

Primordial fluctuations in relativistic nuclear collisions

by

GIULIANO GIACALONE

with :

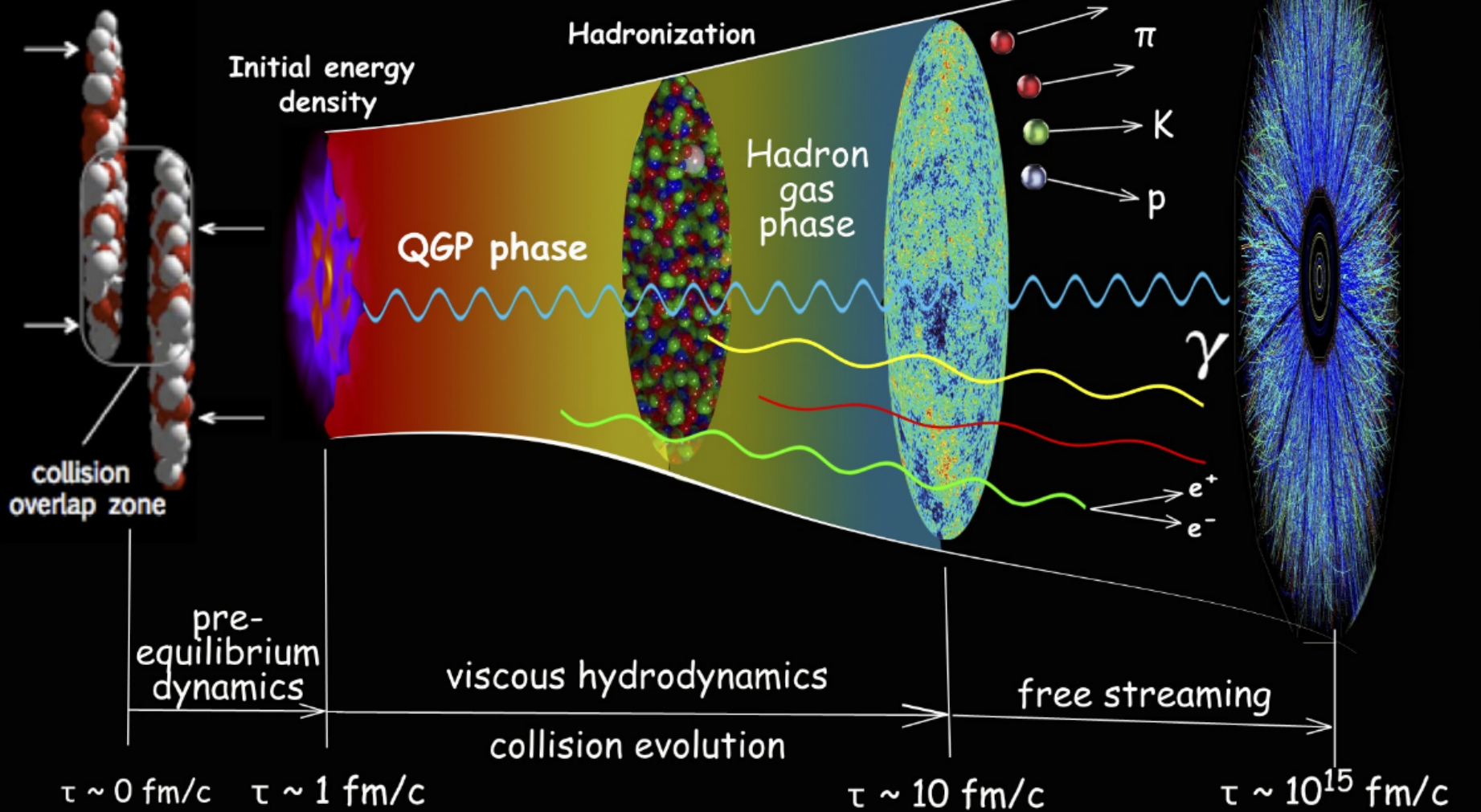
Jean-Yves Ollitrault (IPhT Saclay), François Gelis (IPhT Saclay),
Cyrille Marquet (CPHT), Pablo Guerrero-Rodríguez (Jyväskylä / CPHT)



Relativistic Heavy-Ion Collisions

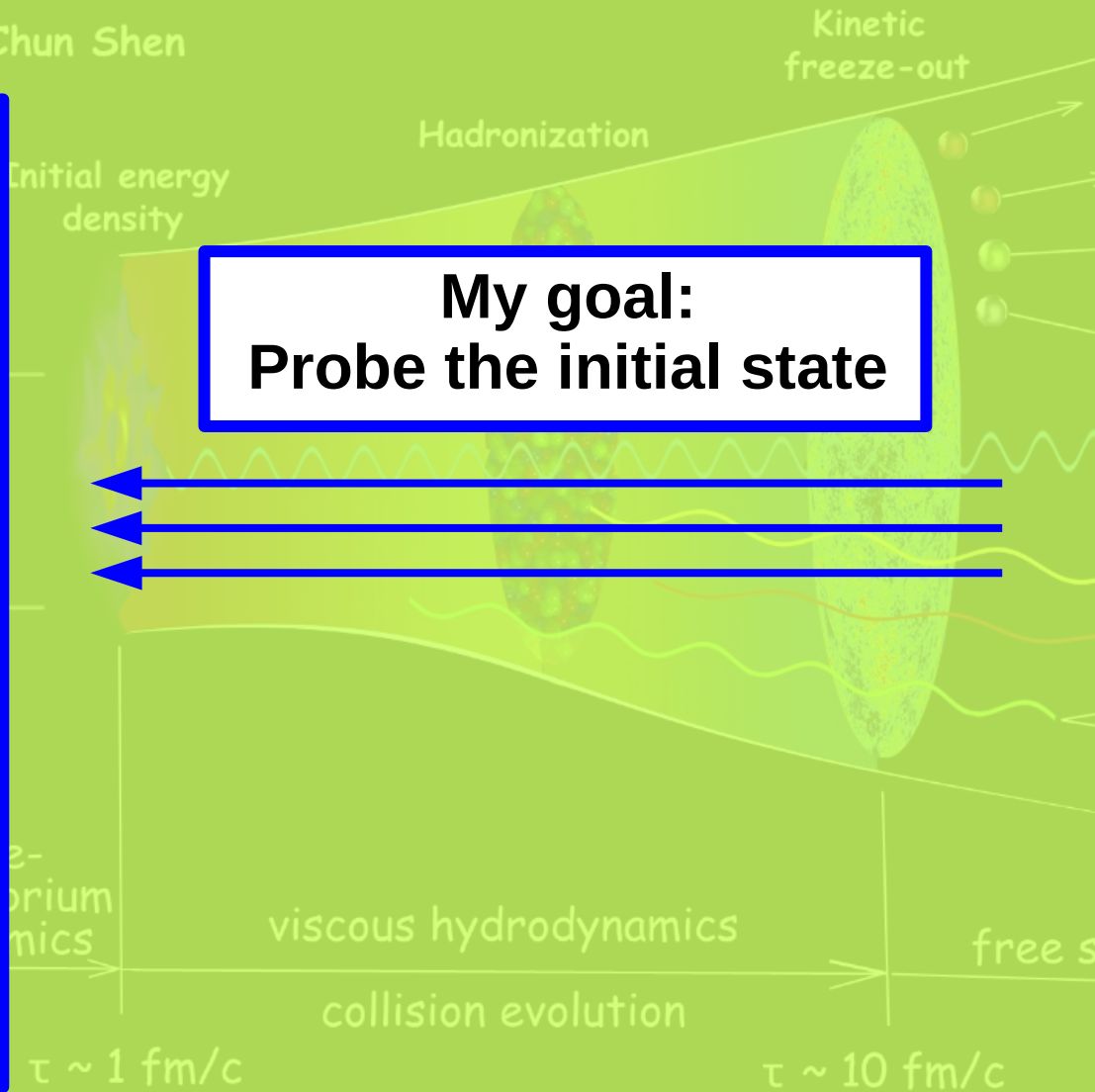
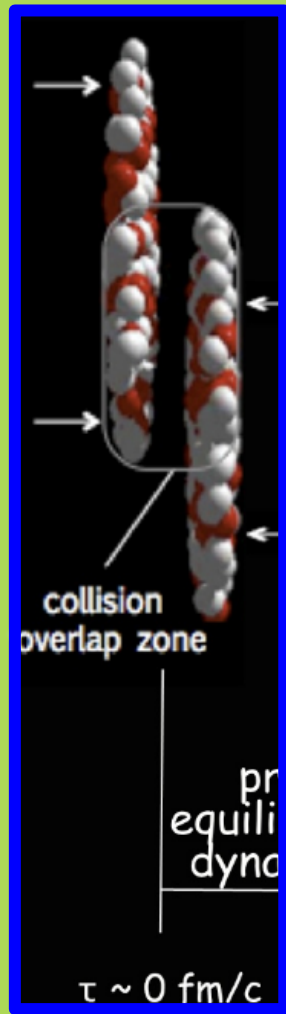
made by Chun Shen

final detected particle distributions

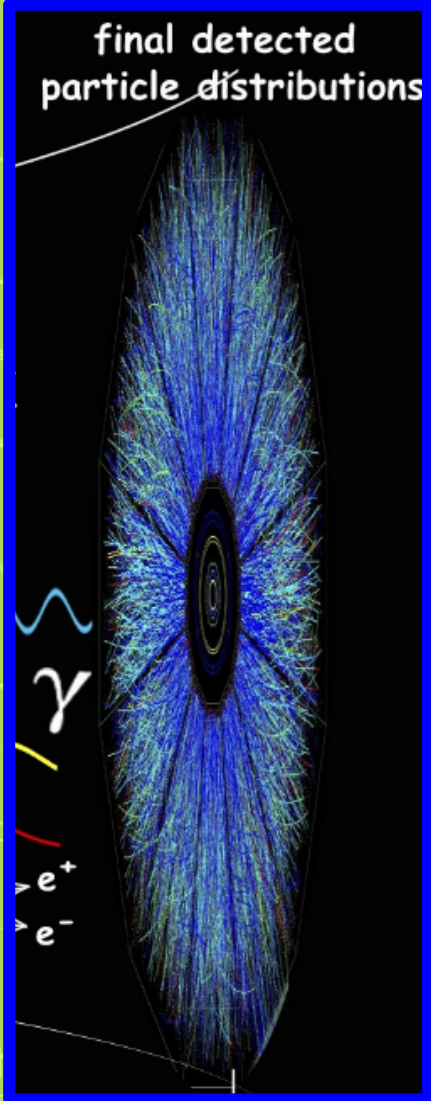


Relativistic Heavy-Ion Collisions

made by Chun Shen



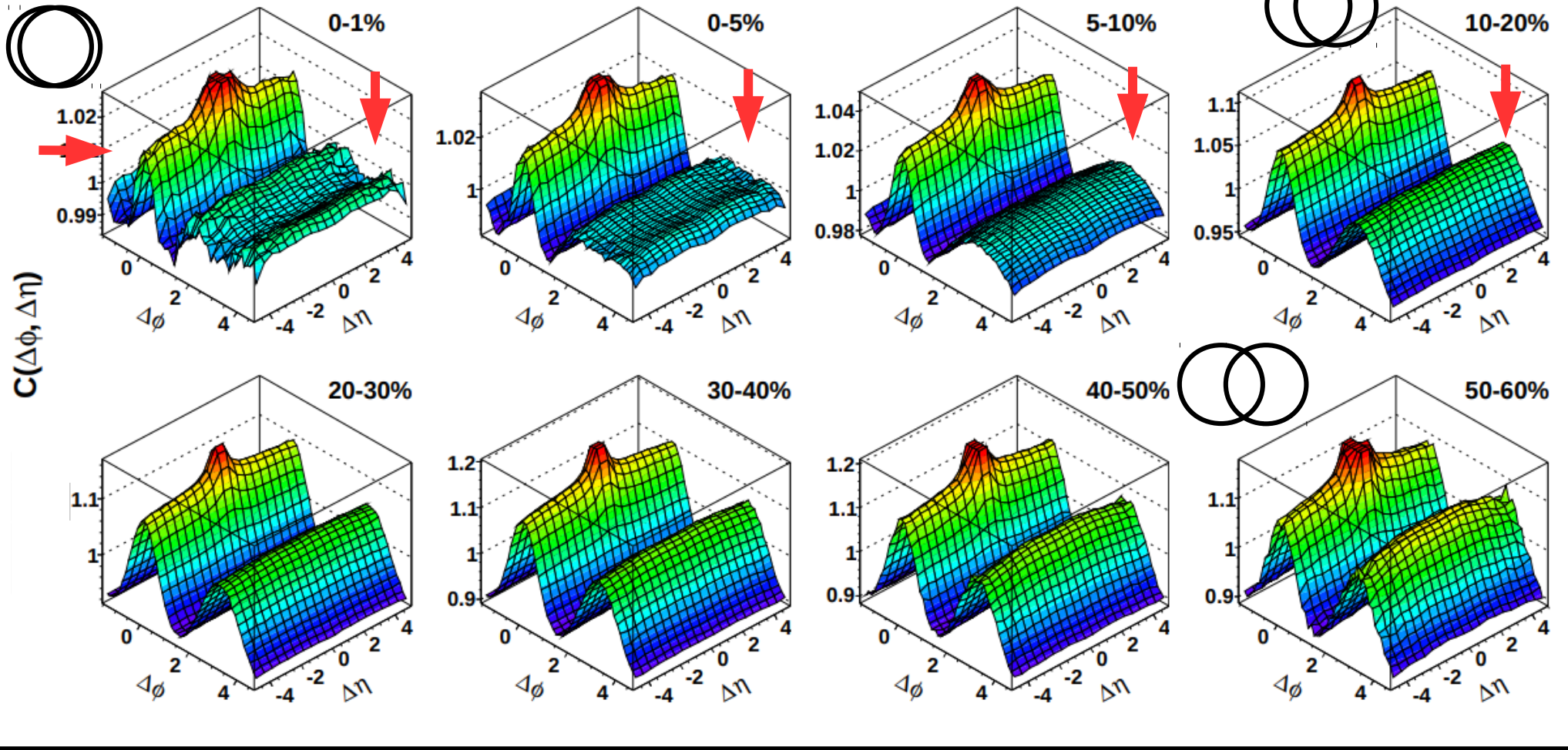
**My goal:
Probe the initial state**



How?

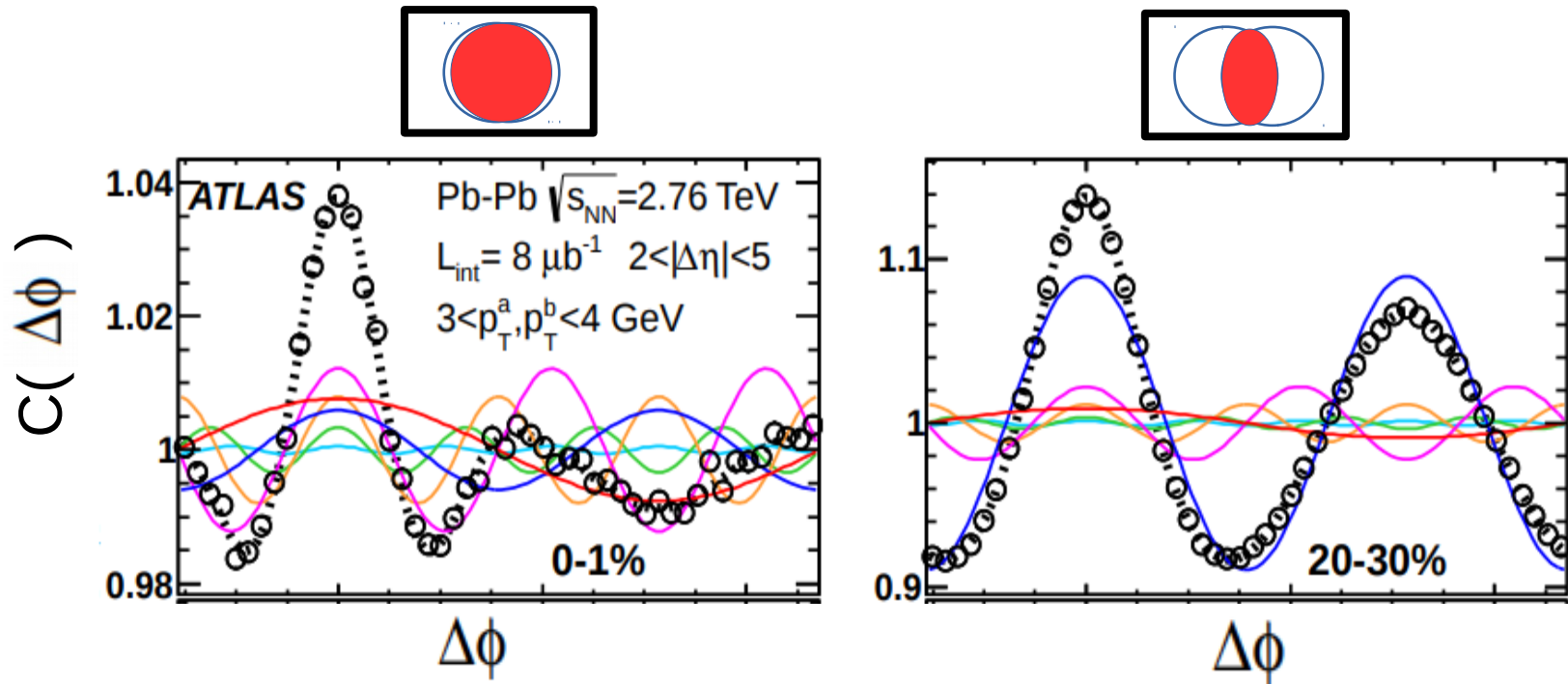
ATLAS Collaboration
[1203.3087]

Pb-Pb $\sqrt{s_{NN}}=2.76$ TeV $L_{int}=8 \mu\text{b}^{-1}$ $2 < p_T^a, p_T^b < 3$ GeV



Long-range azimuthal correlations: A new feature of high-energy physics.

Integrated over eta.

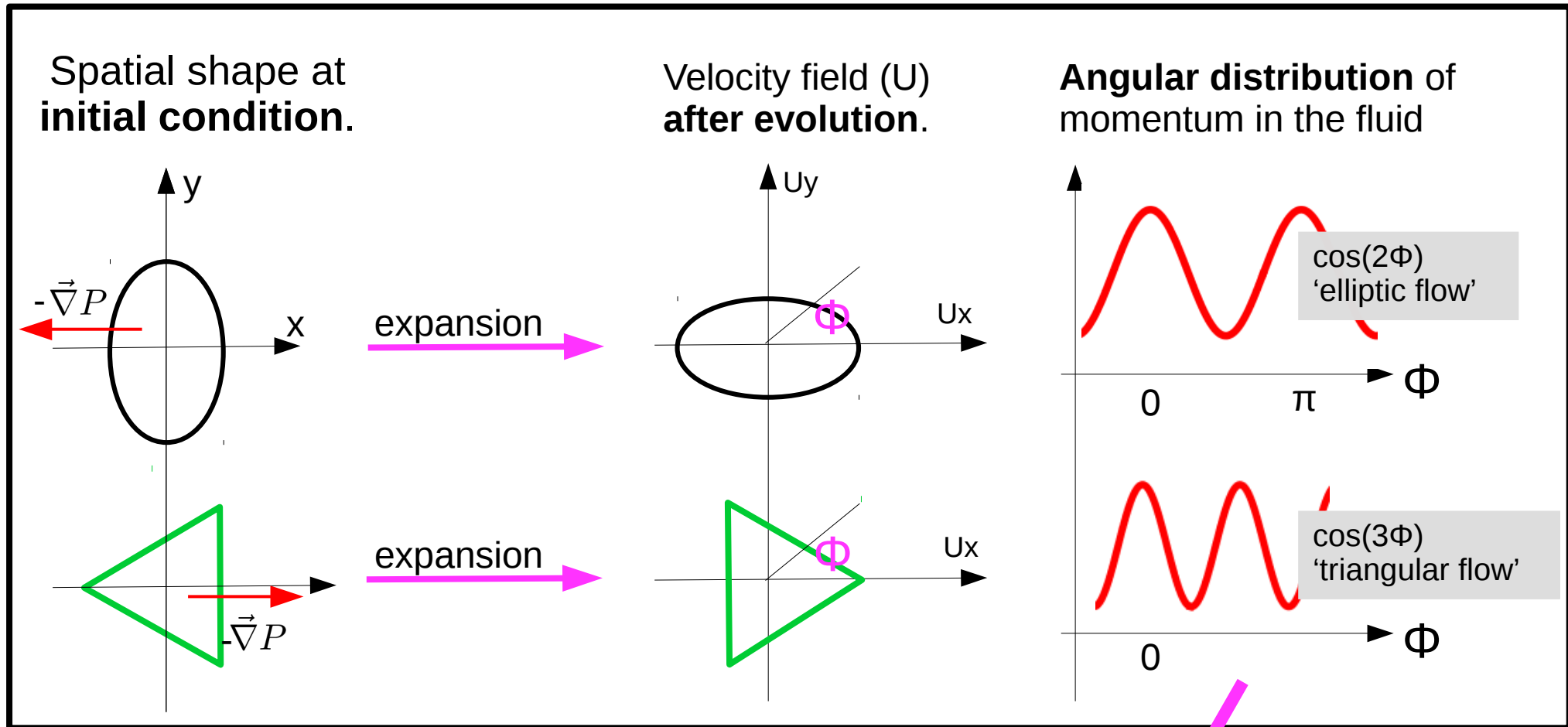


Harmonic decomposition:

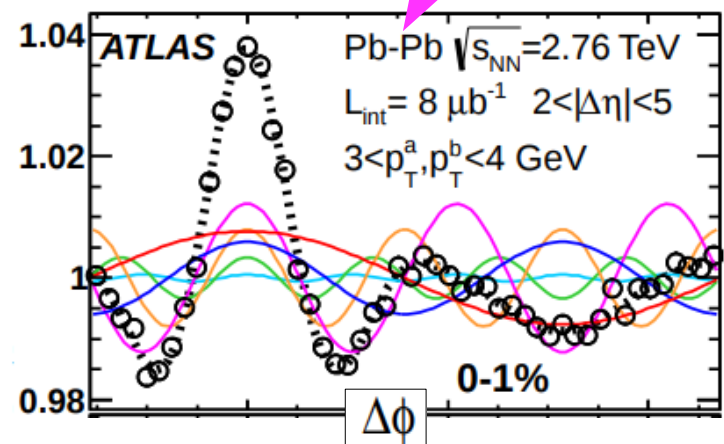
$$\frac{dN}{p_t dp_t d\phi} = \frac{dN}{p_t dp_t} \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} V_n e^{in\phi}, \quad V_{-n} = V_n^*$$

“anisotropic flow” coefficients

Natural explanation in a fluid paradigm. $F = -\nabla P$ [Ollitrault, 1992]



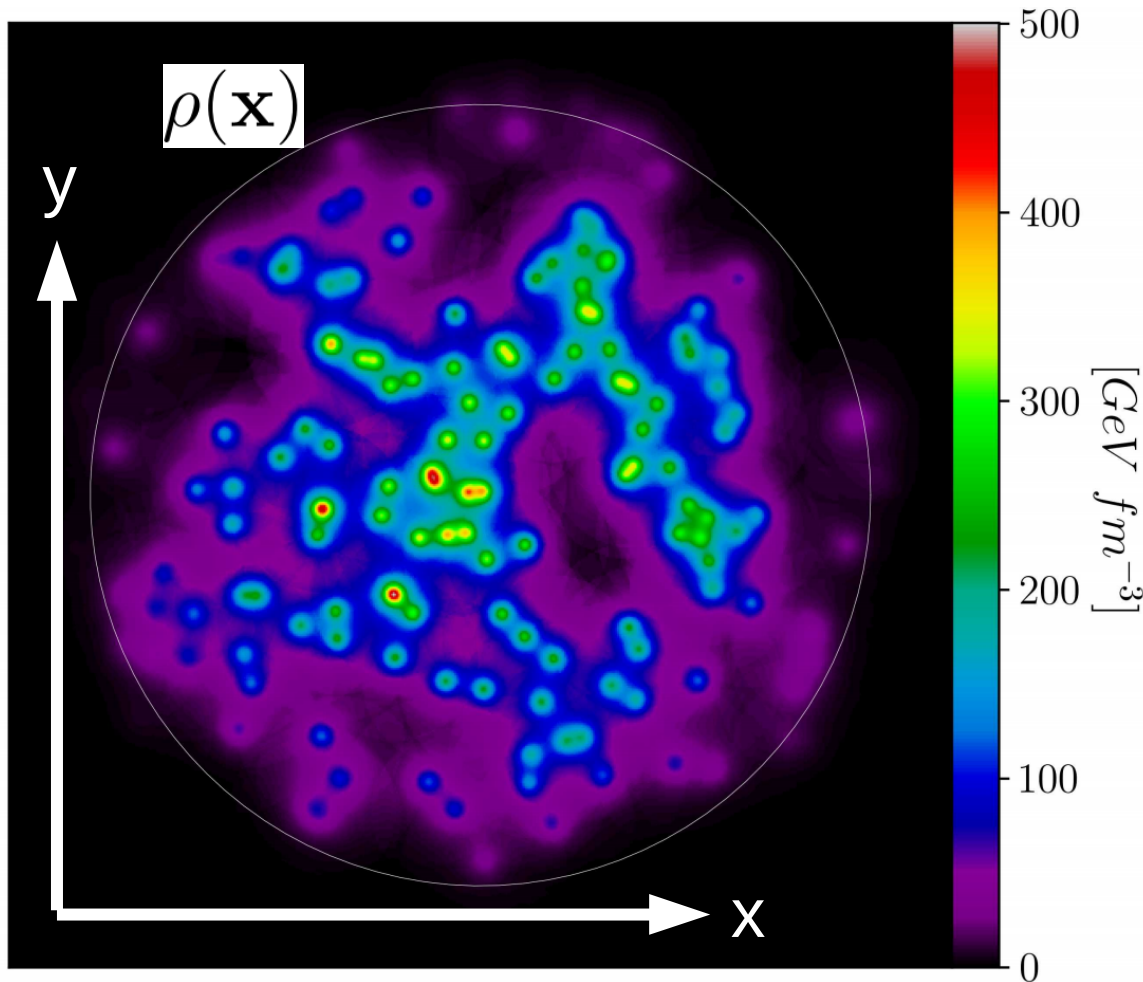
ANISOTROPIC FLOW



Initial-state (primordial) anisotropy. A 2D analysis.

Fluctuations generate anisotropy to all orders!

[Alver, Roland [1003.0194](#)]



Anisotropy of large scale structures.

[Teaney, Yan [1010.1876](#)]

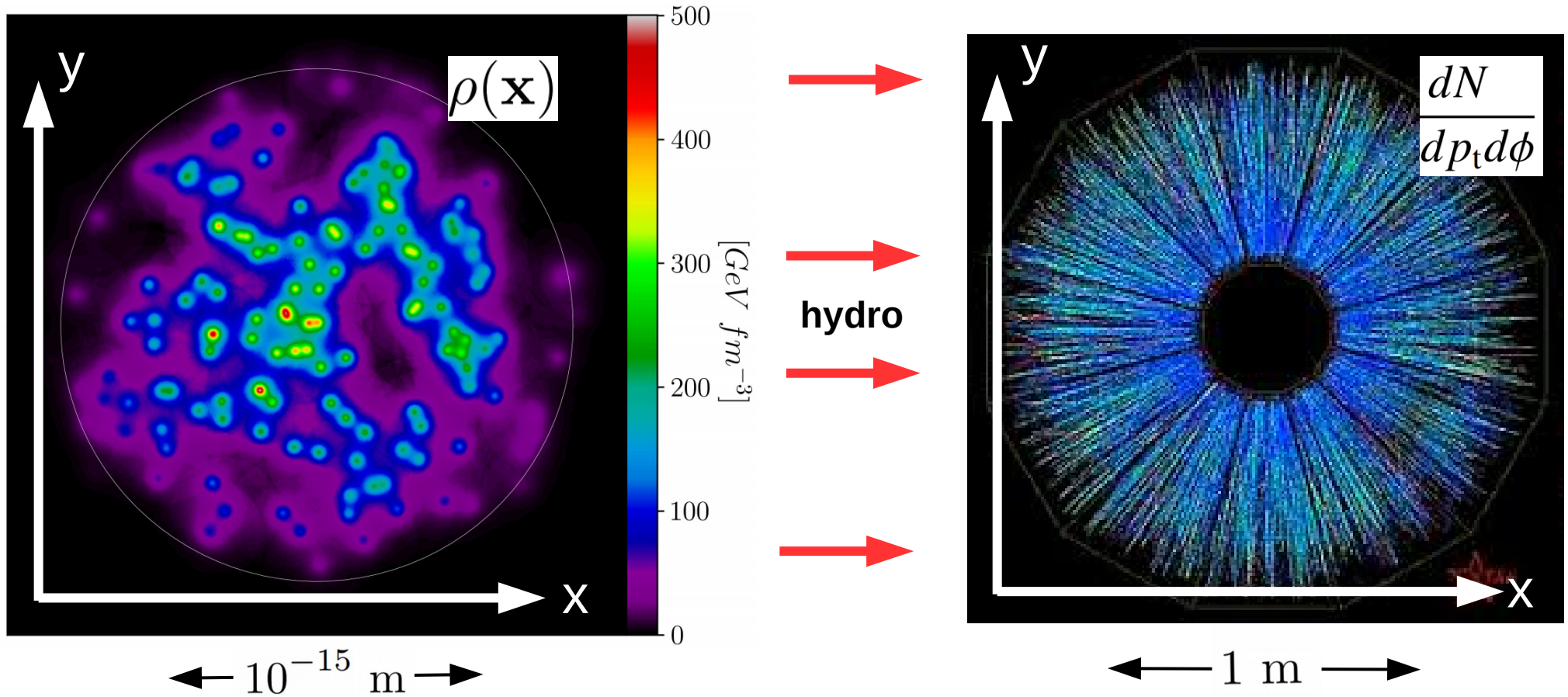
$$\mathcal{E}_n \equiv \frac{\int_{\mathbf{x}} |\mathbf{x}|^n e^{in\phi} \rho(\mathbf{x})}{\int_{\mathbf{x}} |\mathbf{x}|^n \rho(\mathbf{x})}$$

n=2 ellipse

n=3 triangle

[Gelis, Giacalone, Guerrero-Rodriguez, Marquet, Ollitrault, [1907.10948](#)]

Final-state anisotropy from primordial fluctuations.



$$\boxed{\mathcal{E}_n} \equiv \frac{\int_{\mathbf{x}} |\mathbf{x}|^n e^{in\phi} \rho(\mathbf{x})}{\int_{\mathbf{x}} |\mathbf{x}|^n \rho(\mathbf{x})}$$



$$\frac{dN}{p_t dp_t} \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \boxed{V_n e^{in\phi}}$$

The relation is remarkably simple.

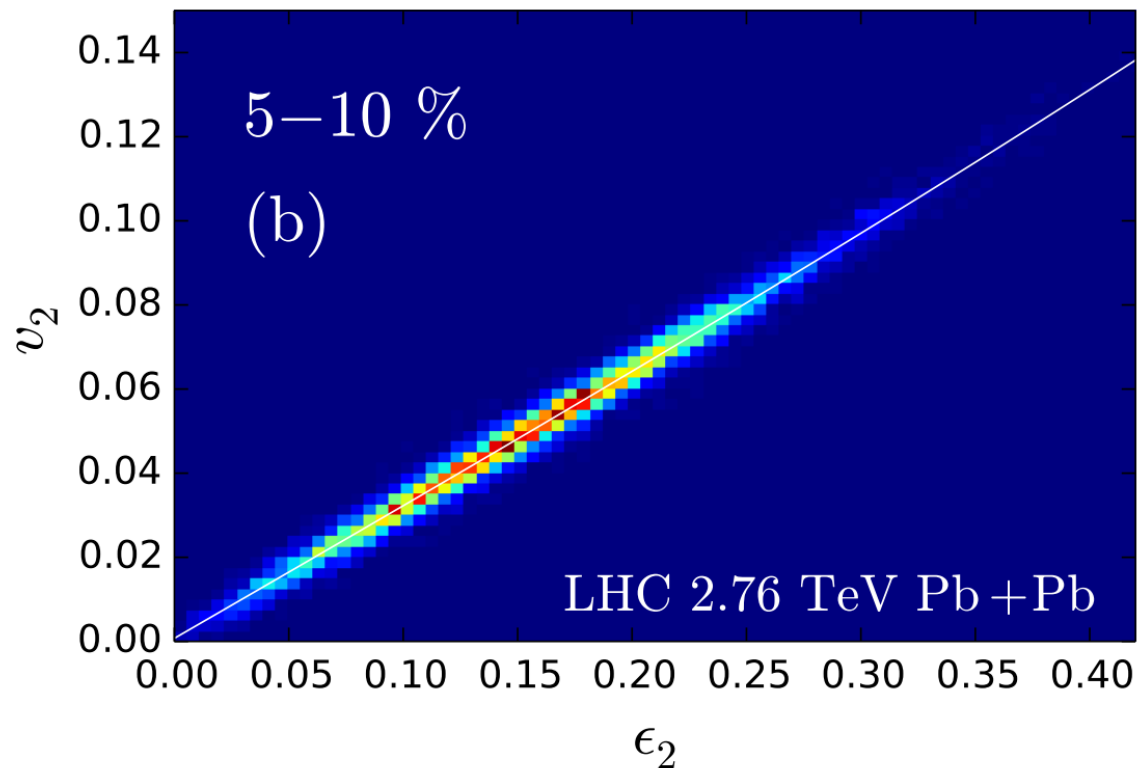
$$V_n = \kappa_n \mathcal{E}_n + \dots$$

“hydro”

(smaller scale structures
allowed by symmetry)

Confirmed by theoretical calculations.

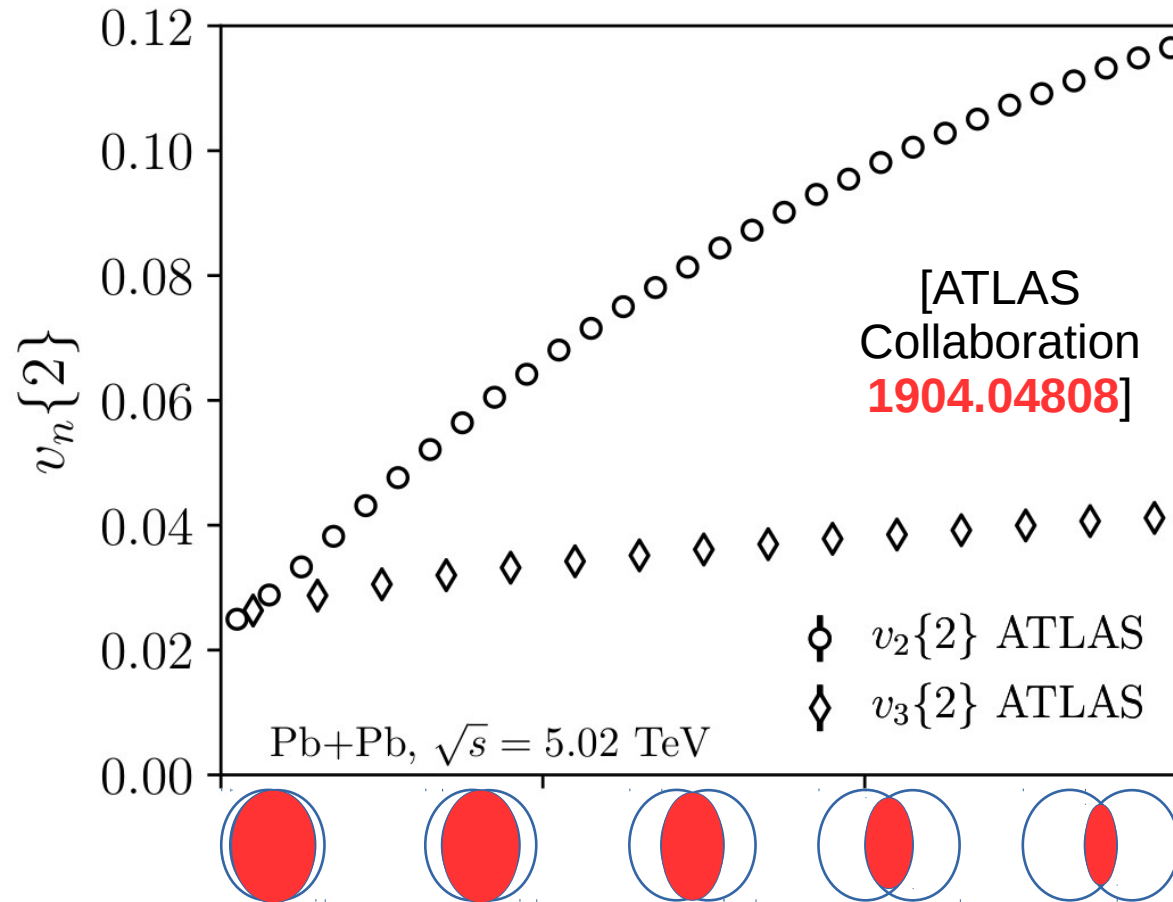
[Niemi, Eskola, Patelaainen, [1505.02677](#)]



Experimentally one can only measure averaged quantities:

$$\langle V_n V_n^* \rangle = \langle \cos n(\phi_1 - \phi_2) \rangle \equiv v_n \{2\}^2$$

average over
a class of events



We need a **statistical theory of anisotropy**. We follow Blaizot.

$$\langle V_n V_n^* \rangle \propto \langle \mathcal{E}_n \mathcal{E}_n^* \rangle \quad [\text{Blaizot, Broniowski, Ollitrault, } \mathbf{1405.3572}]$$

1) In heavy-ion collisions, **large-scale fluctuations are small.**

$$\rho(\mathbf{x}) = \langle \rho(\mathbf{x}) \rangle + \delta\rho(\mathbf{x}), \quad \langle \rho(\mathbf{x}) \rangle \gg \delta\rho(\mathbf{x})$$

2) The **correlation length is typically negligible** compared to the size of the large scale structures.

$$\langle \rho(\mathbf{x}_1)\rho(\mathbf{x}_2) \rangle - \langle \rho(\mathbf{x}_1) \rangle \langle \rho(\mathbf{x}_2) \rangle \approx \xi(\mathbf{x})\delta(\mathbf{r})$$

$$\mathbf{x} = \frac{\mathbf{x}_1 + \mathbf{x}_2}{2}, \quad \mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$$

3) The **observable** that we need becomes:

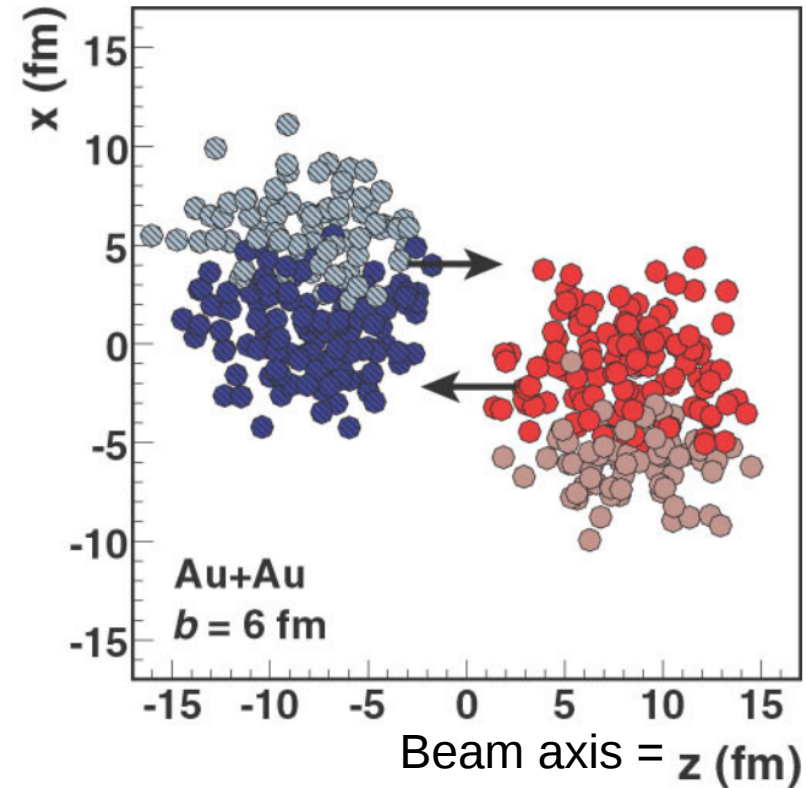
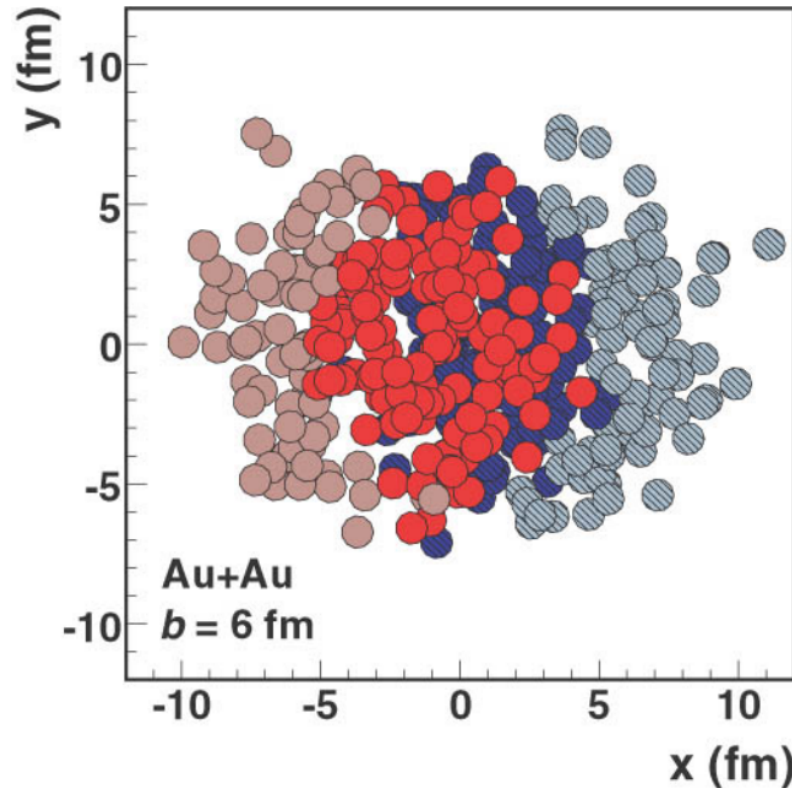
$$v_n \{2\}^2 = \langle V_n V_n^* \rangle \propto \langle \mathcal{E}_n \mathcal{E}_n^* \rangle = \frac{\int_{\mathbf{x}} |\mathbf{x}|^{2n} \xi(\mathbf{x})}{\left[\int_{\mathbf{x}} |\mathbf{x}|^n \langle \rho(\mathbf{x}) \rangle \right]^2}$$

How do we calculate these correlation functions?

The 'standard model' : Glauber Monte Carlo generators

[Miller, Reygers, Sanders, Steinberg, [nucl-ex/0701025](#)]

~1200 citations on INSPIRE

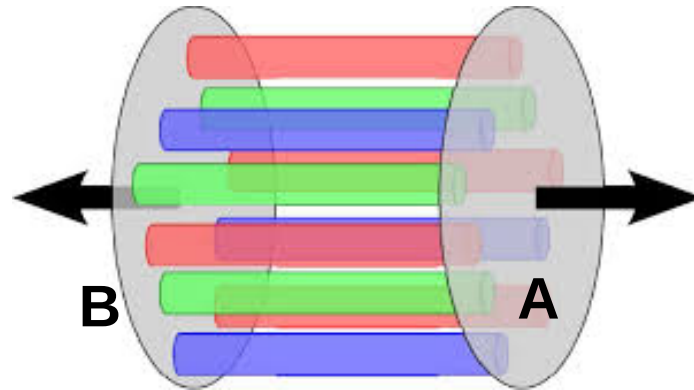


A 10-year old recipe:

1. Sample nucleons according to the ground-state nuclear density.
2. Determine those nucleons that interact (participants).
3. Map the participants into an energy density $\rho(\mathbf{x})$.

A different approach within the **C**olor **G**lass **C**ondensate.

[Giacalone, Guerrero-Rodriguez, Luzum, Marquet, Ollitrault, [1902:07168](#)]



Stress-energy tensor right after the overlap:

[Lappi, McLerran, [hep-ph/0602189](#)]

$$T^{\mu\nu}(\mathbf{x}, \tau = 0^+) = \rho(\mathbf{x})[\text{diag}(1, 1, 1, -1)]^{\mu\nu}$$

Using the **McLerran-Venugopalan model** one can

evaluate correlation functions: [McLerran, Venugopalan, [hep-ph/9309289](#)]

$$\langle T^{00}(\mathbf{x}, \tau = 0^+) \rangle, \quad \langle T^{00}(\mathbf{x}, \tau = 0^+) T^{00}(\mathbf{y}, \tau = 0^+) \rangle$$

Precisely what we need!

The 1-point function has been known for a long time.

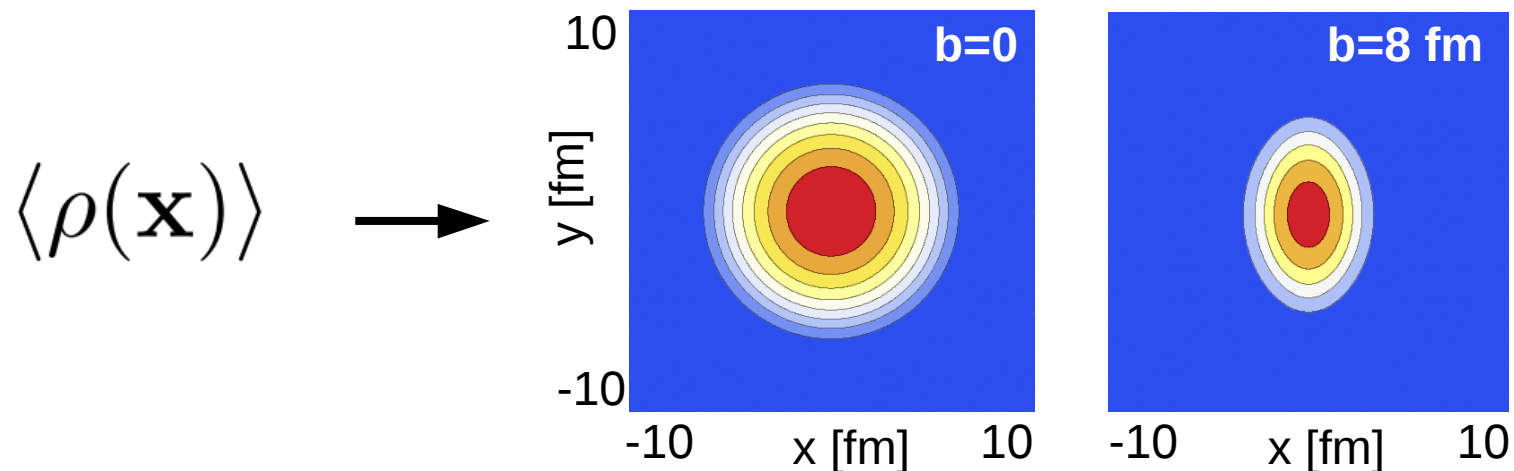
[Lappi, [hep-ph/0606207](#)]

$$\langle \rho(\mathbf{x}) \rangle = \frac{4}{3g^2} Q_A^2(\mathbf{x}) Q_B^2(\mathbf{x})$$

Nuclear saturation scale:

$$Q_s^2(\mathbf{x}) = \left[Q_{s0}^2 \sim O(1 \text{ GeV}^2) \right] \times \frac{[\text{density of gluons at } \mathbf{x}]}{[\text{density of gluons at } \mathbf{0}]}$$

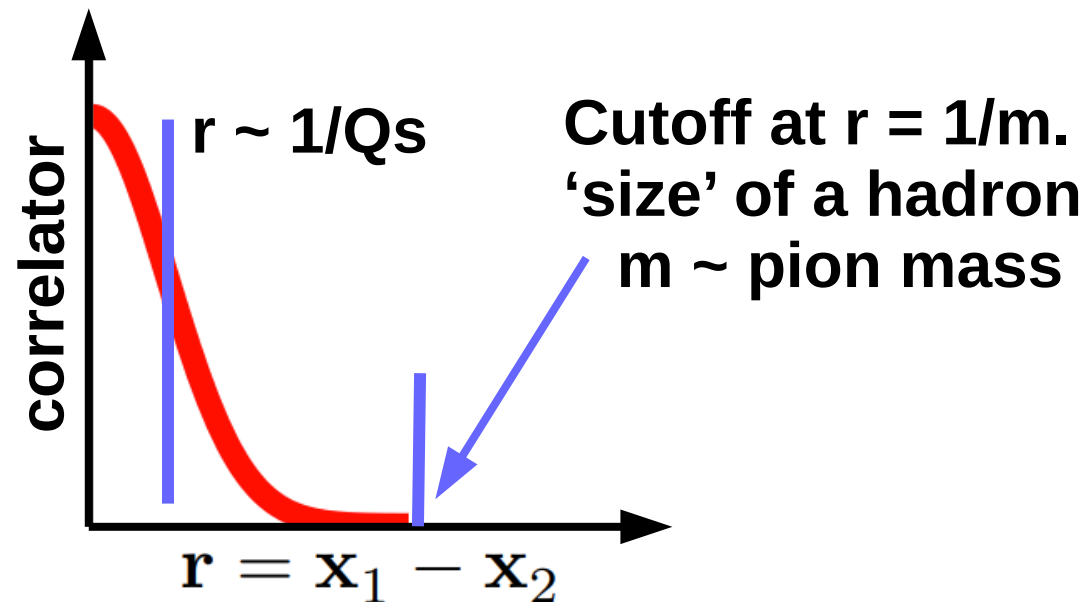
It gives the average energy density at a given b .



The 2-point function has been evaluated recently.

[Albacete, Guerrero-Rodríguez, Marquet **1808.00795**]

$$\langle \rho(\mathbf{x}_1) \rho(\mathbf{x}_2) \rangle - \langle \rho(\mathbf{x}_1) \rangle \langle \rho(\mathbf{x}_2) \rangle \approx \xi(\mathbf{x}) \delta(\mathbf{r})$$



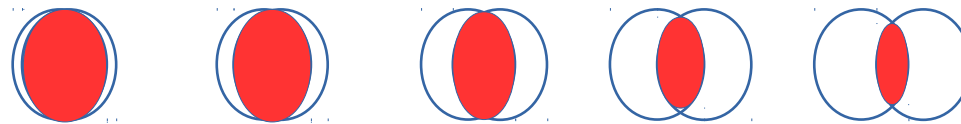
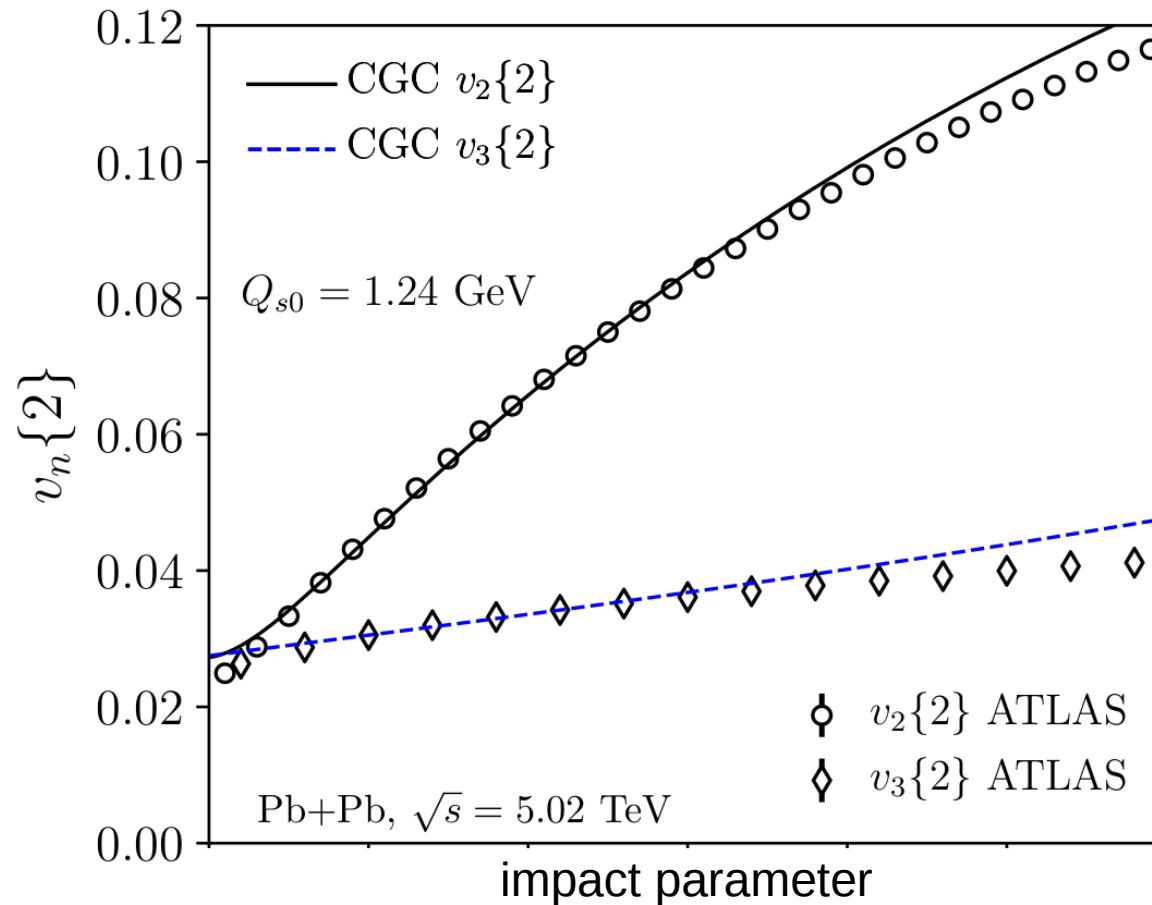
To leading logarithmic accuracy:

[Giacalone, Guerrero-Rodríguez, Luzum, Marquet, Ollitrault, **1902.07168**]

$$\xi(\mathbf{x}) = \frac{16\pi}{9g^4} Q_A^2(\mathbf{x}) Q_B^2(\mathbf{x}) \left[Q_A^2(\mathbf{x}) \ln \left(\frac{Q_B^2(\mathbf{x})}{m^2} \right) + Q_B^2(\mathbf{x}) \ln \left(\frac{Q_A^2(\mathbf{x})}{m^2} \right) \right]$$

RESULTS

$$v_n\{2\}^2 = \langle V_n V_n^* \rangle \propto \langle \mathcal{E}_n \mathcal{E}_n^* \rangle = \frac{\int_{\mathbf{x}} |\mathbf{x}|^{2n} \xi(\mathbf{x})}{\left[\int_{\mathbf{x}} |\mathbf{x}|^n \langle \rho(\mathbf{x}) \rangle \right]^2}$$



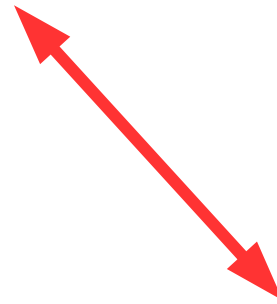
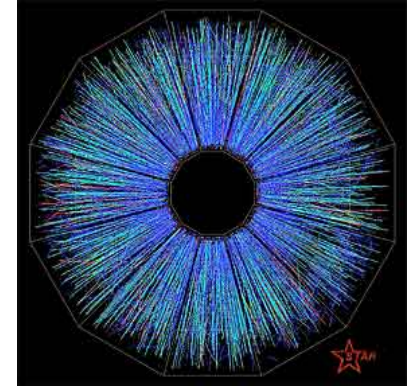
[Giacalone, Guerrero-Rodriguez, Luzum, Marquet, Ollitrault, [1902:07168](#)]

HEP-PHENOMENOLOGY FOR HEAVY IONS?

FINAL-STATE ANISOTROPHY

Multi-particle azimuthal correlations

$$\langle e^{in(\phi_1 - \phi_2)} \rangle, \langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle, \dots$$



PRIMORDIAL ANISOTROPHY

Multi-point correlations of the density field

$$\langle \rho(\mathbf{x}) \rangle, \langle \rho(\mathbf{x}_1)\rho(\mathbf{x}_2) \rangle, \langle \rho(\mathbf{x}_1)\rho(\mathbf{x}_2)\rho(\mathbf{x}_3) \rangle \dots$$

