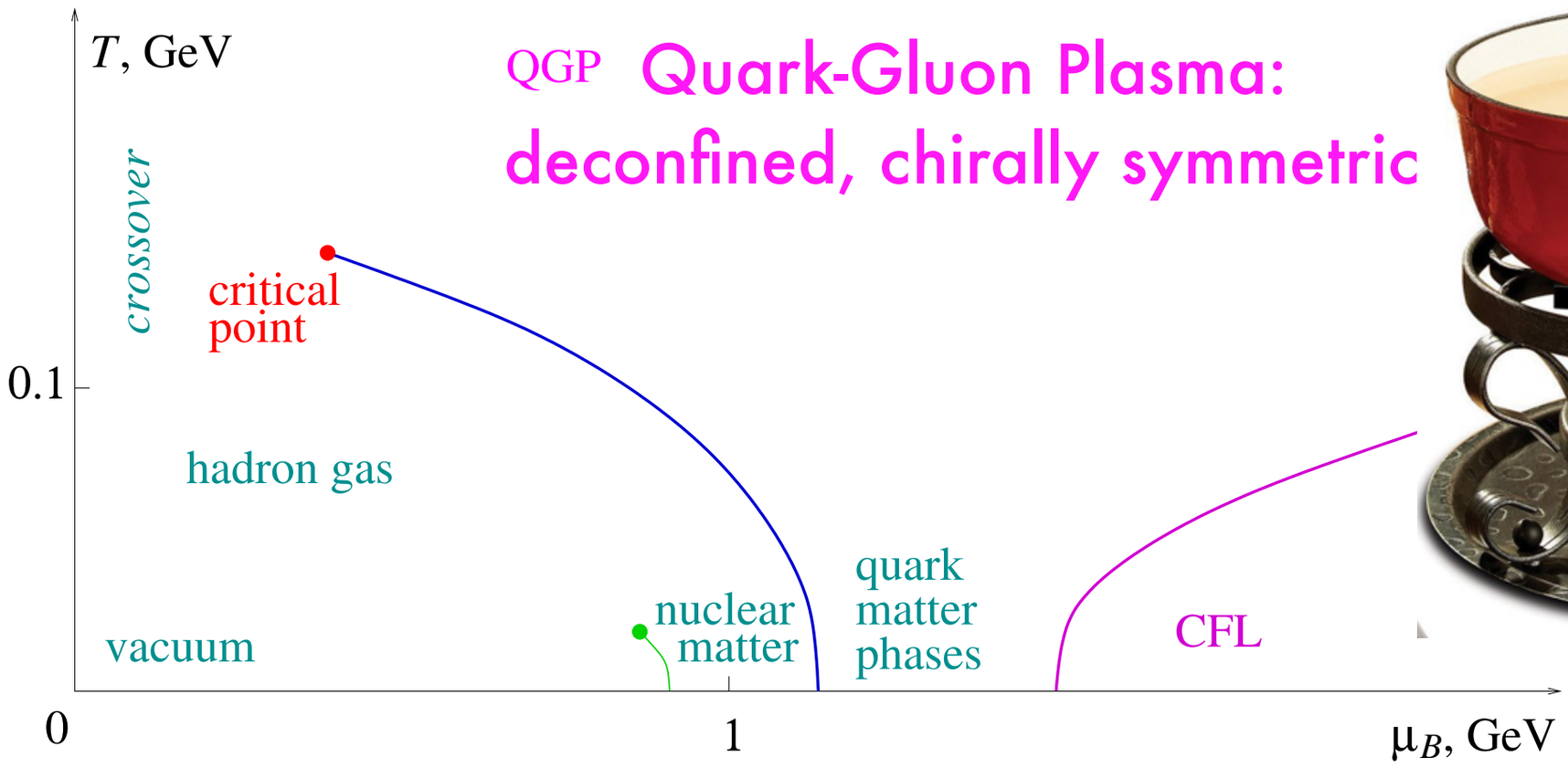

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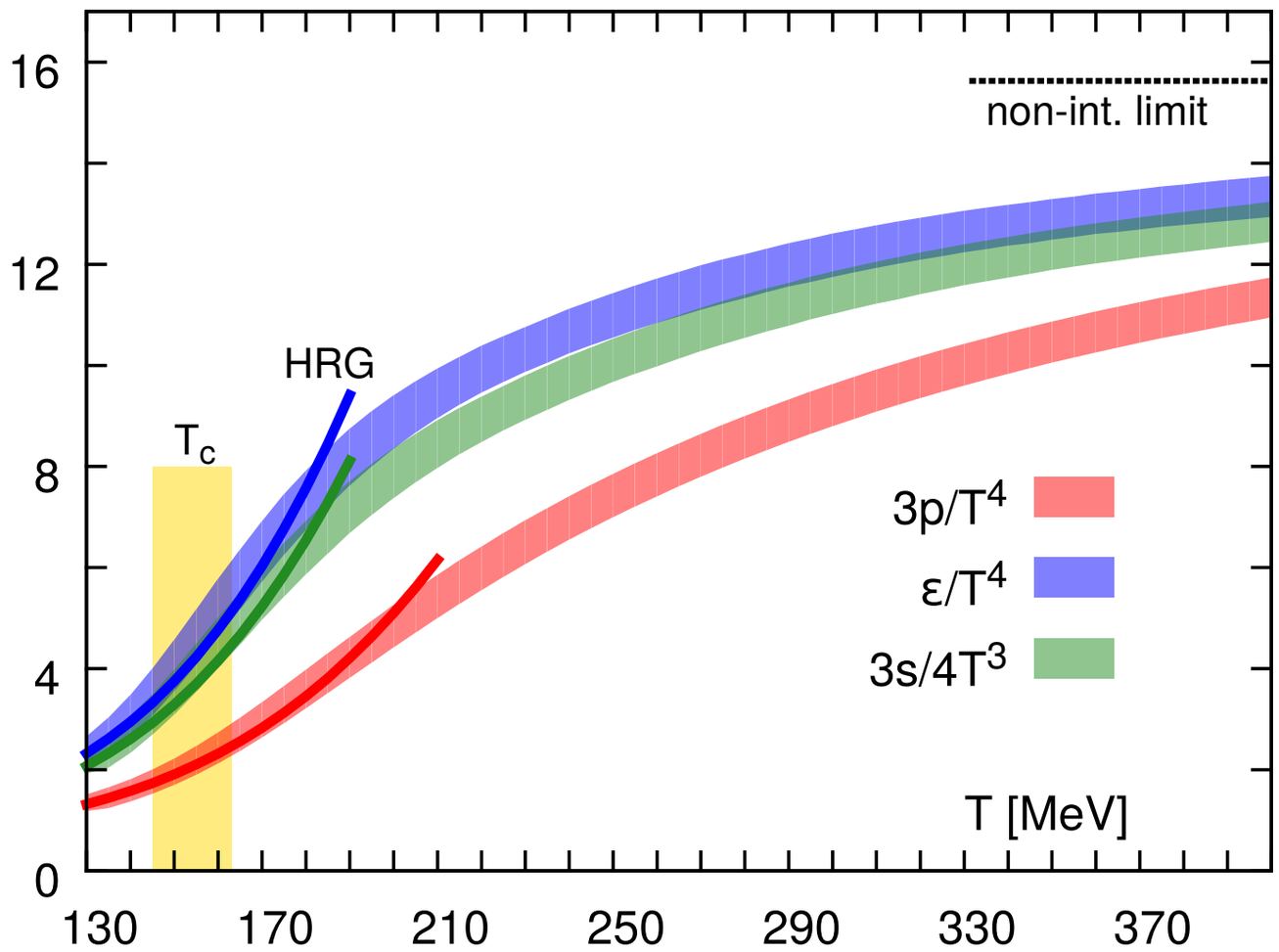
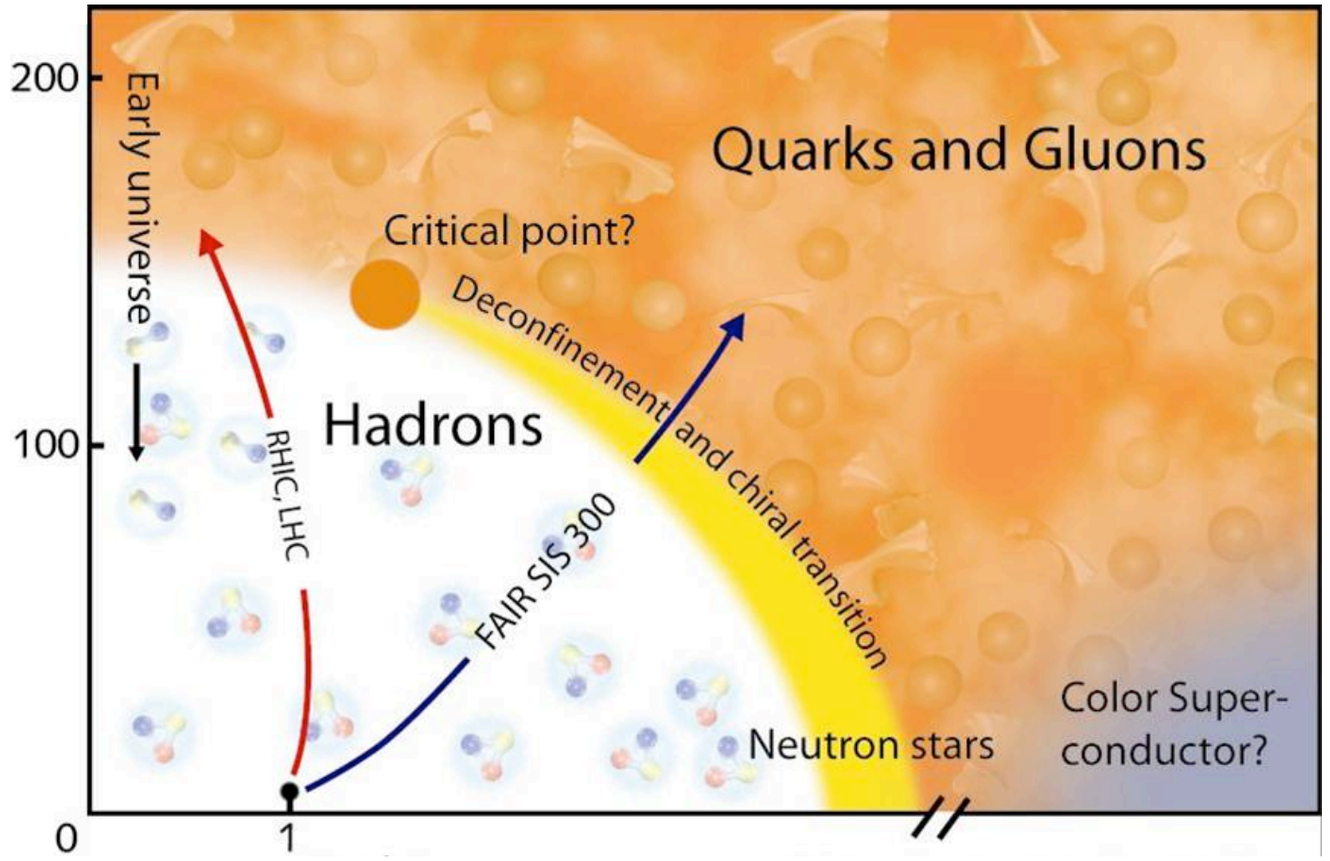


HotQCD coll. **PRD90** (2014)

$$\varepsilon \overset{\text{non-int}}{=} \frac{N_{\text{dof}}\pi^2}{15}T^4 \overset{T\approx 400\,\text{MeV}}{=} 50\frac{\text{GeV}}{\text{fm}^3}$$

Heavy-ion collisions

- A (transient) QGP can be formed in heavy ion collision experiments. RHIC (@BNL), up to $\sqrt{s_{NN}}$=200GeV. LHC up to $\sqrt{s_{NN}}$=5.5 TeV (5 so far).



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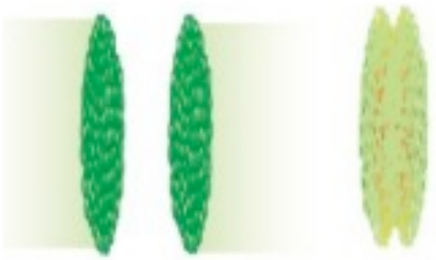

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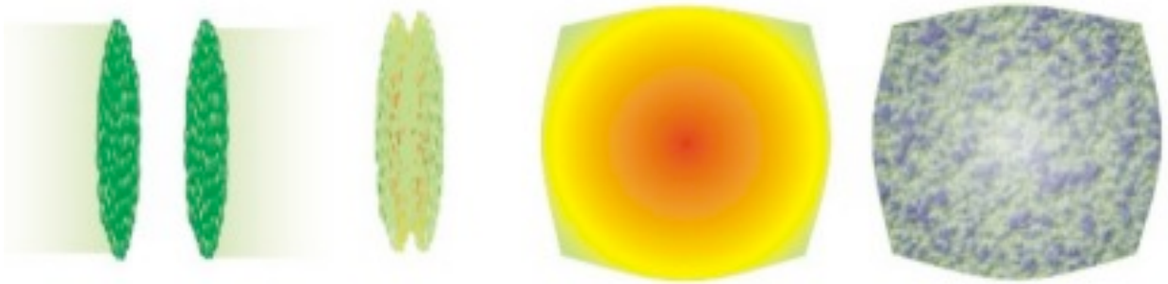

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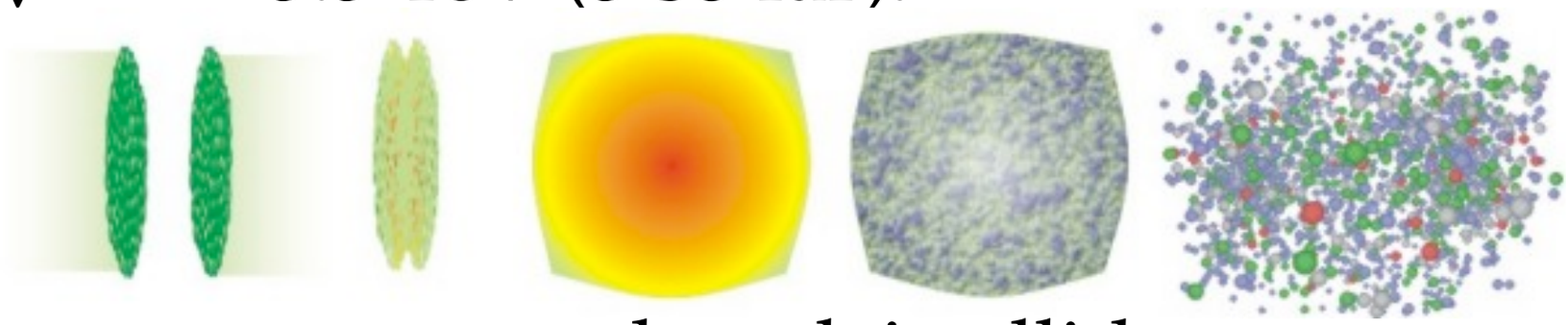

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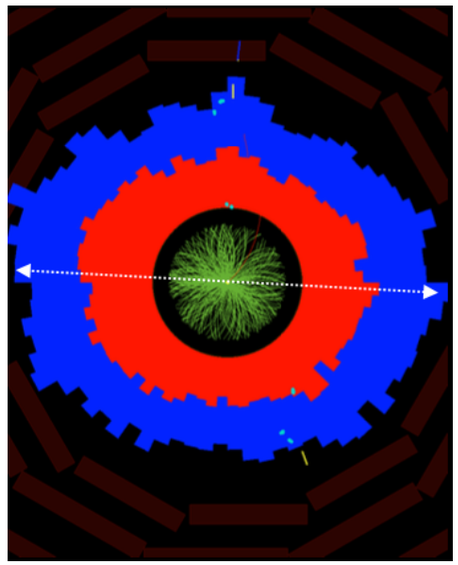
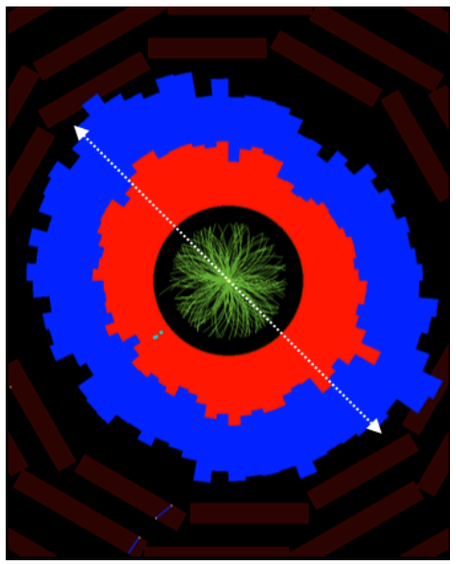
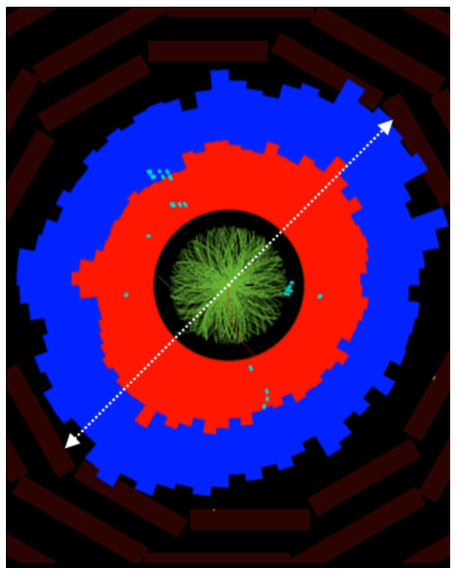
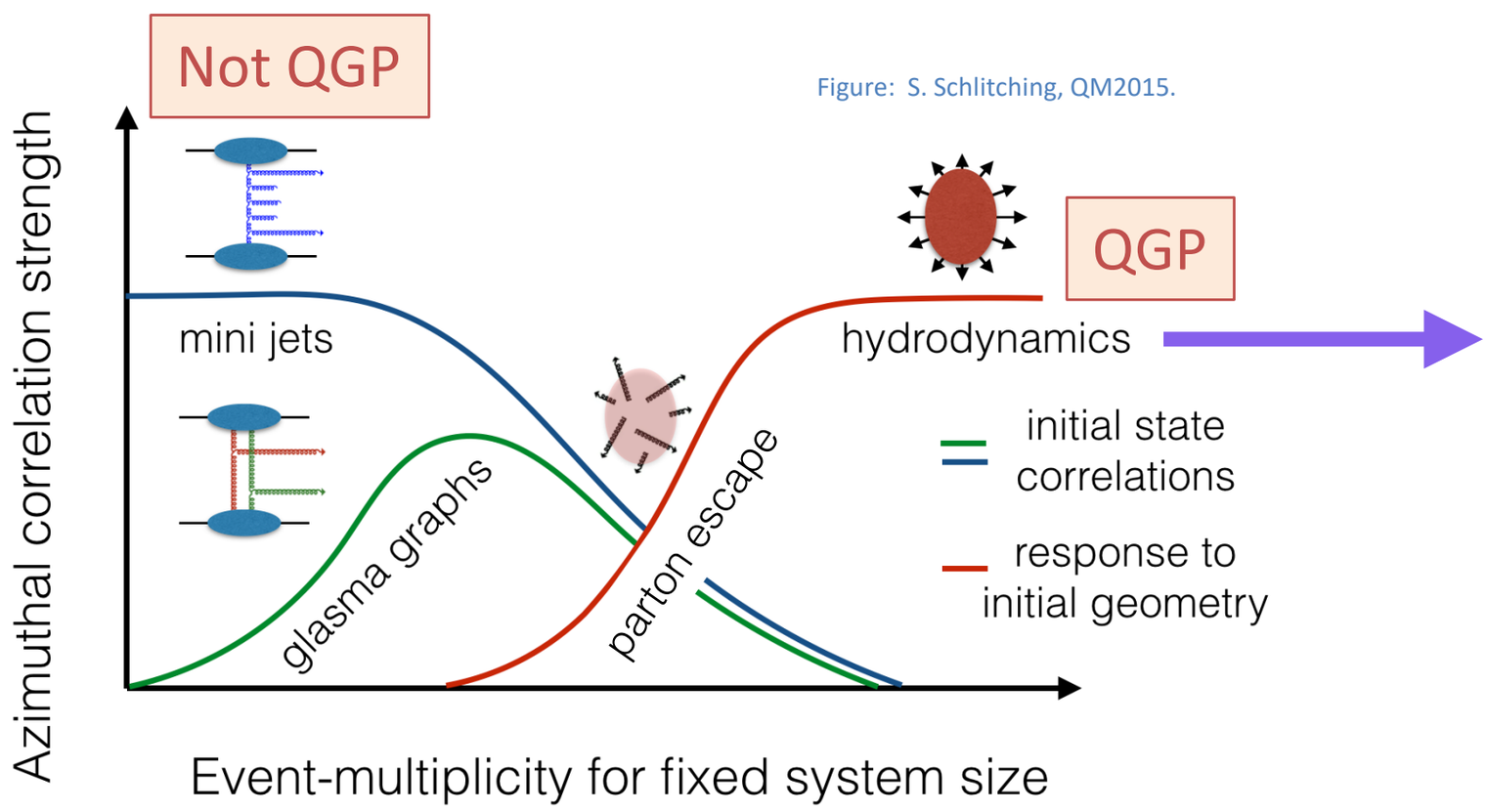
- Rapid formation of a near-thermal QGP (~1 fm/c)
- Expansion and cooling for up to 5-10 fm/c, then
- Hadronizazion
- Lots of particles (dN_{ch}/dy $O(1000)$) stream to the detectors

Azimuthal anisotropies





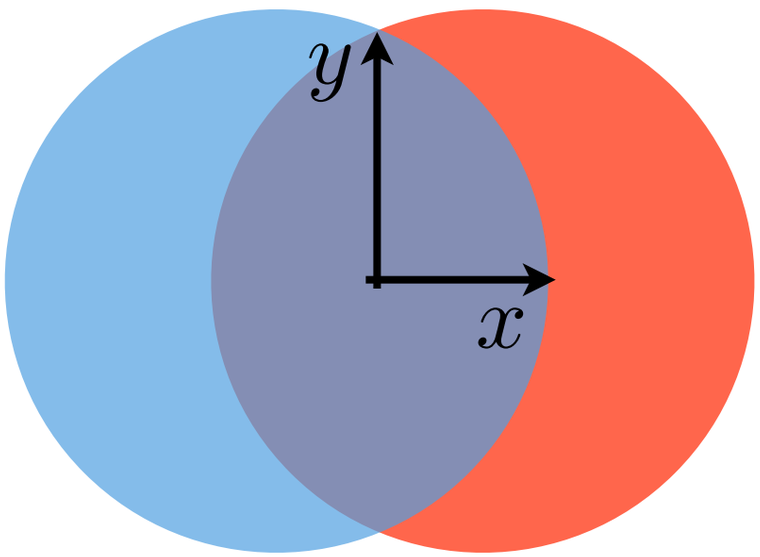




Expanding around infinite collisions in final state

A famous example:elliptic flow

Initial asymmetry



Beam along z

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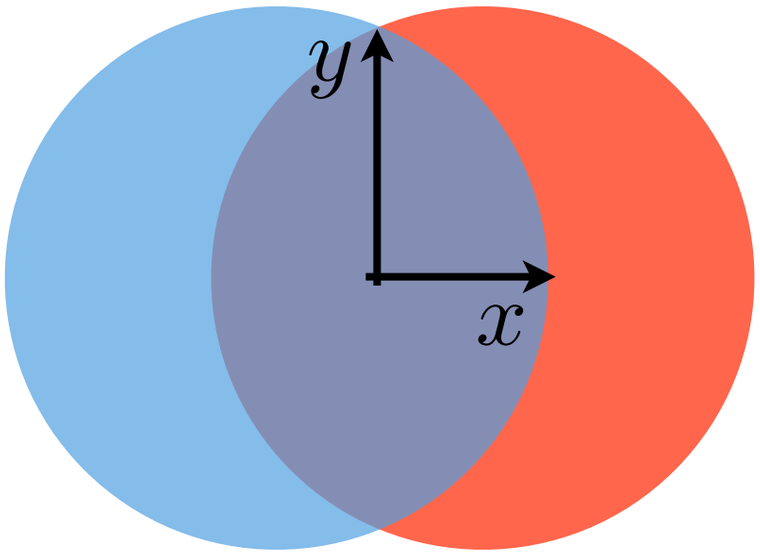
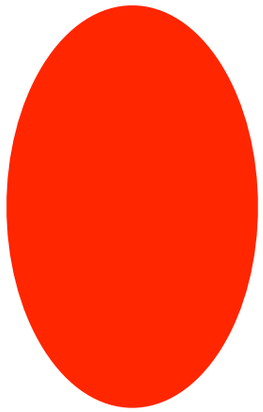


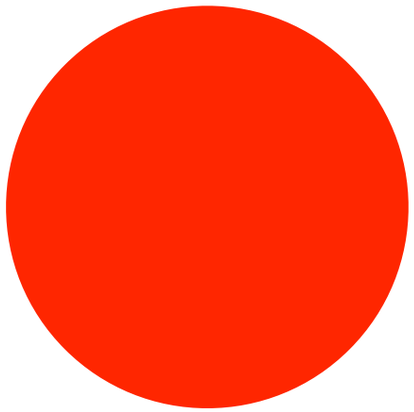
Beam along z

Position space



Large pressure gradients

Momentum space



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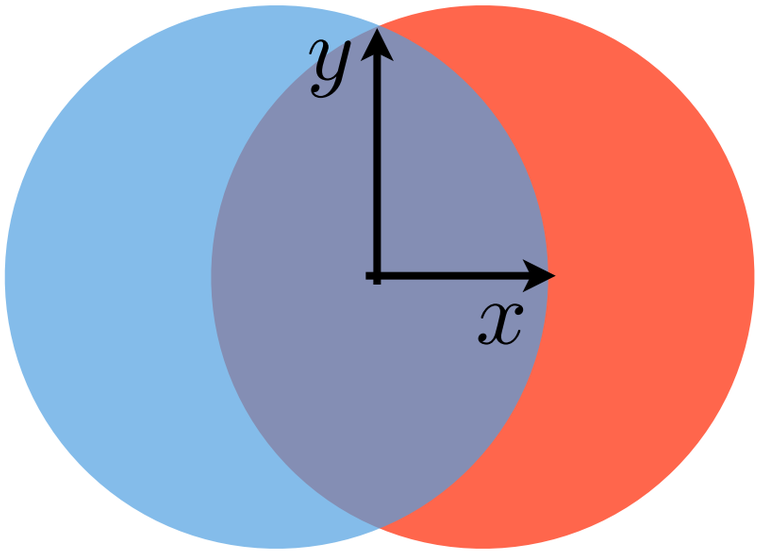

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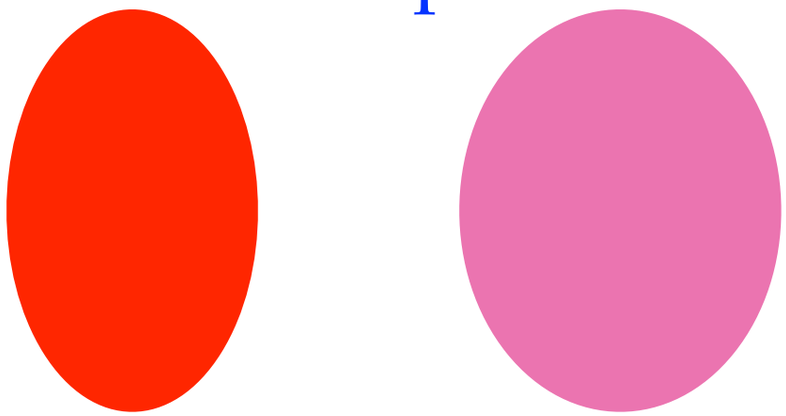

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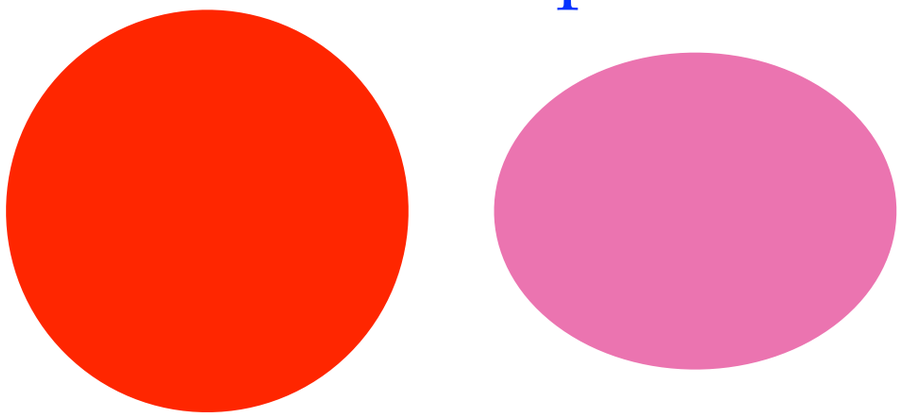

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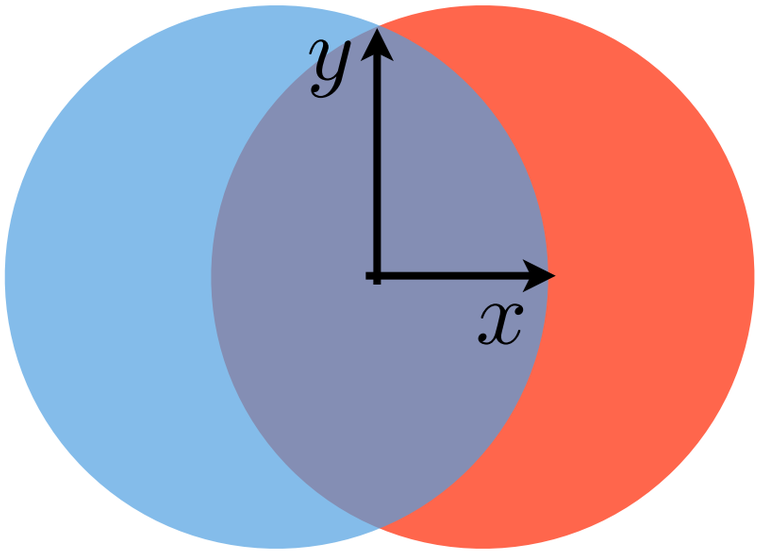

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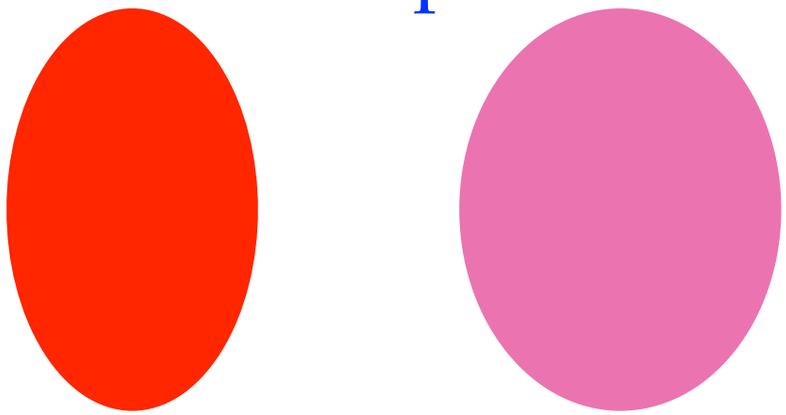

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No more flow

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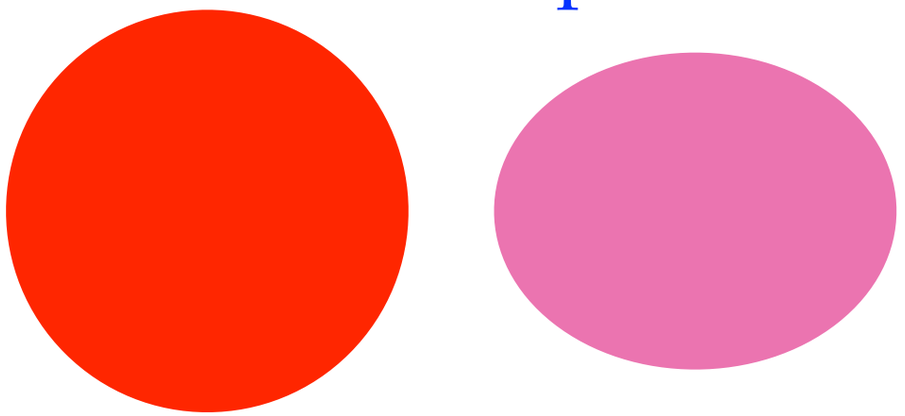

From initial symmetry

to final fixed anisotropy

A famous example:elliptic flow

Position space

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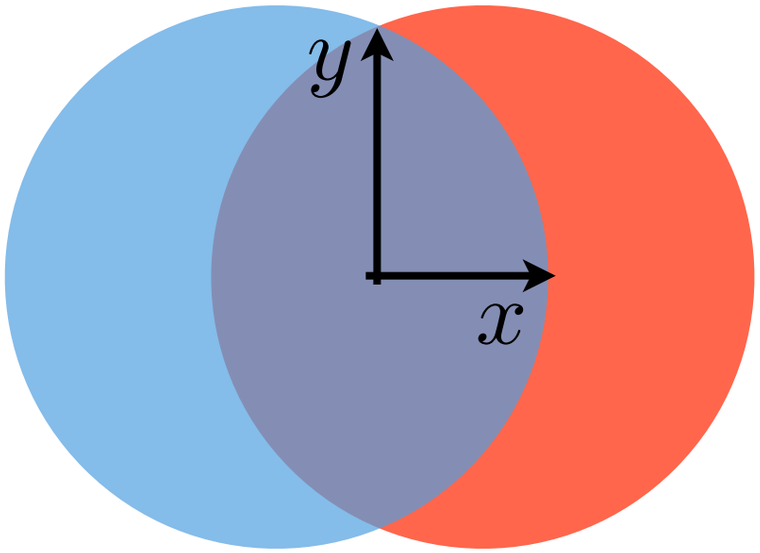

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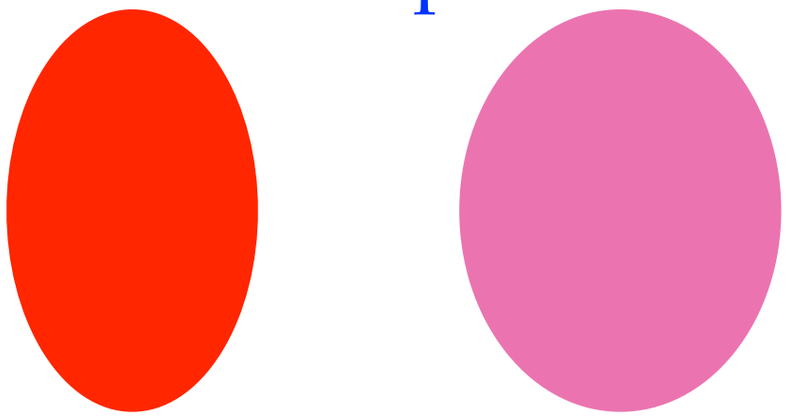

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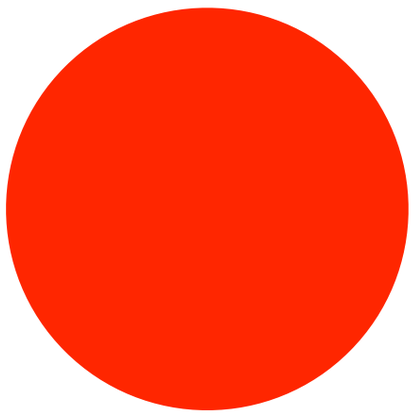

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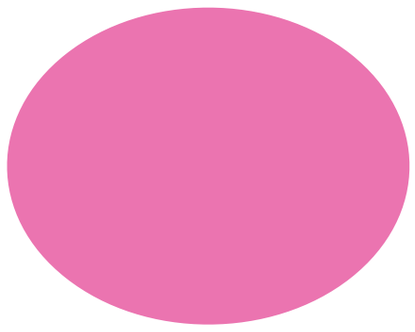
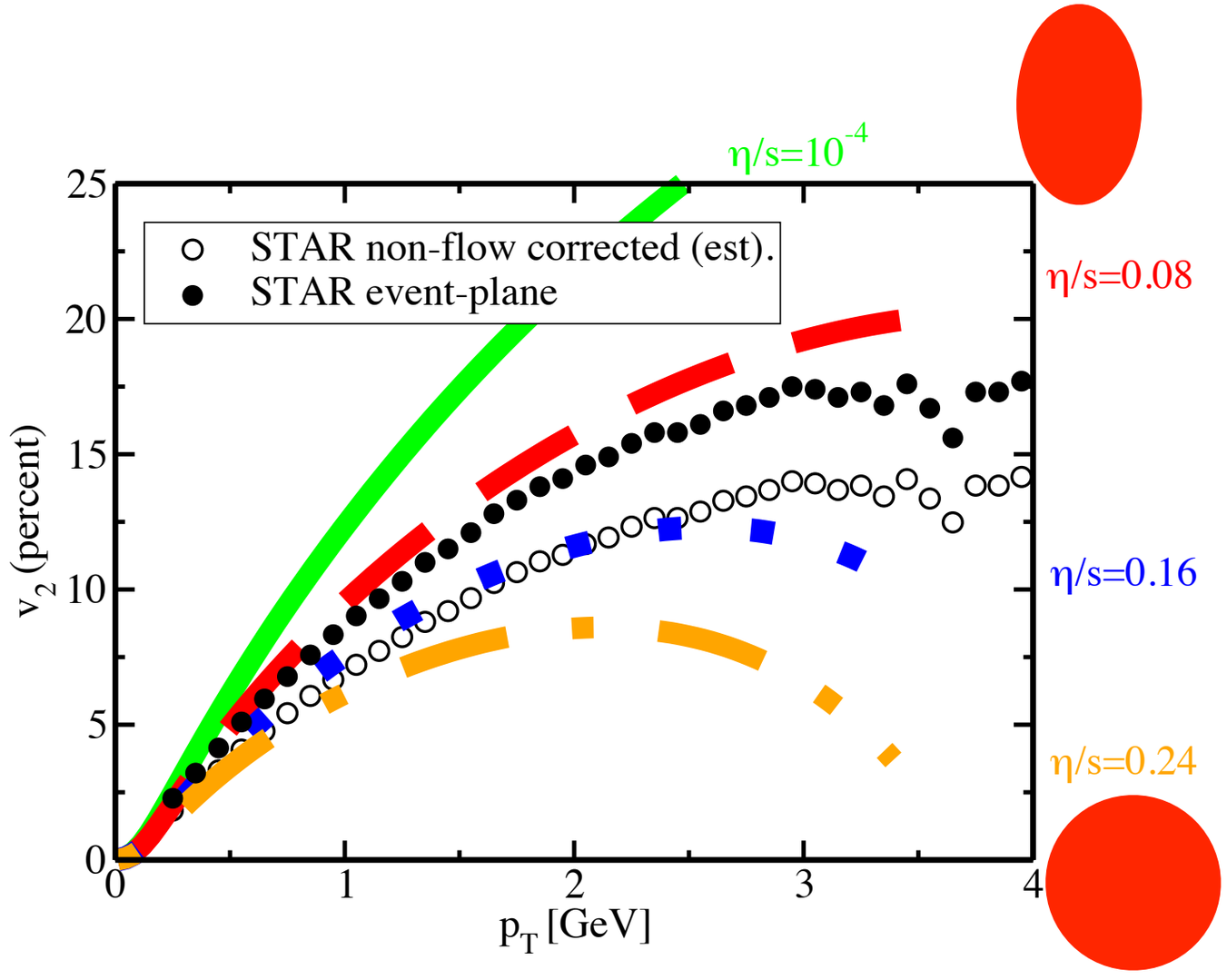


to final fixed anisotropy

- Hydrodynamics describes the buildup of flow. The shear viscosity parametrizes the efficiency of the conversion

A famous example:elliptic flow

Hydrodynamics parametrized by *specific shear viscosity* η/s



Luzum Romatschke **PRC78** (2008)

Hydrodynamics

Hydrodynamics

- Field theories admit a IR limit: Hydrodynamics. Effective Theory based on a gradient expansion of the flow velocity

- For hydro fluctuations with local flow velocity **v** around an equilibrium state (with temp. T), at **first order** in the gradients and in **v**

$$T^{00} = e, \qquad T^{0i} = (e+p)v^i$$

$$T^{ij} = (p - \zeta \nabla\cdot\mathbf{v})\delta^{ij} - \eta\left(\partial_i v^j + \partial_j v^i - \frac{2}{3}\delta^{ij}\nabla\cdot\mathbf{v}\right)$$

Navier-Stokes hydro, two *transport coefficients*: bulk and shear viscosity. Need also the (local) equilibrium Equation of State to relate e, p and T

Estimating η
(or why is η/s natural)

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- Rewrite the first-order T as

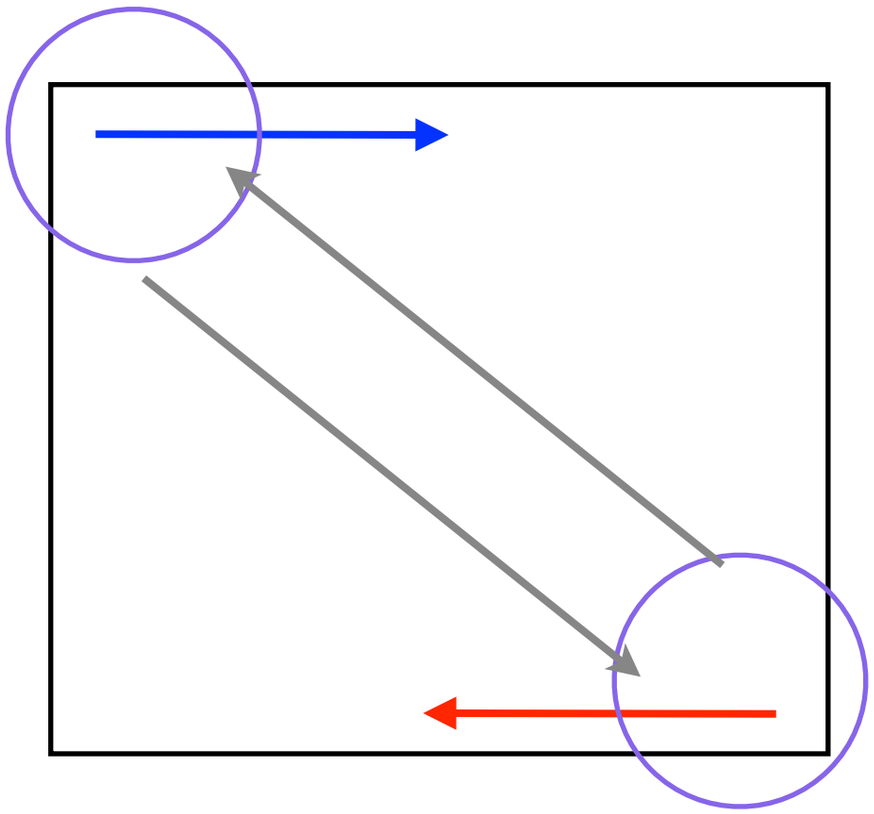
$$T^{xy} = -\frac{\eta}{e+p} \nabla_x T^{0y}$$

- $\eta/(e+p)$ is a (first-order) relaxation timescale. With $e+p=sT$ one gets to the natural, dimensionless η/s (transport / thermodynamics)

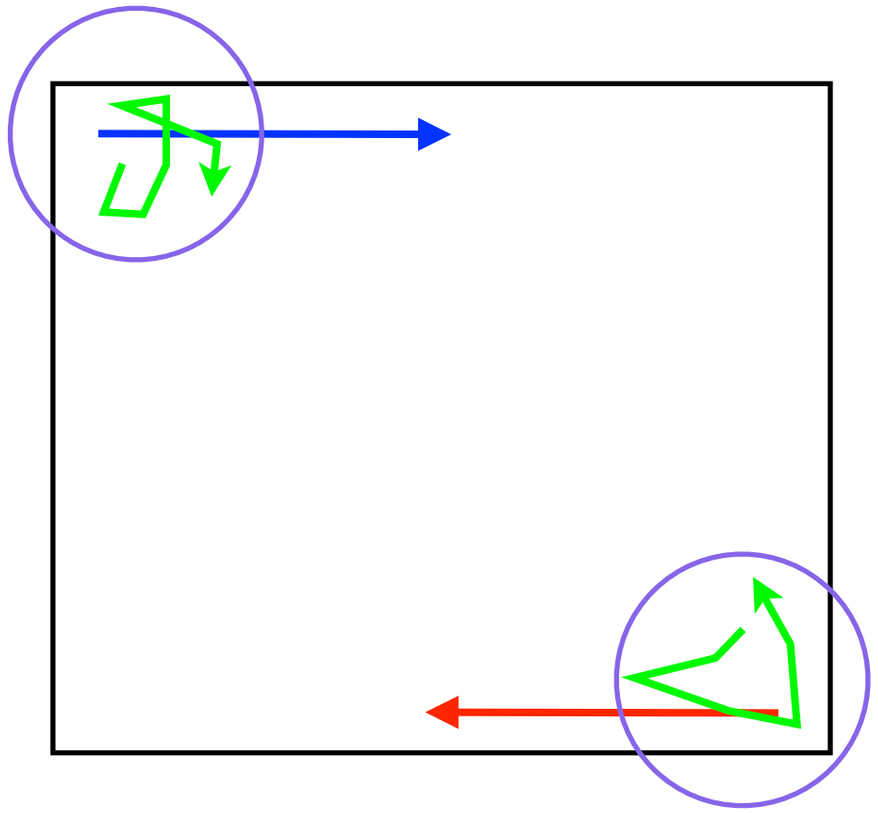
- In cases with well-defined quasi-particles one has naturally

$$\frac{\eta}{s} \sim T l_{\mathrm{mfp}}$$

Estimating η
(or why is η/s natural)



- Weak coupling: long distances between collisions, easy diffusion. **Large** η/s



- Strong coupling: short distances between collisions, little diffusion. **Small** η/s

Estimating η
(or why is η/s natural)

- (Mean free path)$^{-1}$~ cross section x density

$$\frac{\eta}{s} \sim T l_{\rm mfp} \sim \frac{T}{n\sigma} \sim \frac{1}{T^2\sigma}$$

- Cross section in a perturbative gauge theory (T only scale*)

$$\sigma \sim \frac{g^4}{T^2} \qquad \frac{\eta}{s} \sim \frac{1}{g^4}$$

* Coulomb divergences and screening scales ($m_D \sim gT$) in gauge theories

$$\sigma \sim \frac{g^4}{T^2}\ln(1/g) \qquad \frac{\eta}{s} \sim \frac{1}{g^4\ln(1/g)}$$

- From holography one instead has $\eta/s = 1/(4\pi)$ (for $\mathcal{N}=4$ SYM) and a conjectured lower limit

Kovtun Son Starinets Policastro **PRL87** (2001) **PLR94** (2004)

Theory approaches to hot QCD

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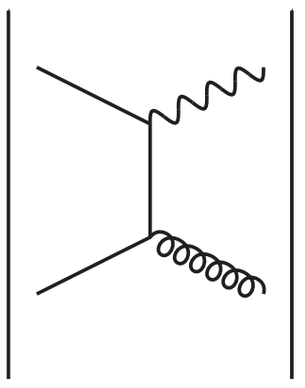
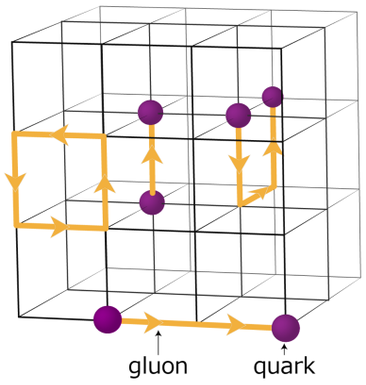
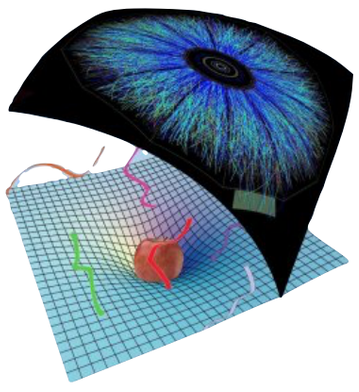


pQCD: QCD action (and EFTs and kinetic theories thereof). Real world: extrapolate from $g \ll 1$ to $\alpha_s \sim 0.3$



lattice QCD: Euclidean QCD action. Real world: analytically continue to Minkowskian domain, sign problem at finite density



AdS/CFT: $\mathcal{N}=4$ action, weak and strong coupling. Real world: extrapolate to QCD

In this talk

- Recent perturbative approaches to thermodynamics and transport coefficients
  - Weak-coupling picture and scale separation
  - Thermodynamics
  - Kinetics and transport coefficients
- Self-advertisement

  Perturbative Thermal QCD: Formalism and Applications
  JG Kurkela Strickland Vuorinen, to be submitted to *Physics Reports,* on the arXiv in February

The weak coupling picture

The weak-coupling picture

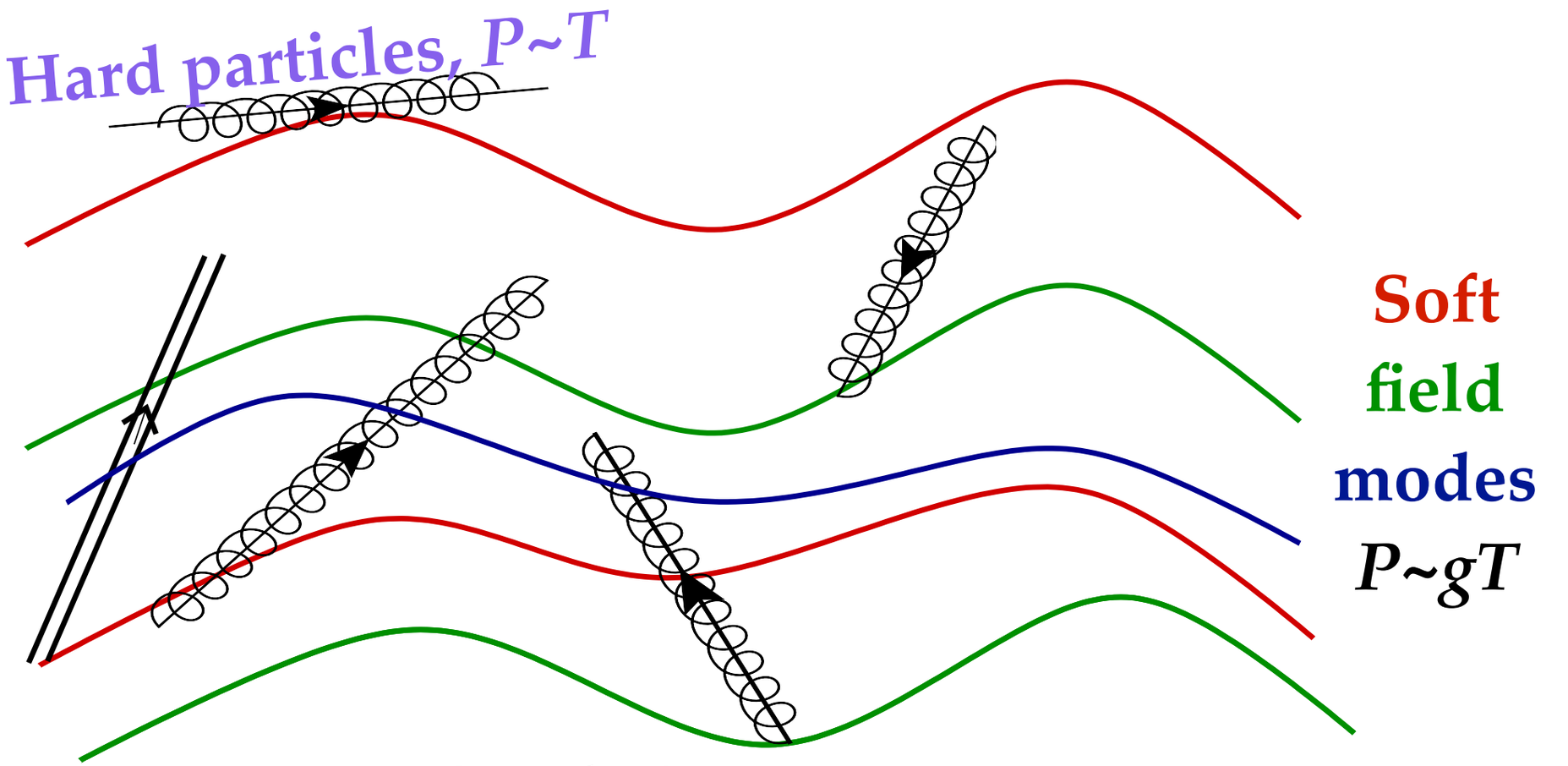


Figure by D. Teaney

- Hard (quasi)-particles carry most of the stress-energy tensor. (Parametrically) largest contribution to thermodynamics

The weak-coupling picture

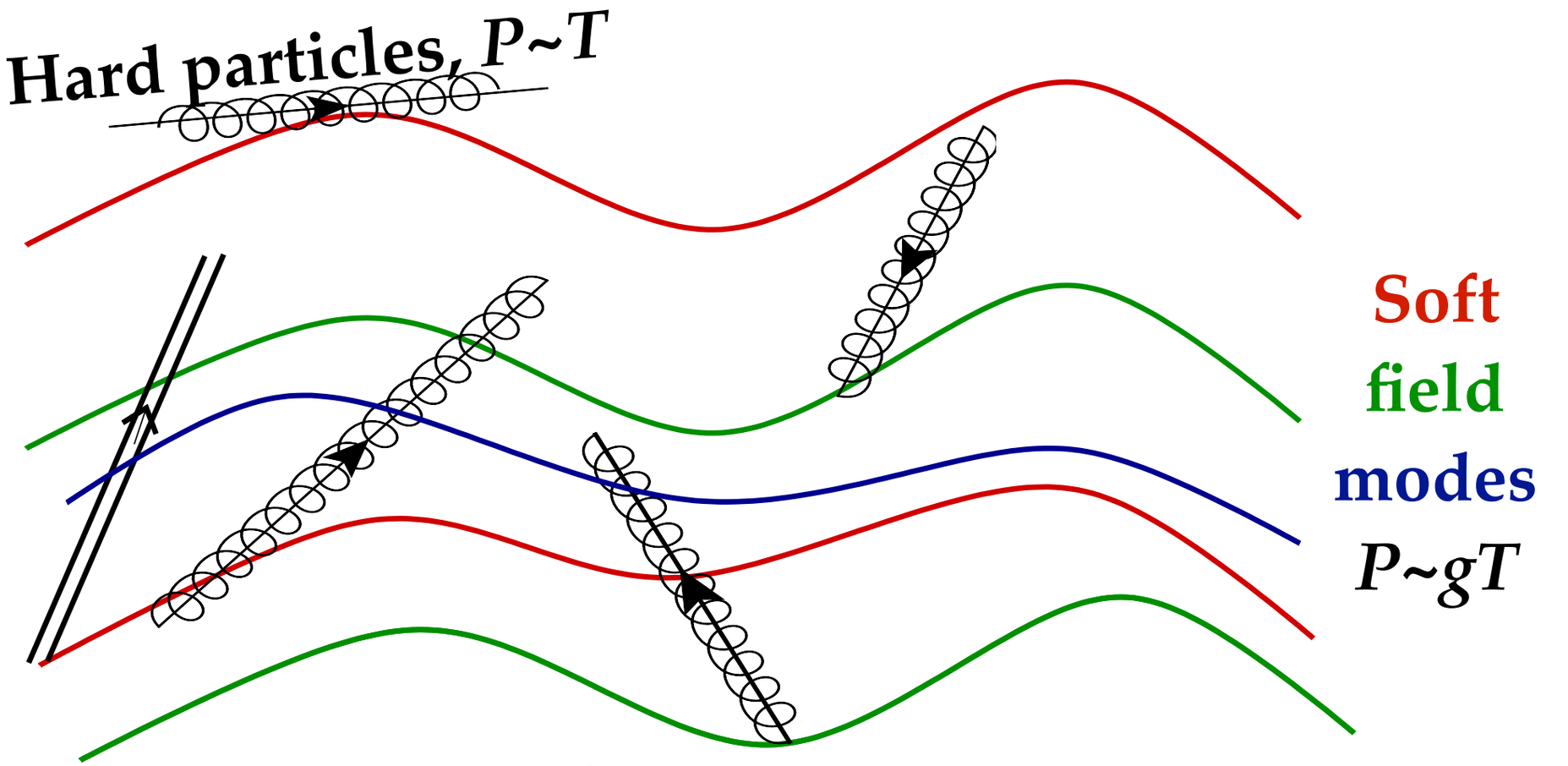


Figure by D. Teaney

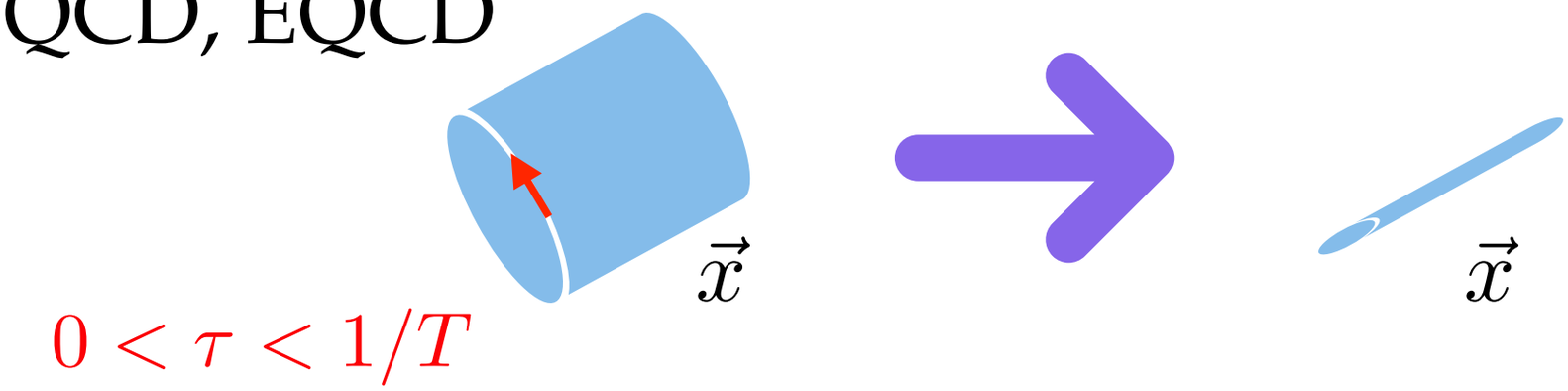
- The gluonic soft fields have large occupation numbers ⇒ they can be treated classically

$$n_{\rm B}(\omega) = \frac{1}{e^{\omega/T}-1} \overset{\omega\sim gT}{\simeq} \frac{T}{\omega} \sim \frac{1}{g}$$

Weak-coupling thermodynamics:
two approaches

1) Dimensional reduction (DR)

- Integrate out the temperature scale to obtain a 3D EFT of QCD, EQCD



d.o.f.s: 3D Yang-Mills and A_0 massive (~gT) adjoint scalar

- Integrate out gT to obtain MQCD, 3D Yang-Mills only, non-perturbative. Thermodynamics receives non-perturbative contribution at $O(g^6)$. EQCD also solvable on the lattice

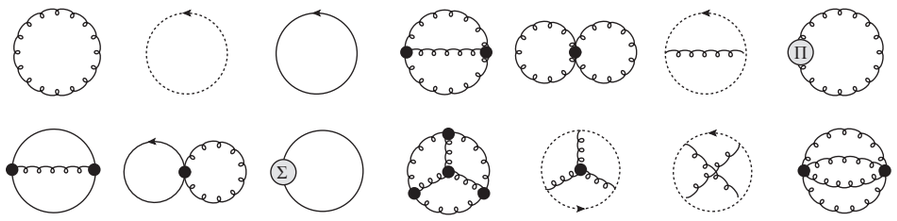
Braaten Nieto, Kajantie Laine Rummukainen Shaposhnikov (1995-97)

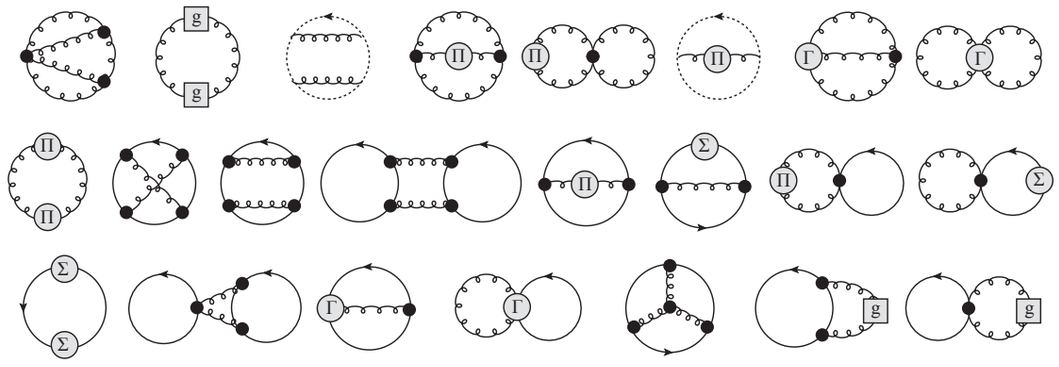
Weak-coupling thermodynamics:
two approaches

2) Hard Thermal Loop perturbation theory (HTLpt)

- Change picture: shift from expanding around an ideal gas of massless quarks and gluons, to around massive quasiparticles, incorporating the dispersion corrections from Hard Thermal Loops

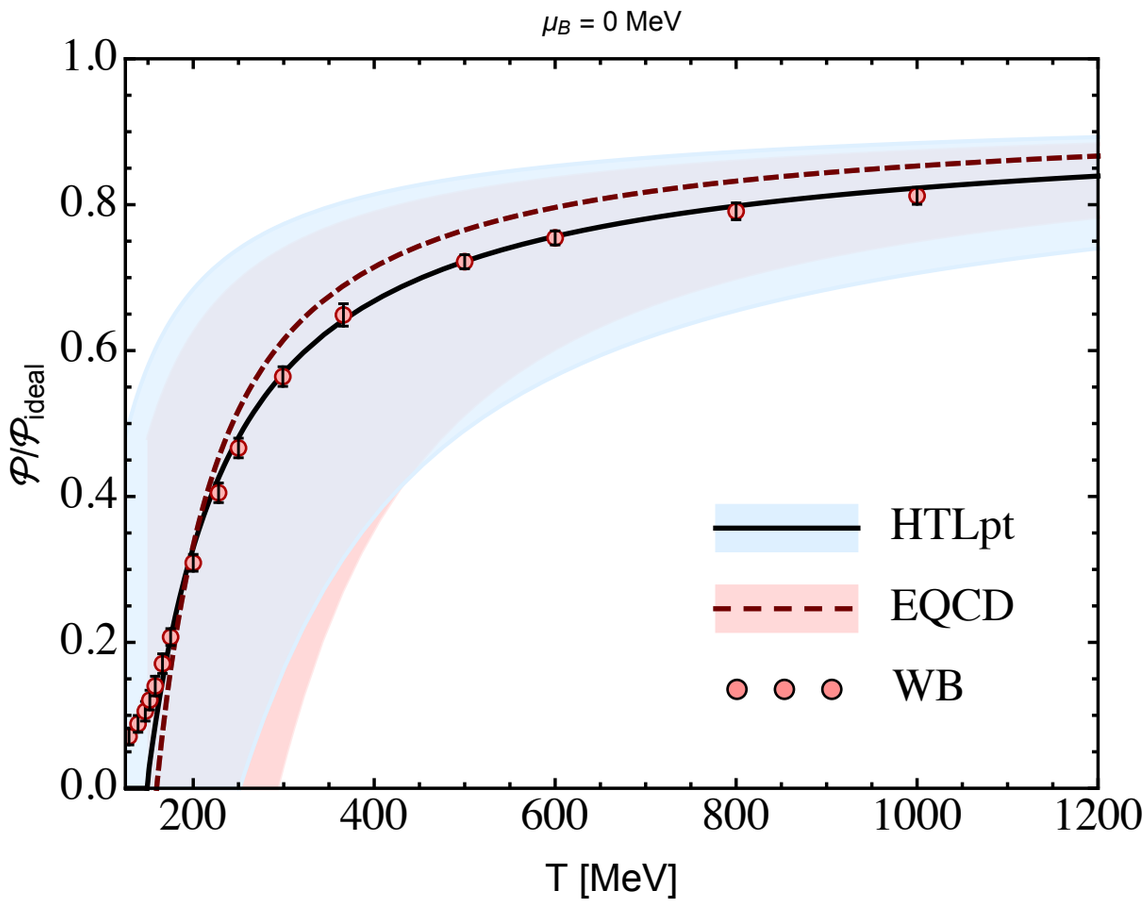
- Hard Thermal Loops (Braaten Pisarski ,1989): gauge-invariant thermal amplitudes for soft (gT) external legs

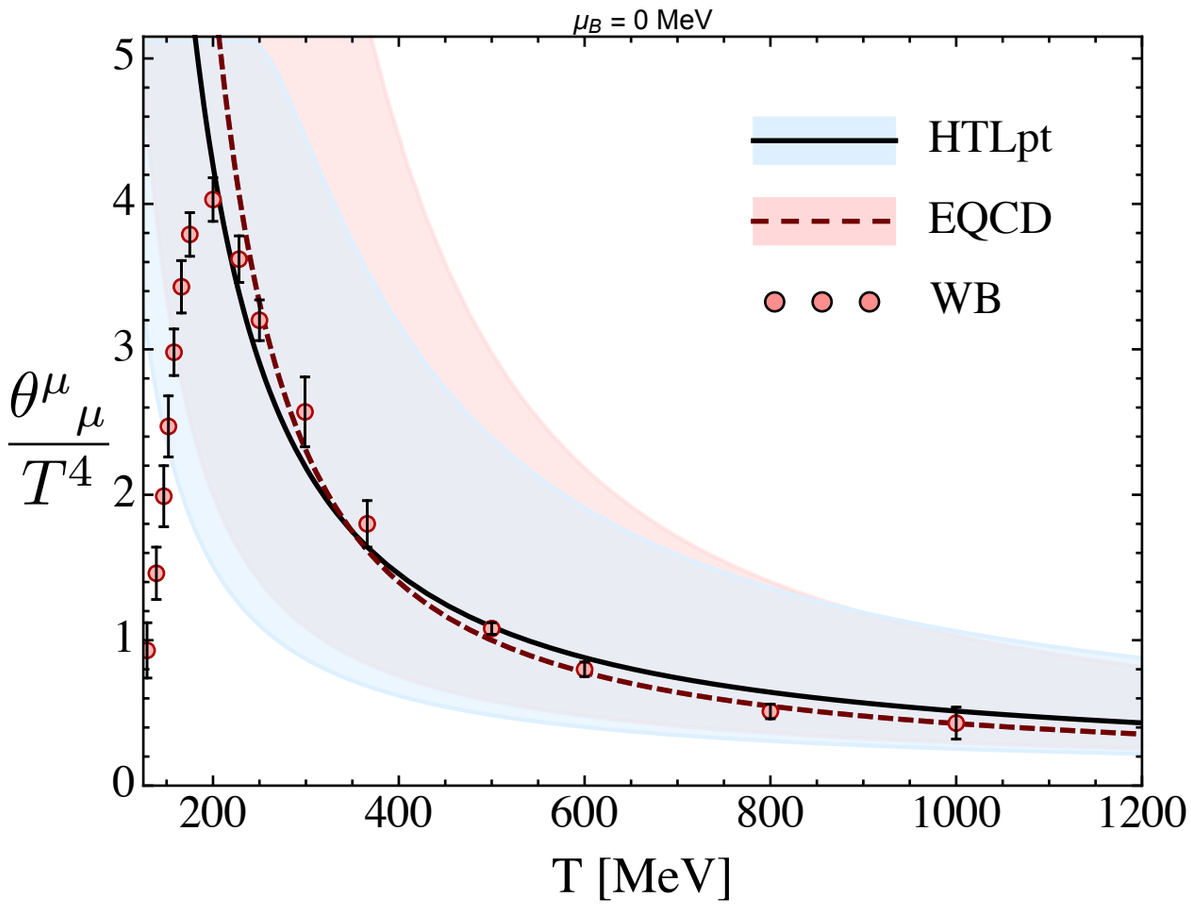




Andersen Braaten Strickland (1999), Andersen Strickland *et al* (2000-2015)

Weak-coupling thermodynamics





Review: JG Kurkela Strickland Vuorinen **Phys. Rep.** (in preparation)

- Both HTLpt and DR (EQCD) have reached 3-loop order. Can we understand how to set up similar resummation programs for kinetics (transport)?

Weak coupling transport and kinetics

- How to study the weak-coupling picture when real (Minkowski) time matters?
- One main tool: the Effective Kinetic Theory
- A long story with many motivations and applications

Baym Braaten Pisarski Arnold Moore Yaffe Baier Dokshitzer Mueller Schiff Son Peigné Armesto Salgado Wiedemann Gyulassy Wang Aurenche Gelis Zaraket Blaizot Iancu . . .

The effective kinetic theory

- The effective theory is obtained by integrating out (off-shell) quantum fluctuations

- Boltzmann equation for the single-particle phase space-distribution: its convective derivative equals a collision operator $(\partial_t + \mathbf{v_p} \cdot \boldsymbol{\nabla}) f(\mathbf{p}, \mathbf{x}, t) = C[f]$

- In other words at weak coupling the underlying QFT has well-defined quasi-particles. These are weakly interacting with a mean free time $(1/g^4 T)$ larger than the actual duration of an individual collision $(1/T)$

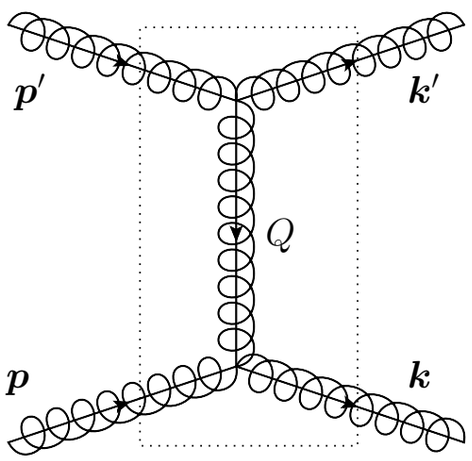
- Justified at weak coupling, can be extended to factor in non-perturbative contributions

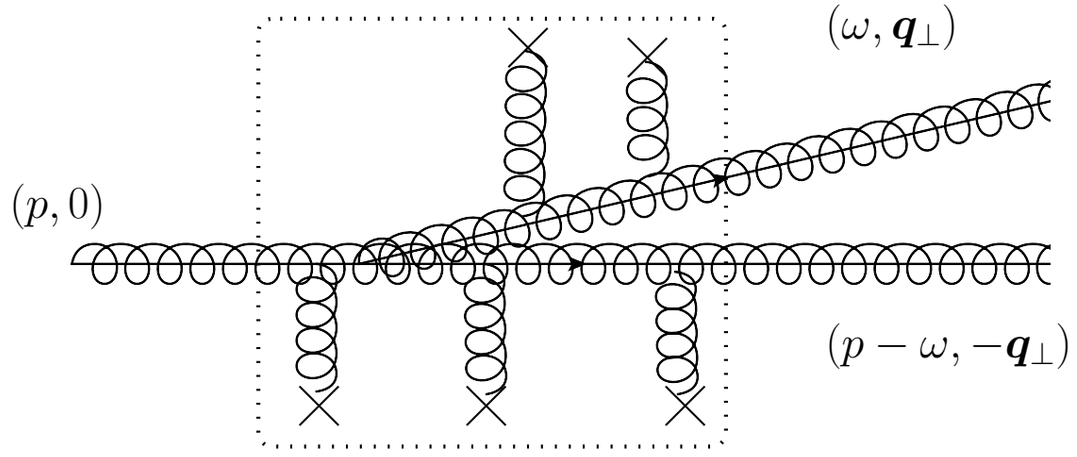
The AMY kinetic theory

- Effective Kinetic Theory (EKT) for the phase space density of quarks and gluons

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\right) f(\mathbf{p}) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$

- At **leading order**: elastic, number-preserving 2↔2 processes and collinear, number-changing effective 1↔2 processes





Arnold Moore Yaffe (AMY) **JHEP** (2002-2003)

Transport coeffs from the EKT

Transport coeffs from the EKT

- To obtain the transport coefficients linearize the theory

$$f(\mathbf{p}) = f_{\mathrm{EQ}}(\mathbf{p}) + \sum_{\ell} \delta f_{\ell}(\mathbf{p})$$

- Source term equates linearized collision operator

$$\mathcal{S}_{\ell} = \mathcal{C}\delta f_{\ell}$$

$$\mathcal{S}_{\ell} \equiv \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} \right) f_{\mathrm{EQ}}(\mathbf{p}, u, \beta, \mu)$$

- Since $\langle T^{i \neq j} \rangle \propto \eta$ η requires ℓ=2
- Transport coefficients obtained by the kinetic theory definitions of T once δf_{ℓ} has been obtained

Transport coeffs from the EKT

- To obtain the transport coefficients linearize the theory

$$f(\mathbf{p}) = f_{\rm EQ}(\mathbf{p}) + \sum_\ell \delta f_\ell(\mathbf{p})$$

- Source term equates linearized collision operator

$$\mathcal{S}_\ell = \mathcal{C}\delta f_\ell$$

- To solve the linear equation, introduce the inner product

$$(f,g) \equiv \int_{\mathbf{p}} f(\mathbf{p})\, g(\mathbf{p})$$

and minimize

$$(\delta f_\ell, \mathcal{S}_\ell) - \tfrac{1}{2}(\delta f_\ell, \mathcal{C} f_\ell)$$

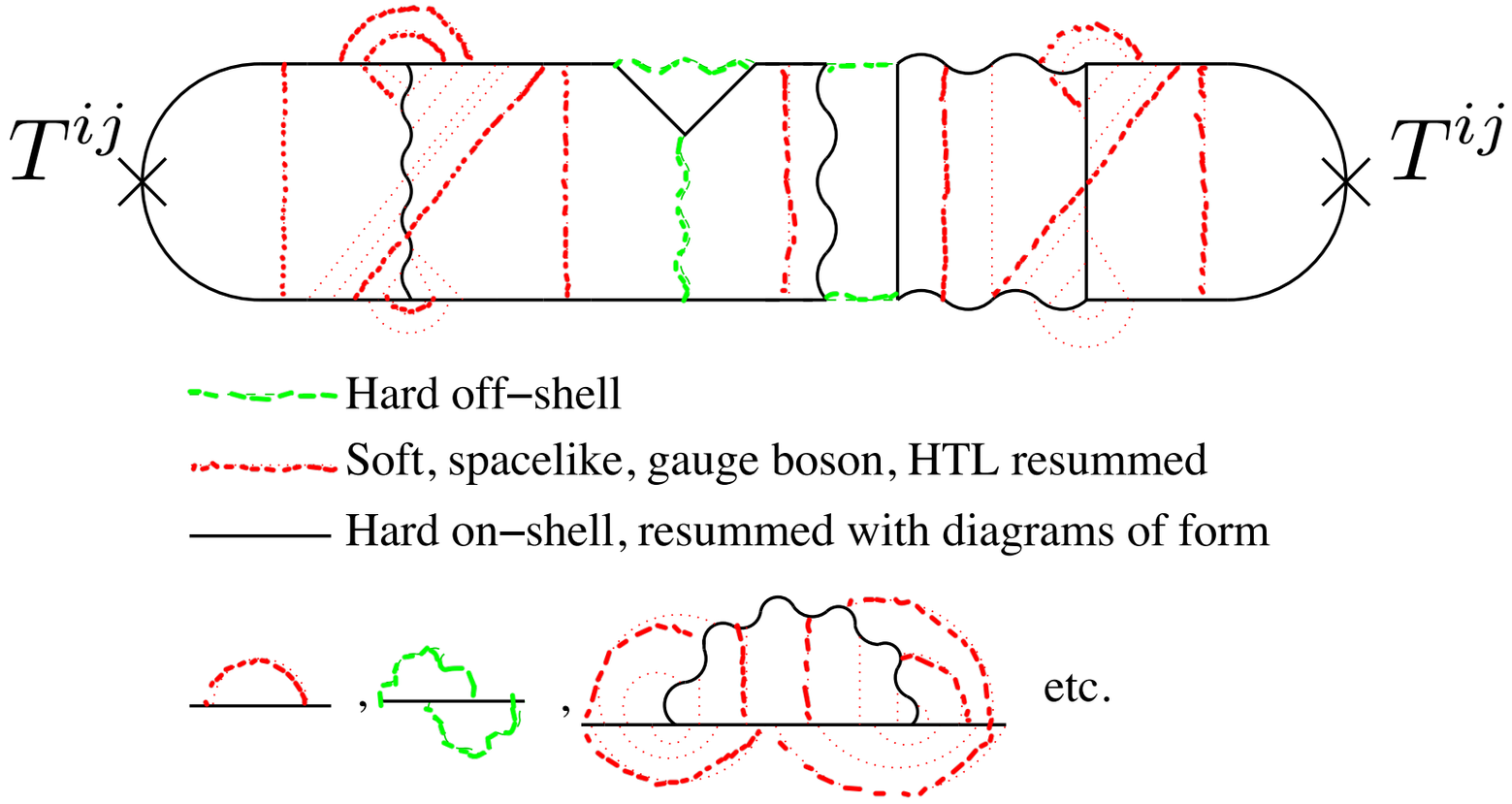
AMY (2000-03)

Transport coeffs from the EKT

- Linearized EKT equivalent to Kubo formula

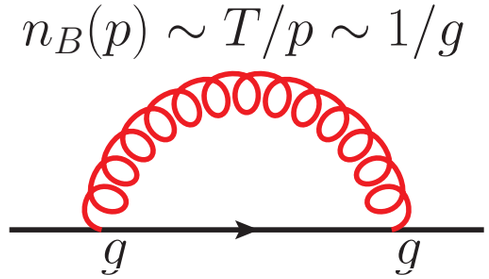
$$\eta = \frac{1}{20} \lim_{\omega \to 0} \frac{1}{\omega} \int d^4x \, e^{i\omega t} \left\langle \left[T^{ij}(t, \mathbf{x}), T^{ij}(0, \mathbf{0}) \right] \right\rangle \theta(t)$$

- Not practical at weak coupling: loop expansion breaks down AMY (2000-2003)

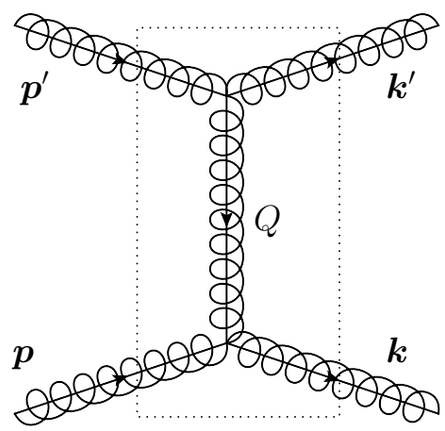


NLO kinetics and transport

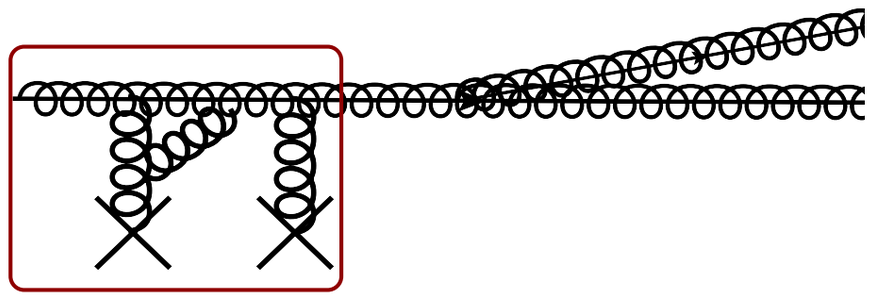
- The NLO $O(g)$ corrections come from soft gluons



- 2↔2 processes with soft gluon loop or soft gluon legs: in soft approx ($Q \ll p,k$) these are encoded in longitudinal and transverse momentum diffusion (up to a term that is unknown at NLO)



- 1↔2 processes with one-loop soft scatterings from the medium



JG Moore Teaney (2015-18)

NLO kinetics and transport

- In principle these are horrible brute-force Hard Thermal Loop computations

- Key advancement over the past decade: analytical properties of soft thermal amplitudes at light-like separations. Heuristically, the hard, light-like parton sees undisturbed soft modes, which *"can't keep up"* with it Caron-Huot **PRD82** (2008) *Review*: JG Teaney **QGP5** (2015)

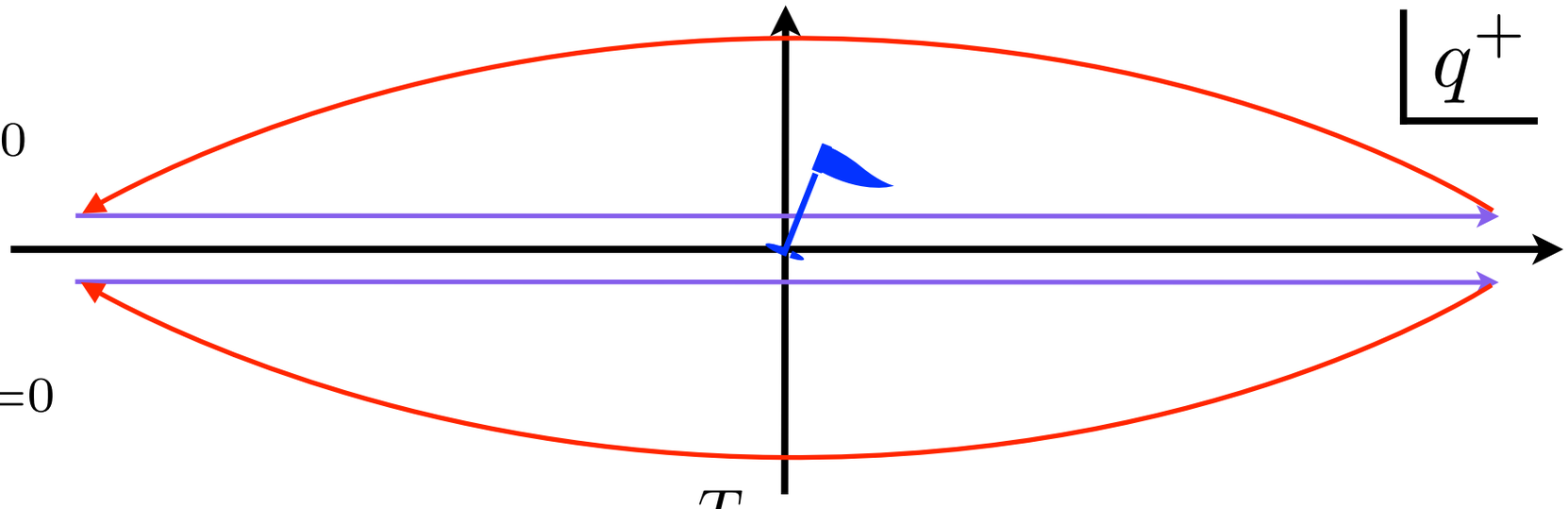
- In practice: tremendous simplification, analytical closed forms and possibility of non-perturbative input

JG Moore Teaney (2015-18)

Light-cone techniques

$$\int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^{-}_{\perp} \rangle_{q^-=0}$$

$$\int \frac{dq^+}{2\pi} \langle F^{-+}(Q) F^{-+} \rangle_{q^-=0}$$

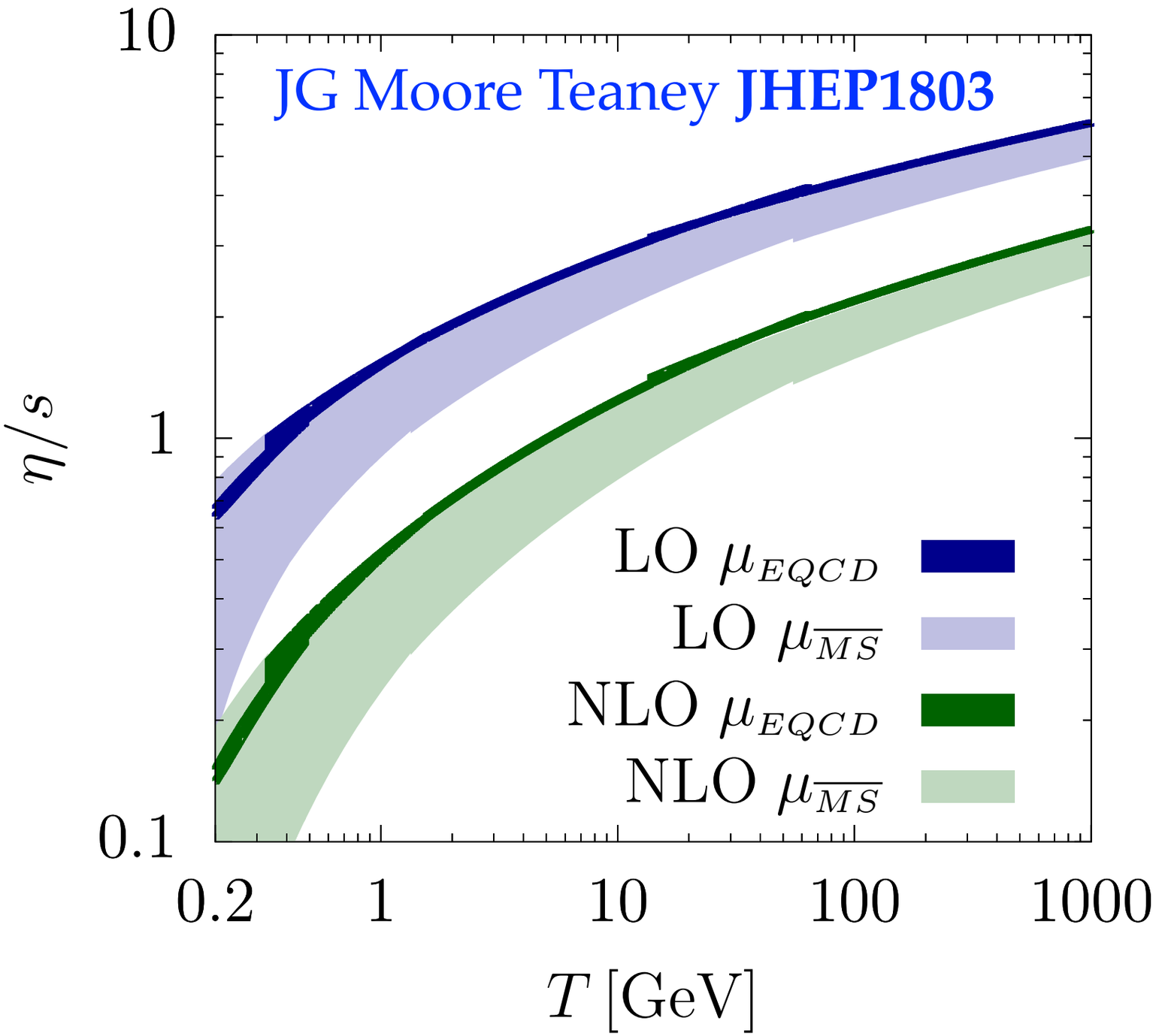


$$\langle F^{-\mu}(Q) F^{-\nu} \rangle_{q^-=0} \propto \frac{T}{q^+} (G_R^{-\mu;-\nu}(Q) - G_A^{-\mu;-\nu}(Q))$$

- Only non-analytic feature in each half-plane: **Matsubara zero mode** ($q^+=0$, all other frequencies far away)
  $\Rightarrow$ Transverse function ($\perp\perp$) becomes **Euclidean (EQCD)**

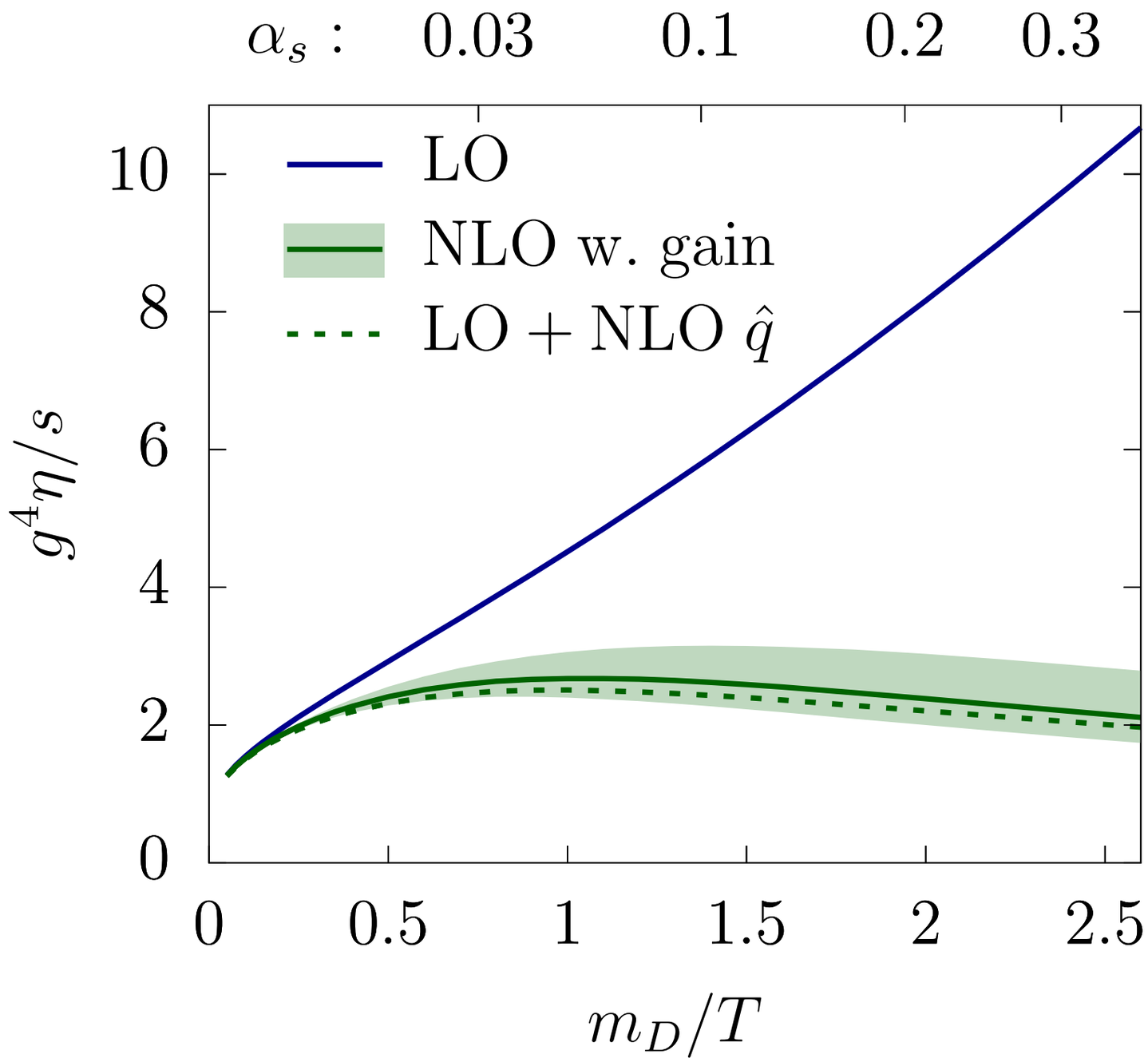
- The longitudinal function ($++$) does not feel the pole, but it does not vanish fast enough on the **arcs at large** $|q^+|$
  $\Rightarrow$ Only sensitive to the **light-cone dispersion relation**

η/s(T) of QCD



- (almost) NLO has larger effect than running (NNLO)

η/*s* convergence



- The ~entirety of the downward shift comes from NLO $\hat{q}$

Second-order relaxation

- To be causal and stable, second order in the gradients is required. There, lots of work on writing down/computing all the new transport coefficients that pop up
  Baier *et al.* **JHEP0804** (2007), Denicol *et al.* **PRD85** (2012), Battacharyya *et al.* **JHEP0802** (2007)

- We look at the second-order relaxation τ_π of the shear stress tensor to its Navier-Stokes form

$$\tau_\pi \partial_t \pi^{ij} = \pi_1^{ij} - \pi^{ij}$$

π^{ij}: shear stress, dissipative (non-ideal), traceless part of T^{ij}

Second-order relaxation

- In the kinetic theory, second-order coefficients require second-order expansion of f. τ_π obtained from first-order δf acting as source term

- From the spectral properties of the collision operator and inner product one can derive a triangular inequality

$$\frac{\tau_\pi}{\eta/(e+p)} \geq 5$$

- Generic bound in **any kinetic theory**, as long as enthalpy determined consistently within it

JG Ghiglieri Moore **PRL121** (2018)

Second-order relaxation

- We have all the spectral ingredients for a triangular inequality

$$\frac{\tau_\pi}{\eta/(e+p)} \geq 5$$

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- At infinite coupling in $\mathcal{N}=4$ (finite coupling correction known)

$$\frac{\tau_\pi}{\eta/(e+p)} = 4 - 2\ln(2) \approx 2.6$$

Baier *et al* **JHEP0804** (2007)

- Strongly-coupled holographic theories are **very far** from having a kinetic quasiparticle description

JG Ghiglieri Moore **PRL121** (2018)

Second-order relaxation

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- NLO kinetic theory and holography can give comparable 1st order coefficients, but they differ fundamentally at 2nd order

JG Ghiglieri Moore **PRL121** (2018)

Summary

- Weak-coupling approach to thermodynamics and kinetics based on (scale) separation between hard and soft modes
- Sophisticated resummation schemes pushed to high orders for thermodynamics, good agreement with lattice
- NLO possible for transport coefficients, large corrections that reduce η/s
- Second-order τ_π is bounded from below in kinetic theory. Strong-coupling holography violates the bounds: disambiguation tool?

Conclusions and outlook

- Corrections to η/s dominated by NLO $\hat{q}$. Can we identify the physics responsible for this and reorganize the perturbative expansion, as in the case of thermodynamics?

- NLO kinetic theory and holography can give comparable 1st order coefficients, but they differ fundamentally at 2nd order: strongly-coupled thus very far from having quasi-particles

Backup

Euclideanization of light-cone soft physics

- For $t/x_z = 0$: equal time Euclidean correlators.

$$G_{rr}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\int_p G_E(\omega_n,p)e^{i\mathbf{p}\cdot\mathbf{x}}$$

Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics

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- Consider the more general case $|t/x^z| < 1$

$$G_{rr}(t,\mathbf{x}) = \int dp^0 dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp - p^0x^0)}\left(\frac{1}{2}+n_{\rm B}(p^0)\right)(G_R(P)-G_A(P))$$

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- Change variables to $\tilde{p}^z = p^z - p^0(t/x^z)$

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- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

Caron-Huot **PRD79** (2009)

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- Soft physics dominated by $n=0$ (and t-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics

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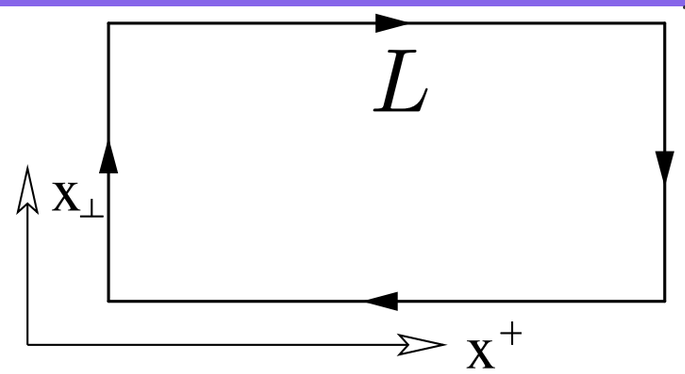
- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

$$G_{rr}(t,\mathbf{x})_{\rm soft} = T\int d^3p\, e^{i\mathbf{p}\cdot\mathbf{x}}\, G_E(\omega_n=0,\mathbf{p})$$

- Soft physics dominated by $n=0$ (and t-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

LPM resummation



$$\propto e^{\mathcal{C}(x_\perp)L}$$

BDMPS-Z, Wiedemann, Casalderrey-Solana Salgado, D'Eramo Liu Rajagopal, Benzke Brambilla Escobedo Vairo

- All points at spacelike or lightlike separation, only preexisting correlations
- Soft contribution becomes Euclidean! Caron-Huot **PRD79** (2008)
  - Can be "easily" computed in perturbation theory
  - Possible lattice measurements Laine **EPJC72** (2012) Laine Rothkopf **JHEP1307** (2013) Panero Rummukainen Schäfer **1307.5850**

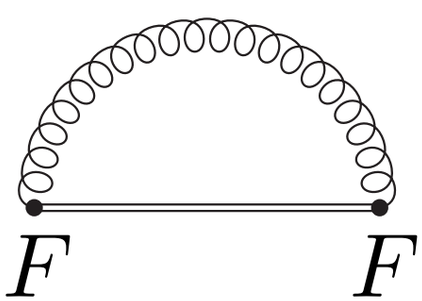
Light-cone techniques

- Key advancement over the past decade: analytical properties of soft thermal amplitudes at light-like separations. Heuristically, the hard, light-like parton sees undisturbed soft modes, which *"can't keep up"* with it
  Caron-Huot **PRD82** (2008) *Review*: JG Teaney **QGP5** (2015)

- In a nutshell, retarded functions are analytical in the upper half plane in **any** time-like variable. In the soft sector to NLO also for **light-like variables** ($q^+=(q^0+q^z)/2$).

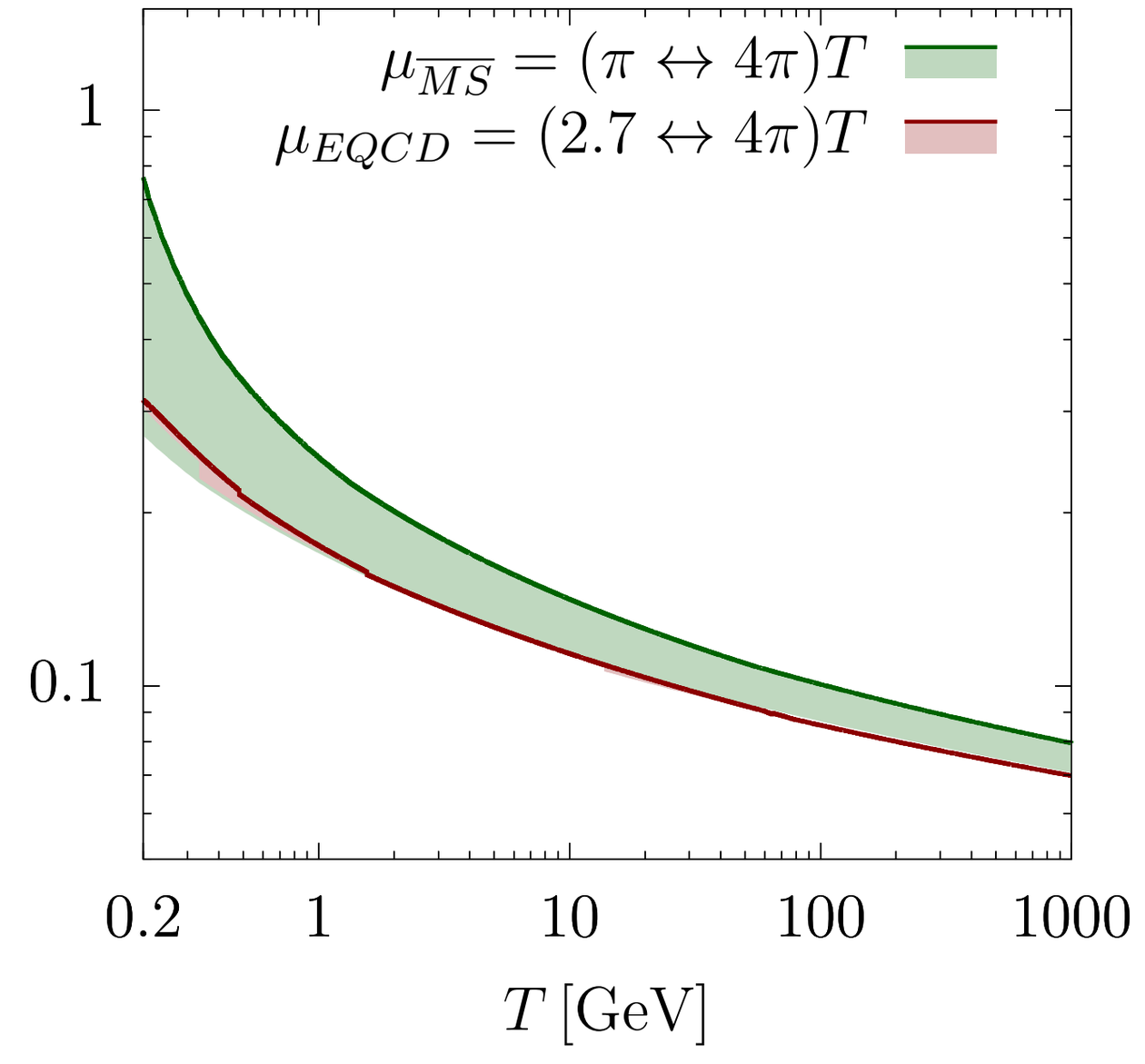
$$\hat{q}(\mu_\perp) = g^2 C_A \int^{\mu_\perp} \frac{d^2q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^-_\perp \rangle_{q^-=0}$$

$$\hat{q}_L(\mu_\perp) = g^2 C_A \int^{\mu_\perp} \frac{d^2q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-+}(Q) F^{-+} \rangle_{q^-=0}$$



Results (and their fine print) (and their fine print)

- A plot of $\eta(T)$ needs $g(T)$. But kinetic theory with massless quarks is conformal to NLO: no UV divergences and no guidance from the calculation on how to set the scale



- Fix $g(T)$ as
  - Two-loop EQCD as in Laine Schröder **JHEP0503** (2005)
  - Simple two-loop $\overline{MS}$ with various μ/T

JG Ghiglieri Moore **JHEP1803** (2018)