Origin of CP and Flavor

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Do we need CP as a Symmetry?

CP (and its violation) is relevant for several phenomena:

- CP violation in standard model (CKM phase)
- matter-antimatter asymmetry of the universe
- the strong CP-problem ($\Theta_{QCD}$)

The strong CP-problem requires CP to be a symmetry

- Origin of CP symmetry? How is it broken?
- Is it related to flavour symmetries?

CP: you have to make it and to break it.

Is there a top-down explanation? We use String Theory.
Warm-up: Interval $S_1/Z_2$
Discrete symmetry $D_4$

- bulk and brane fields
- $S_2$ symmetry from interchange of fixed points
- $Z_2 \times Z_2$ symmetry from brane field selection rules

$D_4$ as multiplicative closure of $S_2$ and $Z_2 \times Z_2$

$D_4$ – a non-abelian subgroup of $SU(2)_{\text{flavor}}$

flavor symmetry for the two lightest families

(Kobayashi, Nilles, Ploeger, Raby, Ratz, 2006)
Orbifold $T_2/Z_3$
Discrete symmetry $\Delta(54)$

- untwisted and twisted fields
- $S_3$ symmetry from interchange of fixed points
- $Z_3 \times Z_3$ symmetry from orbifold selection rules
- $\Delta(54)$ as multiplicative closure of $S_3$ and $Z_3 \times Z_3$
- $\Delta(54)$ – a non-abelian subgroup of $SU(3)_{\text{flavor}}$
- flavor symmetry for three families of quarks and leptons

(Kobayashi, Nilles, Ploeger, Raby, Ratz, 2006)
\[ \Delta(54) \text{ group theory} \]

\[ \Delta(54) \text{ is a non-abelian group and has representations:} \]

- One trivial singlet \( 1_0 \) and one nontrivial singlet \( 1_- \)
- Two triplets \( 3_1, 3_2 \) and corresponding anti-triplets \( \bar{3}_1, \bar{3}_2 \)
- Four doublets \( 2_k \) \((k = 1, 2, 3, 4)\)

Some relevant tensor products are:

- \( 3_1 \otimes \bar{3}_1 = 1_0 \oplus 2_1 \oplus 2_2 \oplus 2_3 \oplus 2_4 \)
- \( 2_k \otimes 2_k = 1_0 \oplus 1_- \oplus 2_k \)

\[ \Delta(54) \text{ is a good candidate for a flavour symmetry.} \]

But where is CP?
CP as outer automorphism

Outer automorphisms map the group to itself but are not group elements themselves

- $\Delta(54)$ has outer automorphism group $S_4$
- CP could be $\mathbb{Z}_2$ subgroup of this $S_4$
- Physical CP transforms $(\text{rep})$ to $(\text{rep})^*$

This gives an intimate relation of flavour and CP symmetry

- possible obstructions for a successful definition of CP
- controlled by "representation content" of the symmetry
- could lead to "explicit geometric CP violation"

(Holthausen, Lindner, Schmidt, 2012; Chen, Fallbacher, Mahanthappa, Ratz, Trautner, 2014)
Orbifold $T_2/Z_3$

Orbifold $T_2/Z_3$
$T_2/Z_3$ orbifold example

We label a string state by its constructing element $g = (\theta^k, n_\alpha e_\alpha)$ of the orbifold space group with $SU(3)$ lattice vectors $e_1$ and $e_2$.

- twist $\theta$ (of 120 degrees) with $\theta^3 = 1$

This leads to different classes of closed string states:

- untwisted states closed on the 2d plane
- winding states $(1, e_i)$ closed on the torus
- twisted states $(\theta, e_i)$ closed on the orbifold

How do they transform under $\Delta(54)$ and CP?
Twisted States

While untwisted states transform as singlets, the twisted states transform nontrivially:

- twisted fields \((\theta, 0)\), \((\theta, e_1)\) and \((\theta, e_1 + e_2)\) transform as triplets under \(\Delta(54)\)
- states in the \(\theta^2\) sector are anti-triplets
- they wind around fixed points \(X, Y\) and \(Z\)

CP maps triplets (anti-triplets) to their complex conjugates
Winding states are represented by the geometric elements:

\[ V_1 = (1, e_1), \ V_2 = (1, e_2) \quad \text{and} \quad V_3 = (1, -e_1 - e_2) \]

- the \( V_i \) wind around two fixed points with opposite orientation

- winding states \( \overline{V_i} \)
i = 1, 2, 3 have negative winding number

- the geometric winding states \( V_i \) and \( \overline{V_i} \) do not transform covariantly under \( \Delta (54) \)
Doublets of $\Delta(54)$

We have to consider linear combinations $[n, \gamma]$

\[
[1, \gamma] = V_1 + \exp(-2\pi i \gamma)V_2 + \exp(-4\pi i \gamma)V_3
\]

to obtain covariant states. This leads to doublets of $\Delta(54)$:

- $2_1 = (W_1, \overline{W}_1)$ with $W_1 = [-1, 0]$
- $2_3 = (W_2, \overline{W}_2)$ with $W_2 = \exp(4\pi i/3)[-1, -1/3]$
- $2_4 = (\overline{W}_3, W_3)$ with $W_3 = \exp(2\pi i/3)[-1, 1/3]$

States with positive and negative winding number form the two components of the individual doublets.

Generically, the winding modes are massive. Otherwise we would have symmetry enhancement (Narain lattice).
Examples from MiniLandscape

There are many examples in the heterotic MiniLandscape
(Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingertner, 2006-2008)

- with $T_2/Z_3$ subsectors
- and potential $\Delta(54)$ symmetry

An inspection of the spectrum reveals that the massless modes transform as

- singlets (untwisted sector)
- triplets (anti-triplets) in $\theta$- ($\theta^2$-) twisted sectors
- there are no doublets in the massless spectrum!!!

For an example see: (Carballo-Perez, Peinado, Ramos-Sanchez, 2016)
CP-symmetry and its violation

We consider CP as a subgroup of $S_4$ of the outer automorphisms of $\Delta(54)$

- CP transforms $(\text{rep})$ to $(\text{rep})^*$
- this is possible for singlets and triplets
- possible simultaneously for up to two doublets
- impossible in the presence of three or more doublets

The low-energy effective theory allows CP symmetry

- which is broken in the presence of winding modes
- physical CP-violation can arise if there are at least three doublets (here $2_1$, $2_3$ and $2_4$) (Trautner, 2017)
CP-violation in physics

The relevance for physics includes

- CP-violation in the standard model (Jarlskog angle)
- the $\Theta$-parameter of QCD
- CP violation for baryo/lepto-genesis

We have special form of of CP symmetry and CP-violation

- "Explicit geometric CP-violation"
- CP as outer automorphism of flavour symmetry
- CP symmetry for the low energy effective theory broken in the presence of (at least three) $\Delta(54)$ doublets
- Example for "CP made and broken"
Signals of CP-violation

The specific signals of CP-violation are strongly model dependent. We consider as a (toy) example the explicit model of

(Carballo-Perez, Peinado, Ramos-Sanchez, 2016)

- it contains singlets, triplets and anti-triplets of $\Delta(54)$
- quarks and leptons as triplets
- Higgs as singlet
- right handed neutrinos as anti-triplets
- SM singlets as triplets and anti-triplets of $\Delta(54)$

The relevant couplings to the winding modes $2_i$ are

- $3 \otimes \bar{3} \rightarrow 2_i$ and $3 \otimes 3 \otimes 3 \rightarrow 2_i$ \quad (i = 1, 3, 4)
CP violation through decays

CP-violation from the decay of heavy doublets. All three doublets have to appear in the process. CP-violation from the interference of two decay diagrams.

$\chi^{(2_1)} \rightarrow c_1 \rightarrow \phi^{(3_1)} \rightarrow \varphi^{(3_1)}$

$\chi^{(2_1)} \rightarrow c_1 \rightarrow \phi^{(3_1)} \rightarrow \varphi^{(3_1)}$

- $2_3$ and $2_4$ in (non-planar) two-loop diagram
- Decay to right-handed neutrinos and SM singlets as source for lepto-genesis
CP-odd basis invariant
CP-violation in physics

The "CP-odd basis invariants" control all possible CP-violation in physics

- CP-violating decay of heavy doublets
- CP violation in the standard model (Jarlskog determinant)
- QCD $\Theta$-angle

We need explicit model building to study these effects (coupling of doublets to CKM matrix and $\Theta_{QCD}$)
Intermediate Conclusions

Discussion of CP requires

- the origin of the symmetry ("Make It")
- and its violation ("Break It")

String theory could provide such a mechanism through

- "Explicit geometric CP-violation"
- Unification of flavour symmetry and CP
- CP symmetry for the low energy effective theory
- broken in the presence of heavy string modes

It provides calculable effects for CP-violating decay of heavy particles and a solution to the strong CP-problem
General discussion of CP and Flavor

We have seen that even in simple systems we obtain sizeable flavor groups

- $D_4$ for the interval
- $\Delta(54)$ for the 2-dimensional $\mathbb{Z}_3$ orbifold

Did we find the complete flavor symmetry in these cases?

In reality we have even six compact dimensions

- variety of complicated group structures possible
- dependence on localisation of fields in extra dimensions

A general mechanism to find the complete set of flavor symmetries is based on the outer automorphisms of the Narain space group.

(Baur, Nilles, Trautner, Vaudrevange, 2019)
The Narain Lattice

In the string there are \( D \) right- and \( D \) left-moving degrees of freedom \( Y = (y_R, y_L) \). \( Y \) compactified on a \( 2D \) torus

\[
Y = \begin{pmatrix} y_R \\ y_L \end{pmatrix} \sim Y + E \hat{N} = \begin{pmatrix} y_R \\ y_L \end{pmatrix} + E \begin{pmatrix} n \\ m \end{pmatrix}
\]

defines the Narain lattice with

- the string’s winding and Kaluza-Klein quantum numbers \( n \) and \( m \)
- the Narain vielbein matrix \( E \) that depends on the moduli of the torus: radii, angles and anti-symmetric tensor fields \( B \).
A $Z_K$ orbifold with twist $\Theta$ leads to the identification

$$Y \sim \Theta^k Y + E \hat{N} \text{ where } \Theta = \begin{pmatrix} \theta_R & 0 \\ 0 & \theta_L \end{pmatrix} \text{ and } \Theta^K = 1$$

with $\theta_L, \theta_R$ elements of $SO(D)$. For a symmetric orbifold $\theta_L = \theta_R$ (we do not include roto-translations here).

The Narain space group $g = (\Theta^k, E \hat{N})$ is then generated by twists $(\Theta, 0)$ and shifts $(1, E_i)$ for $i = 1 \ldots 2D$.

Outer automorphisms map the group to itself but are not elements of the group.
Modular transformations (dualities) exchange windings and momenta and act nontrivially on the moduli of the torus. In $D = 2$ these transformations are connected to the group $SL(2, \mathbb{Z})$ acting on Kähler and complex structure moduli.

The group $SL(2, \mathbb{Z})$ is generated by two elements

$$S, T : \quad \text{with} \quad S^4 = 1 \quad \text{and} \quad S^2 = (ST)^3$$

On a modulus $M$ we have the transformations

$$S : \quad M \rightarrow -\frac{1}{M} \quad \text{and} \quad T : \quad M \rightarrow M + 1$$

Further transformations might include $M \rightarrow -\bar{M}$ and mirror symmetry between Kähler and complex structure moduli.
Candidate symmetries

As outer automorphisms of the Narain space group we can identify

- traditional flavor symmetries which are universal in moduli space
- a subset of the modular transformations that act as symmetries at specific "points" in moduli space
- at these "points" we shall have an enhanced symmetry that combines the traditional flavor symmetry with some of the modular symmetries

The full flavor symmetry is non-universal in moduli space. At generic points in moduli space we have the universal traditional flavor symmetry.
Orbifold $T_2/Z_3$
Example: $T_2/Z_3$ Orbifold

On the orbifold some of the moduli are frozen
- lattice vectors $e_1$ and $e_2$ have the same length
- angle is 120 degrees

Modular transformations form a subgroup of $SL(2, \mathbb{Z})$
- $\Gamma(3)$ as a mod(3) subgroup of $SL(2, \mathbb{Z})$; ($\Gamma(3) = A_4$)
- $\Gamma(3)$ (12 elements) acts on the moduli $M$

Twisted fields transform under a bigger group $T'$,
(similar to enhancement of $SO(3)$ to $SU(2)$ for spinors)

(Lauer, Mas, Nilles, 1989; Lerche, Lüst, Warner, 1989)

Transformation $M \rightarrow -\overline{M}$ completes the picture

Full group is $SG(48,29)$ with 48 elements
Moduli space of $\Gamma(3)$
Generic point in moduli space.

Outer automorphisms of the Narain space group are

- shift $A = (1_4; \frac{1}{3}, \frac{2}{3}, 0, 0)$
- and shift $B = (1_4; 0, 0, \frac{1}{3}, \frac{1}{3})$
- a left-right symmetric rotation $C = (-1_4; 0, 0, 0, 0)$

Multiplicative closure of $A$, $B$ and $C$ leads to $\Delta(54)$.

- the earlier guesswork gave the correct result!
- but the new method produces the result automatically
- can be generalised easily to more complicated situations (like, e.g. six dimensions)
Moduli space of flavour groups

Re M

Im M

D(0)
D(1)
A, B, C
Δ(54)

E
SG(216,87)
SG(324,39)
SG(108,17)

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Fixed lines and points

\[ S : M \rightarrow -\frac{1}{M}, \quad T : M \rightarrow M + 1 \quad \text{and} \quad U : M \rightarrow -M \]
Duality transformations might become symmetries!

The red lines:
These are fixed lines under $T$ and $U$. We have
- again $A$, $B$, $C$ and a left-right symmetric reflection $D$

Multiplicative closure leads to $SG(108, 17)$. This includes the formerly discussed CP-transformation! Unification of flavor and CP (spontaneous breakdown away from the line).

The circles: e.g. fixed lines under $S$ and $U$
- new asymmetric reflection $E$ (instead of $D$)
- again $SG(108, 17)$ but differently aligned
- enhanced with different $Z_2$ from $S_4 = \text{Out}(\Delta(54))$
Moduli space of flavour groups
Flavor Symmetries III

Blue squares: two lines meet
- enhancement to $SG(216, 87)$

The small circles: three lines meet
- maximum enhancement to $SG(324, 39)$

The modular group $T'$ has 24 elements, but not all of them lead to an enhancement of the flavor group $\Delta(54)$.

Only the elements within $S_4$ of the outer automorphisms of $\Delta(54)$ are relevant
- this leads to unification of flavour and CP
- CP exact at those fixed lines and points
Messages

We have designed a generic method to find all flavor symmetries (based on the Narain space group)

- unification of traditional (discrete) flavor, CP and modular symmetries
- traditional flavor symmetry is universal in moduli space
- there are non-universal enhancements (including CP at some places (broken in generic moduli space))
- CP is a consequence of the duality symmetries of string theory
- the potential flavor groups are large and non-universal (in our example already up to $SG(324, 39)$ for two extra dimensions)
Consequences

This opens a new arena for flavor model building

- a new look at CP as discrete gauge symmetry (Nilles, Ratz, Trautner, Vaudrevange, 2018)

- modular symmetries for flavor (Altarelli, Feruglio, 2006; Feruglio, 2017)

- groups are large and allow for flexibility (Hagedorn, König, 2018)

- the concept of local flavor symmetries allows different flavor groups for different sectors of the theory (Baur, Nilles, Trautner, Vaudrevange, 2019)

- non-universal structure from duality symmetries (there is still the traditional universal flavor group)

- different flavor symmetries for quarks and leptons are no surprise