Supersymmetry, g-2, gauge coupling unification

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In collaboration with:

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JHEP 1904 (2019) 161 [arXiv:1902.06093]

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- Gauge Coupling Unification
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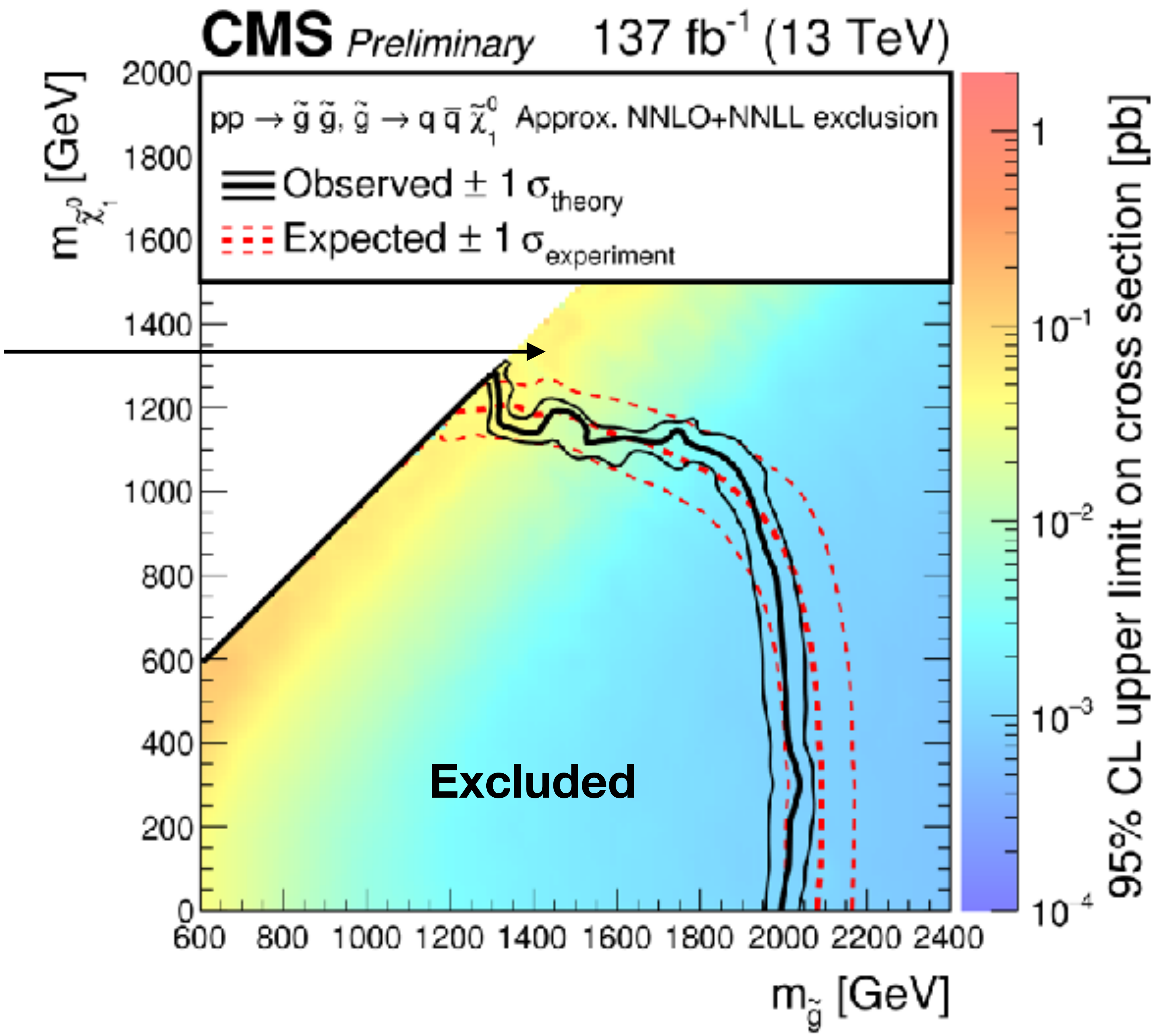
Tension with LHC results

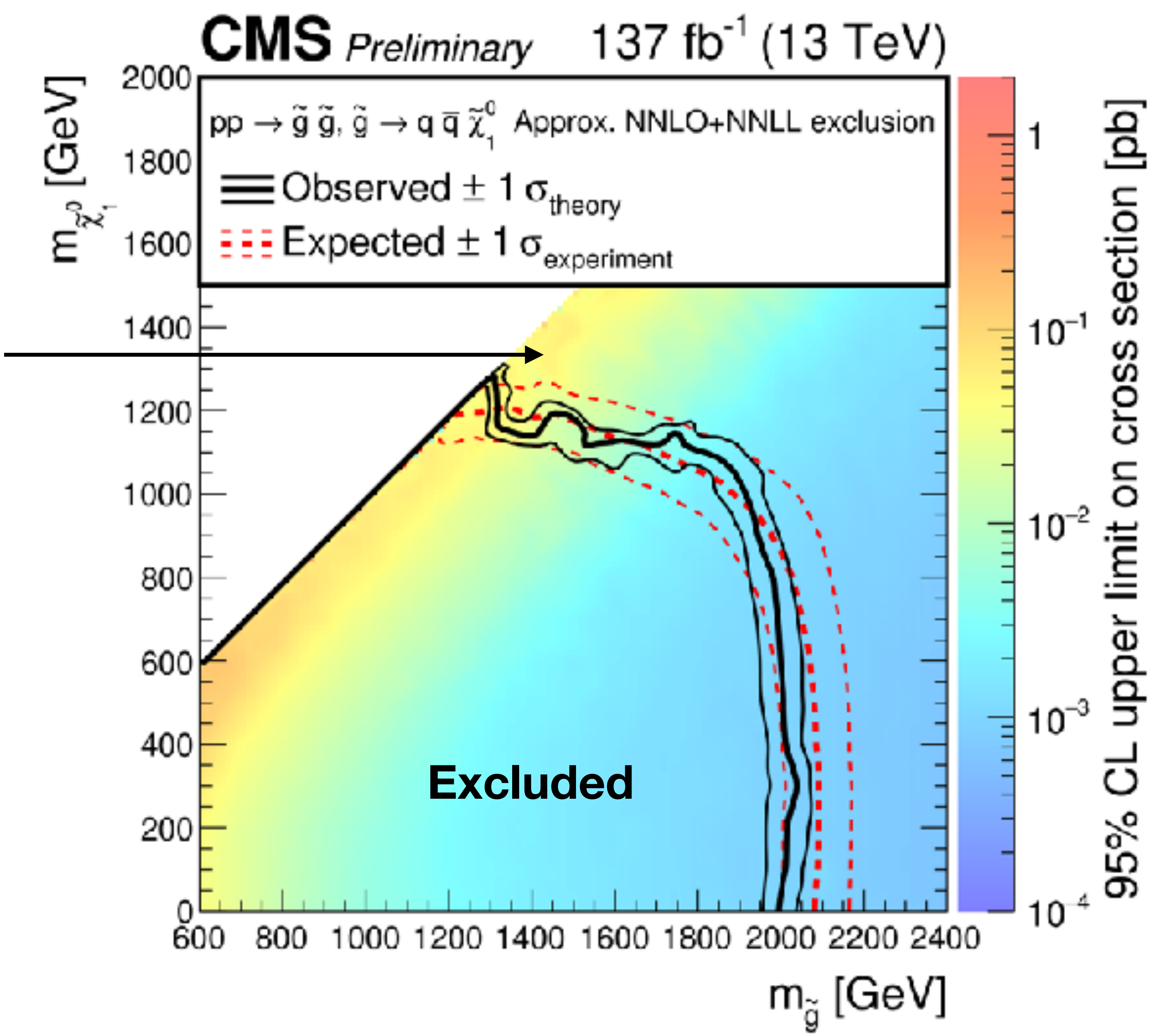
gluino can be ~ 1.4TeV



$$\Delta m_h^2 \sim M_{\rm SUSY}^2 \ln\left(\frac{\Lambda}{M_{\rm SUSY}}\right)$$

Tension with LHC results

gluino can be
~ 1.4TeV

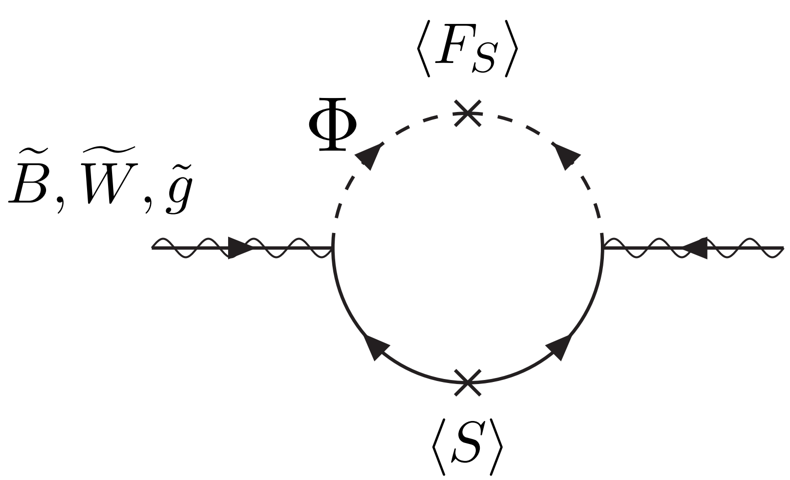


$$\Delta m_h^2 \sim M_{\rm SUSY}^2 \ln\left(\frac{\Lambda}{M_{\rm SUSY}}\right)$$

it helps fine-tuning if the log factor is small

Gravitino LSP in GMSB

- **Gauge mediation:** SUSY breaking field (singlet) $\longrightarrow$ $S\Phi\overline{\Phi}$ $\longleftarrow$ messenger field (gauged)



integrating out Φ at the scale Λ = <S>

$$\longrightarrow \quad m_{\tilde{g}} = \frac{\alpha_3}{4\pi}\frac{F_S}{\Lambda}$$

(gluino mass)

Fine-tuning: $$\Delta m_h^2 \sim M_{\rm SUSY}^2 \ln\left(\frac{\Lambda}{M_{\rm SUSY}}\right)$$

If both F and Λ are lowered, one can reduce the fine-tuning while keeping the SUSY mass

(gravitino mass) $$m_{3/2} = \frac{F_S}{\sqrt{3}M_P} \quad \longrightarrow \quad \frac{m_{3/2}}{m_{\tilde{g}}} \propto \frac{\Lambda}{M_P}$$

natural gauge mediation predicts gravitino LSP

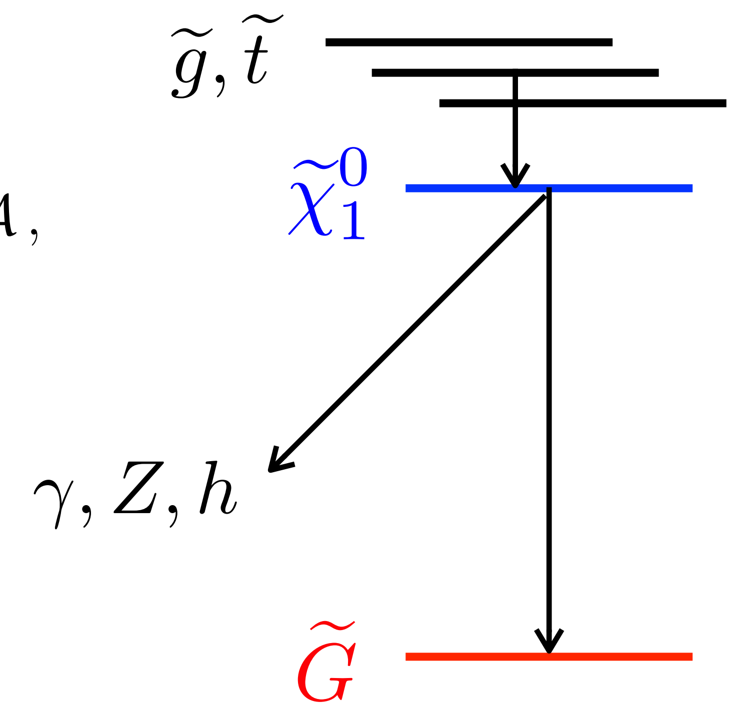
Neutralino (NLSP) decay

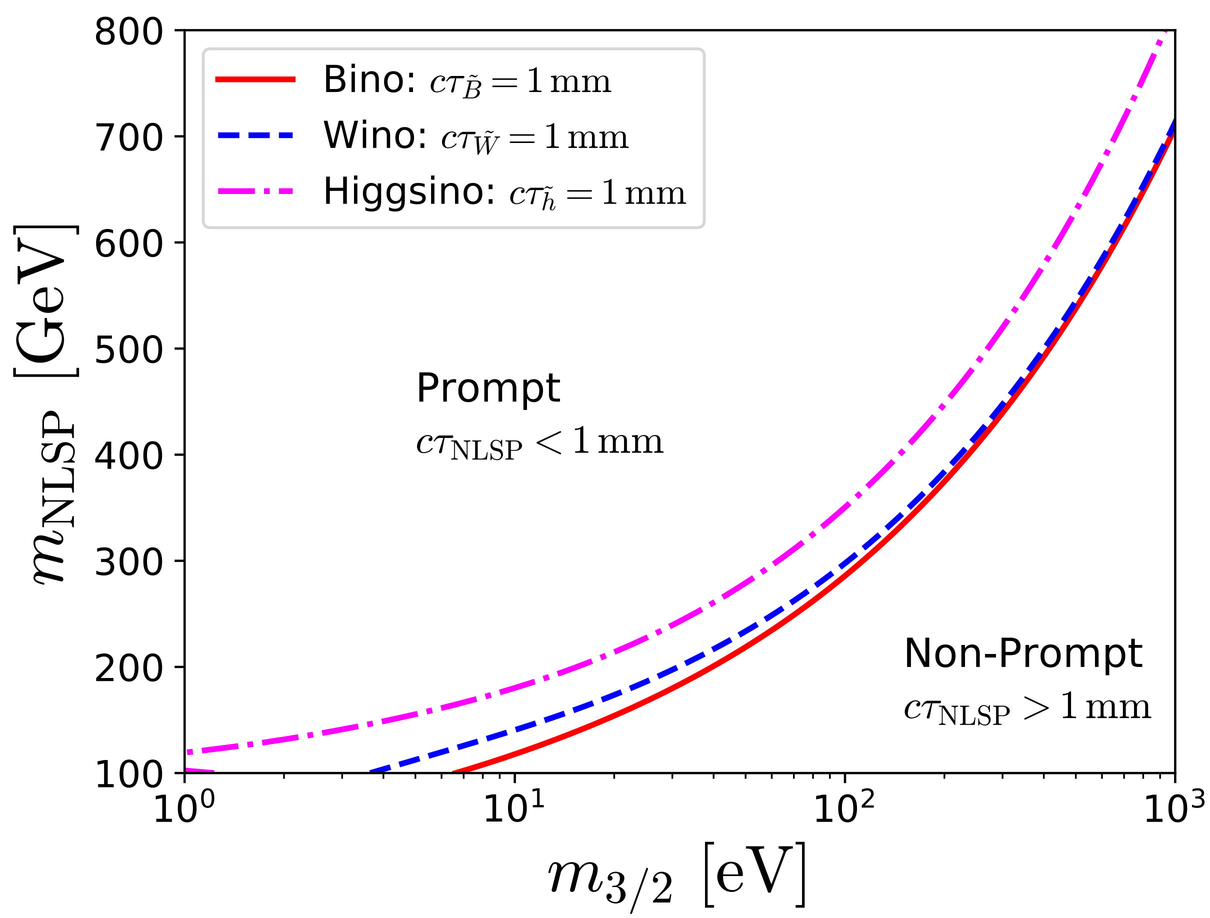
$$\Gamma(\tilde{\chi}_1^0 \to \tilde{G}\gamma) = \left|N_{11}c_W + N_{12}s_W\right|^2 \mathcal{A}\,,$$

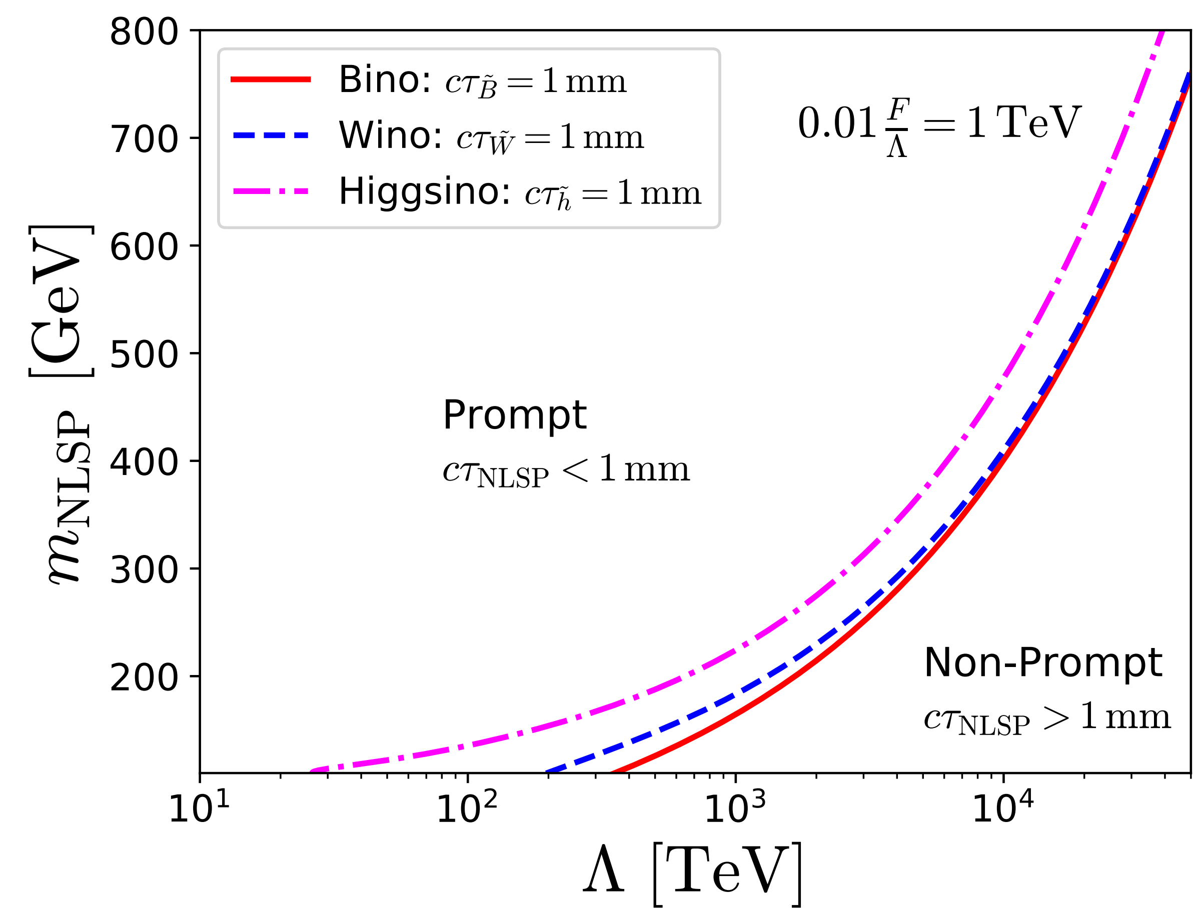
$$\Gamma(\tilde{\chi}_1^0 \to \tilde{G}Z) = \left(\left|N_{12}c_W - N_{11}s_W\right|^2 + \frac{1}{2}\left|N_{13}c_\beta - N_{14}s_\beta\right|^2\right)\left(1 - \frac{m_Z^2}{m_{\tilde{\chi}_1^0}^2}\right)^4 \mathcal{A}\,,$$

$$\Gamma(\tilde{\chi}_1^0 \to \tilde{G}h) = \frac{1}{2}\left|N_{13}c_\beta + N_{14}s_\beta\right|^2\left(1 - \frac{m_h^2}{m_{\tilde{\chi}_1^0}^2}\right)^4 \mathcal{A}\,,$$

$$\mathcal{A} = \frac{m_{\tilde{\chi}_1^0}^5}{16\pi m_{3/2}^2 M_{\rm pl}^2} \sim \frac{1}{0.3\,\mathrm{mm}}\left(\frac{m_{\tilde{\chi}_1^0}}{100\,\mathrm{GeV}}\right)^5\left(\frac{m_{3/2}}{10\,\mathrm{eV}}\right)^{-2}$$







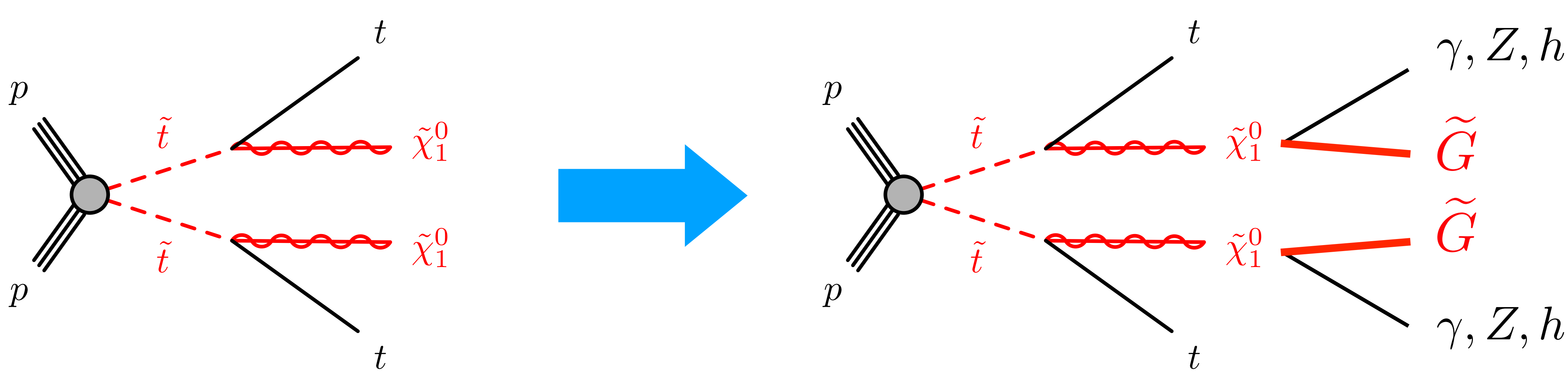
How the LHC limit on gluino and stop masses change if gravitino is the LSP?

J-S.Kim, S.Pokorski, K.Rolbiecki, KS, 1905.05648

higgsino **stop** **gluino**

$$\delta M_B \propto M_{\rm SUSY} \ln\left(\frac{\Lambda_{\rm UV}}{M_{\rm SUSY}}\right) \longrightarrow m_h^2 \sim \underbrace{(m_{H_u}^2 + |\mu|^2)}_{\rm tree} - \underbrace{\frac{3y_t^2}{8\pi^2} m_{\rm stop}^2 \log\frac{\Lambda}{Q}}_{1-\rm loop} - \underbrace{\frac{g_3^2 y_t^2}{4\pi^4}|M_3|^2\left(\log\frac{\Lambda}{Q}\right)^2}_{2-\rm loop}$$

neutralino LSP **gravitino LSP**



2 t + MET **less MET, extra γ, Z or h**

Monte Carlo simulation with CheckMATE

CheckMATE identifier	Description	L [fb^{-1}]
atlas_1709_04183	$\tilde{t}$, b-jets + $E_T^{\rm miss}$	36.1
atlas_1712_02332	$\tilde{q}$, $\tilde{g}$, jets + $E_T^{\rm miss}$	36.1
atlas_1710_11412	dark matter with t or b, b-jets, leptons	36.1
atlas_1802_03158	GMSB, $\gamma(\geq 1)$ + $E_T^{\rm miss}$	36.1
atlas_conf_2017_019	$\tilde{t}$, Z or h + $E_T^{\rm miss}$	36.1
cms_1801_03957	electroweak, diboson final states (W, Z, h)	35.9
cms_sus_16_046	GMSB, $\gamma(\geq 1)$ + $E_T^{\rm miss}$	35.9
atlas_conf_2018_041	$\tilde{g}$, b-jets + $E_T^{\rm miss}$	79.9

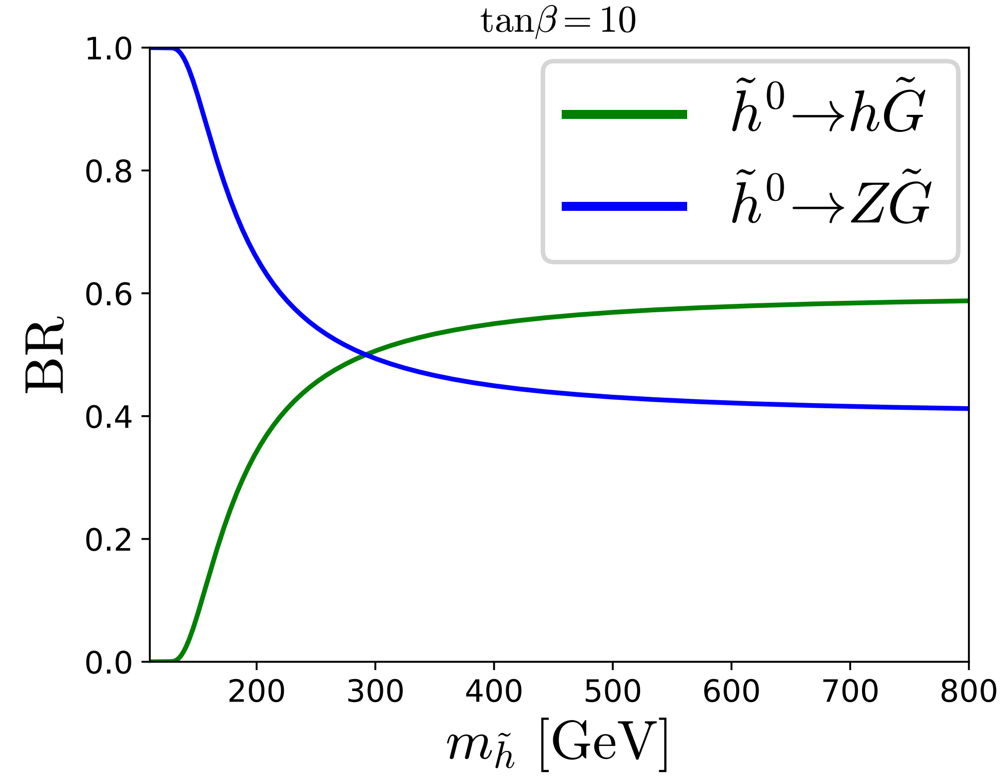
J-S.Kim, S.Pokorski, K.Rolbiecki, KS, 1905.05648

Stop-Higgsino

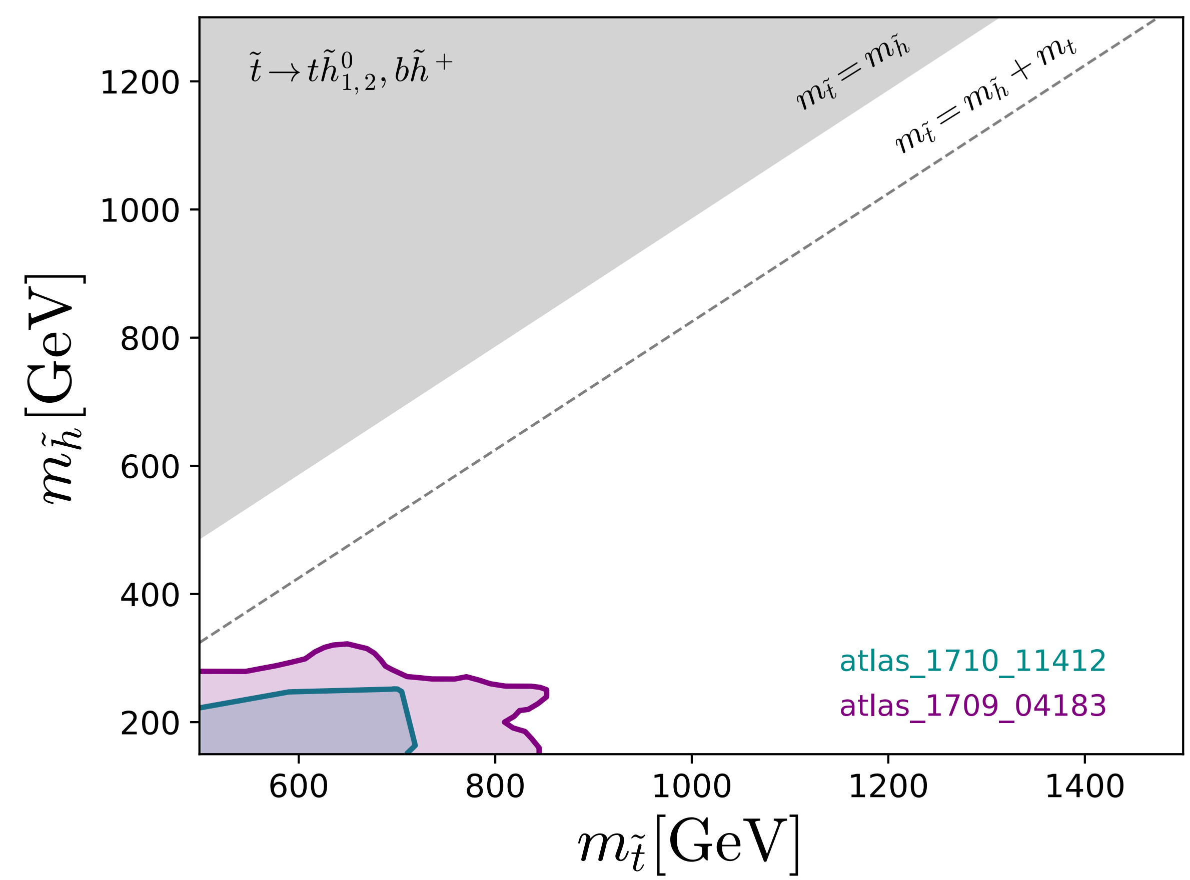
$$\widetilde{t}_1 \to t\widetilde{h}_1^0 \quad (\text{BR} \simeq 25\%)$$
$$\widetilde{t}_1 \to b\widetilde{h}^+ \quad (\text{BR} \simeq 50\%)$$
$$\widetilde{t}_1 \to t\widetilde{h}_2^0 \quad (\text{BR} \simeq 25\%)$$

$$\text{BR}(\widetilde{h}_1^0 \to \gamma\widetilde{G}) = 0$$

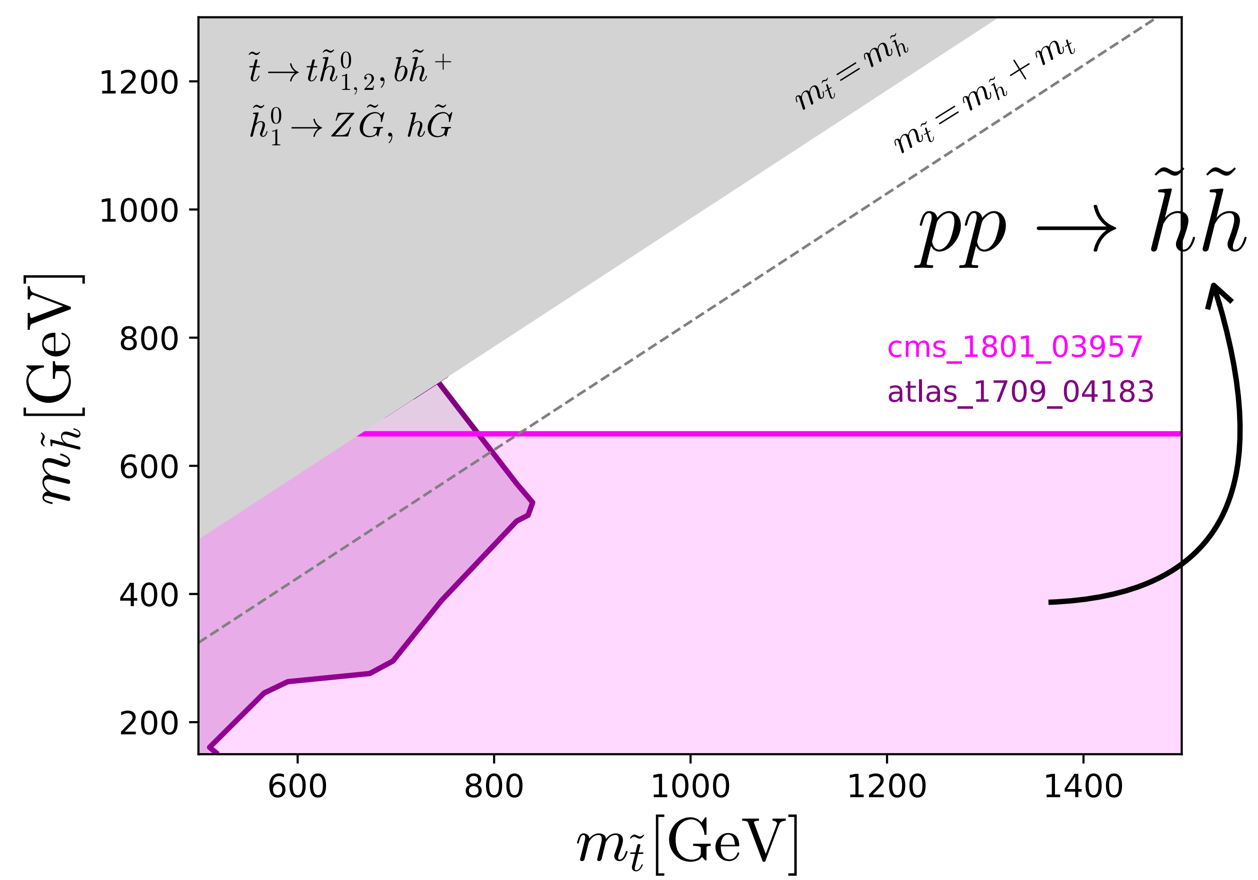
$$\frac{\Gamma(\widetilde{h}_1^0 \to Z\widetilde{G})}{\Gamma(\widetilde{h}_1^0 \to h\widetilde{G})} \simeq \frac{|c_\beta + s_\beta|^2\left(1 - m_Z^2/m_{\widetilde{h}_1^0}^2\right)^4}{|c_\beta - s_\beta|^2\left(1 - m_h^2/m_{\widetilde{h}_1^0}^2\right)^4}$$



neutralino LSP



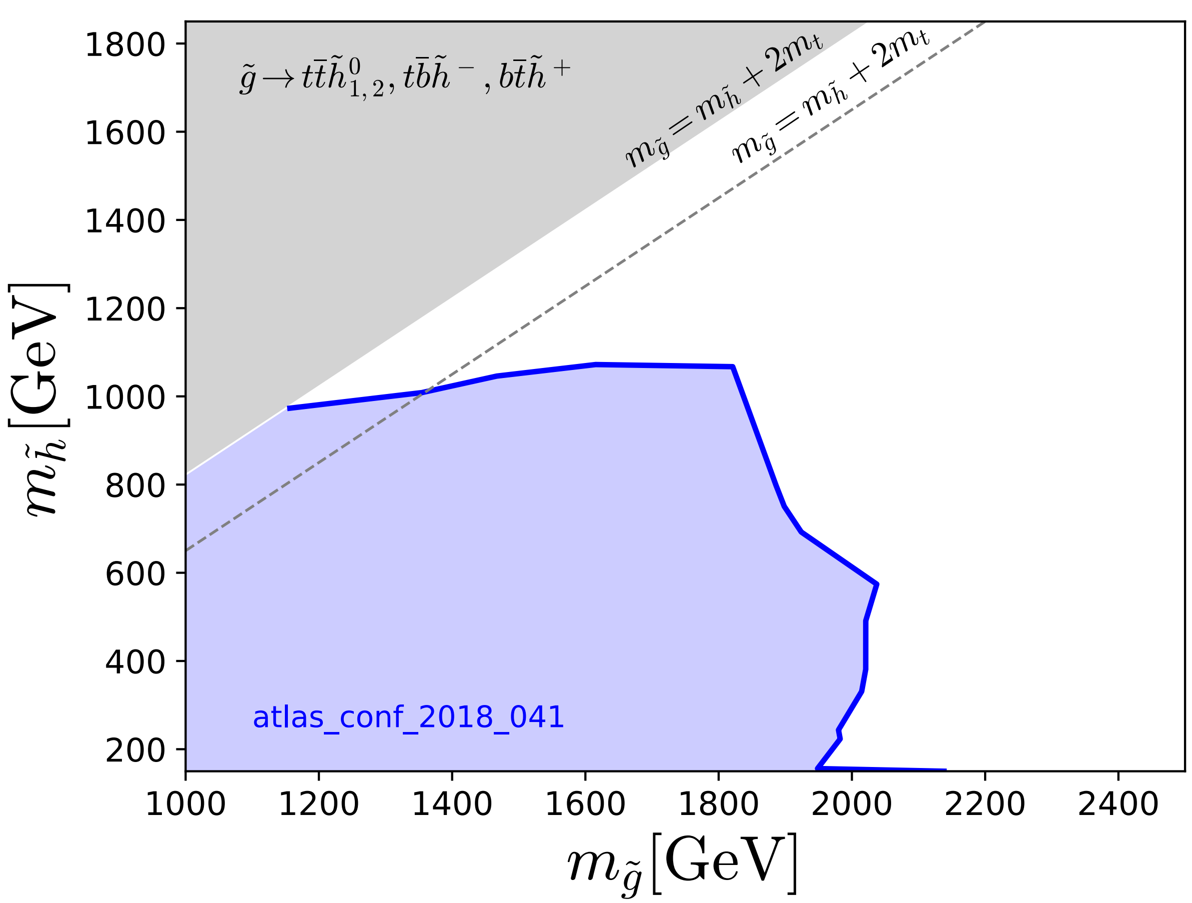
gravitino LSP



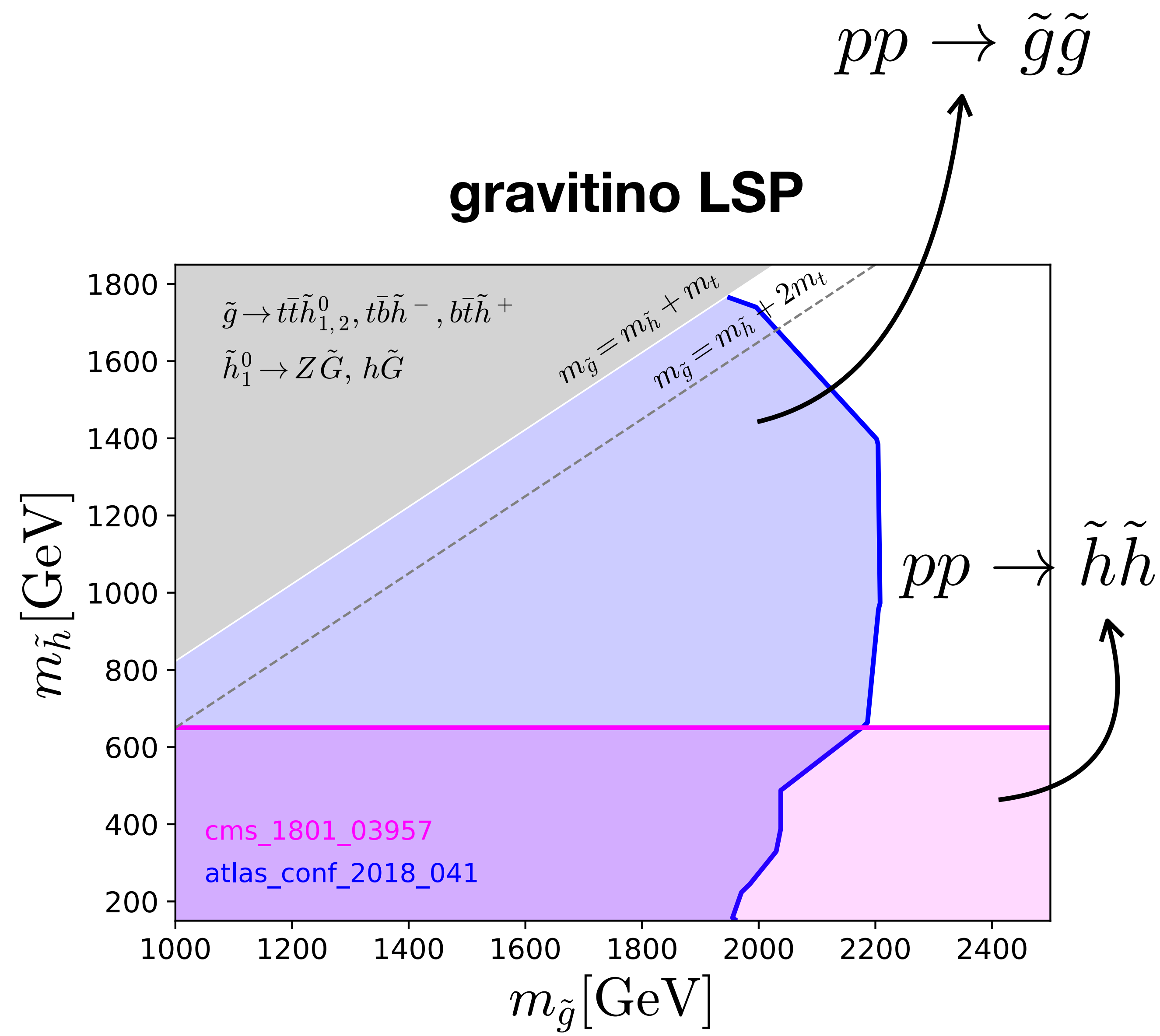
Gluino-Higgsino

$$\widetilde{g} \to t\bar{t}\widetilde{\chi}_1^0 \quad (\mathrm{BR} = 25\%),$$
$$\widetilde{g} \to t\bar{t}\widetilde{\chi}_2^0 \quad (\mathrm{BR} = 25\%),$$
$$\widetilde{g} \to t\bar{b}\widetilde{\chi}_1^- \; + \; \mathrm{c.c} \quad (\mathrm{BR} = 50\%)$$

neutralino LSP



gravitino LSP



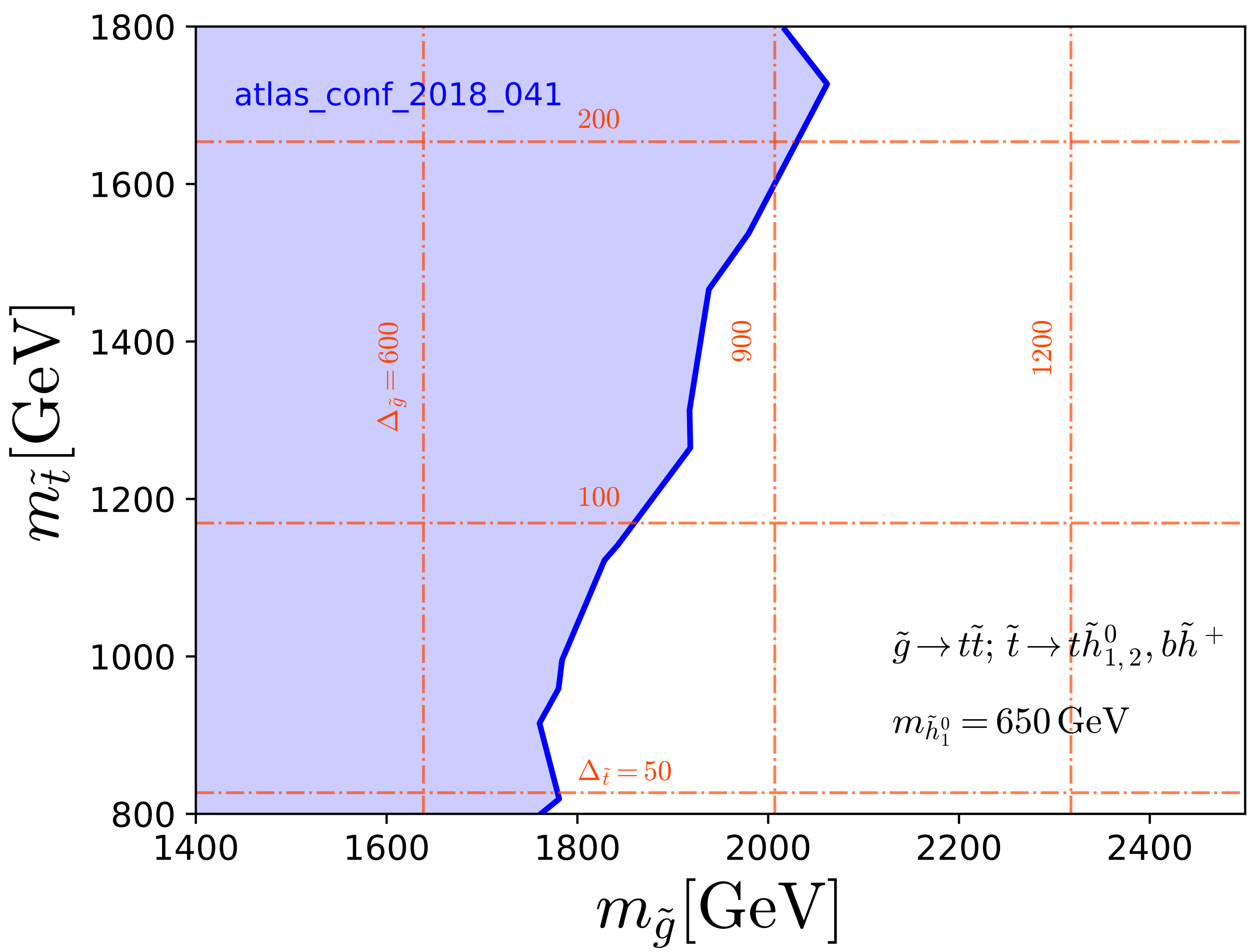
Gluino-Stop-Higgsino

fine-tuning: $\Delta = \max_x \left[\frac{\delta m_h^2}{\delta m_x^2} \frac{m_x^2}{m_h^2} \right]$

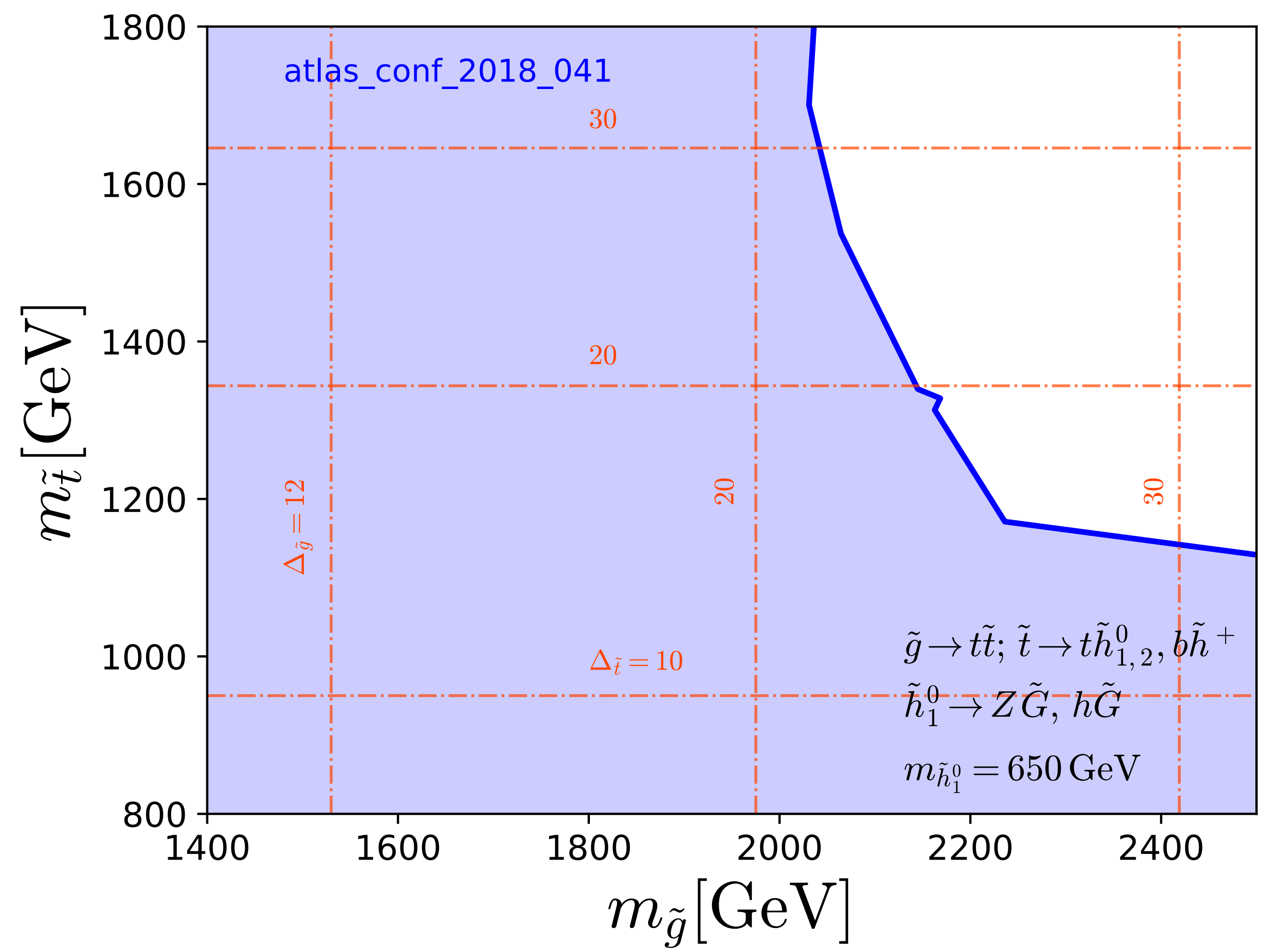
$$m_h^2 \sim \underbrace{(m_{H_u}^2 + |\mu|^2)}_{\text{tree}} - \underbrace{\frac{3y_t^2}{8\pi^2} m_{\text{stop}}^2 \log \frac{\Lambda}{Q}}_{1-\text{loop}} - \underbrace{\frac{g_3^2 y_t^2}{4\pi^4} |M_3|^2 \left(\log \frac{\Lambda}{Q} \right)^2}_{2-\text{loop}}$$

The SUSY limit gets stronger in gauge mediation with gravitino LSP, but the fine-tuning is still relaxed compared to ordinary neutralino LSP scenarios.

neutralino LSP



gravitino LSP $\Delta_{\tilde{h}} \simeq 27$



Muon and Electron g-2

Muon and Electron g-2

$$\vec{\mu} = g\left(\frac{e}{2m}\right)\vec{s}$$

$$a_\ell = \frac{g-2}{2}$$

- **3.7-σ anomaly in muon g-2**

$$\Delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM} = (2.74 \pm 0.73) \times 10^{-9}$$

T. Blum, et al '18
BNL '06

$$\mathcal{L}_{\rm eff} = i\frac{a_\ell}{m_\ell} \cdot \bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}$$

- **2.4-σ anomaly in electron g-2**

$$\Delta a_e \equiv a_e^{\rm exp} - a_e^{\rm SM} = -(8.8 \pm 3.6) \times 10^{-13}$$

R. H. Parker, C. Yu, W. Zhong, B. Estey and H. Mller '18
D. Hanneke, S. Fogwell and G. Gabrielse '08

Any flavour blind new physics predict: $$\frac{m_\mu^2}{m_e^2}\frac{a_e^{\rm NP}}{a_\mu^{\rm NP}} \sim 1$$

Observation tells: $$\frac{m_\mu^2}{m_e^2}\frac{\Delta a_e}{\Delta a_\mu} \sim -14$$

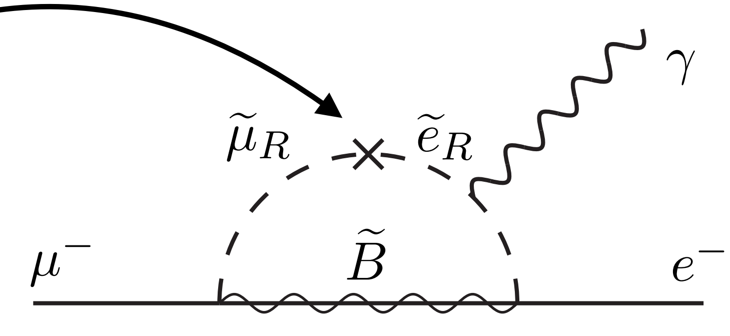
- **magnitude**
- **sign**

flavour violating NP is necessary

- Smuon-Selectron mass splitting is difficult

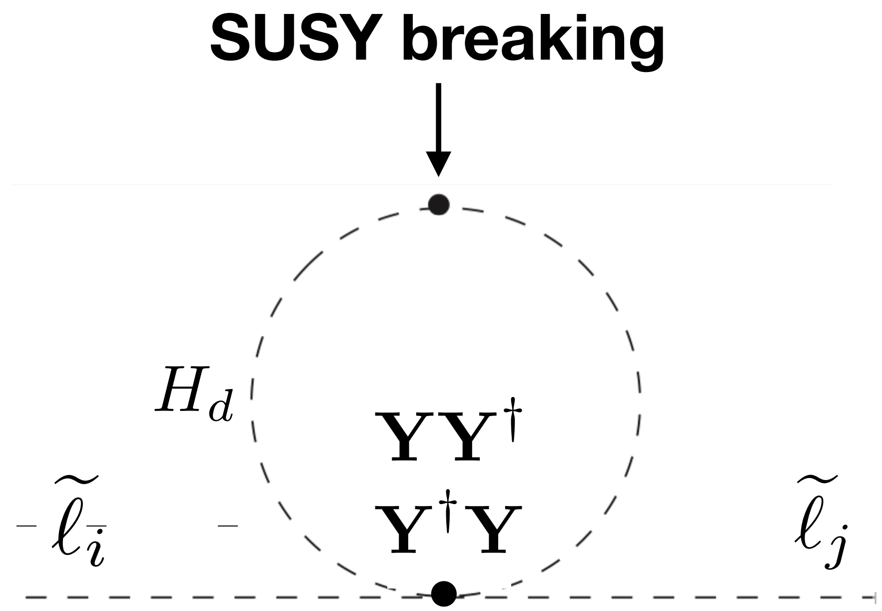
$$\mathbf{Y} \longrightarrow \mathbf{Y}^{\mathrm{diag}} = V_L^\dagger \mathbf{Y} V_R$$

$$\begin{pmatrix} m_1^2 & \\ & m_2^2 \end{pmatrix} \xrightarrow{V^\dagger \mathbf{m}^2 V} \begin{pmatrix} m_1'^2 & \Delta \\ \Delta & m_2'^2 \end{pmatrix}$$



too large contribution to μ→eγ

Higgs mediation



usual diagonal contribution (gravity, GMSB, AMSB, ..)

Higgs mediation contribution

$$\widetilde{\mathbf{m}}_L^2 = m_L^2 \cdot \mathbf{1} + m_H^2 \cdot Y_\mu^{-2} \cdot \mathbf{Y}\mathbf{Y}^\dagger$$

$$\widetilde{\mathbf{m}}_R^2 = m_R^2 \cdot \mathbf{1} + m_H^2 \cdot Y_\mu^{-2} \cdot \mathbf{Y}^\dagger\mathbf{Y}$$

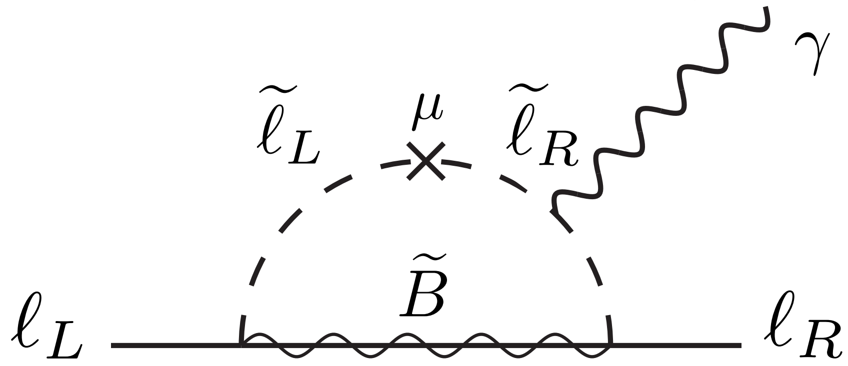
$$\mathbf{Y} \longrightarrow \mathbf{Y}^{\mathrm{diag}} = V_L^\dagger \mathbf{Y} V_R$$

$$\widetilde{\mathbf{m}}_L \longrightarrow (\widetilde{\mathbf{m}}_L^2)^{\mathrm{diag}} = V_L^\dagger \widetilde{\mathbf{m}}_L^2 V_L$$

$$\widetilde{\mathbf{m}}_R^2 \longrightarrow (\widetilde{\mathbf{m}}_R^2)^{\mathrm{diag}} = V_R^\dagger \widetilde{\mathbf{m}}_R^2 V_R$$

soft mass matrices can be diagonalised together with Yukawa

Bino contribution;



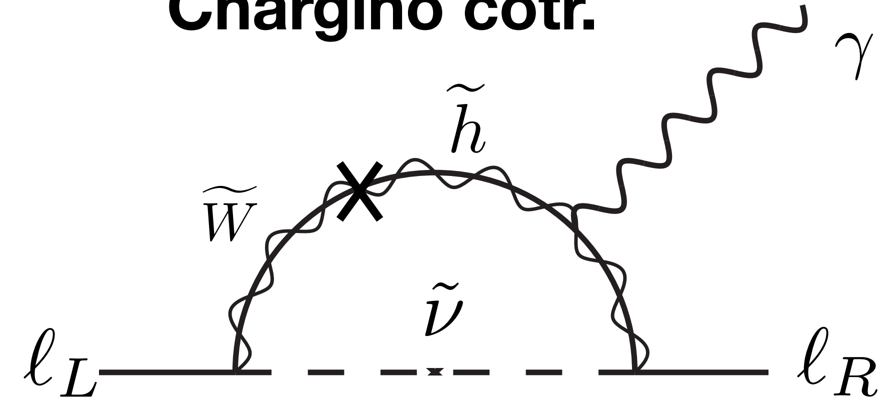
$a_\ell^0 \propto \mathrm{sign}(\mu M_1)$ **= -1**

for electron

$\gg$

(for light sleptons)

Chargino cotr.



$a_\ell^\pm \propto \mathrm{sign}(\mu M_2)$ **= +1**

for muon

- For light selectron + large tanβ, $a_\ell^0 \gg a_\ell^\pm$ ← take this for electron $a_e \sim a_e^0$

- If sleptons are heavy, $a_\mu^{\chi^0} \sim \dfrac{\mu M_1}{m^2_{\tilde{\mu}_L} m^2_{\tilde{\mu}_R}}$ $a_\mu^{\chi^\pm} \sim \dfrac{\mu M_2}{m^2_{\tilde{\chi}_2^\pm} m^2_{\tilde{\nu}_\mu}}$ $m_{\tilde{\chi}_2^\pm} = \max(|\mu|, |M_2|)$

bino contribution decouple faster than chargino
for heavier slepton

M. Badziak, KS, 1908.03607

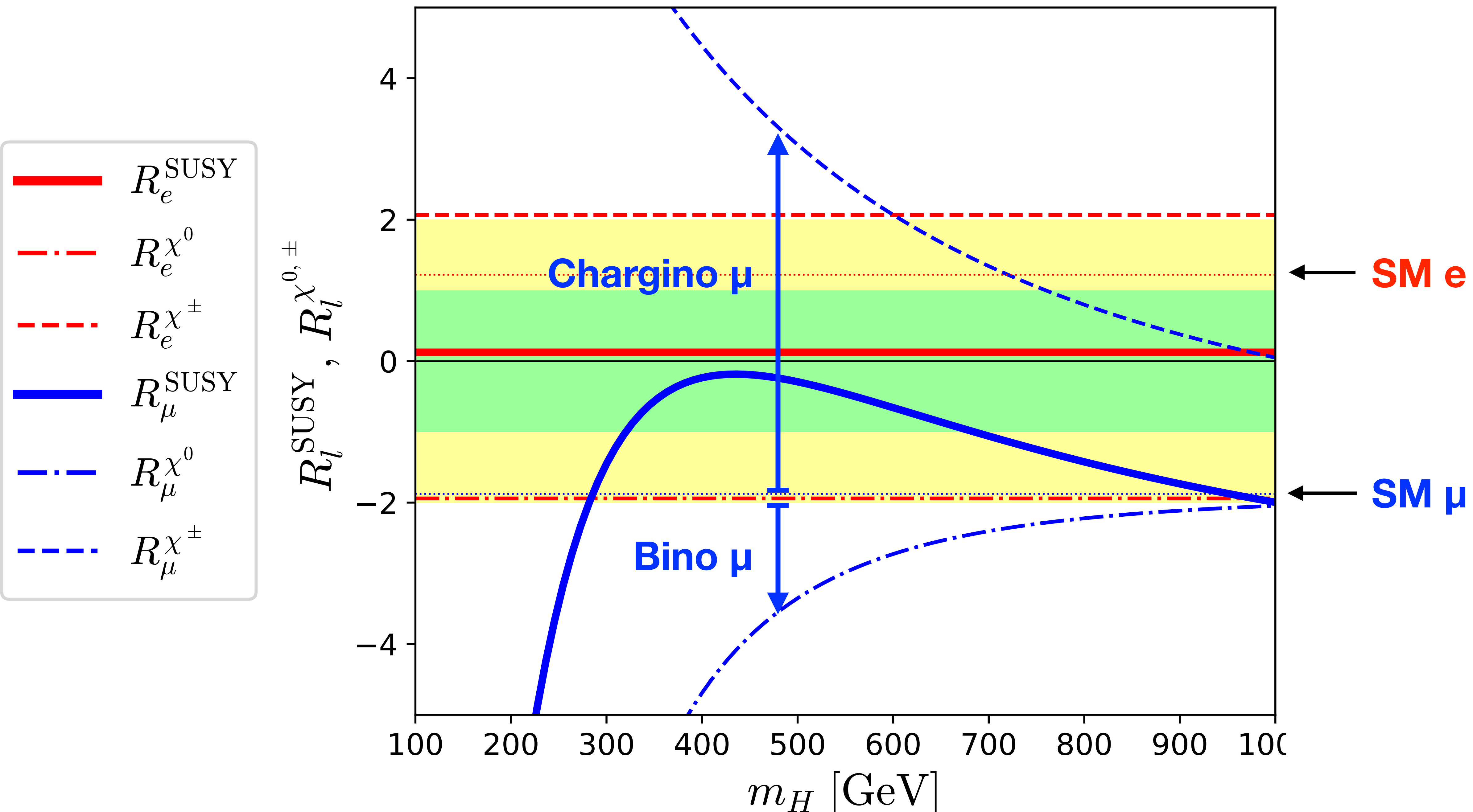
$$R_l^{\chi^\pm/\chi^0} = \frac{2a_l^{\chi^\pm/\chi^0} - \Delta a_l}{2\sigma_l}$$

$$R_l^{\mathrm{SUSY}} = \frac{a_l^{\mathrm{SUSY}} - \Delta a_l}{\sigma_l} = R_l^{\chi^\pm} + R_l^{\chi^0}$$

$$m_{\tilde{E}_1}^2 = m_R^2\,, \qquad m_{\tilde{L}_1}^2 = m_L^2\,,$$

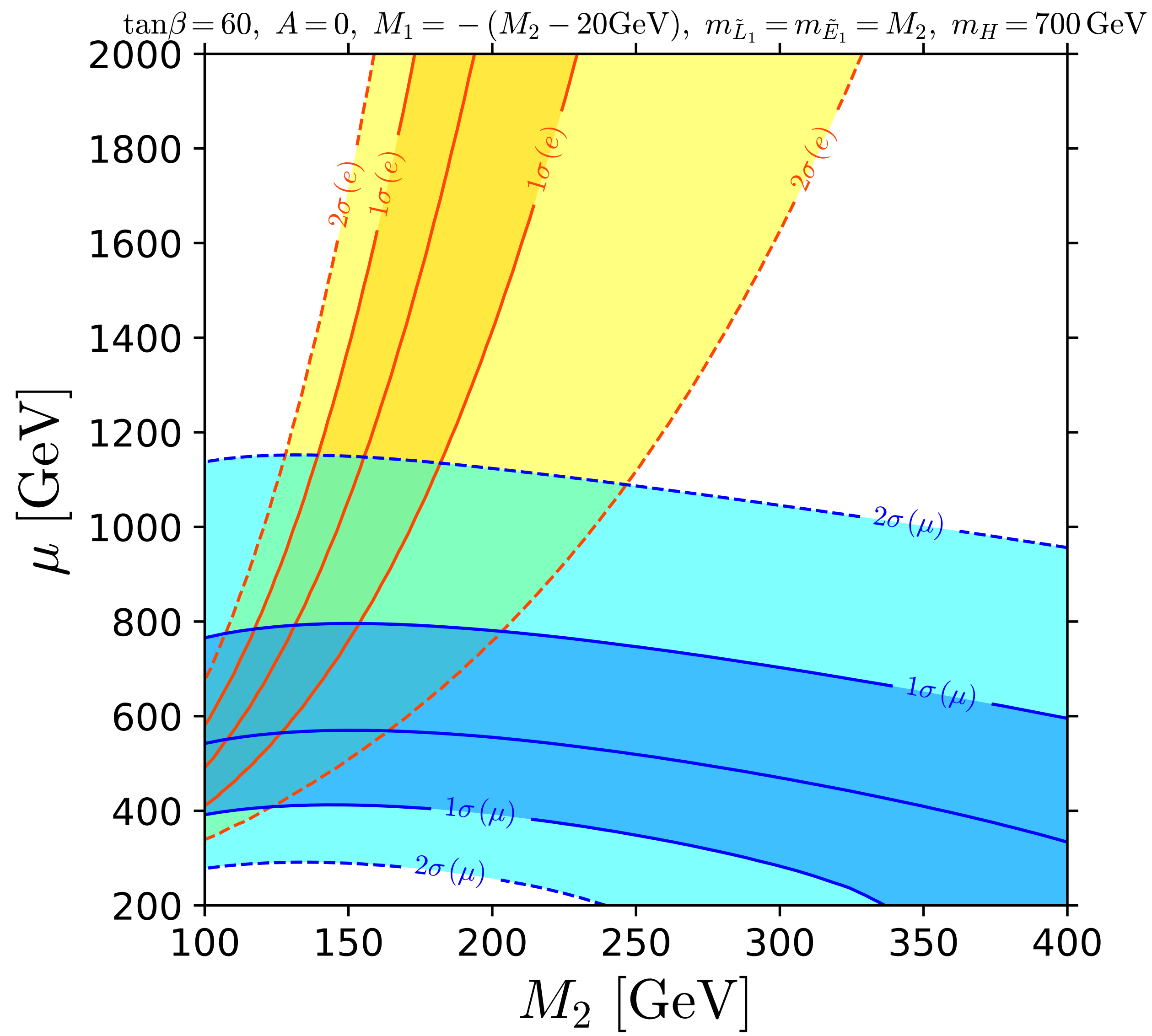
$$m_{\tilde{E}_2}^2 = m_R^2 + m_H^2 \qquad m_{\tilde{L}_2}^2 = m_L^2 + m_H^2$$

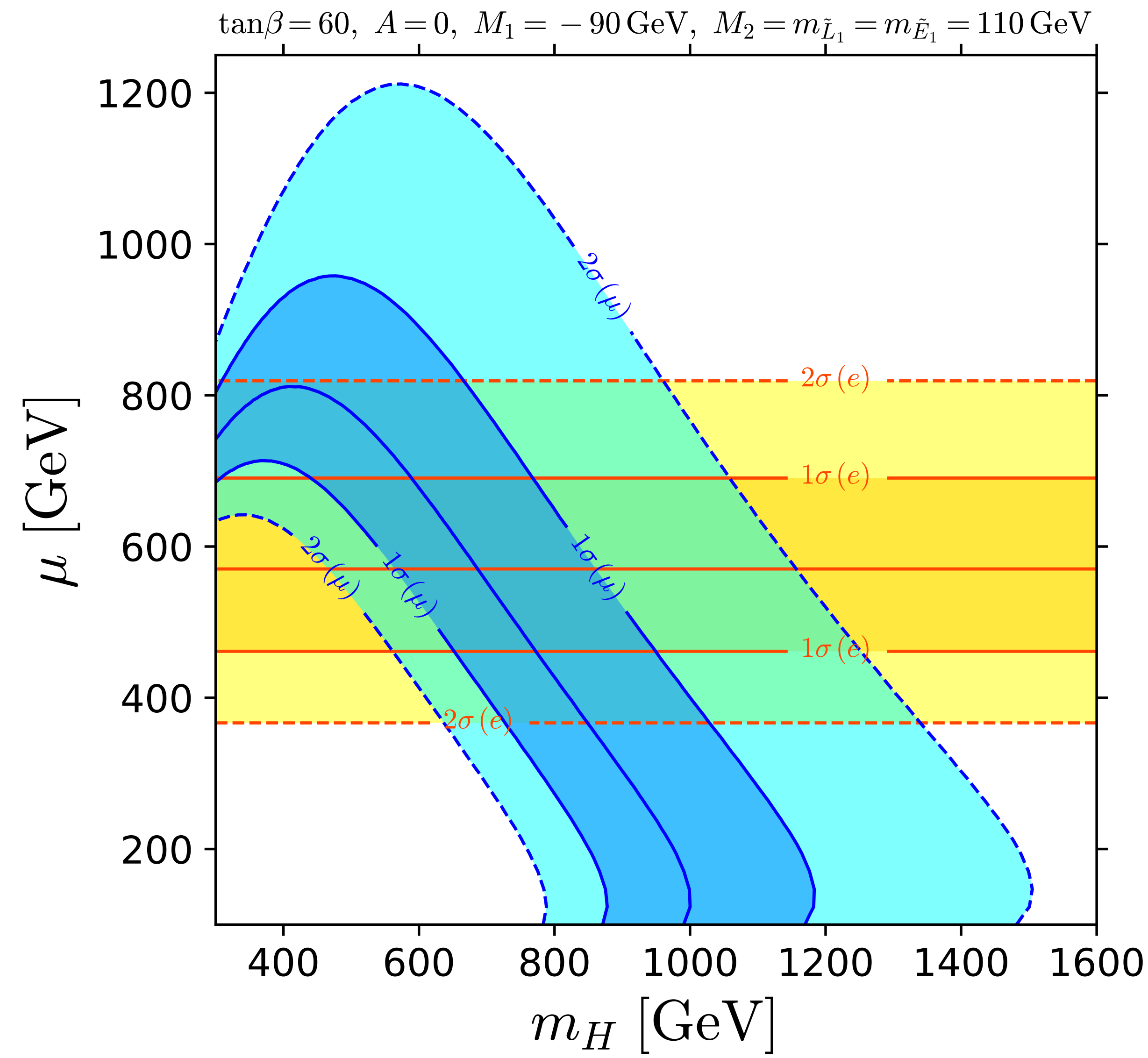
$\tan\beta = 60,\ M_1 = -130\,\mathrm{GeV},\ M_2 = 130\,\mathrm{GeV},\ \mu = 800\,\mathrm{GeV},\ m_L = 130\,\mathrm{GeV},\ m_R = 130\,\mathrm{GeV},$



M. Badziak, KS, 1908.03607

Favoured Regions





M. Badziak, KS, 1908.03607

LHC constraint

- physical masses

$\tilde{\chi}_1^0$	121.2	$\tilde{\nu}_e$	124.7	$\tilde{\nu}_\mu$	711	$\tilde{\chi}_3^0$	711.9
$\tilde{\chi}_2^0$	123.5	$\tilde{e}_L$	147.3	$\tilde{\mu}_L$	715.3	$\tilde{\chi}_4^0$	713.3
$\tilde{\chi}_1^\pm$	123.5	$\tilde{e}_R$	127.9	$\tilde{\mu}_R$	711.6	$\tilde{\chi}_2^\pm$	715.7

- benchmark point

$$M_1 = -125\,\mathrm{GeV}, \quad M_2 = 118\,\mathrm{GeV}, \quad m_R = 120\,\mathrm{GeV}, \quad m_L = 140\,\mathrm{GeV},$$

$$m_H = 700\,\mathrm{GeV}, \quad \mu = 700\,\mathrm{GeV}, \quad A = 0, \quad \tan\beta = 60,$$

- g-2 prediction

$$a_e^{\mathrm{SUSY}} = -6.71 \cdot 10^{-13}, \quad R_e^{\mathrm{SUSY}} = 0.58, \quad (a_e^{\tilde{\chi}_1^0}, a_e^{\tilde{\chi}_1^\pm}) = (-10.170,\ 3.459) \cdot 10^{-13},$$

$$a_\mu^{\mathrm{SUSY}} = \ 2.21 \cdot 10^{-9}, \quad R_\mu^{\mathrm{SUSY}} = -0.73, \quad (a_\mu^{\tilde{\chi}_1^0}, a_\mu^{\tilde{\chi}_1^\pm}) = (-0.336,\ 2.544) \cdot 10^{-9},$$

M. Badziak, KS, 1908.03607

LHC constraint

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$\tilde{\chi}_1^0$	121.2	$\tilde{\nu}_e$	124.7	$\tilde{\nu}_\mu$	711	$\tilde{\chi}_3^0$	711.9
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mode		BR [%]
$\tilde{\chi}_1^\pm$	$\to \tilde{\chi}_1^0 \nu_e e^\pm$	100
$\tilde{\chi}_2^0$	$\to \tilde{\chi}_1^0 \gamma$	12
	$\to \tilde{\chi}_1^0 \nu_e \bar{\nu}_e$	88

invisible

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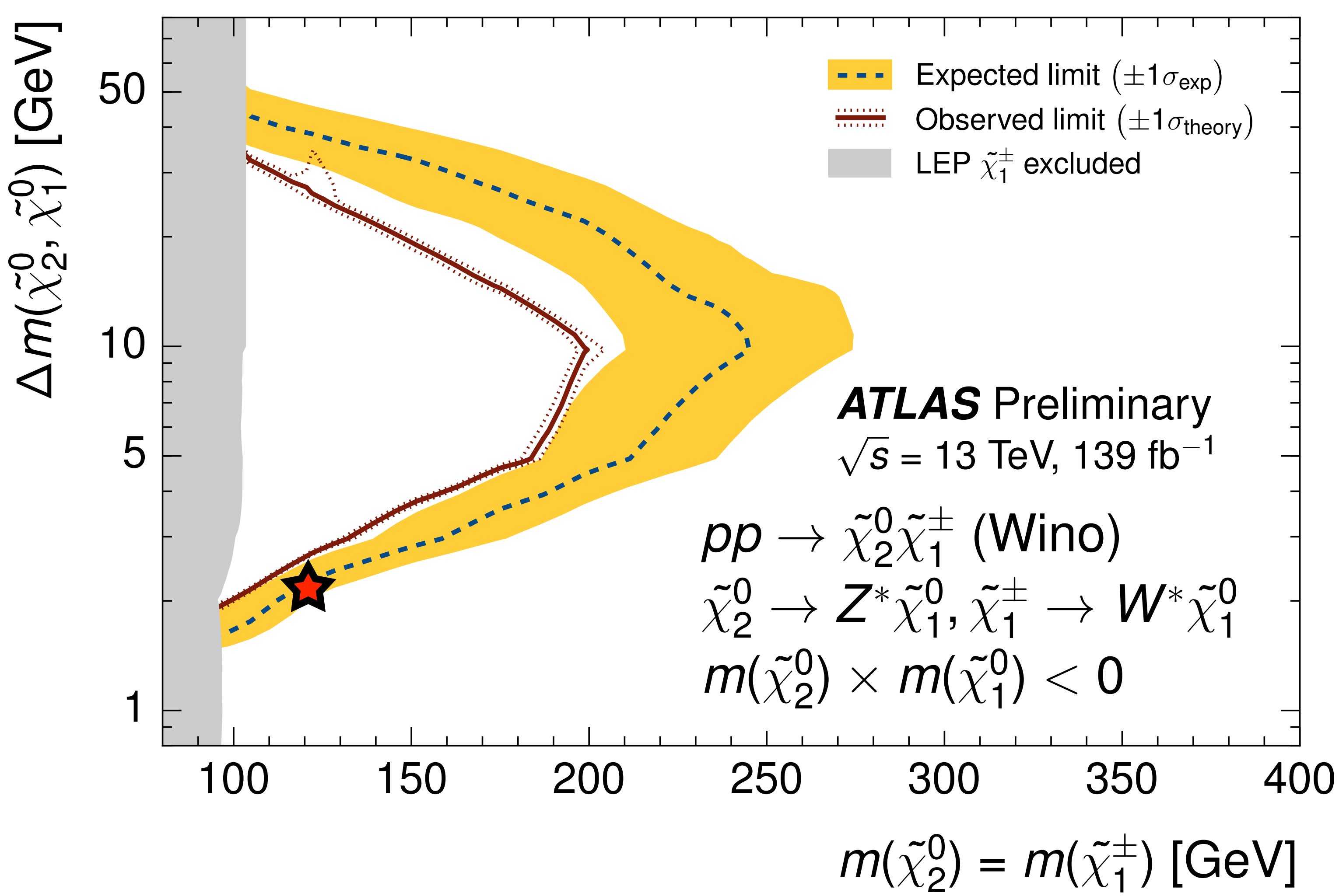

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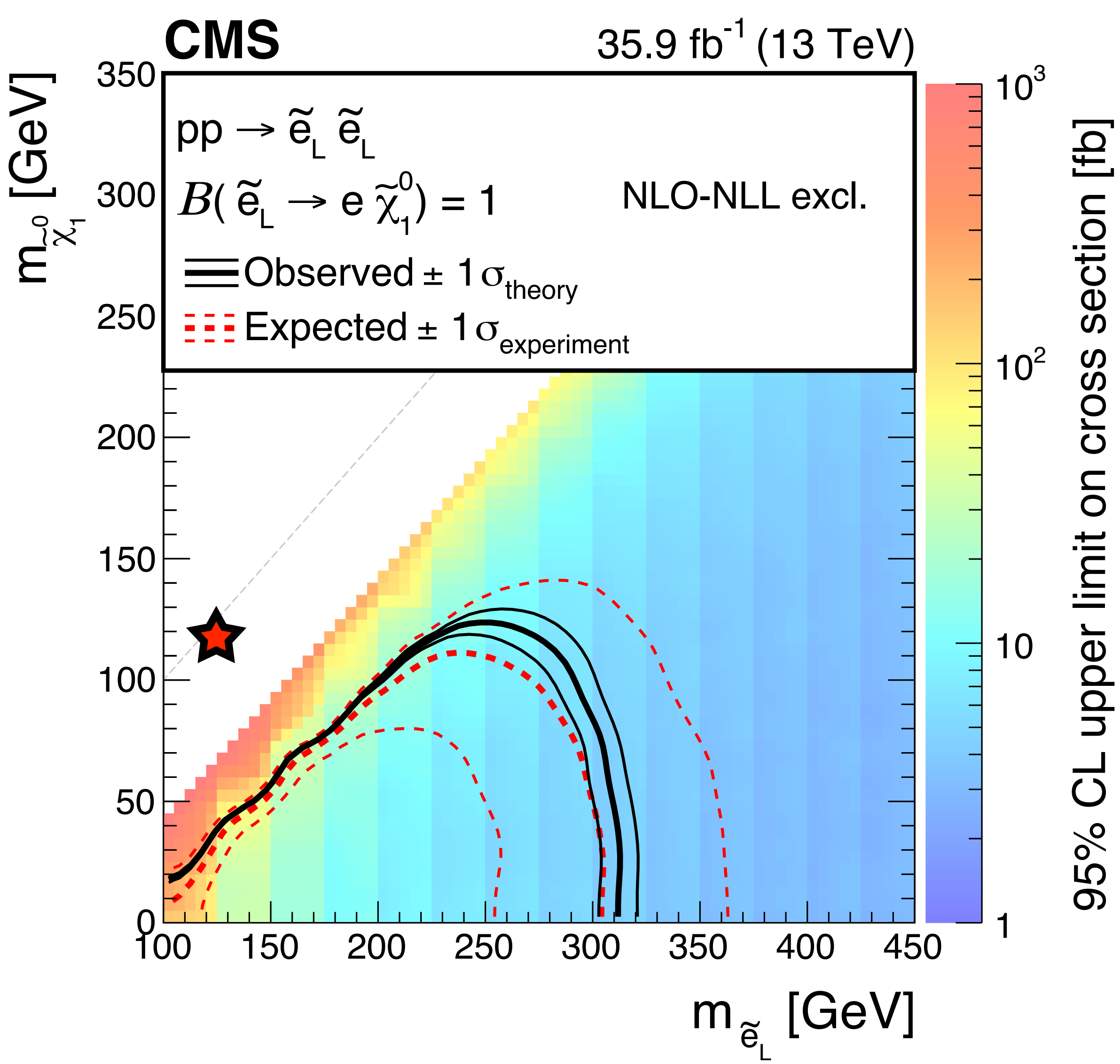
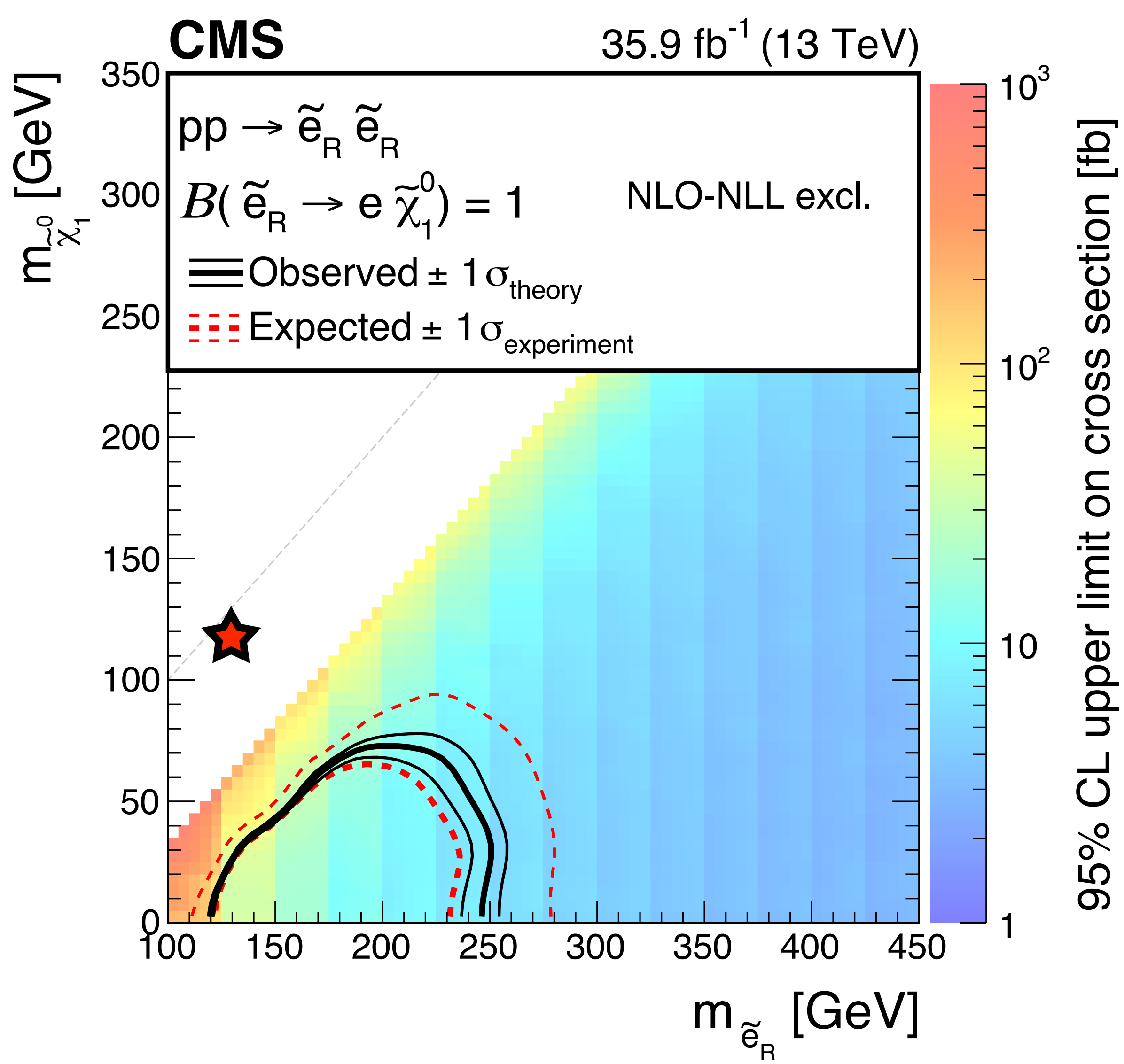
$\tilde{e}_R^\pm$	$\to \tilde{\chi}_1^0 e^\pm$	100
$\tilde{e}_L^\pm$	$\to \tilde{\chi}_1^\pm \nu_e$	59
	$\to \tilde{\chi}_2^0 e^\pm$	30
	$\to \tilde{\chi}_1^0 e^\pm$	11
$\tilde{\nu}_e$	$\to \tilde{\chi}_1^\pm e^\mp$	36
	$\to \tilde{\chi}_2^0 \nu_e$	18
	$\to \tilde{\chi}_1^0 \nu_e$	47





LHC constraint

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$\tilde{\mu}_R^\pm$	$\to \tilde{\chi}_1^0 \mu^\pm$	100
$\tilde{\mu}_L^\pm$	$\to \tilde{\chi}_1^\pm \nu_\mu$	60
	$\to \tilde{\chi}_2^0 \mu^\pm$	30
	$\to \tilde{\chi}_1^0 \mu^\pm$	10
$\tilde{\nu}_\mu$	$\to \tilde{\chi}_1^\pm \mu^\mp$	61
	$\to \tilde{\chi}_2^0 \nu_\mu$	30
	$\to \tilde{\chi}_1^0 \nu_\mu$	10

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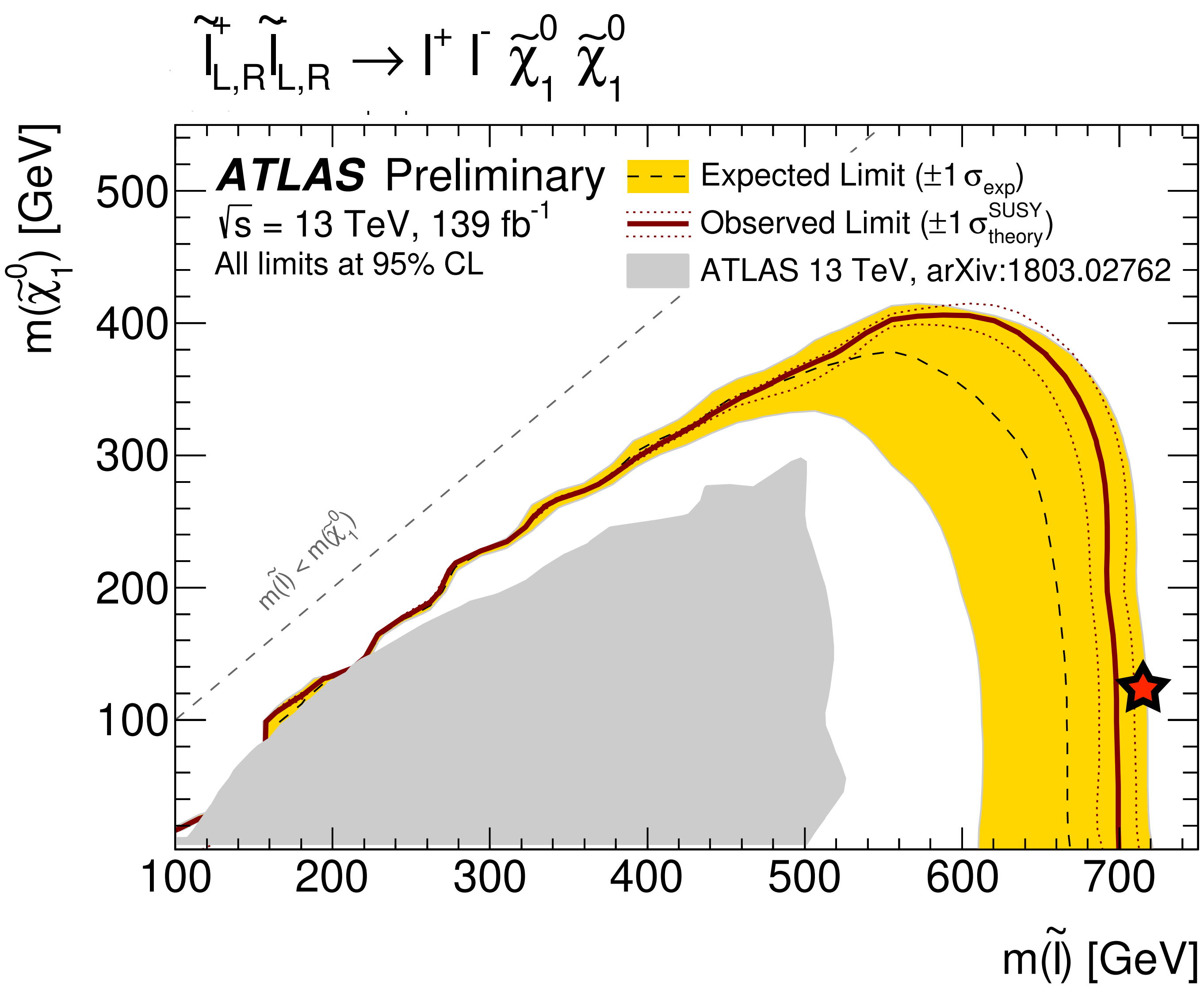

LHC constraint

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mode		BR [%]
$\tilde{\chi}_3^0$	$\to \tilde{\chi}_1^\pm W^\mp$	59
	$\to \tilde{\chi}_2^0 Z$	20
	$\to \tilde{\chi}_1^0 Z$	3
	$\to \tilde{\chi}_2^0 h$	9
	$\to \tilde{\chi}_1^0 h$	6
$\tilde{\chi}_4^0$	$\to \tilde{\chi}_1^\pm W^\mp$	59
	$\to \tilde{\chi}_2^0 h$	20
	$\to \tilde{\chi}_1^0 h$	3
	$\to \tilde{\chi}_2^0 Z$	10
	$\to \tilde{\chi}_1^0 Z$	6
$\tilde{\chi}_2^\pm$	$\to \tilde{\chi}_1^\pm Z$	30
	$\to \tilde{\chi}_1^\pm h$	29
	$\to \tilde{\chi}_2^0 W^\pm$	29
	$\to \tilde{\chi}_1^0 W^\pm$	9
	$\to \tilde{e}^\pm \nu_e$	1.5

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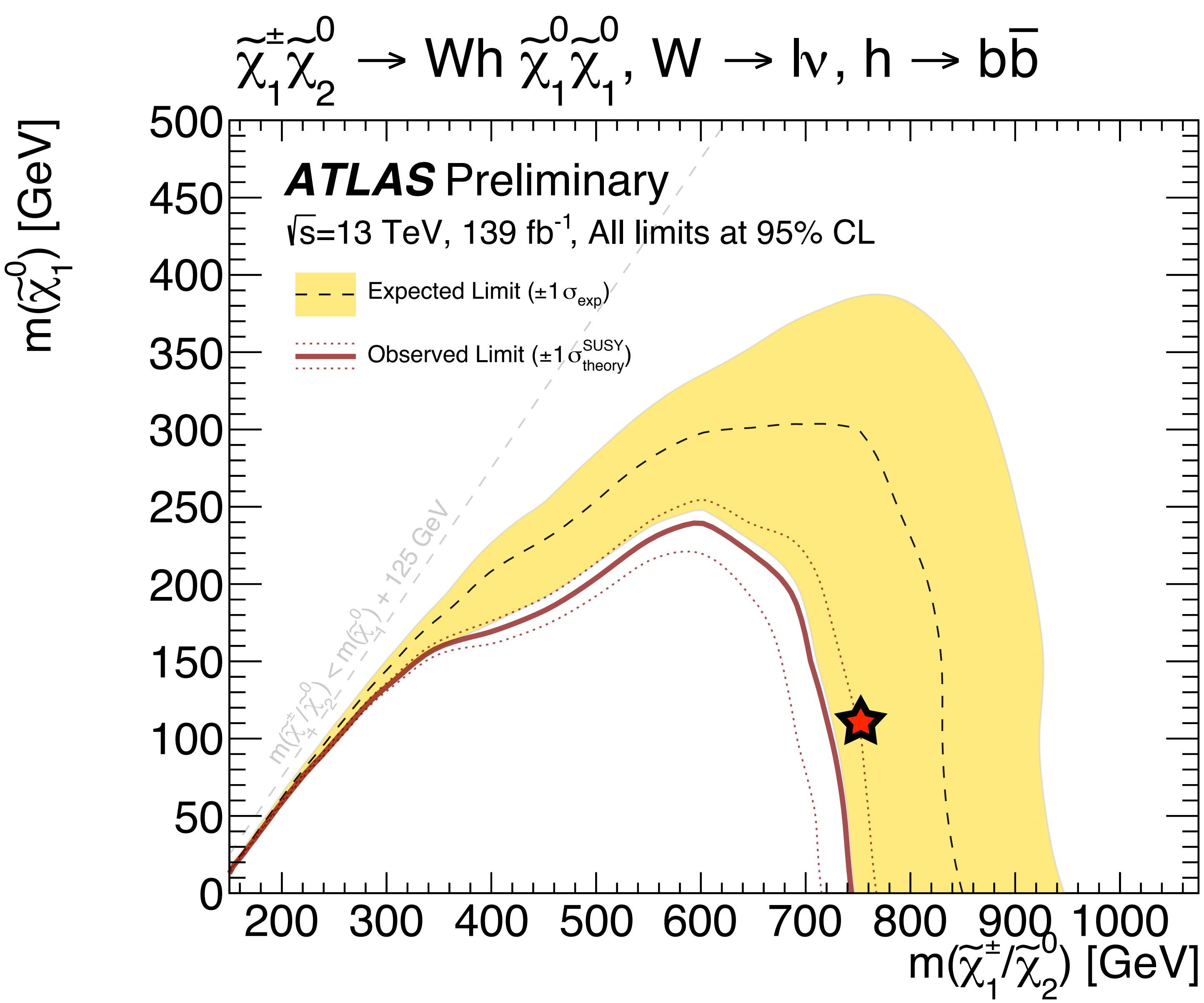

LHC constraint

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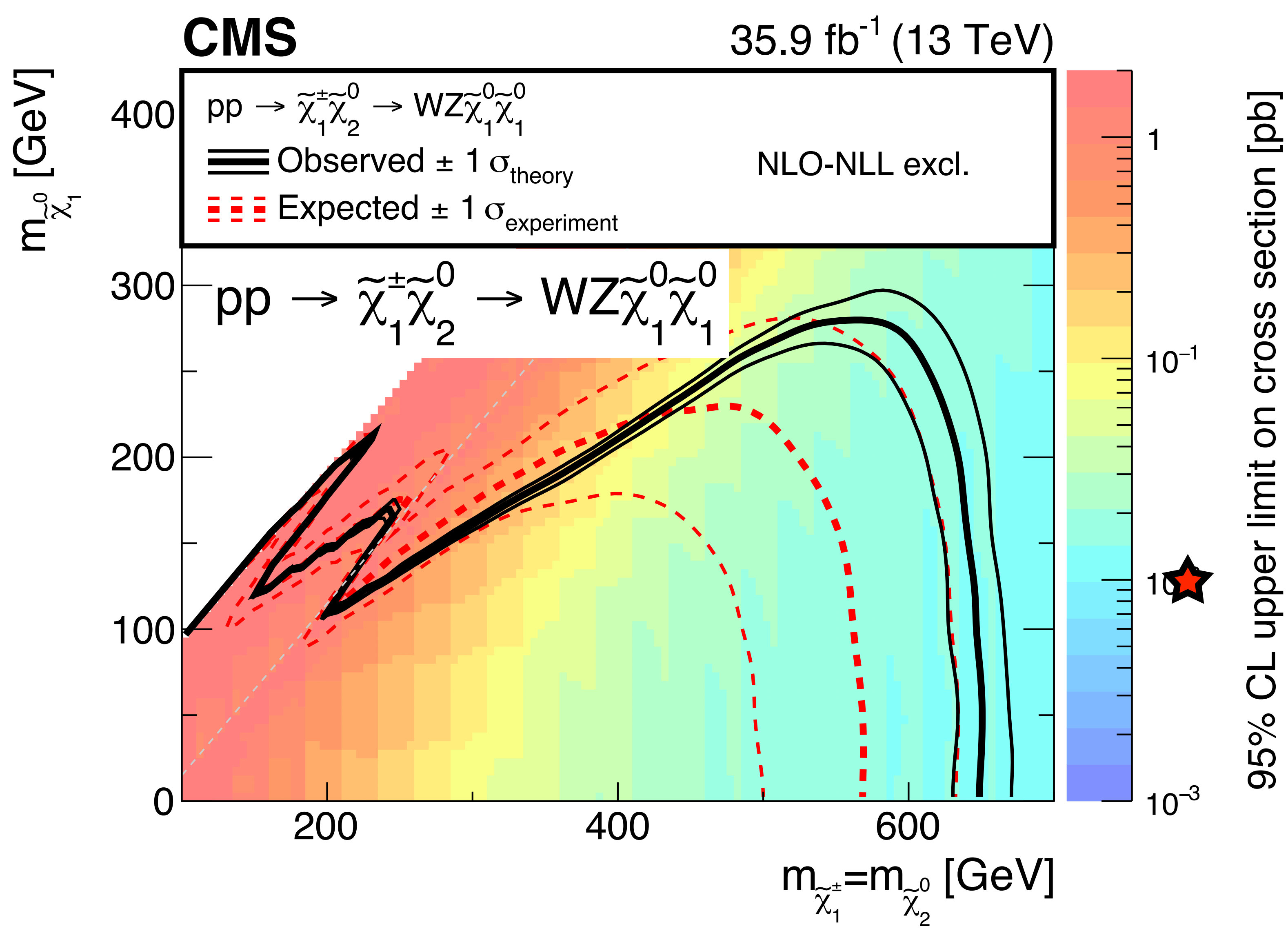
$\tilde{\chi}_1^\pm$	123.5	$\tilde{e}_R$	127.9	$\tilde{\mu}_R$	711.6	$\tilde{\chi}_2^\pm$	715.7

mode		BR [%]
$\tilde{\chi}_3^0$	$\to \tilde{\chi}_1^\pm W^\mp$	59
	$\to \tilde{\chi}_2^0 Z$	20
	$\to \tilde{\chi}_1^0 Z$	3
	$\to \tilde{\chi}_2^0 h$	9
	$\to \tilde{\chi}_1^0 h$	6
$\tilde{\chi}_4^0$	$\to \tilde{\chi}_1^\pm W^\mp$	59
	$\to \tilde{\chi}_2^0 h$	20
	$\to \tilde{\chi}_1^0 h$	3
	$\to \tilde{\chi}_2^0 Z$	10
	$\to \tilde{\chi}_1^0 Z$	6
$\tilde{\chi}_2^\pm$	$\to \tilde{\chi}_1^\pm Z$	30
	$\to \tilde{\chi}_1^\pm h$	29
	$\to \tilde{\chi}_2^0 W^\pm$	29
	$\to \tilde{\chi}_1^0 W^\pm$	9
	$\to \tilde{e}^\pm \nu_e$	1.5

1801.03957



Gauge Coupling Unification

Gauge Coupling Unification

$$\frac{d}{dt}\tilde{\alpha}_i^{-1} = b_i$$

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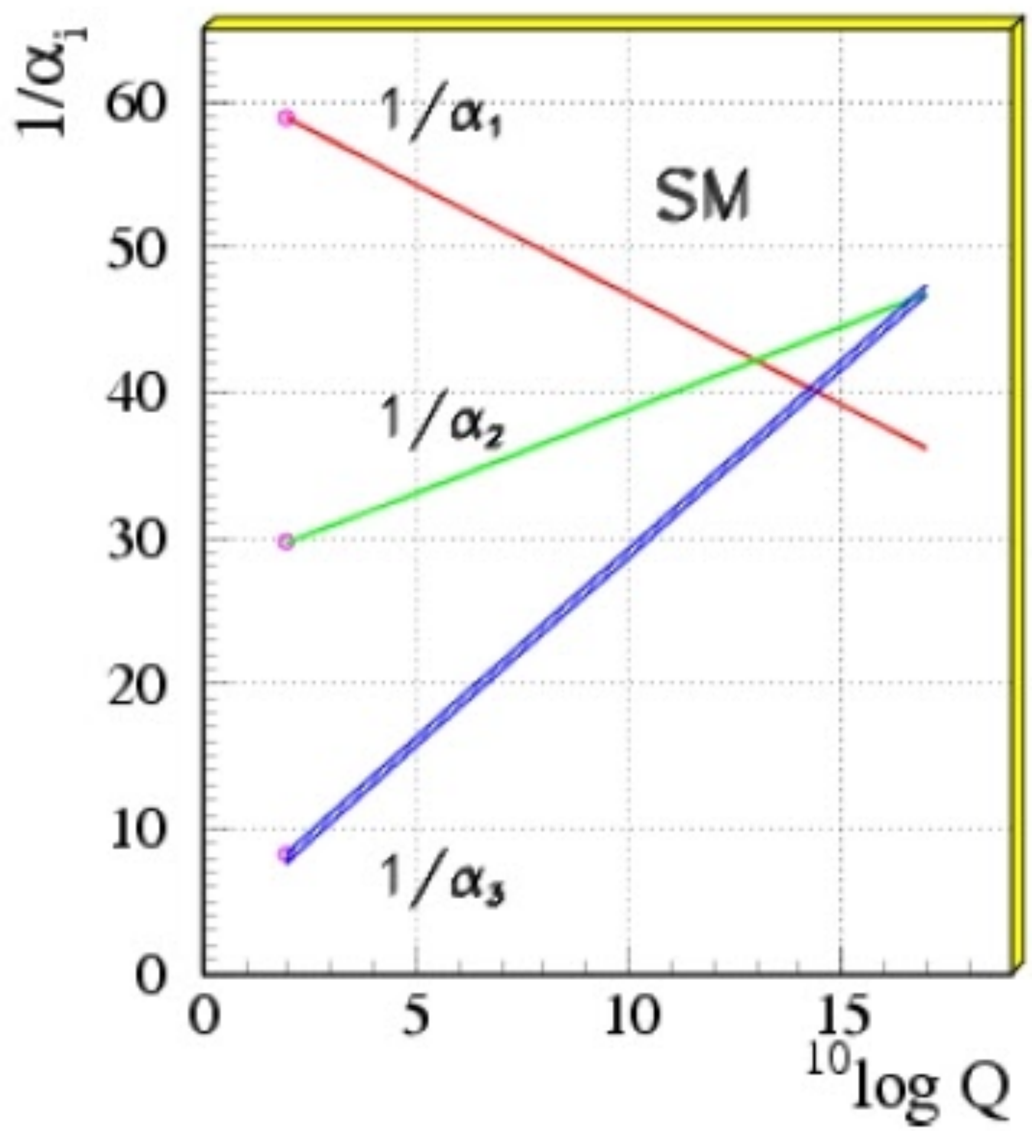
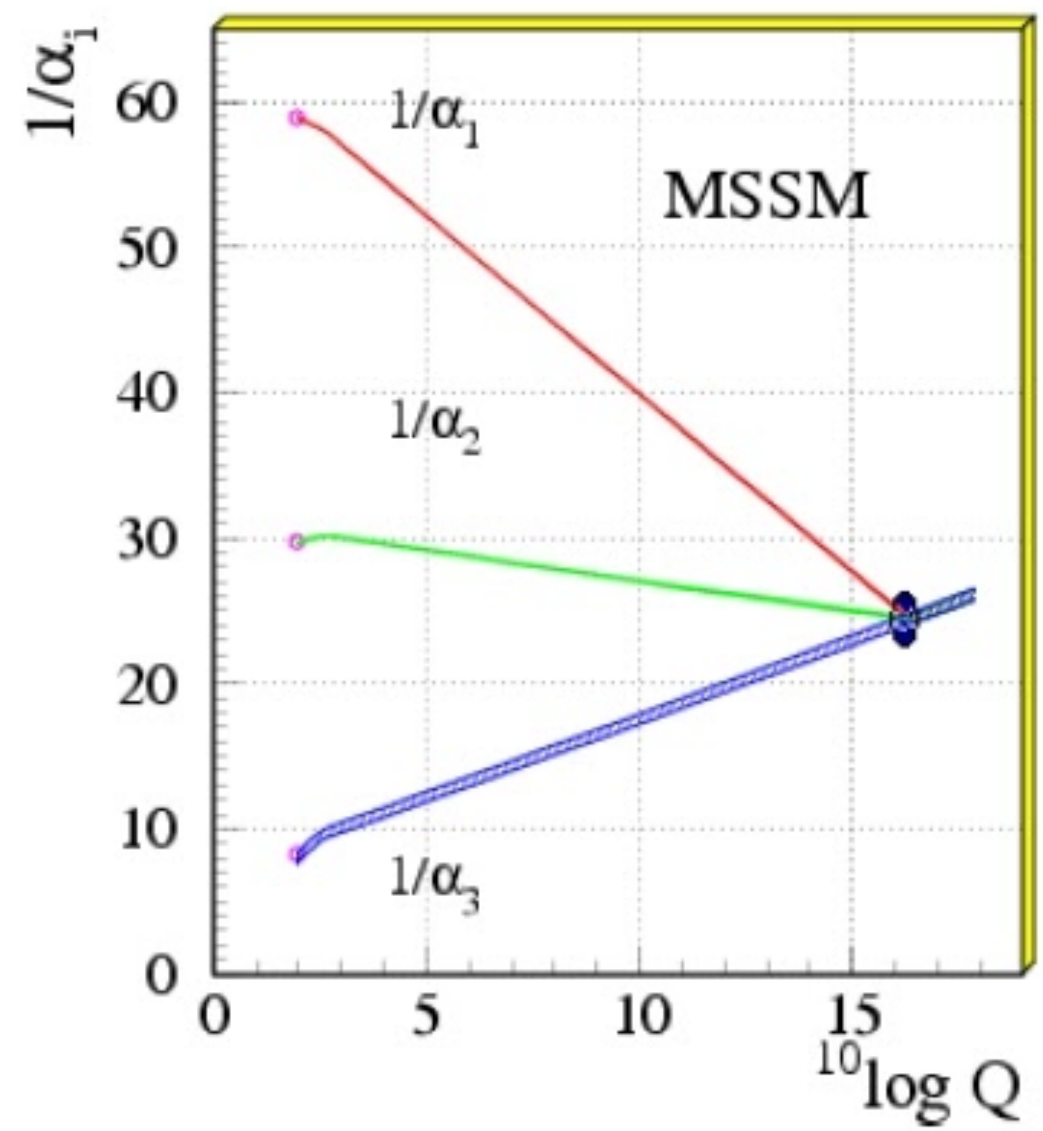
$$t \equiv \ln\left(Q/Q_0\right) > 0$$





$$b_i = \left(\tfrac{41}{10},\ -\tfrac{19}{6},\ -7\right)$$

$$b_i = \left(\tfrac{33}{5},\ 1,\ -3\right)$$

Grand Unification

Fermion	$\mathrm{SU}(3)_C$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$
$Q_{Li} = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$1/6$
$u^\dagger_{Ri}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-2/3$
$d^\dagger_{Ri}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$1/3$
$L_{Li} = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$
$e^\dagger_{Ri}$	$\mathbf{1}$	$\mathbf{1}$	$+1$

$$SU(5) \supset SU(3) \times SU(2) \times U(1)$$

$$\Phi^{\bar{\mathbf{5}}}_{\bar{i}} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{pmatrix}$$

Georgi, Glashow ‘74

Grand Unification

Fermion	SU(3)$_C$	SU(2)$_L$	U(1)$_Y$
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$$\langle \Sigma_{\mathbf{24}} \rangle = \underset{\sim 10^{16}\,\mathrm{GeV}}{V'} \cdot \begin{pmatrix} -\frac{1}{3} & & & & \\ & -\frac{1}{3} & & & \\ & & -\frac{1}{3} & & \\ & & & \frac{1}{2} & \\ & & & & \frac{1}{2} \end{pmatrix}$$

$$SU(5) \to SU(3) \times SU(2) \times U(1)$$

Georgi, Glashow ‘74

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$$\Psi^{\mathbf{10}}_{\langle ij \rangle} = \begin{pmatrix} 0 & \bar{U}_3 & -\bar{U}_2 & U^1 & D^1 \\ -\bar{U}_3 & 0 & \bar{U}_1 & U^2 & D^2 \\ \bar{U}_2 & -\bar{U}_1 & 0 & U^3 & D^3 \\ -U^1 & -U^2 & -U^3 & 0 & \bar{E} \\ -D^1 & -D^2 & -D^3 & -\bar{E} & 0 \end{pmatrix}$$

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Georgi, Glashow '74

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$$\boxed{\bar{\mathbf{5}} + \mathbf{10} + \mathbf{1}_{\nu_R} = \mathbf{16} \quad \text{in SO(10)}}$$

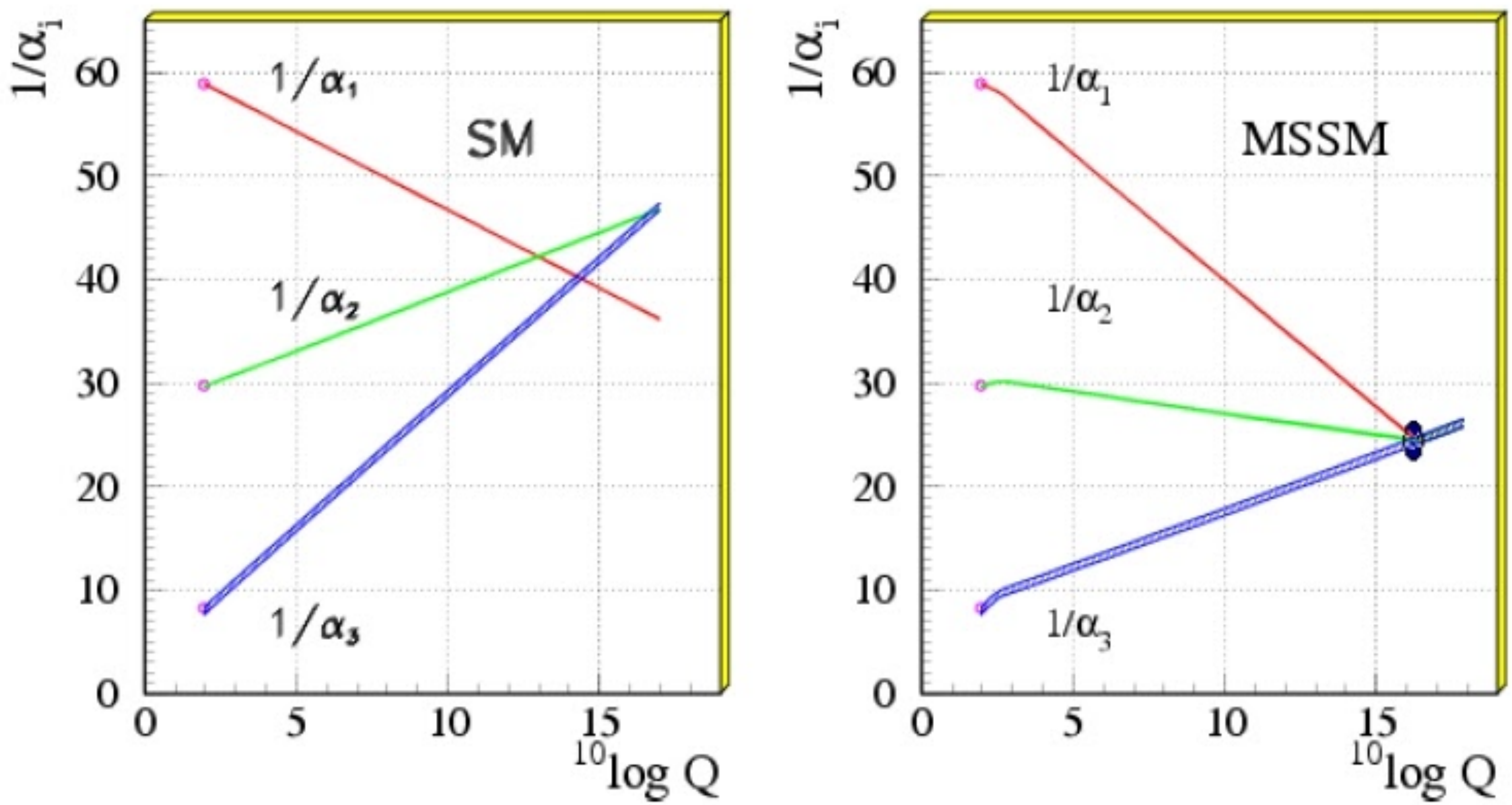
Georgi, Glashow '74

Gauge Coupling Unification

$$\frac{d}{dt}\tilde{\alpha}_i^{-1} = b_i$$

$$\tilde{\alpha}_i^{-1} \equiv 2\pi\alpha_i^{-1}$$

$$t \equiv \ln\left(Q/Q_0\right) > 0$$



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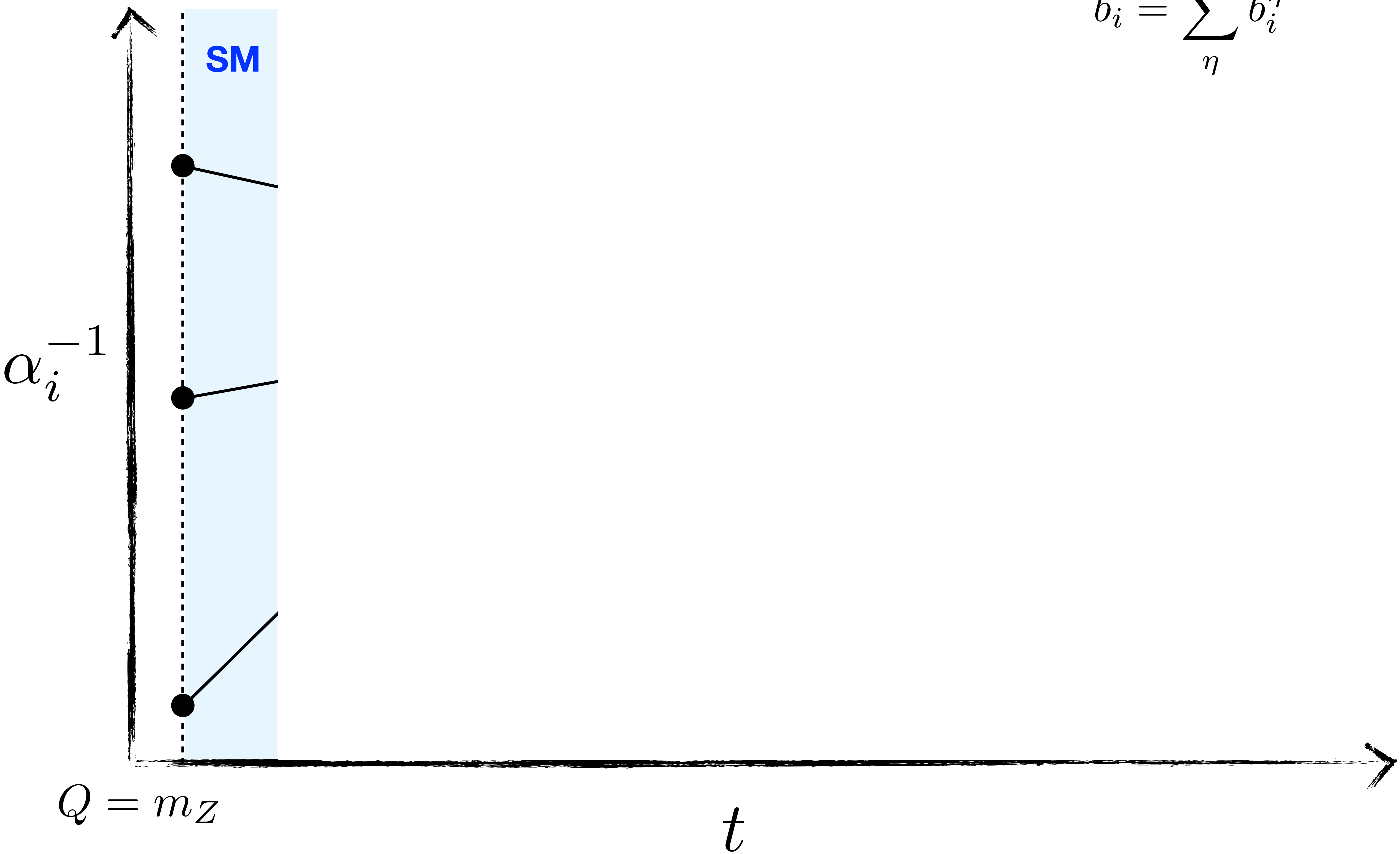

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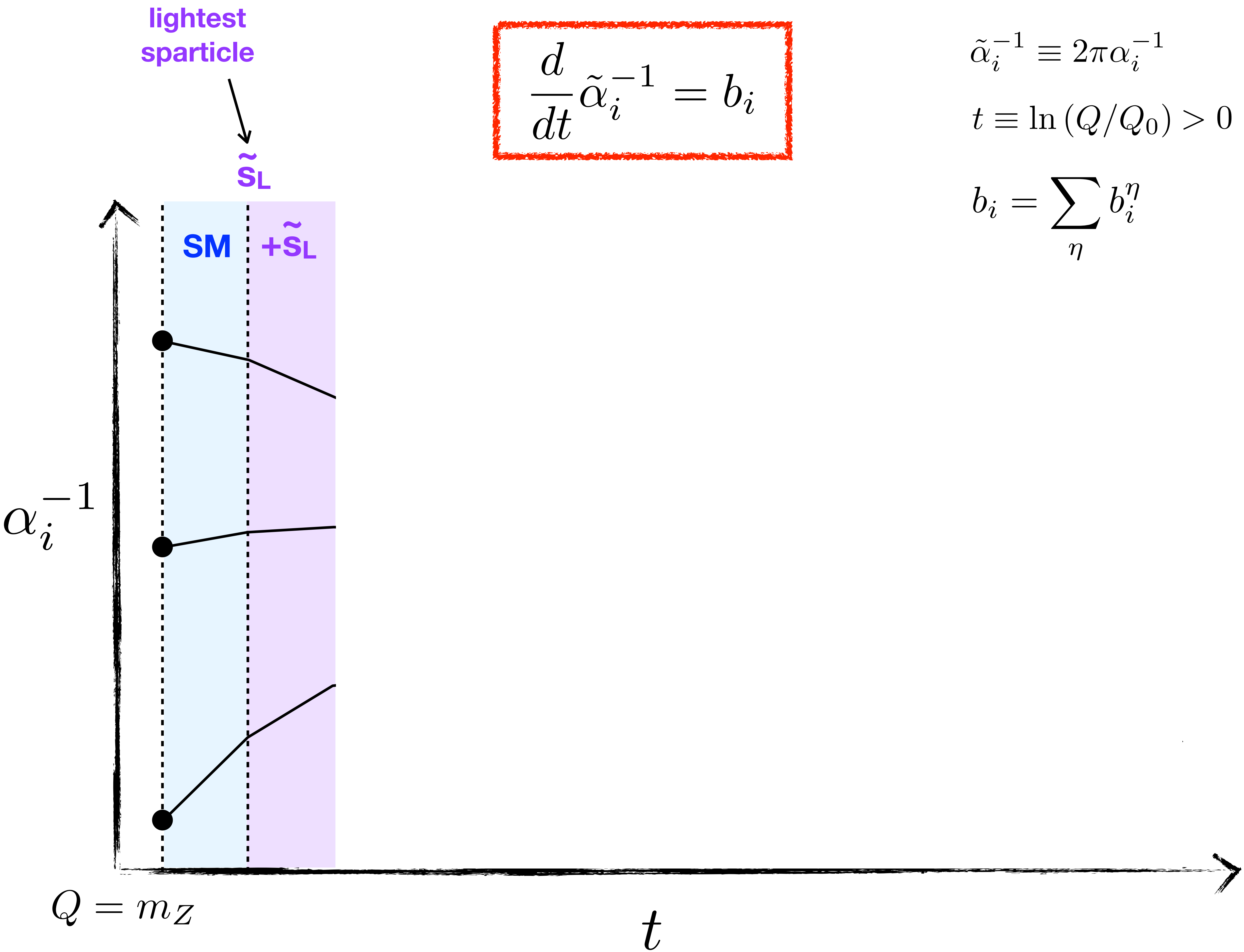

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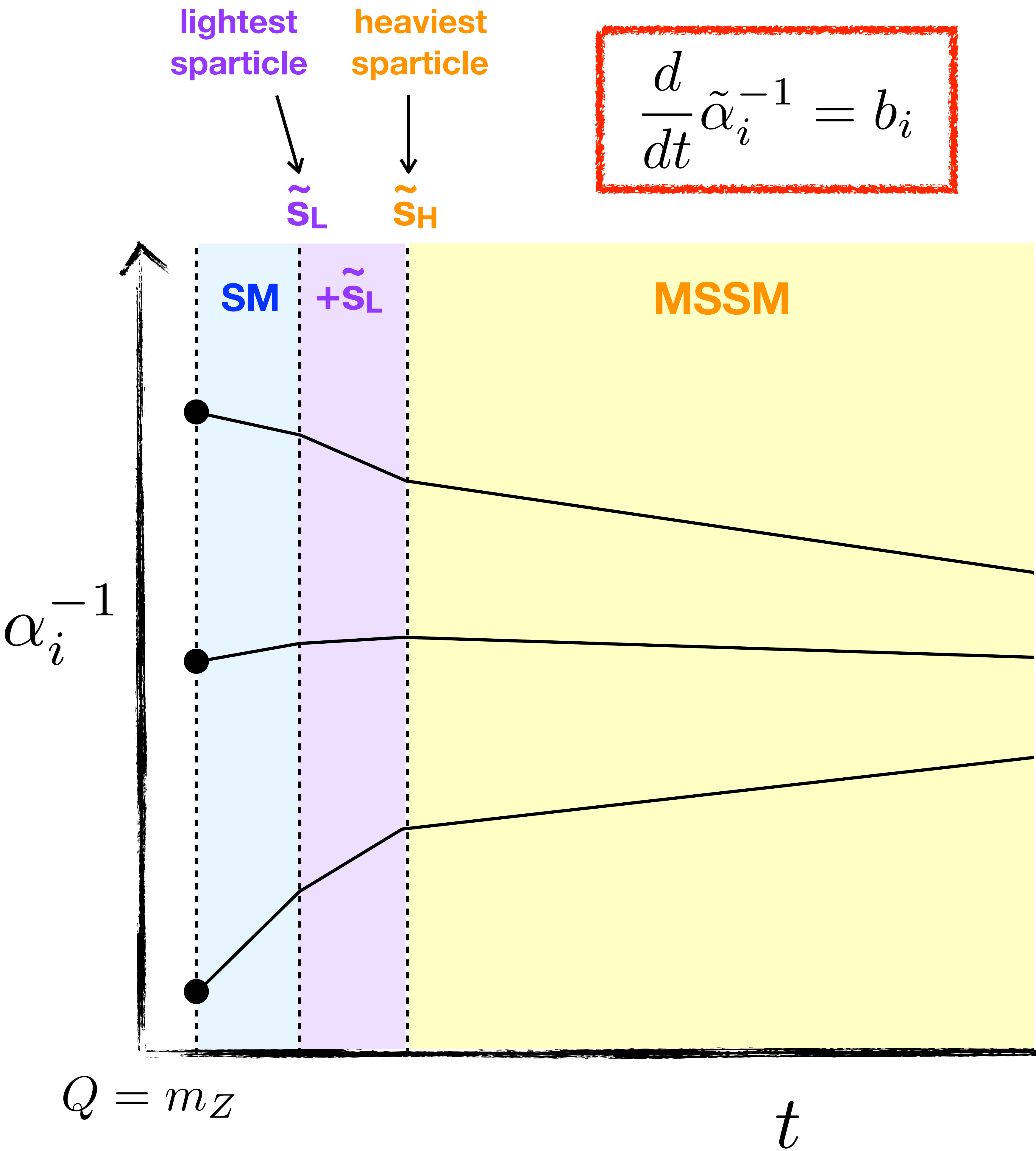

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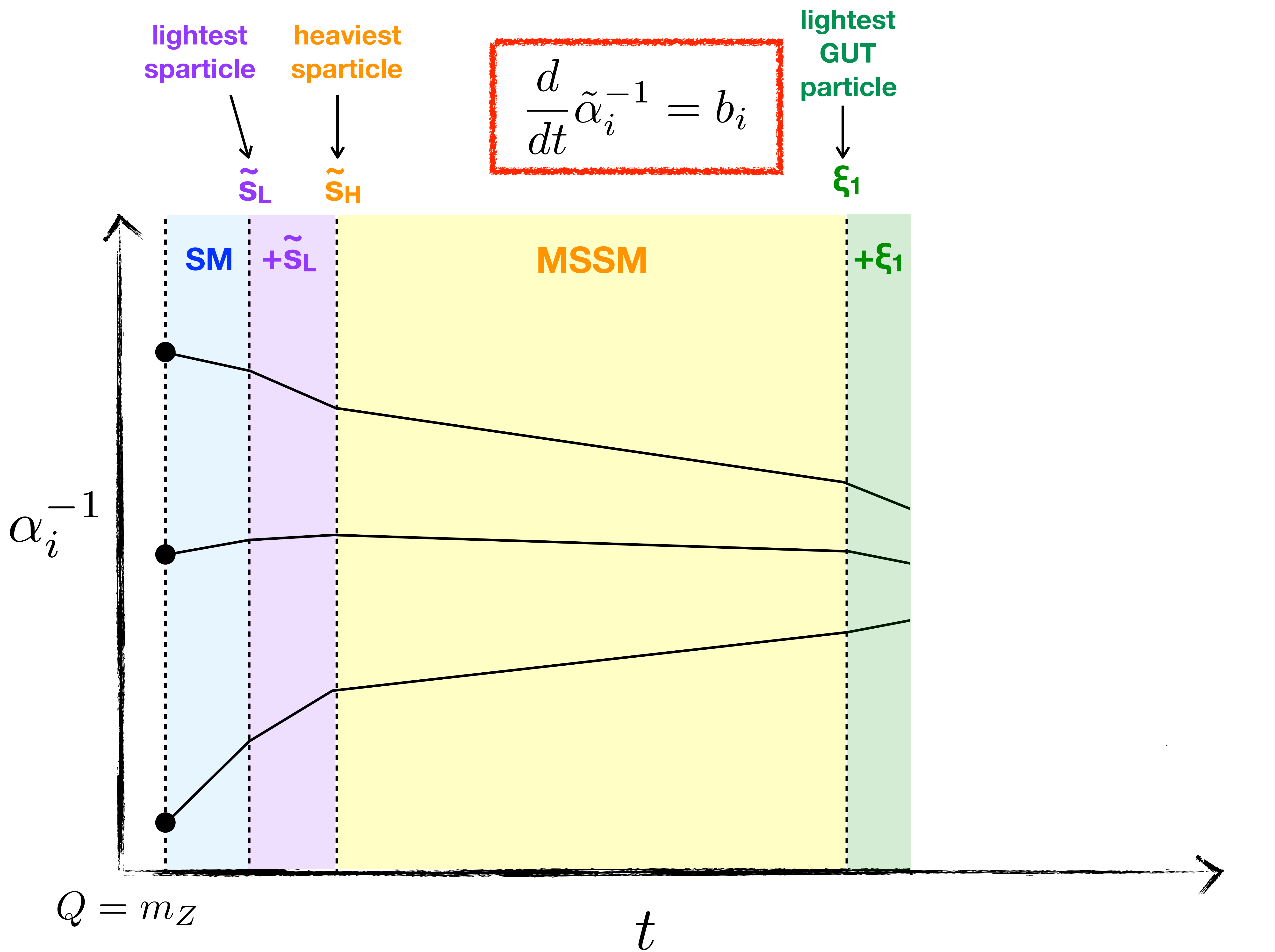
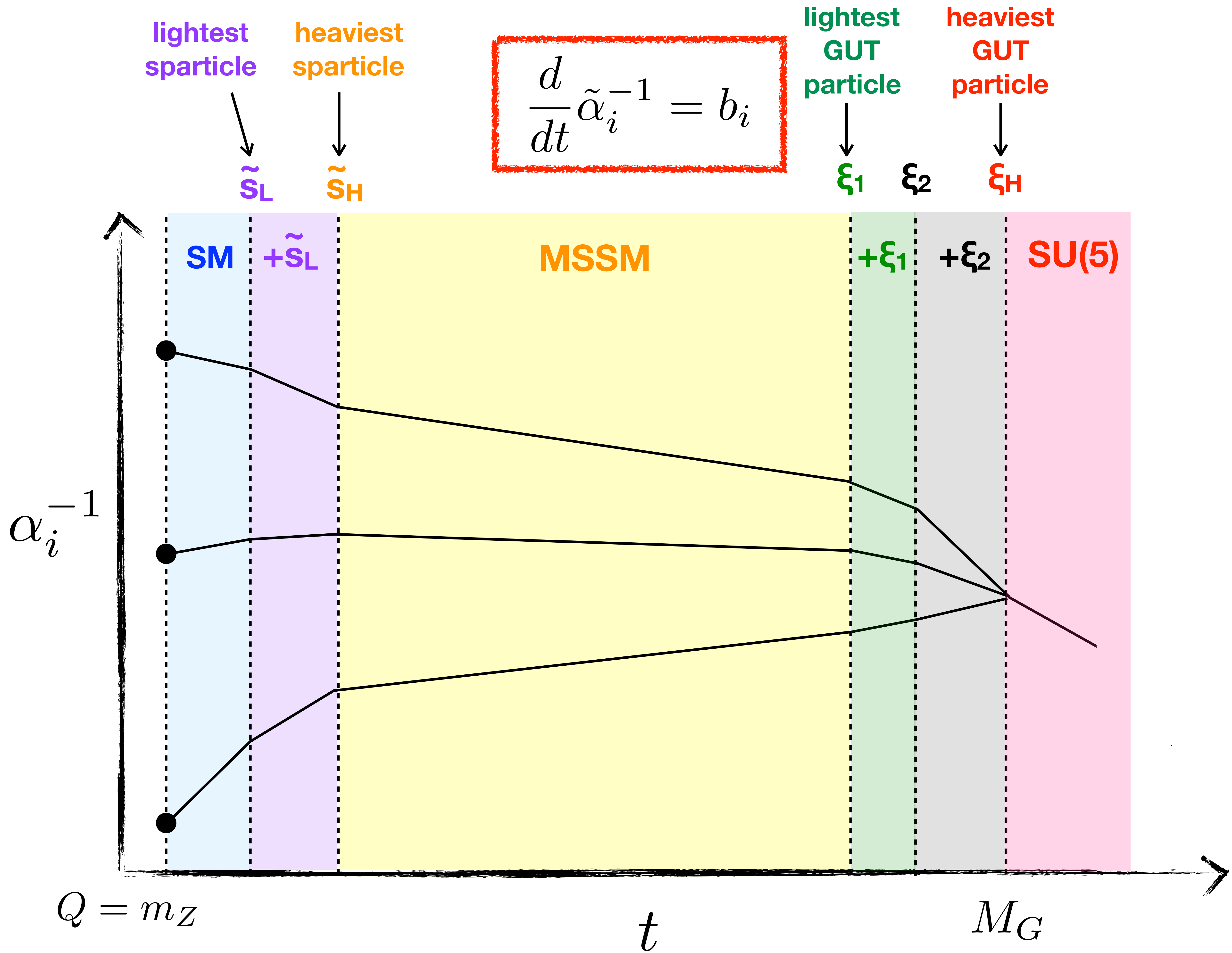


lightest sparticle
heaviest sparticle
$$\boxed{\frac{d}{dt}\tilde{\alpha}_i^{-1} = b_i}$$
lightest GUT particle
$\tilde{s}_L$
$\tilde{s}_H$
ξ_1
SM
$+\tilde{s}_L$
MSSM
$+\xi_1$
α_i^{-1}
$Q = m_Z$
t


lightest sparticle
heaviest sparticle
$\frac{d}{dt}\tilde{\alpha}_i^{-1} = b_i$
lightest GUT particle
heaviest GUT particle
$\tilde{s}_L$
$\tilde{s}_H$
ξ_1
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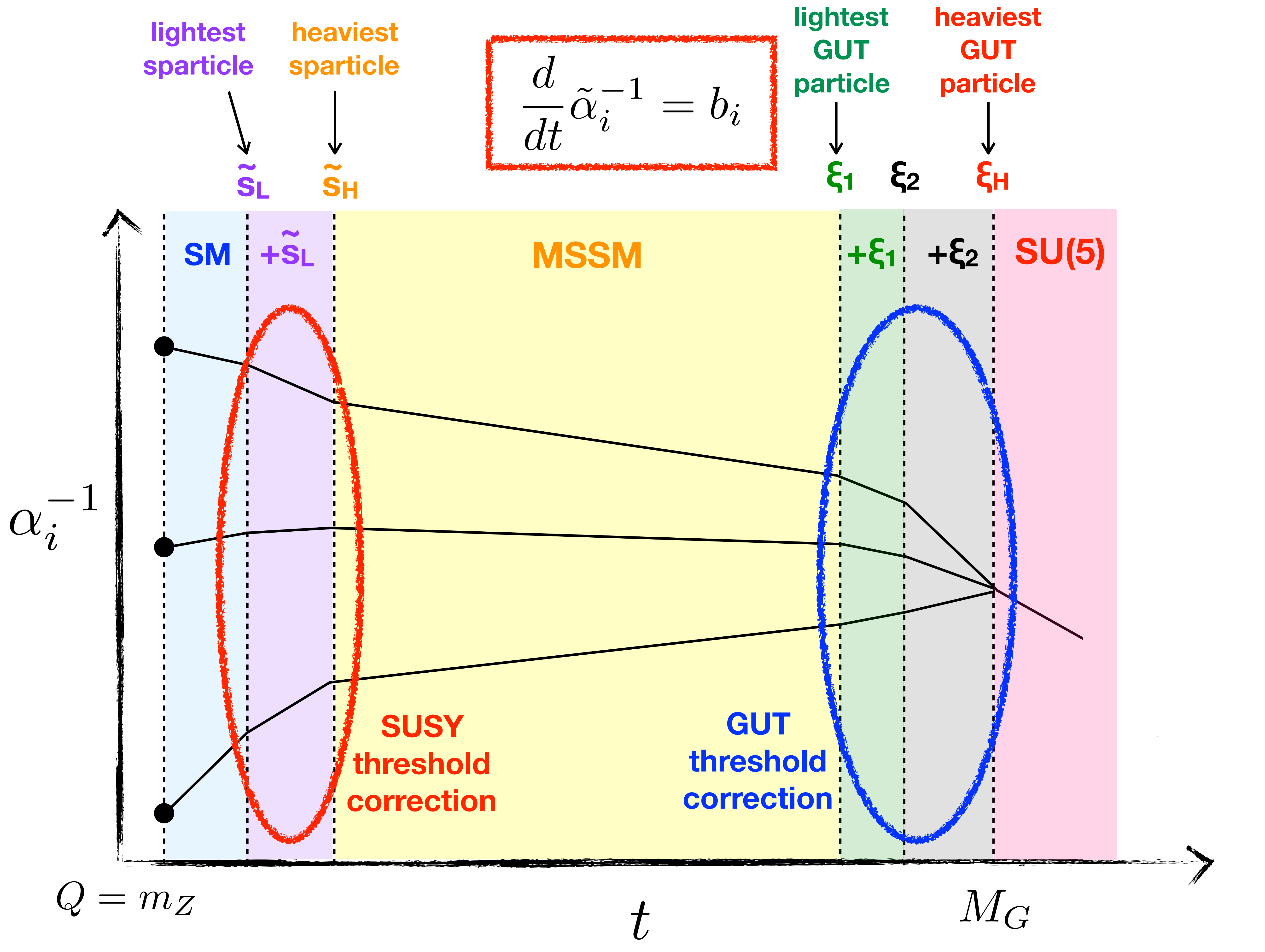
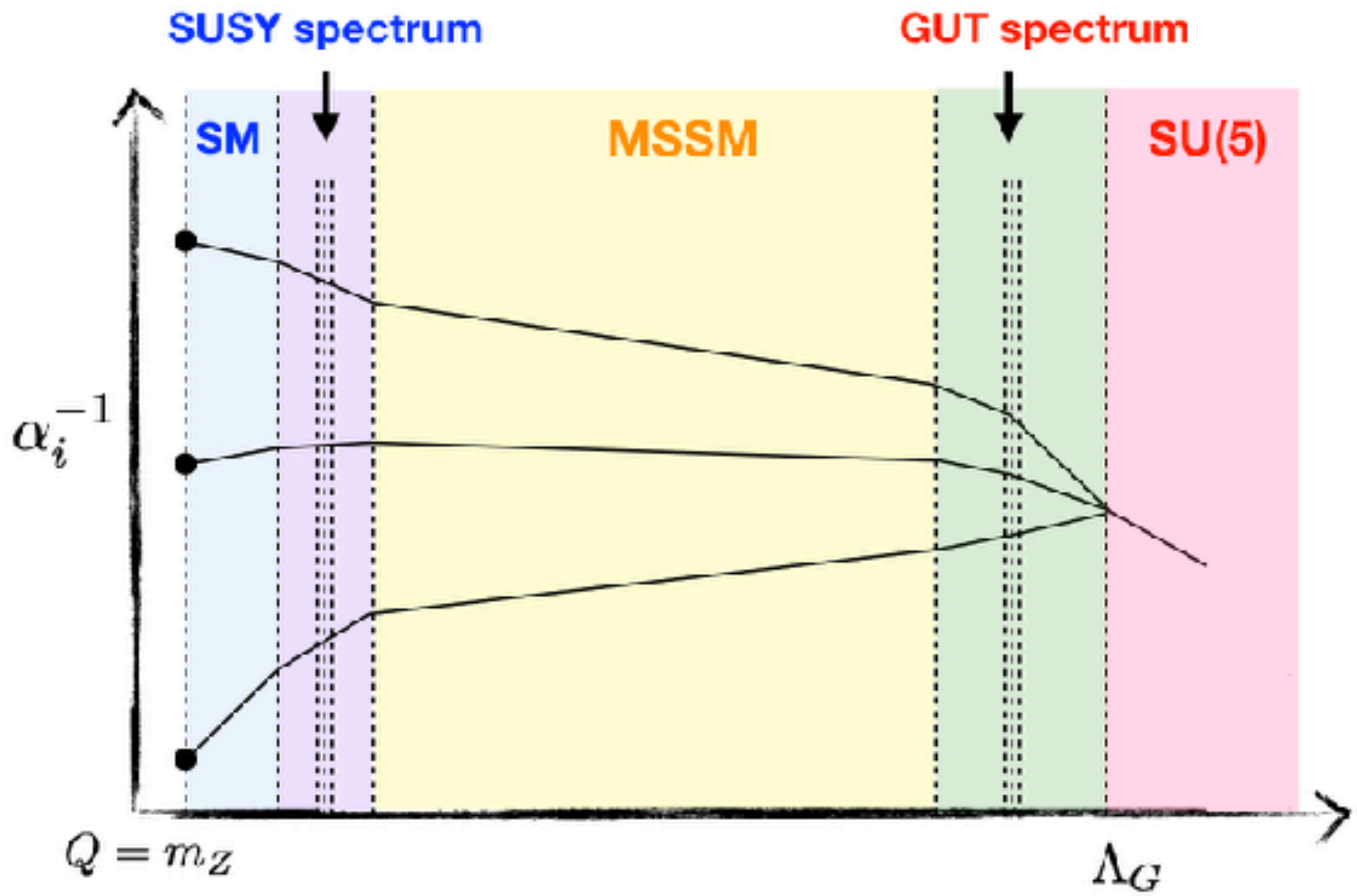
$+\xi_1$
$+\xi_2$
SU(5)
α_i^{-1}
$Q = m_Z$
t
M_G



lightest sparticle
heaviest sparticle
$$\frac{d}{dt}\tilde{\alpha}_i^{-1} = b_i$$
lightest GUT particle
heaviest GUT particle
$\tilde{s}_L$
$\tilde{s}_H$
ξ_1
ξ_2
ξ_H
SM
$+\tilde{s}_L$
MSSM
$+\xi_1$
$+\xi_2$
SU(5)
α_i^{-1}
SUSY threshold correction
GUT threshold correction
$Q = m_Z$
t
M_G



GCU gives constraints on SUSY and GUT spectrum.

Can we formulate such a constraint analytically?

The condition of GCU (2-loop level)

$$T_S(\mathbf{m}_S) = M_S^*(\alpha_s^{mz})\,\Omega_G(\mathbf{m}_\xi) \quad \bigcap \quad T_G(\mathbf{m}_\xi) = M_G^*(\alpha_s^{mz})\,\Omega_S(\mathbf{m}_S)$$

dim-1 function of **SUSY masses**

dim-0 function of **GUT masses**

dim-1 function of **GUT masses**

dim-0 function of **SUSY masses**

$$M_S^* = 2.08\,\mathrm{TeV} + \epsilon\,(\alpha_s^{mz})$$

$$M_G^* = 1.27 \cdot 10^{16}\,\mathrm{GeV} + \epsilon'\,(\alpha_s^{mz})$$

The condition of GCU (2-loop level)

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gluino

wino

higgsino

heavy Higgs

sfermions

$$T_S = \left[M_3^{-28} M_2^{32} \mu^{12} m_A^3 X_T\right]^{\frac{1}{19}}$$

$$\Omega_S = \left[M_3^{-100} M_2^{60} \mu^{32} m_A^8 X_\Omega\right]^{\frac{1}{288}}$$

$$X_T \equiv \prod_{i=1\ldots3}\left(\frac{m_{\tilde{l}_i}^3}{m_{\tilde{d}_{Ri}}^3}\right)\left(\frac{m_{\tilde{q}_i}^7}{m_{\tilde{e}_{Ri}}^2 m_{\tilde{u}_{Ri}}^5}\right)$$

$$X_\Omega \equiv \prod_{i=1\ldots3}\left(\frac{m_{\tilde{l}_i}^8}{m_{\tilde{d}_{Ri}}^8}\right)\left(\frac{m_{\tilde{q}_i}^6 m_{\tilde{e}_{Ri}}}{m_{\tilde{u}_{Ri}}^7}\right)$$

The condition of GCU (2-loop level)

$$T_S(\mathbf{m}_S) = M_S^*(\alpha_s^{mz})\,\Omega_G(\mathbf{m}_\xi) \quad \bigcap \quad T_G(\mathbf{m}_\xi) = M_G^*(\alpha_s^{mz})\,\Omega_S(\mathbf{m}_S)$$

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$$M_G^* = 1.27\cdot 10^{16}\,\mathrm{GeV} + \epsilon'\,(\alpha_s^{mz})$$

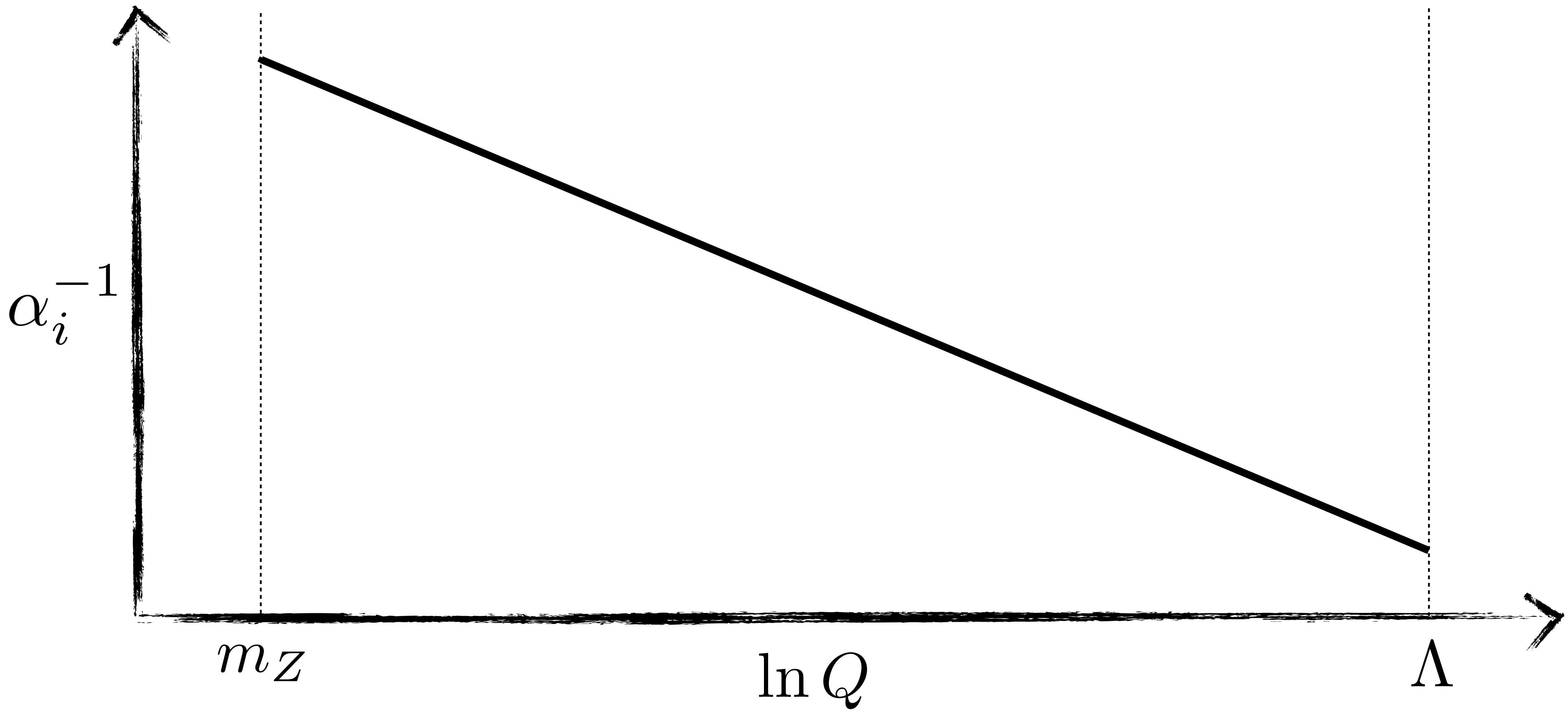
$$\ln\left(\frac{T_G}{\Lambda}\right) = \sum_\xi \left(-\frac{5}{288}b_1^\xi - \frac{15}{76}b_2^\xi + \frac{25}{114}b_3^\xi\right)\ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$\ln \Omega_G = \sum_\xi \left(\frac{10}{19}b_1^\xi - \frac{24}{19}b_2^\xi + \frac{14}{19}b_3^\xi\right)\ln\left(\frac{m_\xi}{\Lambda}\right)$$

GUT mass spectrum

Unification scale

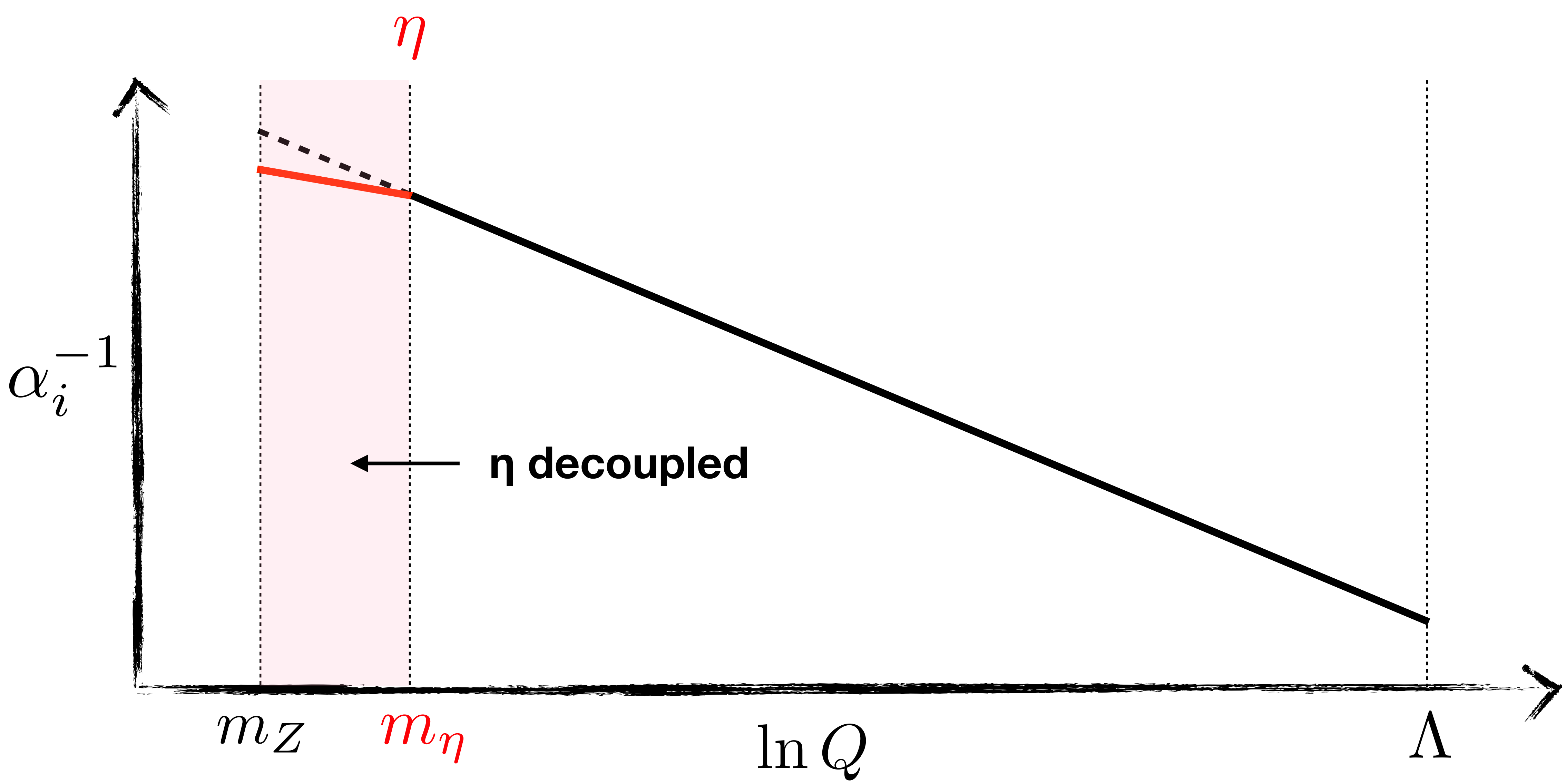
b_i^ξ **: contribution to the β-coefficient from ξ**



$$\frac{2\pi}{\alpha_i(m_Z)} = \frac{2\pi}{\alpha_i(\Lambda)} + b_i \ln\left(\frac{\Lambda}{m_Z}\right)$$

full MSSM

$$b_i = (\frac{33}{5}, 1, -3)$$

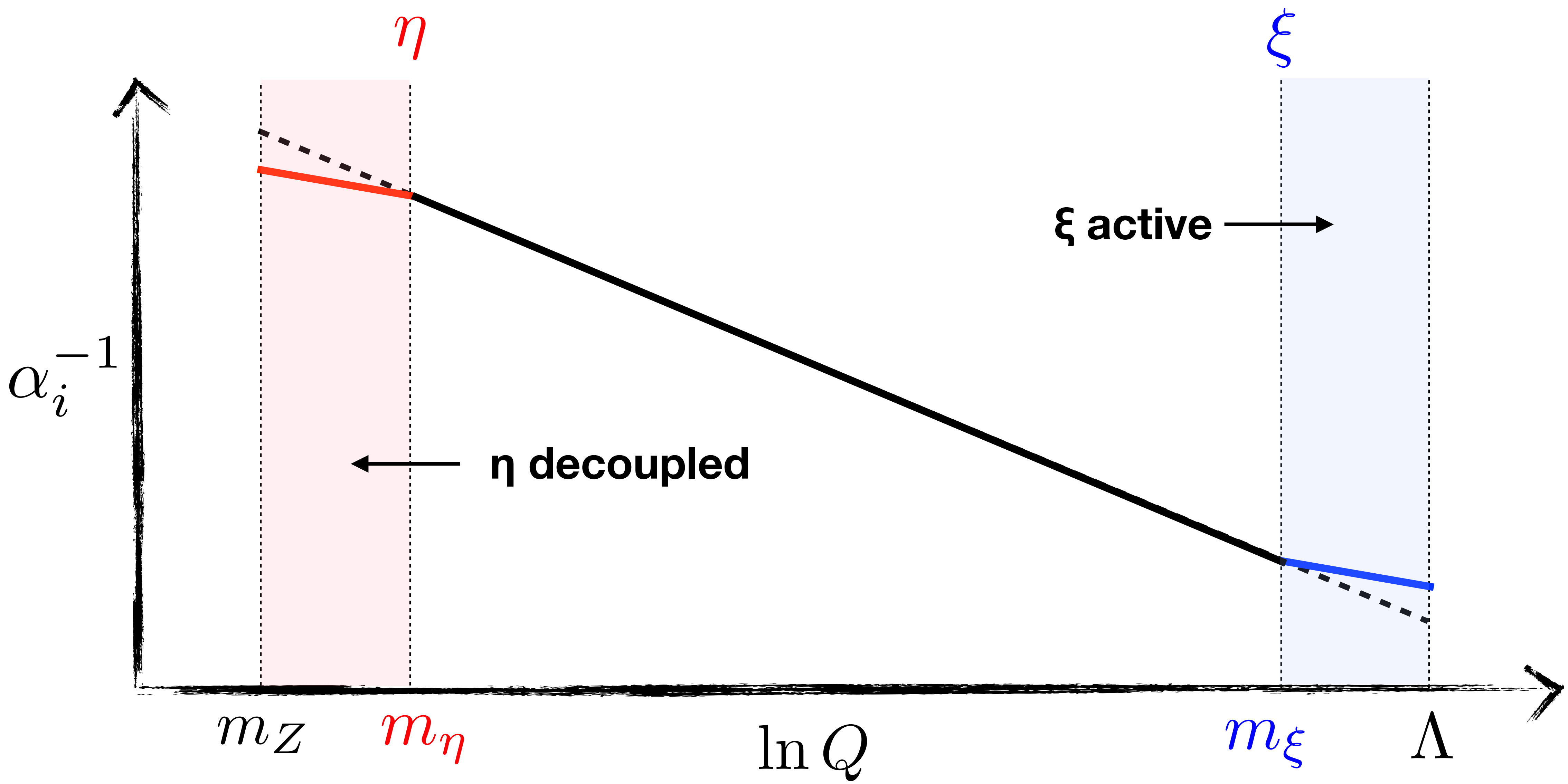


$$\frac{2\pi}{\alpha_i(m_Z)} = \frac{2\pi}{\alpha_i(\Lambda)} + b_i \ln\left(\frac{\Lambda}{m_Z}\right) - b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right)$$

full MSSM

$$b_i = (\frac{33}{5}, 1, -3)$$

threshold corr. from η



$$\frac{2\pi}{\alpha_i(m_Z)} = \frac{2\pi}{\alpha_i(\Lambda)} + b_i \ln\left(\frac{\Lambda}{m_Z}\right) - b_i^{\eta} \ln\left(\frac{m_\eta}{m_Z}\right) + b_i^{\xi} \ln\left(\frac{\Lambda}{m_\xi}\right)$$

full MSSM

$$b_i = (\frac{33}{5}, 1, -3)$$

threshold corr. from η

threshold corr. from ξ

General solution to RGE

full MSSM

SUSY threshold

$$\frac{2\pi}{\alpha(\Lambda)} = \frac{2\pi}{\alpha_i(m_Z)} - b_i \ln\left(\frac{\Lambda}{m_Z}\right) + s_i + r_i$$

unified coupling

experimental input

GUT threshold

$$s_i = \sum_\eta b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right)$$

$$r_i = \sum_\xi b_i^\xi \ln\left(\frac{m_\xi}{\Lambda}\right)$$

General solution to RGE

$$\frac{2\pi}{\alpha(\Lambda)} = \frac{2\pi}{\alpha_i(m_Z)} - b_i \ln\left(\frac{\Lambda}{m_Z}\right) + s_i + r_i + \gamma_i + \Delta_i.$$

$\frac{2\pi}{\alpha(\Lambda)}$: unified coupling

$\frac{2\pi}{\alpha_i(m_Z)}$: experimental input

$b_i \ln\left(\frac{\Lambda}{m_Z}\right)$: full MSSM

s_i: SUSY threshold

r_i: GUT threshold

γ_i: 2-loop contribution

Δ_i: top-quark threshold, MSbar-DRber conversion

$$s_i = \sum_\eta b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right)$$

$$r_i = \sum_\xi b_i^\xi \ln\left(\frac{m_\xi}{\Lambda}\right)$$

[2-loop contribution]

$$b_{ij} = \begin{pmatrix} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{pmatrix}$$

$$\gamma_i = -\frac{1}{2}\sum_j \frac{b_{ij}}{b_j} \ln\left(\frac{\alpha_j(\Lambda)}{\alpha_j(m_Z)}\right)$$

$$\simeq -\frac{1}{2}\sum_j \frac{b_{ij}}{b_j} \ln\left(1 + \frac{b_j\alpha(\Lambda)}{2\pi} \ln\frac{\Lambda}{m_Z}\right)$$

one can find γ_i by iteratively updating $\alpha(\Lambda)$ and Λ

General solution to RGE

full MSSM

SUSY threshold

2-loop contribution

$$\frac{2\pi}{\alpha(\Lambda)} = \frac{2\pi}{\alpha_i(m_Z)} - b_i \ln\left(\frac{\Lambda}{m_Z}\right) + s_i + r_i + \gamma_i + \Delta_i$$

$$s_i = \sum_\eta b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right)$$

$$r_i = \sum_\xi b_i^\xi \ln\left(\frac{m_\xi}{\Lambda}\right)$$

unified coupling

experimental input

GUT threshold

top-quark threshold, MSbar-DRber conversion

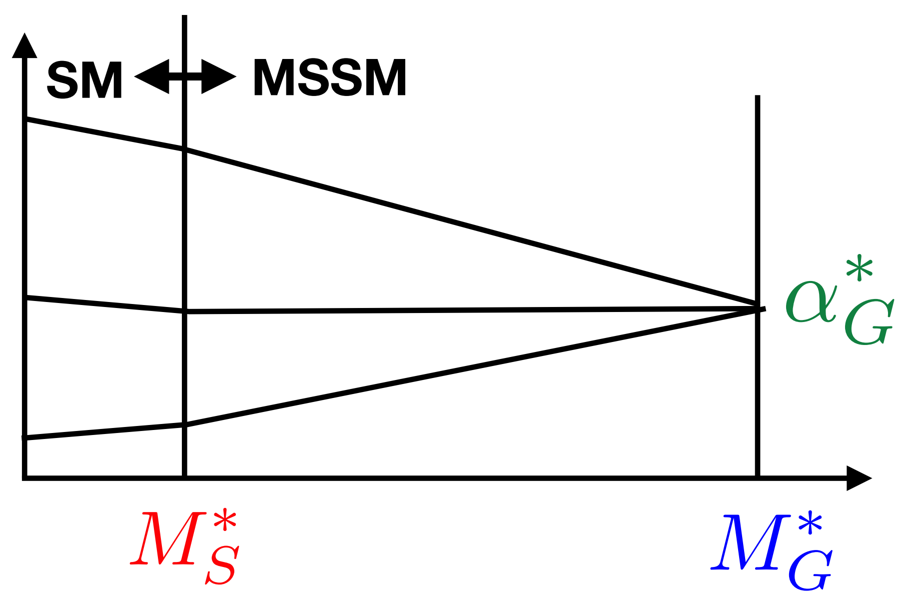
- **We trade** 3 exp inputs **with the *3 constants*:** $[\alpha_1(m_Z), \alpha_2(m_Z), \alpha_3(m_Z)] \longrightarrow [M_S^*, M_G^*, \alpha_G^*]$

$$\frac{2\pi}{\alpha_i(m_Z)} = \frac{2\pi}{\alpha_G^*} + b_i \ln\left(\frac{M_G^*}{m_Z}\right) - \delta_i \ln\left(\frac{M_S^*}{m_Z}\right) - \gamma_i - \Delta_i$$

degenerate SUSY without GUT thres

$$b_i = \left(\frac{33}{5}, 1, -3\right)$$

$$\delta_i \equiv \sum_\eta b_i^\eta = b_i - b_i^{\rm SM}$$
$$= \left(\frac{2}{5}, \frac{25}{6}, 4\right)$$



$$M_S^* = 2.08\ \text{TeV}$$

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$$\alpha_G^{*-1} = 25.5$$

$$\boxed{\frac{2\pi}{\alpha_i(m_Z)}} = \frac{2\pi}{\alpha_G^*} + b_i \ln\left(\frac{M_G^*}{m_Z}\right) - \delta_i \ln\left(\frac{M_s^*}{m_Z}\right) - \gamma_i - \Delta_i$$

degenerate SUSY without GUT thres

$$\frac{2\pi}{\alpha(\Lambda)} = \boxed{\frac{2\pi}{\alpha_i(m_Z)}} - b_i \ln\left(\frac{\Lambda}{m_Z}\right) + s_i + r_i + \gamma_i + \Delta_i$$

general solution

$$= \frac{2\pi}{\alpha_G^*} + b_i \ln\left(\frac{M_G^*}{\Lambda}\right) - \delta_i \ln\left(\frac{M_s^*}{m_Z}\right) + \underset{\text{SUSY threshold}}{s_i} + \underset{\text{GUT threshold}}{r_i}$$

SUSY threshold **GUT threshold**

$M_s^* = 2.08\,\mathrm{TeV}$

$M_G^* = 1.27 \cdot 10^{16}\,\mathrm{GeV}$

$\alpha_G^{*-1} = 25.5$

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degenerate SUSY without GUT thres

$$\Downarrow$$

$$\frac{2\pi}{\alpha(\Lambda)} = \boxed{\frac{2\pi}{\alpha_i(m_Z)}} - b_i \ln\left(\frac{\Lambda}{m_Z}\right) + s_i + r_i + \gamma_i + \Delta_i$$

general solution

$$= \frac{2\pi}{\alpha_G^*} + b_i \ln\left(\frac{M_G^*}{\Lambda}\right) - \delta_i \ln\left(\frac{M_s^*}{m_Z}\right) + \underset{\substack{\uparrow \\ \text{SUSY threshold}}}{s_i} + \underset{\substack{\uparrow \\ \text{GUT threshold}}}{r_i}$$

Any 3D vector can be decomposed into a sum of 3 independent vectors: ***1, b_i, δ_i***

$$\vec{1} = (1,1,1) \qquad \vec{b} = \left(\frac{33}{5}, 1, -3\right) \qquad \vec{\delta} \equiv \vec{b} - \vec{b}_{\rm SM} = \left(\frac{2}{5}, \frac{25}{6}, 4\right)$$

$$s_i = \sum_\eta b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right) = C_S + b_i \ln \Omega_S + \delta_i \ln\left(\frac{T_S}{m_Z}\right)$$

$$r_i = \sum_\xi b_i^\xi \ln\left(\frac{m_\xi}{\Lambda}\right) = C_G - b_i \ln\left(\frac{T_G}{\Lambda}\right) - \delta_i \ln \Omega_G$$

$$\boxed{\frac{2\pi}{\alpha_i(m_Z)}} = \frac{2\pi}{\alpha_G^*} + b_i \ln\left(\frac{M_G^*}{m_Z}\right) - \delta_i \ln\left(\frac{M_s^*}{m_Z}\right) - \gamma_i - \Delta_i$$

degenerate SUSY without GUT thres

$$\frac{2\pi}{\alpha(\Lambda)} = \boxed{\frac{2\pi}{\alpha_i(m_Z)}} - b_i \ln\left(\frac{\Lambda}{m_Z}\right) + s_i + r_i + \gamma_i + \Delta_i$$

general solution

i-independent

$$= \frac{2\pi}{\alpha_G^*} + b_i \ln\left(\frac{M_G^*}{\Lambda}\right) - \delta_i \ln\left(\frac{M_s^*}{m_Z}\right) + s_i + r_i$$

must vanish

$$= \left[\frac{2\pi}{\alpha_G^*} + C_S + C_G\right] + b_i \ln\left(\frac{M_G^*\Omega_S}{T_G}\right) + \delta_i \ln\left(\frac{T_S}{M_s^*\Omega_G}\right)$$

Any 3D vector can be decomposed into a sum of 3 independent vectors:

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degenerate SUSY without GUT thres

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general solution

i-independent

$$= \frac{2\pi}{\alpha_G^*} + b_i \ln\left(\frac{M_G^*}{\Lambda}\right) - \delta_i \ln\left(\frac{M_s^*}{m_Z}\right) + s_i + r_i$$

must vanish

$$= \left[\frac{2\pi}{\alpha_G^*} + C_S + C_G\right] + b_i \ln\left(\frac{M_G^*\Omega_S}{T_G}\right) + \delta_i \ln\left(\frac{T_S}{M_s^*\Omega_G}\right)$$

The condition of gauge coupling unification:

$$T_S = M_s^*\Omega_G \quad \cap \quad T_G = M_G^*\Omega_S$$

$$M_s^* = 2.08\,\mathrm{TeV}$$

$$M_G^* = 1.27\cdot 10^{16}\,\mathrm{GeV}$$

$$\alpha_G^{*-1} = 25.5$$

The unified coupling at Λ

$$\alpha^{-1}(\Lambda) = \alpha_G^{*-1} + \frac{1}{2\pi}(C_S + C_G)$$

$$s_i = \sum_\eta b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right) = C_S + b_i \ln \Omega_S + \delta_i \ln\left(\frac{T_S}{m_Z}\right)$$

$$b_i = (\frac{33}{5}, 1, -3)$$

$$\delta_i = (\frac{2}{5}, \frac{25}{6}, 4)$$

$$\Leftrightarrow \quad \begin{pmatrix} \ln\left(\prod_\eta \left[\frac{m_\eta}{m_Z}\right]^{b_1^\eta}\right) \\ \ln\left(\prod_\eta \left[\frac{m_\eta}{m_Z}\right]^{b_2^\eta}\right) \\ \ln\left(\prod_\eta \left[\frac{m_\eta}{m_Z}\right]^{b_3^\eta}\right) \end{pmatrix} = \begin{pmatrix} 1 & b_1 & \delta_1 \\ 1 & b_2 & \delta_2 \\ 1 & b_3 & \delta_3 \end{pmatrix} \begin{pmatrix} C_S \\ \ln \Omega_S \\ \ln\left(\frac{T_S}{m_Z}\right) \end{pmatrix}$$

$$s_i = \sum_\eta b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right) = C_S + b_i \ln \Omega_S + \delta_i \ln\left(\frac{T_S}{m_Z}\right)$$

$$b_i = (\frac{33}{5}, 1, -3)$$

$$\delta_i = (\frac{2}{5}, \frac{25}{6}, 4)$$

$$\Leftrightarrow \begin{pmatrix} \ln\left(\prod_\eta \left[\frac{m_\eta}{m_Z}\right]^{b_1^\eta}\right) \\ \ln\left(\prod_\eta \left[\frac{m_\eta}{m_Z}\right]^{b_2^\eta}\right) \\ \ln\left(\prod_\eta \left[\frac{m_\eta}{m_Z}\right]^{b_3^\eta}\right) \end{pmatrix} = \begin{pmatrix} 1 & b_1 & \delta_1 \\ 1 & b_2 & \delta_2 \\ 1 & b_3 & \delta_3 \end{pmatrix} \begin{pmatrix} C_S \\ \ln \Omega_S \\ \ln\left(\frac{T_S}{m_Z}\right) \end{pmatrix}$$

$$\Rightarrow$$

$$T_S = \left[M_3^{-28} M_2^{32} \mu^{12} m_A^3 X_T\right]^{\frac{1}{19}}$$

$$\Omega_S = \left[M_3^{-100} M_2^{60} \mu^{32} m_A^8 X_\Omega\right]^{\frac{1}{288}}$$

$$X_T \equiv \prod_{i=1\ldots3} \left(\frac{m_{\tilde{l}_i}^3}{m_{\tilde{d}_{Ri}}^3}\right)\left(\frac{m_{\tilde{q}_i}^7}{m_{\tilde{e}_{Ri}}^2 m_{\tilde{u}_{Ri}}^5}\right)$$

$$X_\Omega \equiv \prod_{i=1\ldots3} \left(\frac{m_{\tilde{l}_i}^8}{m_{\tilde{d}_{Ri}}^8}\right)\left(\frac{m_{\tilde{q}_i}^6 m_{\tilde{e}_{Ri}}}{m_{\tilde{u}_{Ri}}^7}\right)$$

$$C_S = \frac{125}{19}\ln M_3 - \frac{113}{19}\ln M_2 - \frac{40}{19}\ln \mu - \frac{10}{19}\ln m_A + \sum_{i=1\ldots3}\left[\frac{79}{114}\ln m_{\tilde{d}_{Ri}} - \frac{10}{19}\ln m_{\tilde{l}_i} - \frac{121}{114}\ln m_{\tilde{q}_i} + \frac{257}{228}\ln m_{\tilde{u}_{Ri}} + \frac{33}{76}\ln m_{\tilde{e}_{Ri}}\right]$$

$$r_i = \sum_\xi b_i^\xi \ln\left(\frac{m_\xi}{\Lambda}\right) = C_G - b_i \ln\left(\frac{T_G}{\Lambda}\right) - \delta_i \ln \Omega_G$$

GUT particle masses

$$\ln\left(\frac{T_G}{\Lambda}\right) = \sum_\xi \left(-\frac{5}{288}b_1^\xi - \frac{15}{76}b_2^\xi + \frac{25}{114}b_3^\xi\right) \ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$\ln \Omega_G = \sum_\xi \left(\frac{10}{19}b_1^\xi - \frac{24}{19}b_2^\xi + \frac{14}{19}b_3^\xi\right) \ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$C_G = \sum_\xi \left(\frac{165}{76}b_1^\xi - \frac{339}{76}b_2^\xi + \frac{125}{38}b_3^\xi\right) \ln\left(\frac{m_\xi}{\Lambda}\right)$$

The condition of gauge coupling unification:

$$T_S = M_s^* \Omega_G \ \cap \ T_G = M_G^* \Omega_S$$

$$M_s^* = 2.08\ \text{TeV}$$

$$M_G^* = 1.27 \cdot 10^{16}\ \text{GeV}$$

$$\alpha_G^{*-1} = 25.5$$

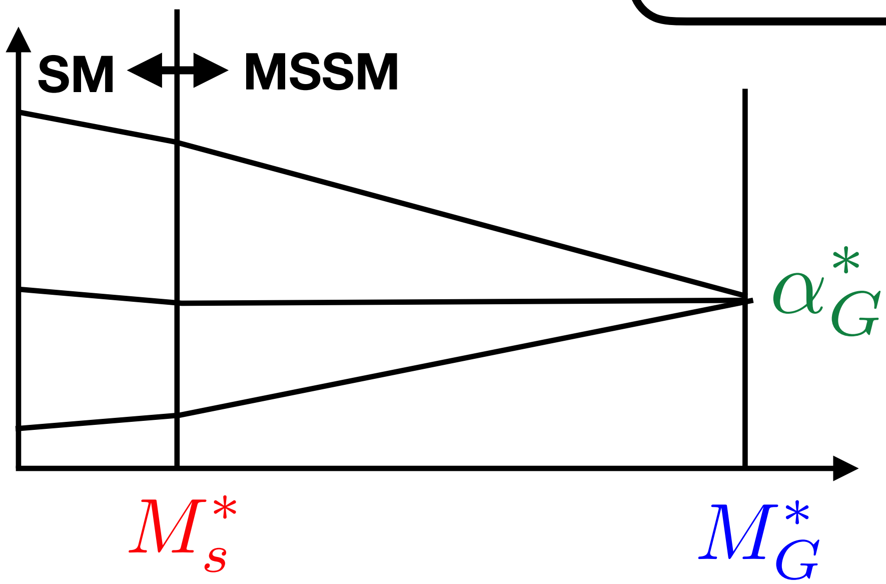
The unified coupling at Λ

$$\alpha^{-1}(\Lambda) = \alpha_G^{*-1} + \frac{1}{2\pi}(C_S + C_G)$$

Uncertainty of $\alpha_s(m_Z)$

$$\alpha_s(m_Z) = \overset{\alpha_s^0}{0.1183} \pm \overset{\Delta\alpha_s}{0.0008}$$

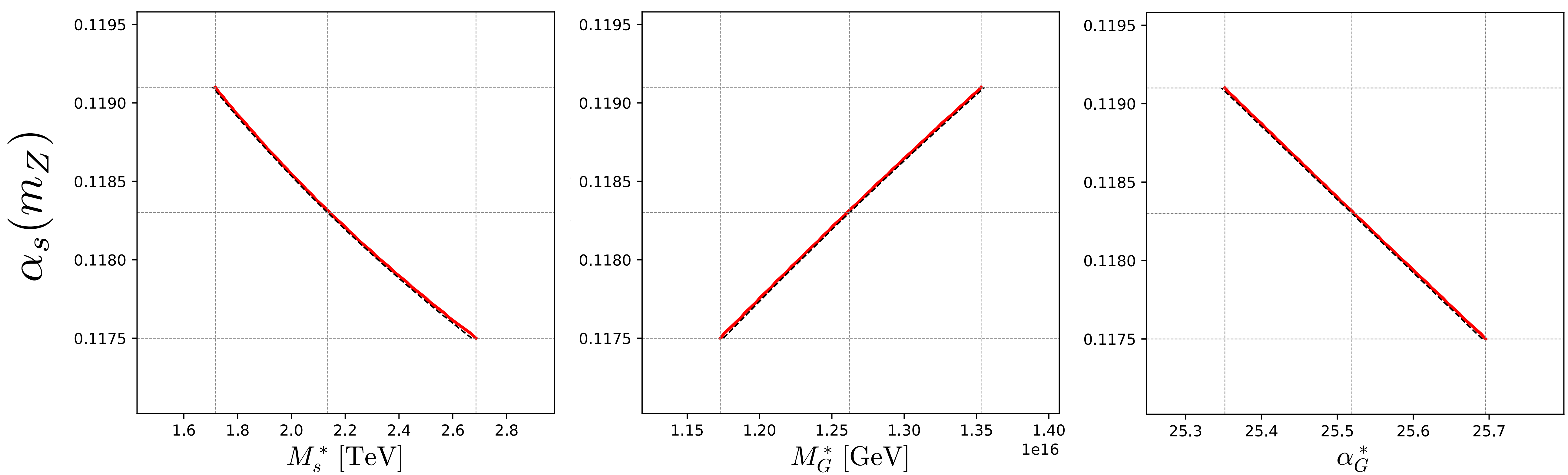
D. d'Enterria [1806.06156]



$$\frac{M_s^*}{\mathrm{TeV}} = \frac{2.08}{\mathrm{TeV}} \cdot \exp\left[-0.224\left(\frac{\alpha_s - \alpha_s^0}{\Delta\alpha_s}\right)\right],$$

$$\frac{M_G^*}{\mathrm{GeV}} = \frac{1.27\cdot 10^{16}}{\mathrm{GeV}} \cdot \exp\left[0.0715\left(\frac{\alpha_s - \alpha_s^0}{\Delta\alpha_s}\right)\right]$$

$$\alpha_G^{*-1} = 25.5 - 0.172\left(\frac{\alpha_s - \alpha_s^0}{\Delta\alpha_s}\right).$$



$M_s^* \in [2.69,\ 1.72]$ TeV $\quad$ $M_G^* \in [1.17,\ 1.35]\cdot 10^{16}$ GeV $\quad$ $\alpha_G^{*-1} \in [25.7,\ 25.4]$

Application to Minimal SU(5)

Minimal SUSY SU(5)

$$H(\mathbf{5}) = (\boxed{H_C}, H_u) \qquad \Sigma(\mathbf{24}) = (\boxed{\Sigma_8, \Sigma_3}, \Sigma_1, \Sigma_{(2,3)}, \Sigma_{(2,3^*)})$$

$$\overline{H}(\overline{\mathbf{5}}) = (\boxed{\overline{H}_C}, H_d) \qquad \mathcal{V}(\mathbf{24}) = (G, W, B, \boxed{(X,Y),(X,Y)^\dagger})$$

$$\boxed{(H_C, \overline{H}_C)} \rightarrow M_{H_C} = 5\lambda_H V$$

$$\boxed{(X,Y),(X,Y)^\dagger} \rightarrow M_V = 5\sqrt{2} g_5 V$$

$$\boxed{(\Sigma_8, \Sigma_3)} \rightarrow M_\Sigma = \frac{2}{5}\lambda_\Sigma V$$

mass	$(U(1)\times SU(2)\times SU(3))$	(b_1, b_2, b_3)
M_{H_C}	$(-\frac{1}{3}, \mathbf{1}, \mathbf{3}),\ (\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$	$(\frac{2}{5}, 0, 1)$
M_V	$(-\frac{5}{6}, \mathbf{2}, \mathbf{3}),\ (\frac{5}{6}, \mathbf{2}, \overline{\mathbf{3}})$	$(-10, -6, -4)$
M_Σ	$(0, \mathbf{3}, \mathbf{1}),\ (0, \mathbf{1}, \mathbf{8})$	$(0, 2, 3)$

Minimal SUSY SU(5)

$$H(\mathbf{5}) = (\boxed{H_C}, H_u) \qquad \Sigma(\mathbf{24}) = (\boxed{\Sigma_8, \Sigma_3}, \Sigma_1, \Sigma_{(2,3)}, \Sigma_{(2,3^*)})$$

$$\overline{H}(\overline{\mathbf{5}}) = (\boxed{\overline{H}_C}, H_d) \qquad \mathcal{V}(\mathbf{24}) = (G, W, B, \boxed{(X,Y), (X,Y)^\dagger})$$

$$\boxed{(H_C, \overline{H}_C)} \rightarrow M_{H_C} = 5\lambda_H V$$

$$\boxed{(X,Y), (X,Y)^\dagger} \rightarrow M_V = 5\sqrt{2} g_5 V$$

$$\boxed{(\Sigma_8, \Sigma_3)} \rightarrow M_\Sigma = \frac{2}{5}\lambda_\Sigma V$$

mass	$(U(1)\times\mathrm{SU}(2)\times\mathrm{SU}(3))$	(b_1, b_2, b_3)
M_{H_C}	$(-\frac{1}{3}, \mathbf{1}, \mathbf{3}),\ (\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$	$(\frac{2}{5}, 0, 1)$
M_V	$(-\frac{5}{6}, \mathbf{2}, \mathbf{3}),\ (\frac{5}{6}, \mathbf{2}, \overline{\mathbf{3}})$	$(-10, -6, -4)$
M_Σ	$(0, \mathbf{3}, \mathbf{1}),\ (0, \mathbf{1}, \mathbf{8})$	$(0, 2, 3)$

$$\ln\left(\frac{T_G}{\Lambda}\right) = \sum_\xi \left(-\frac{5}{288} b_1^\xi - \frac{15}{76} b_2^\xi + \frac{25}{114} b_3^\xi\right) \ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$\ln \Omega_G = \sum_\xi \left(\frac{10}{19} b_1^\xi - \frac{24}{19} b_2^\xi + \frac{14}{19} b_3^\xi\right) \ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$\Rightarrow$$

$$T_G = M_{H_C}^{\frac{4}{19}} \left(M_V^2 M_\Sigma\right)^{\frac{5}{19}}$$

$$\Omega_G = M_{H_C}^{\frac{18}{19}} \left(M_V^2 M_\Sigma\right)^{-\frac{6}{19}}$$

Minimal SUSY SU(5)

$$H(\mathbf{5}) = (\boxed{H_C}, H_u) \qquad \Sigma(\mathbf{24}) = (\boxed{\Sigma_8, \Sigma_3}, \Sigma_1, \Sigma_{(2,3)}, \Sigma_{(2,3^*)})$$

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mass	$(U(1)\times SU(2)\times SU(3))$	(b_1, b_2, b_3)
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M_V	$(-\frac{5}{6}, \mathbf{2}, \mathbf{3})$, $(\frac{5}{6}, \mathbf{2}, \overline{\mathbf{3}})$	$(-10, -6, -4)$
M_Σ	$(0, \mathbf{3}, \mathbf{1})$, $(0, \mathbf{1}, \mathbf{8})$	$(0, 2, 3)$

$$\ln\left(\frac{T_G}{\Lambda}\right) = \sum_\xi \left(-\frac{5}{288} b_1^\xi - \frac{15}{76} b_2^\xi + \frac{25}{114} b_3^\xi\right) \ln\left(\frac{m_\xi}{\Lambda}\right)$$

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$$\Omega_G = M_{H_C}^{\frac{18}{19}} \left(M_V^2 M_\Sigma\right)^{-\frac{6}{19}}$$

GCU condition

$$T_S = M_s^* \Omega_G \quad \cap \quad T_G = M_G^* \Omega_S$$

$$\Rightarrow$$

$$M_{H_C} = M_G^* \Omega_S \left(\frac{T_S}{M_s^*}\right)^{\frac{5}{6}}$$

$$(M_V^2 M_\Sigma)^{\frac{1}{3}} = M_G^* \Omega_S \left(\frac{T_S}{M_s^*}\right)^{-\frac{2}{9}}$$

Minimal SUSY SU(5)

D=5 proton decay rate can be calculated from the SUSY masses!

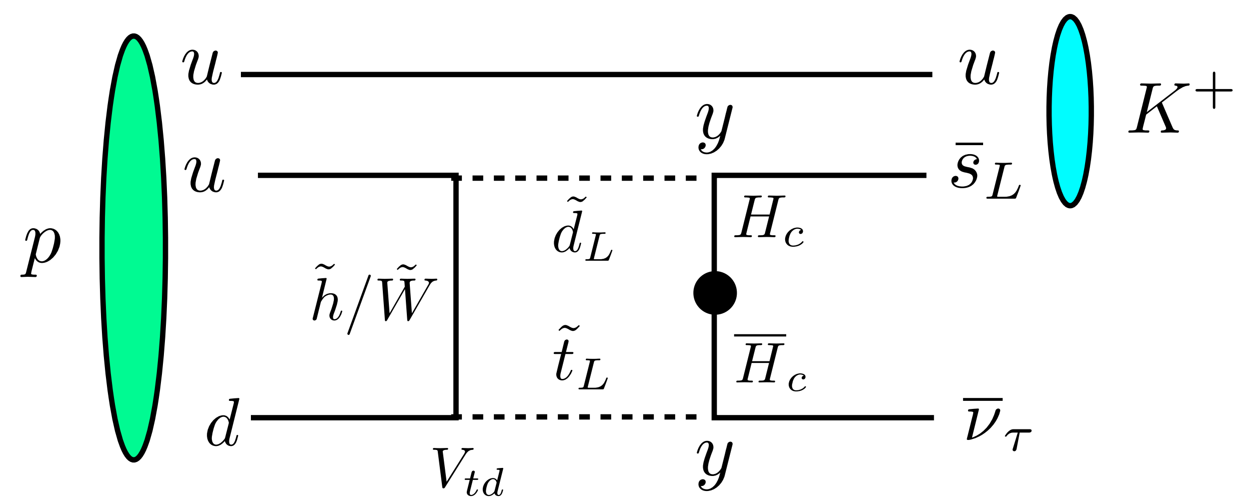
$$T_S = \left[M_3^{-28} M_2^{32} \mu^{12} m_A^3 X_T\right]^{\frac{1}{19}}$$

$$\Omega_S = \left[M_3^{-100} M_2^{60} \mu^{32} m_A^8 X_\Omega\right]^{\frac{1}{288}}$$

$$M_s^* = 2.08\,\mathrm{TeV}$$

$$M_G^* = 1.27 \cdot 10^{16}\,\mathrm{GeV}$$

D=5 proton decay



GCU condition

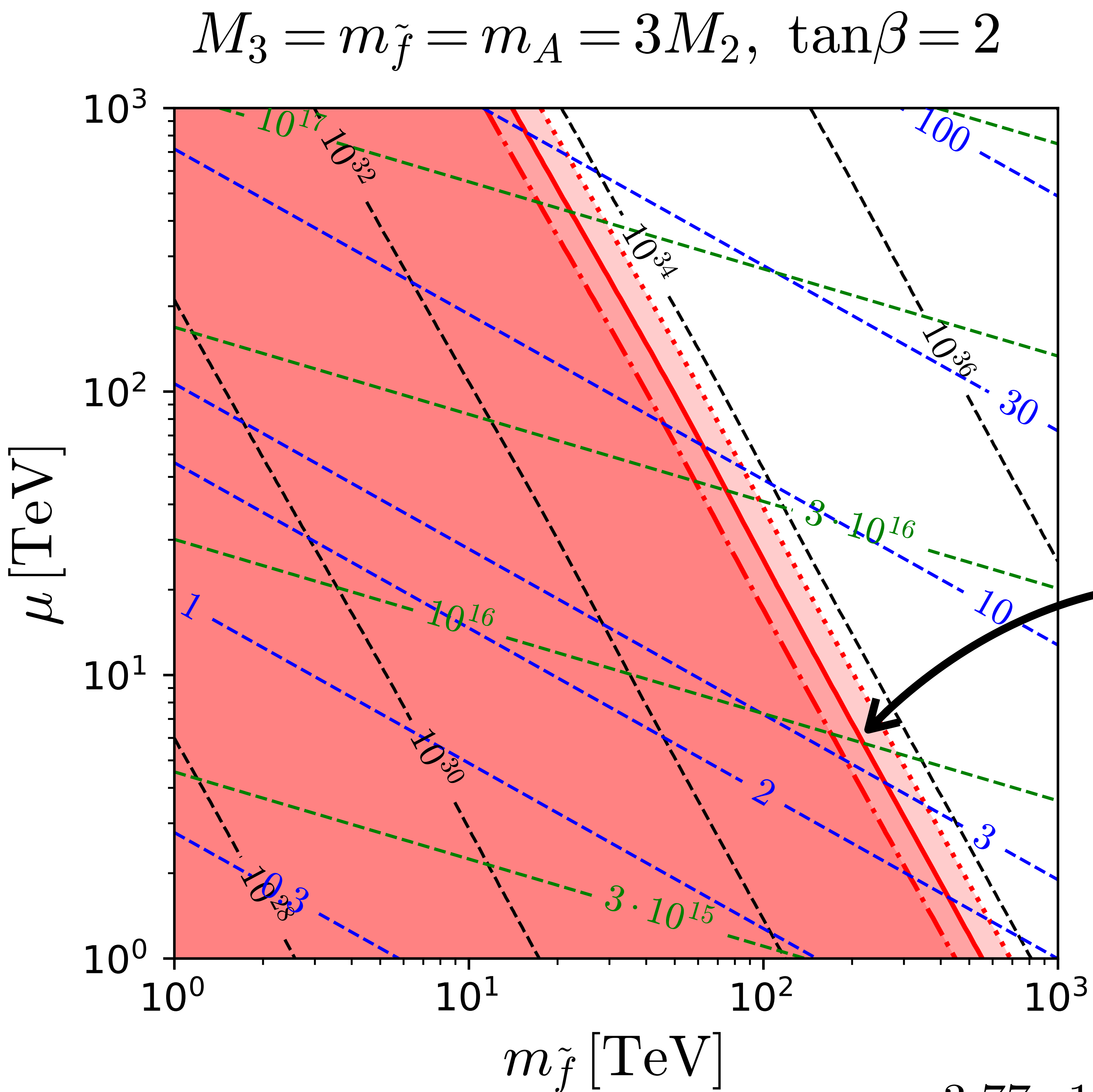
$$T_S = M_s^* \Omega_G \ \cap \ T_G = M_G^* \Omega_S$$

$$\boxed{M_{H_C} = M_G^* \Omega_S \left(\frac{T_S}{M_s^*}\right)^{\frac{5}{6}}}$$

$$(M_V^2 M_\Sigma)^{\frac{1}{3}} = M_G^* \Omega_S \left(\frac{T_S}{M_s^*}\right)^{-\frac{2}{9}}$$

Vanilla SUSY

$(M_V = M_\Sigma)$



$\tau(p\to K^+\bar\nu) > 4.0\times10^{33}\,\mathrm{yrs}$

$\alpha_s(m_Z) = 0.1183 \pm 0.0008$

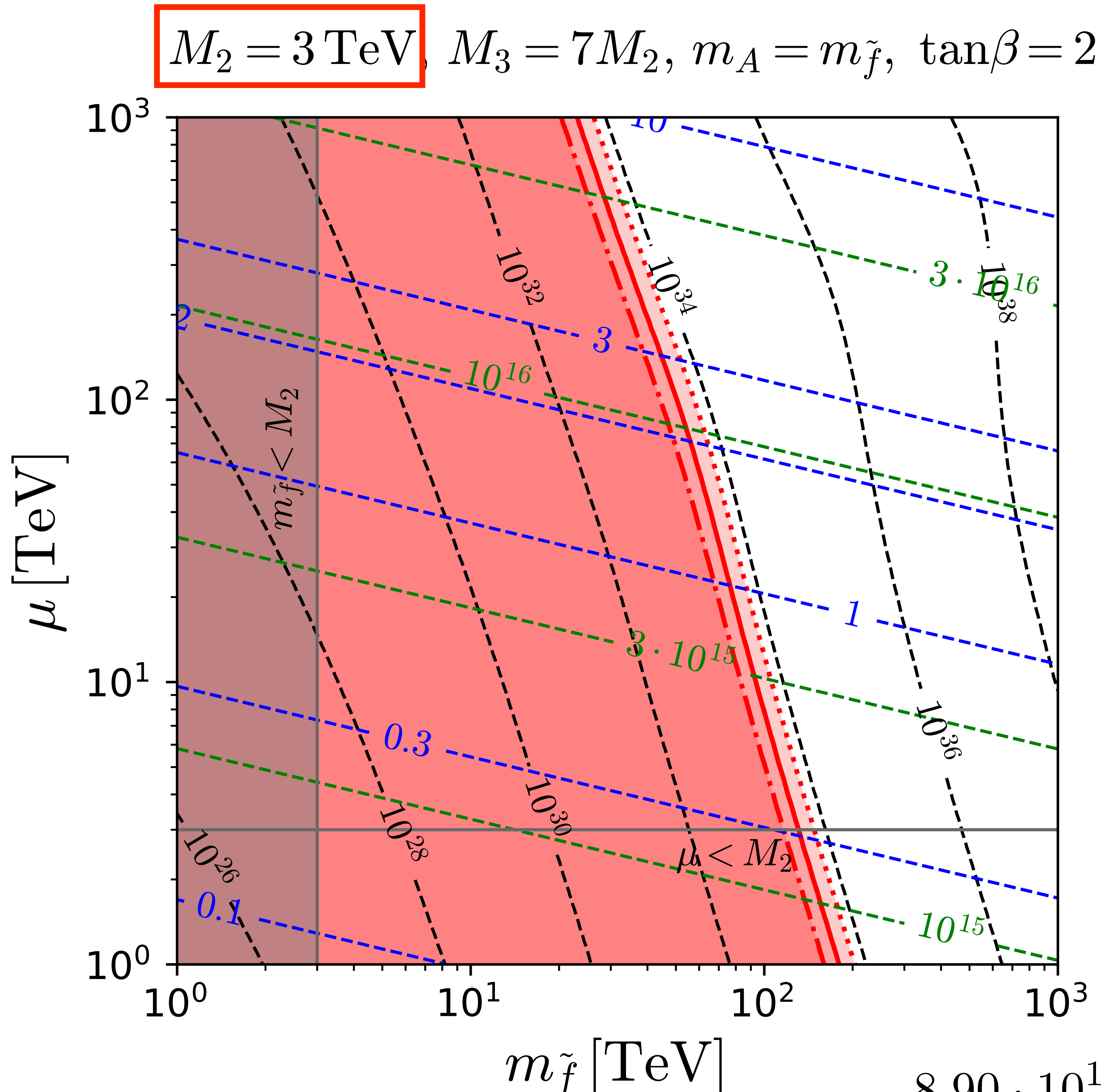
$\Lambda \equiv \max\{M_{H_C}, M_V\}$

$24.7 < \alpha^{-1}(\Lambda) < 31.8\ \ (M_V = M_\Sigma)$

$3.77\cdot10^{15}\,\mathrm{GeV} < (M_V^2 M_\Sigma)^{\frac{1}{3}} < 1.83\cdot10^{16}\,\mathrm{GeV}$

S. Pokorski, K. Rolbiecki, G.Ross, KS, 1902.06093

AMSB with Wino DM



$$\tau(p \to K^+\bar\nu) > 4.0 \times 10^{33}\,\text{yrs}$$

$$\alpha_s(m_Z) = 0.1183 \pm 0.0008$$

$$\Lambda \equiv \max\{M_{H_C}, M_V\}$$

$$25.3 < \alpha^{-1}(\Lambda) < 29.5 \;\; (M_V = M_\Sigma)$$

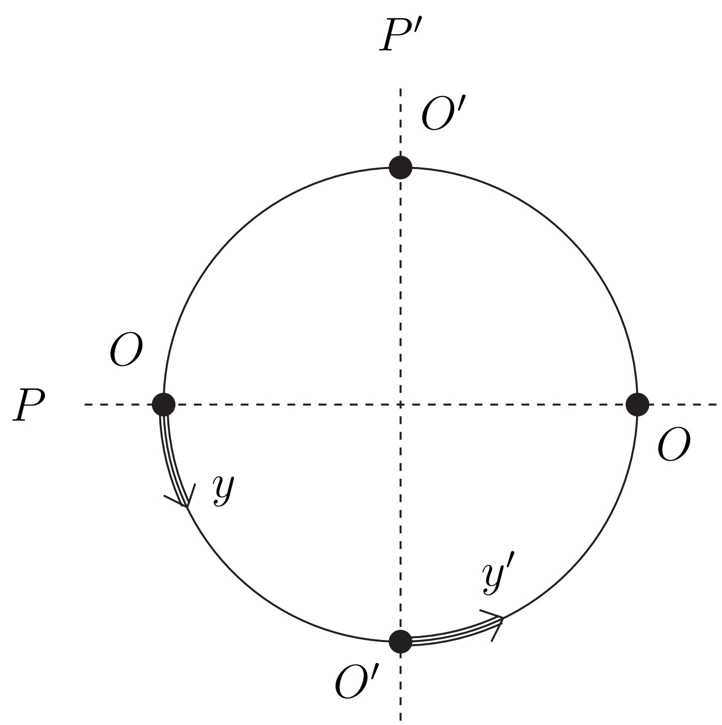
$$8.90 \cdot 10^{15}\,\text{GeV} < (M_V^2 M_\Sigma)^{\frac{1}{3}} < 1.19 \cdot 10^{16}\,\text{GeV}$$

S. Pokorski, K. Rolbiecki, G.Ross, KS, 1902.06093

Orbifold SUSY SU(5) GUT

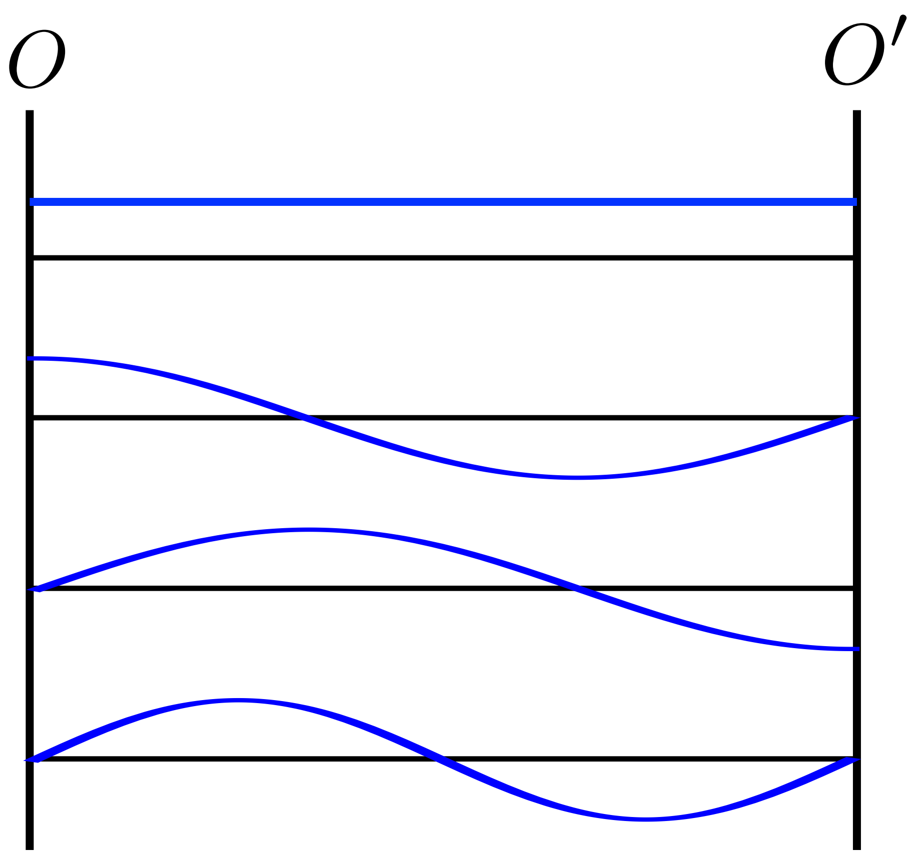
$S^1/(Z_2 \times Z_2')$ orbifold in the fifth dimension

[Hall, Nomura '01]



two orbifold parities (P, P')

$$
\begin{aligned}
\phi(x^\mu, y) \to \phi(x^\mu, -y) &= P\phi(x^\mu, y), \\
\phi(x^\mu, y') \to \phi(x^\mu, -y') &= P'\phi(x^\mu, y'),
\end{aligned}
$$



(P, P')

$(+,+) \longrightarrow 2n/R$ ← **zero mode only here**

$(+,-) \longrightarrow (2n+1)/R$

$(-,+) \longrightarrow (2n+1)/R$

$(-,-) \longrightarrow (2n+2)/R$

$$n = 0, 1, 2, \cdots$$

$a\ =$ unbroken generators $\quad \hat{a}\ =$ broken generators $\qquad n = 0, 1, 2, \cdots$

KK mode	m_ξ	(P, P')	4d fields	$\sum(b_1, b_2, b_3)$
zero	0	$(+,+)$	$V^a,\ H_F,\ H_{\overline{F}}$	
even	$(2n+2)/R$	$(+,+)$	$V^a,\ H_F,\ H_{\overline{F}}$	$(\frac{6}{5}, -2, -6)$
		$(-,-)$	$\Sigma^a,\ H^c_F,\ H^c_{\overline{F}}$	
odd	$(2n+1)/R$	$(+,-)$	$V^{\hat{a}},\ H_C,\ H_{\overline{C}}$	$(-\frac{46}{5}, -6, -2)$
		$(-,+)$	$\Sigma^{\hat{a}},\ H^c_C,\ H^c_{\overline{C}}$	

a = unbroken generators $\hat{a}$ = broken generators $n = 0, 1, 2, \cdots$

KK mode	m_ξ	(P, P')	4d fields	$\sum(b_1, b_2, b_3)$
zero	0	$(+,+)$	$V^a,\ H_F,\ H_{\overline{F}}$	
even	$(2n+2)/R$	$(+,+)$	$V^a,\ H_F,\ H_{\overline{F}}$	$(\frac{6}{5}, -2, -6)$
		$(-,-)$	$\Sigma^a,\ H_F^c,\ H_{\overline{F}}^c$	
odd	$(2n+1)/R$	$(+,-)$	$V^{\hat{a}},\ H_C,\ H_{\overline{C}}$	$(-\frac{46}{5}, -6, -2)$
		$(-,+)$	$\Sigma^{\hat{a}},\ H_C^c,\ H_{\overline{C}}^c$	

$(r \equiv R\Lambda)$

$$\ln\left(\frac{T_G}{\Lambda}\right) = \sum_\xi \left(-\frac{5}{288} b_1^\xi - \frac{15}{76} b_2^\xi + \frac{25}{114} b_3^\xi\right) \ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$\ln \Omega_G = \sum_\xi \left(\frac{10}{19} b_1^\xi - \frac{24}{19} b_2^\xi + \frac{14}{19} b_3^\xi\right) \ln\left(\frac{m_\xi}{\Lambda}\right)$$

$r \in$	$(1,2]$	$(2,3]$	$(3,4]$	$(4,5]$	$\cdots$
Ω_G	$\left[\frac{1}{r}\right]^{\frac{24}{19}}$	$\left[\frac{1}{2}\right]^{\frac{24}{19}}$	$\left[\frac{1\cdot 3}{2}\frac{1}{r}\right]^{\frac{24}{19}}$	$\left[\frac{1\cdot 3}{2\cdot 4}\right]^{\frac{24}{19}}$	$\cdots$
T_G/Λ	$\left[\frac{1}{r}\right]^{\frac{18}{19}}$	$\left[\frac{1}{2}\right]^{\frac{18}{19}}$	$\left[\frac{1\cdot 3}{2}\frac{1}{r}\right]^{\frac{18}{19}}$	$\left[\frac{1\cdot 3}{2\cdot 4}\right]^{\frac{18}{19}}$	$\cdots$

a = unbroken generators $\quad \hat{a}$ = broken generators $\quad n = 0, 1, 2, \cdots$

KK mode	m_ξ	(P, P')	4d fields	$\sum(b_1, b_2, b_3)$
zero	0	$(+,+)$	$V^a,\ H_F,\ H_{\overline{F}}$	
even	$(2n+2)/R$	$(+,+)$	$V^a,\ H_F,\ H_{\overline{F}}$	$(\frac{6}{5}, -2, -6)$
		$(-,-)$	$\Sigma^a,\ H^c_F,\ H^c_{\overline{F}}$	
odd	$(2n+1)/R$	$(+,-)$	$V^{\hat{a}},\ H_C,\ H_{\overline{C}}$	$(-\frac{46}{5}, -6, -2)$
		$(-,+)$	$\Sigma^{\hat{a}},\ H^c_C,\ H^c_{\overline{C}}$	

$(r \equiv R\Lambda)$

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$$\ln \Omega_G = \sum_\xi \left(\frac{10}{19}b_1^\xi - \frac{24}{19}b_2^\xi + \frac{14}{19}b_3^\xi\right)\ln\left(\frac{m_\xi}{\Lambda}\right)$$

$r \in$	$(1,2]$	$(2,3]$	$(3,4]$	$(4,5]$	$\cdots$
Ω_G	$\left[\frac{1}{r}\right]^{\frac{24}{19}}$	$\left[\frac{1}{2}\right]^{\frac{24}{19}}$	$\left[\frac{1\cdot 3}{2}\frac{1}{r}\right]^{\frac{24}{19}}$	$\left[\frac{1\cdot 3}{2\cdot 4}\right]^{\frac{24}{19}}$	$\cdots$
T_G/Λ	$\left[\frac{1}{r}\right]^{\frac{18}{19}}$	$\left[\frac{1}{2}\right]^{\frac{18}{19}}$	$\left[\frac{1\cdot 3}{2}\frac{1}{r}\right]^{\frac{18}{19}}$	$\left[\frac{1\cdot 3}{2\cdot 4}\right]^{\frac{18}{19}}$	$\cdots$

GCU condition

$$T_S = M_s^* \Omega_G$$

$$T_G = M_G^* \Omega_S$$

$$T_S = \left[M_3^{-28} M_2^{32} \mu^{12} m_A^3 X_T\right]^{\frac{1}{19}} = M_s^* \Omega_G$$

non-trivial constraint on SUSY masses

a = unbroken generators $\quad \hat{a}$ = broken generators $\quad n = 0, 1, 2, \cdots$

KK mode	m_ξ	(P, P')	4d fields	$\sum(b_1, b_2, b_3)$
zero	0	$(+,+)$	$V^a,\ H_F,\ H_{\overline{F}}$	
even	$(2n+2)/R$	$(+,+)$	$V^a,\ H_F,\ H_{\overline{F}}$	$(\frac{6}{5}, -2, -6)$
		$(-,-)$	$\Sigma^a,\ H_F^c,\ H_{\overline{F}}^c$	
odd	$(2n+1)/R$	$(+,-)$	$V^{\hat{a}},\ H_C,\ H_{\overline{C}}$	$(-\frac{46}{5}, -6, -2)$
		$(-,+)$	$\Sigma^{\hat{a}},\ H_C^c,\ H_{\overline{C}}^c$	

$(r \equiv R\Lambda)$

$$\ln\left(\frac{T_G}{\Lambda}\right) = \sum_\xi \left(-\frac{5}{288}b_1^\xi - \frac{15}{76}b_2^\xi + \frac{25}{114}b_3^\xi\right)\ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$\ln\Omega_G = \sum_\xi \left(\frac{10}{19}b_1^\xi - \frac{24}{19}b_2^\xi + \frac{14}{19}b_3^\xi\right)\ln\left(\frac{m_\xi}{\Lambda}\right)$$

$r \in$	$(1,2]$	$(2,3]$	$(3,4]$	$(4,5]$	$\cdots$
Ω_G	$\left[\frac{1}{r}\right]^{\frac{24}{19}}$	$\left[\frac{1}{2}\right]^{\frac{24}{19}}$	$\left[\frac{1\cdot 3}{2}\frac{1}{r}\right]^{\frac{24}{19}}$	$\left[\frac{1\cdot 3}{2\cdot 4}\right]^{\frac{24}{19}}$	$\cdots$
T_G/Λ	$\left[\frac{1}{r}\right]^{\frac{18}{19}}$	$\left[\frac{1}{2}\right]^{\frac{18}{19}}$	$\left[\frac{1\cdot 3}{2}\frac{1}{r}\right]^{\frac{18}{19}}$	$\left[\frac{1\cdot 3}{2\cdot 4}\right]^{\frac{18}{19}}$	$\cdots$

GCU condition

$$\boxed{\begin{array}{c} T_S = M_s^* \Omega_G \\ T_G = M_G^* \Omega_S \end{array}}$$

$$T_S = \left[M_3^{-28} M_2^{32} \mu^{12} m_A^3 X_T\right]^{\frac{1}{19}} = M_s^* \Omega_G$$

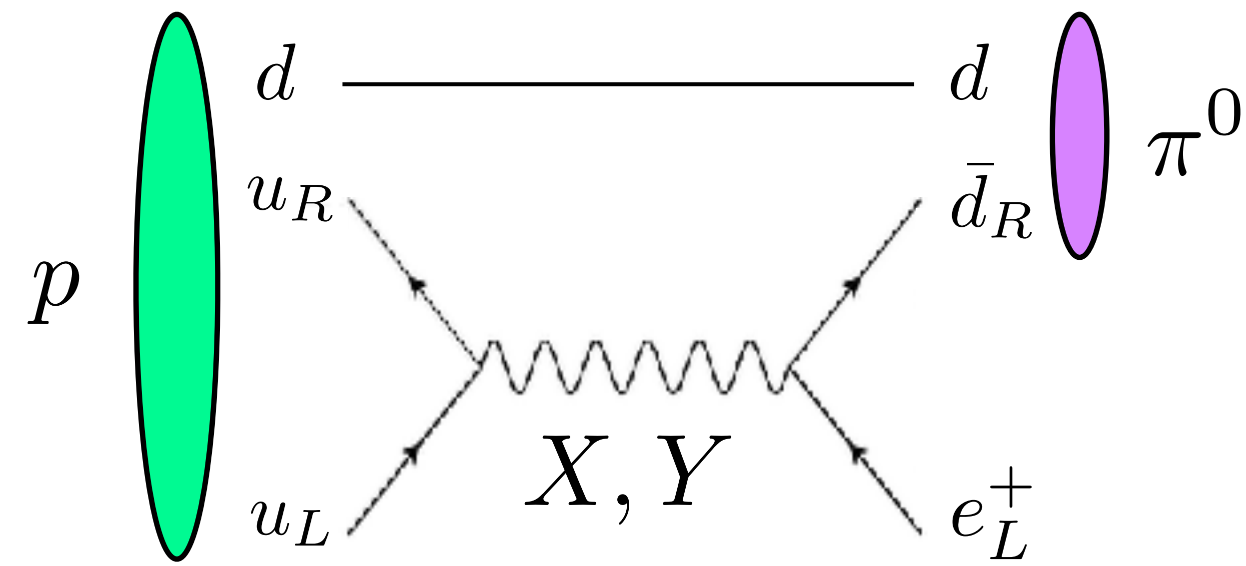
non-trivial constraint on SUSY masses

$$\frac{1}{R} = M_{(X,Y)_1} = M_G^* \Omega_G^{-\frac{3}{4}} \Omega_S / r$$

$$= M_G^* M_s^{*\frac{19}{108}} \Omega_G^{-\frac{31}{54}} M_3^{-\frac{19}{216}} M_2^{-\frac{19}{216}} X_T^{-\frac{1}{108}} X_\Omega^{-\frac{1}{288}} / r$$

M_c = 1/R, (i.e. X,Y boson mass) can be predicted from SUSY spectrum, allowing to predict D=6 proton decay

D=6 proton decay



GCU condition

$$T_S = M_s^* \Omega_G$$

$$T_G = M_G^* \Omega_S$$

$$\frac{1}{R} = M_{(X,Y)_1} = M_G^* \Omega_G^{-\frac{3}{4}} \Omega_S \,/r$$

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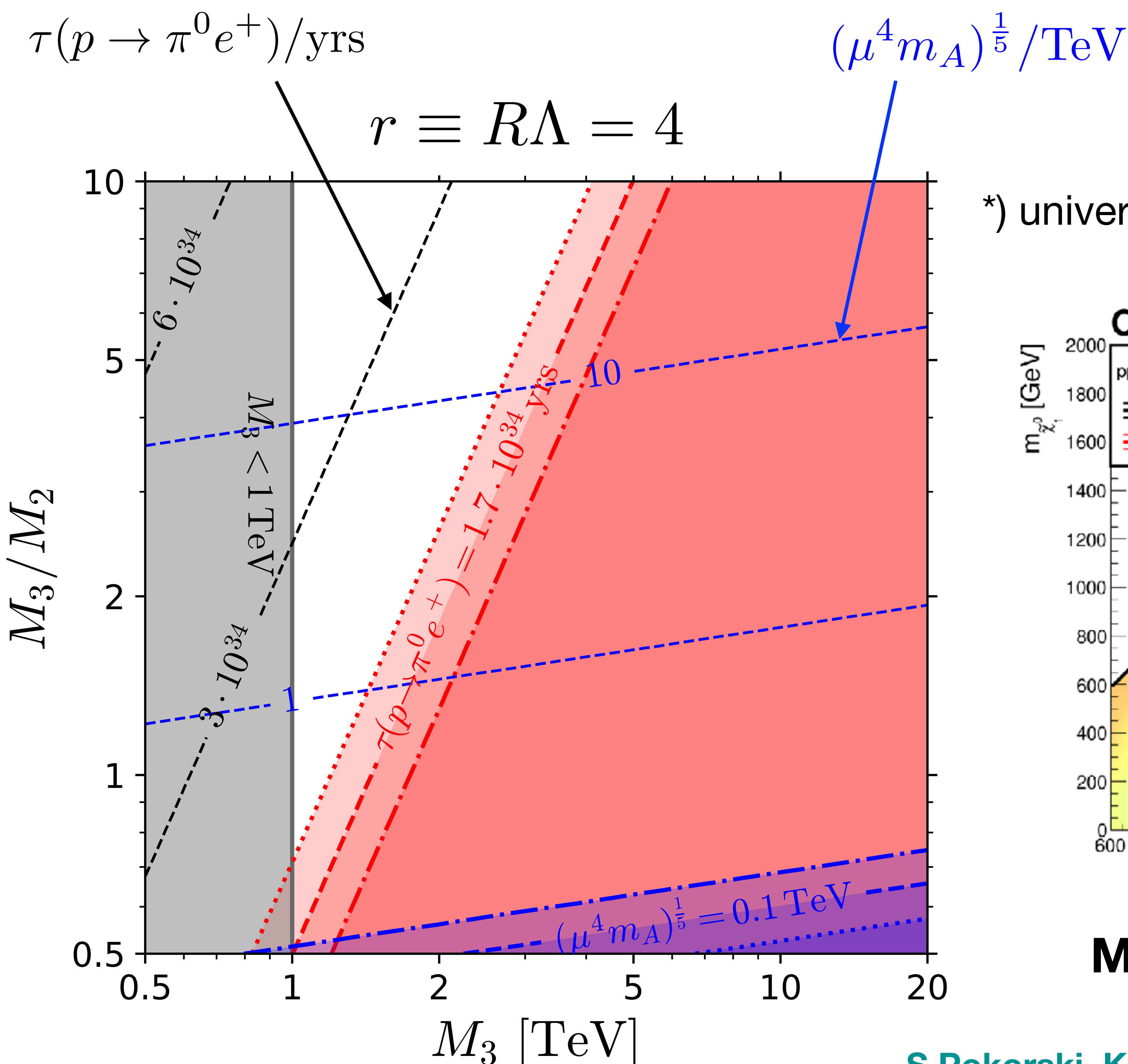
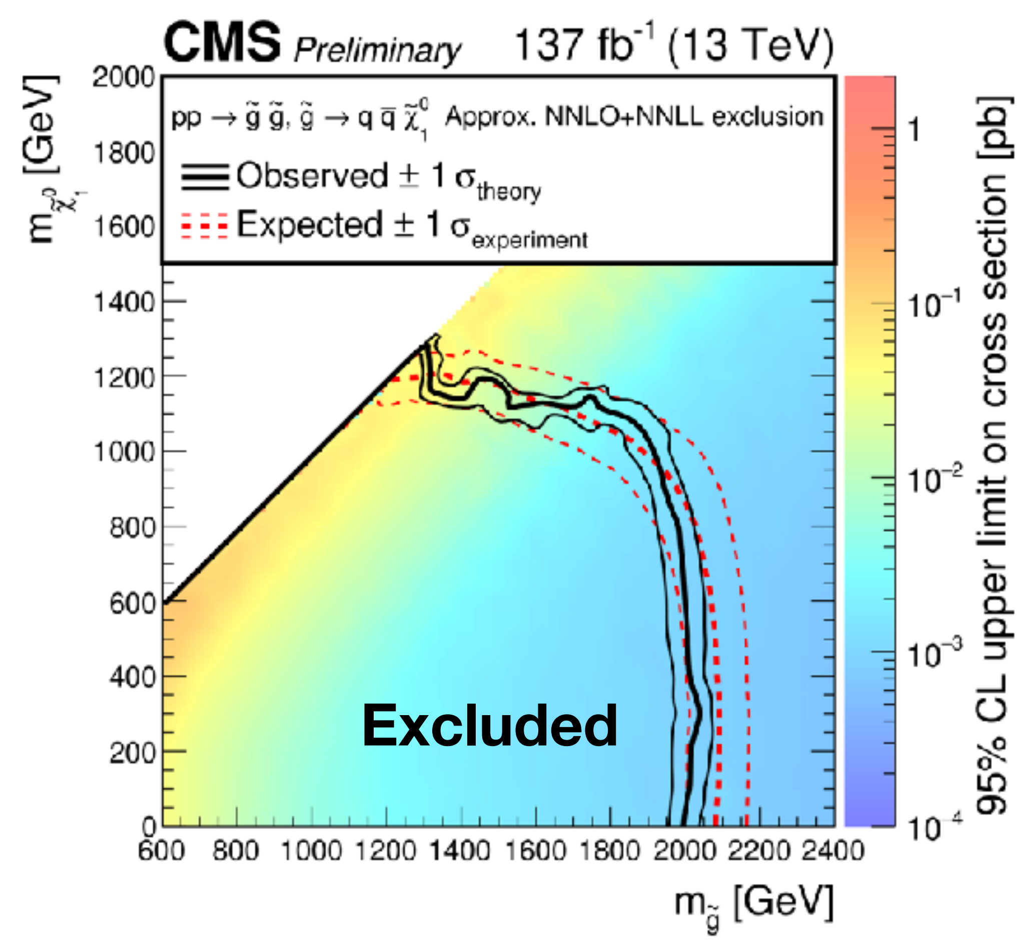
A SUSY plane with GCU



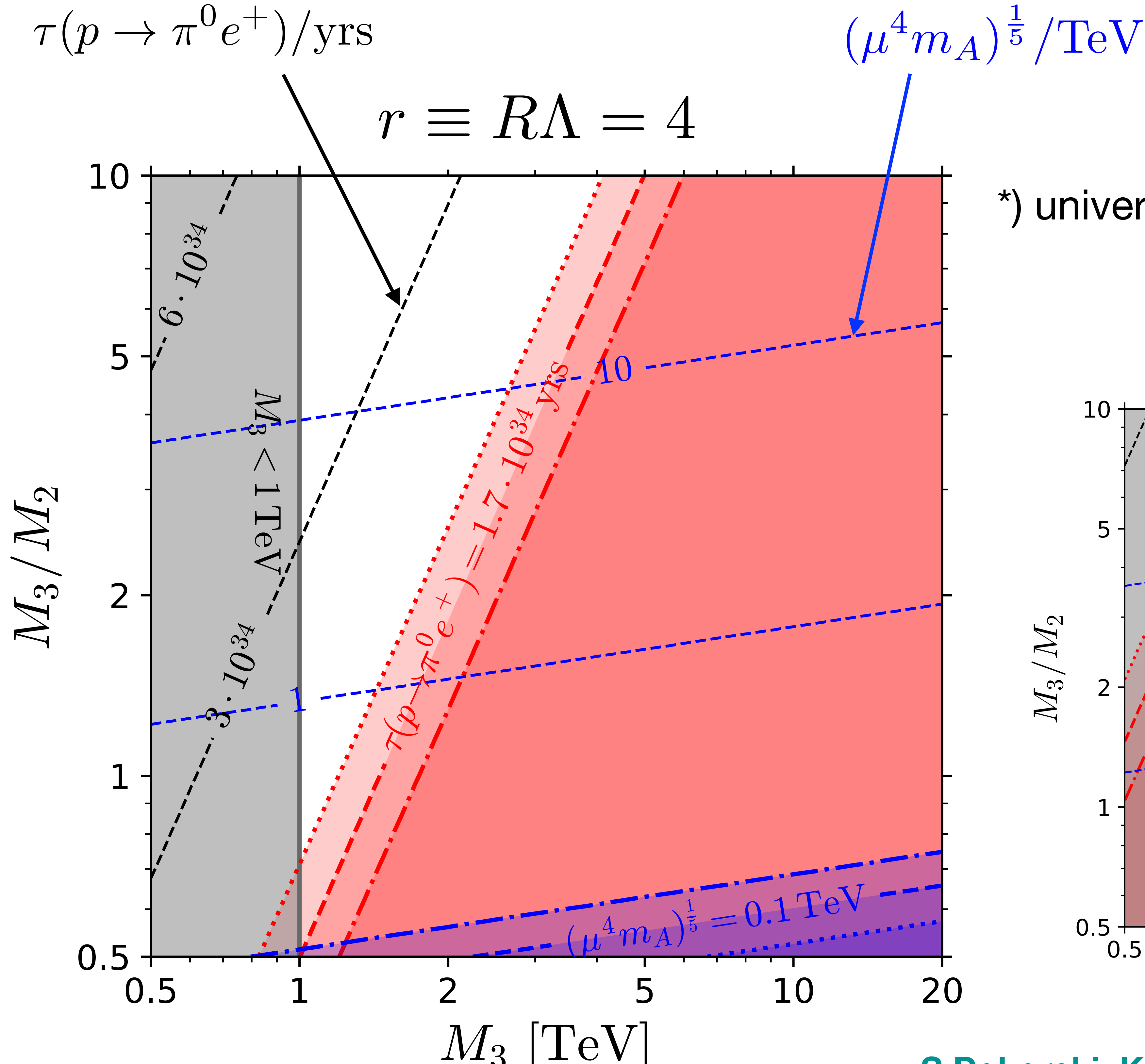
*) universal sfermion mass is assumed



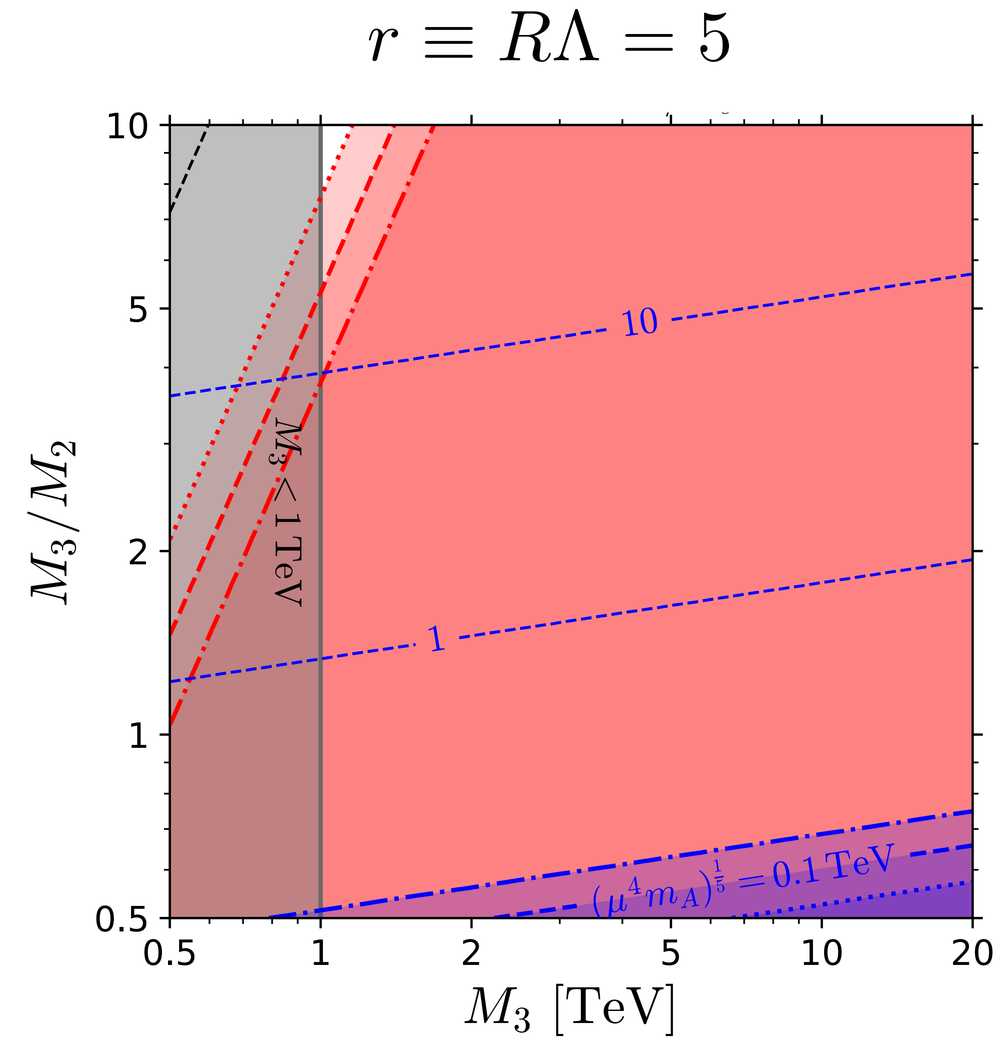
M_3 > 1 - 2 TeV from LHC

S.Pokorski, K.Rolbiecki, G.Ross, KS, 1902.06093

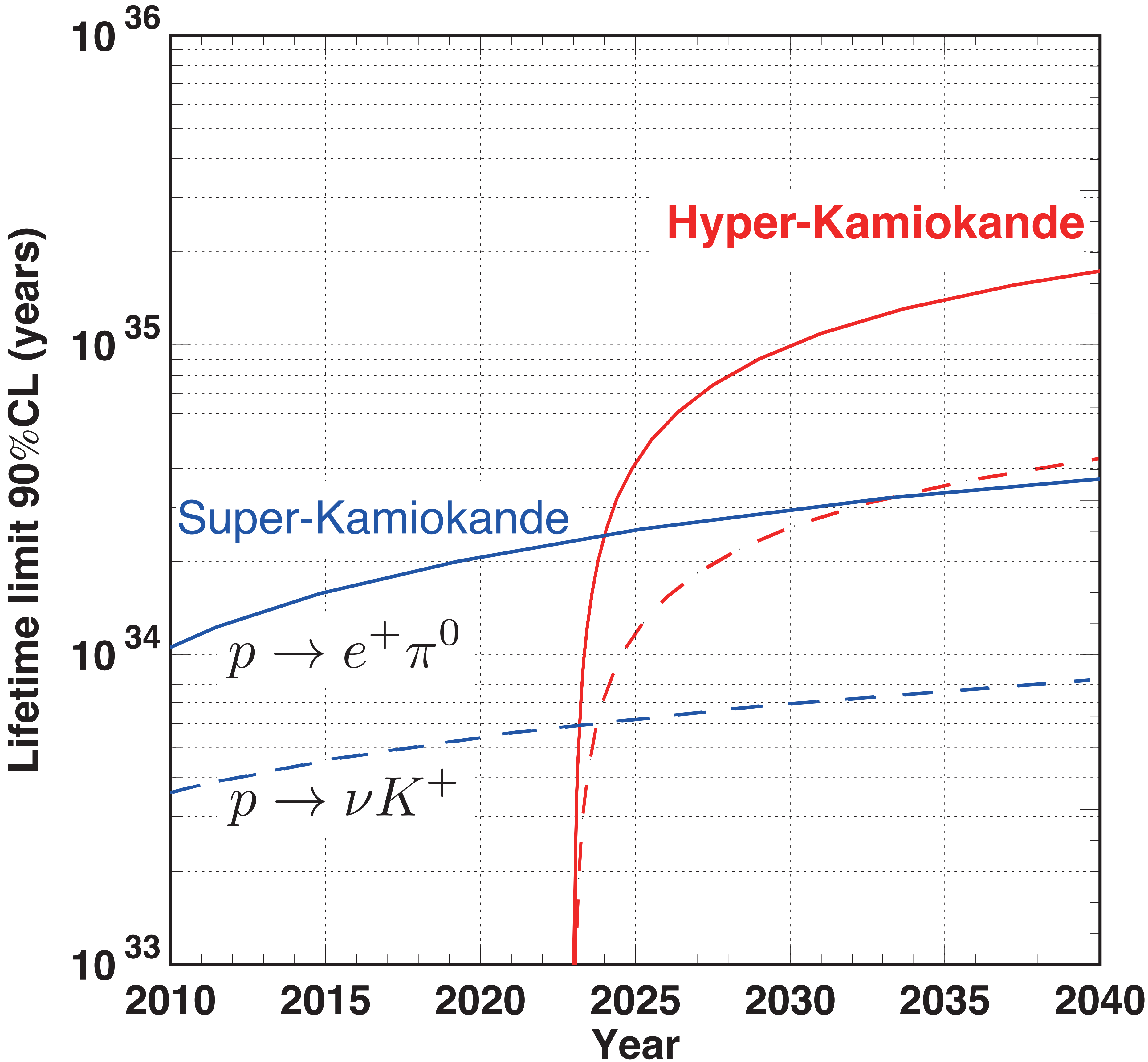
A SUSY plane with GCU



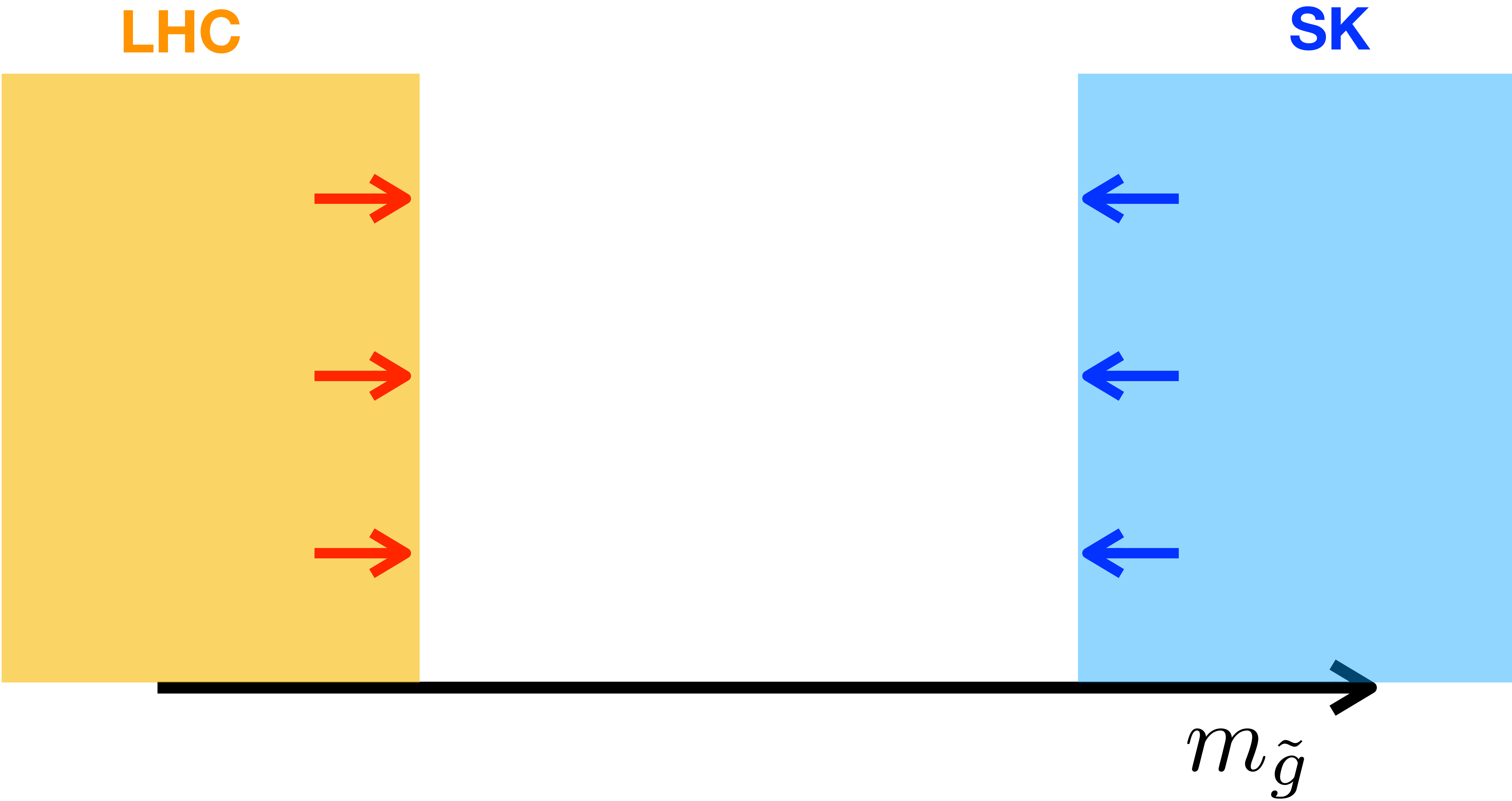
*) universal sfermion mass is assumed

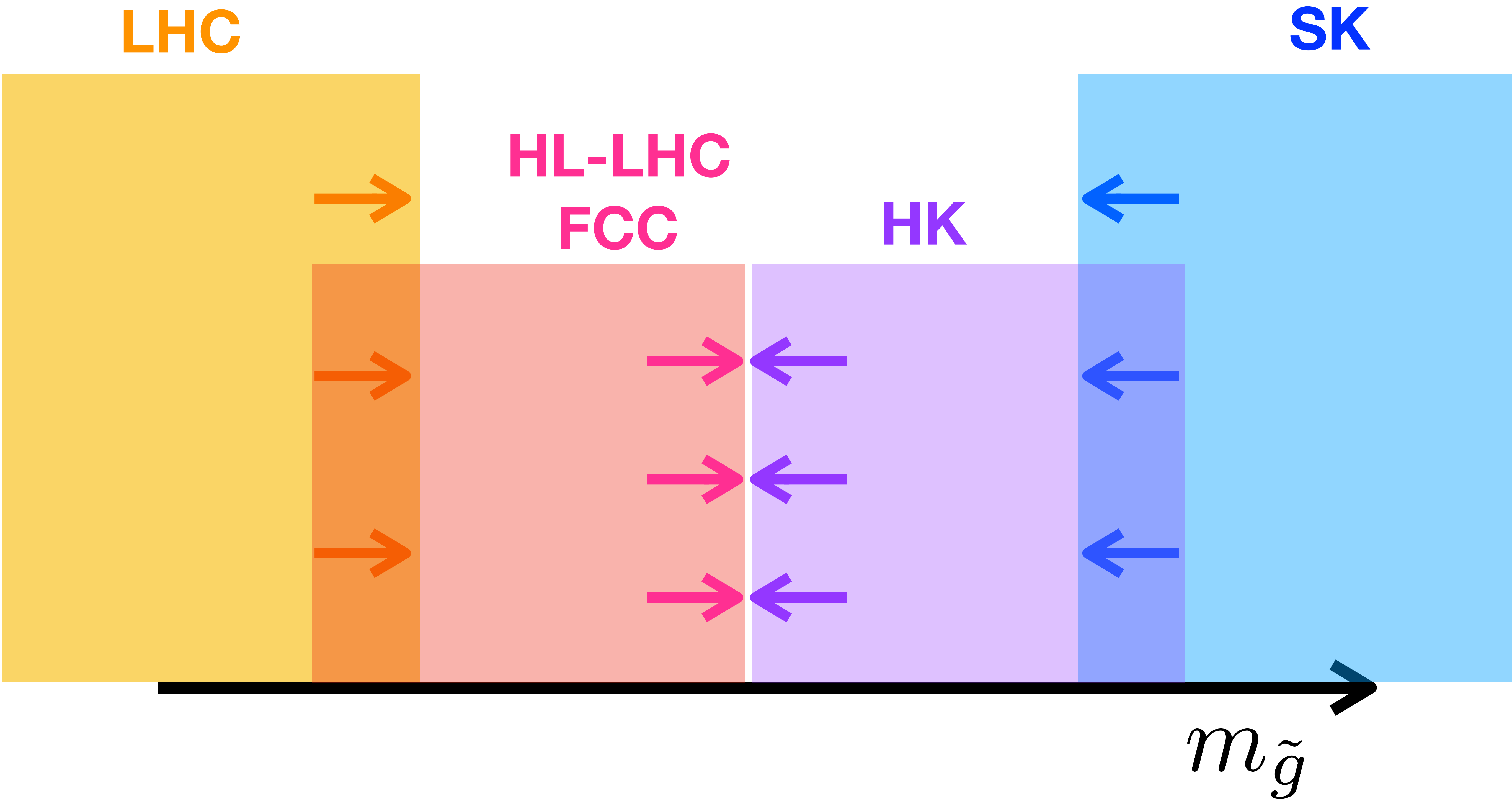


S.Pokorski, K.Rolbiecki, G.Ross, KS, 1902.06093



Hyper-Kamiokande
Super-Kamiokande
$p \to e^+\pi^0$
$p \to \nu K^+$
Lifetime limit 90%CL (years)
10^{36}
10^{35}
10^{34}
10^{33}
2010
2015
2020
2025
2030
2035
2040
Year


LHC
SK
$m_{\tilde{g}}$


LHC
SK
HL-LHC
FCC
HK
$m_{\tilde{g}}$

Conclusion

- In gauge mediation with light gravitino models, the LHC constraints gets stronger, but the fine-tuning can nevertheless be relaxed.

- Muon and Electron g-2 can be explained within Higgs mediation SUSY models without introducing flavour problem. The spectrum contains very light EW states, but the LHC constraints can be satisfied due to compressed spectrum.

- Requirement of gauge coupling unification tells us a relation between SUSY and GUT spectra. For minimal SU(5) and orbifold SU(5) models, proton lifetime can be predicted from low energy SUSY spectrum alone. We found a complementarity between collider and proton decay measurements.

a = unbroken generators $\quad \hat{a}$ = broken generators $\quad n = 0, 1, 2, \cdots$

KK mode	mass	(P, P')	4d fields	$\sum(b_1, b_2, b_3)$
zero	0	$(+,+)$	$V^a,\ H_F,\ H_{\overline{F}}$	
even	$(2n+2)/R$	$(+,+)$	$V^a,\ H_F,\ H_{\overline{F}}$	$(\frac{6}{5}, -2, -6)$
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odd	$(2n+1)/R$	$(+,-)$	$V^{\hat{a}},\ H_C,\ H_{\overline{C}}$	$(-\frac{46}{5}, -6, -2)$
		$(-,+)$	$\Sigma^{\hat{a}},\ H^c_C,\ H^c_{\overline{C}}$	

- doublet-triplet splitting can be achieved by the parity assignment
- X,Y fields are absent at the 3-brane at *O'* => SU(5) is explicitly broken at *O'*
- non-universal contribution to the gauge couplings from the 3-brane at *O'*

$$f^a \mathcal{W}^a \mathcal{W}^a \Big|_{O'}$$

the effect is negligible due to the dominant bulk contribution if the extra dimension is large, $R\Lambda > 4$ [Hall, Nomura '01]

- The success of charge quantisation in 4d GUTs can be kept by placing the matter fields at the SU(5) symmetric brane at *O*.

- D=5 proton decay is absent due to an accidental $U(1)_R$ sym.

	Σ	$H_{\mathbf{5}}$	$H_{\bar{\mathbf{5}}}$	$H^c_{\mathbf{5}}$	$H^c_{\bar{\mathbf{5}}}$	$T_{\mathbf{10}}$	$F_{\bar{\mathbf{5}}}$	$N_{\mathbf{1}}$
$U(1)_R$	0	0	0	2	2	1	1	1

- Above the KK mass scale, $M_c = 1/R$, *KK* modes appear and the three gauge couplings approach each other. They finally unify at the cut-off scale, Λ, where the 5d theory is incorporated into a more fundamental theory.

$$\boxed{\begin{aligned} \ln\left(\frac{T_G}{\Lambda}\right) &= \sum_\xi \left(-\frac{5}{288}b_1^\xi - \frac{15}{76}b_2^\xi + \frac{25}{114}b_3^\xi\right)\ln\left(\frac{m_\xi}{\Lambda}\right) \\ \ln \Omega_G &= \sum_\xi \left(\frac{10}{19}b_1^\xi - \frac{24}{19}b_2^\xi + \frac{14}{19}b_3^\xi\right)\ln\left(\frac{m_\xi}{\Lambda}\right) \end{aligned}}$$

the total contributions from (2k+1) and (2k+2) modes

$$c_o \ln \frac{2k+1}{R\Lambda} \qquad c_e \ln \frac{2k+2}{R\Lambda}$$

$$c_o = -c_e = \frac{24}{19} \text{ for } \ln \Omega_G \qquad c_o = -c_e = \frac{18}{19} \text{ for } \ln(T_G/\Lambda) \qquad c_o = \frac{4}{19},\ c_e = -\frac{156}{19} \text{ for } C_G$$

The index k runs from 0 to $k_{\max}^{o/e}$, where $\begin{cases} 2k_{\max}^o + 1 < R\Lambda & \text{for odd modes} \\ 2k_{\max}^e + 2 < R\Lambda & \text{for even modes} \end{cases}$

$$\Omega_G = \left[\frac{\prod_k^{k_{\max}^o}(2k+1)}{\prod_k^{k_{\max}^e}(2k+2)}\left(\frac{1}{r}\right)^{k_{\max}^o - k_{\max}^e}\right]^{\frac{24}{19}},$$

$$\frac{T_G}{\Lambda} = \left[\frac{\prod_k^{k_{\max}^o}(2k+1)}{\prod_k^{k_{\max}^e}(2k+2)}\left(\frac{1}{r}\right)^{k_{\max}^o - k_{\max}^e}\right]^{\frac{18}{19}}, \qquad (r \equiv R\Lambda)$$

$$C_G = \frac{4}{19}\ln\left[\prod_{k=0}^{k_{\max}^o}\frac{2k+1}{r}\right] - \frac{156}{19}\ln\left[\prod_{k=0}^{k_{\max}^e}\frac{2k+2}{r}\right]$$