# The $B_{(s)} \to D_{(s)}^{(*)}\ell\nu$ semileptonic decay at non-zero recoil in lattice QCD


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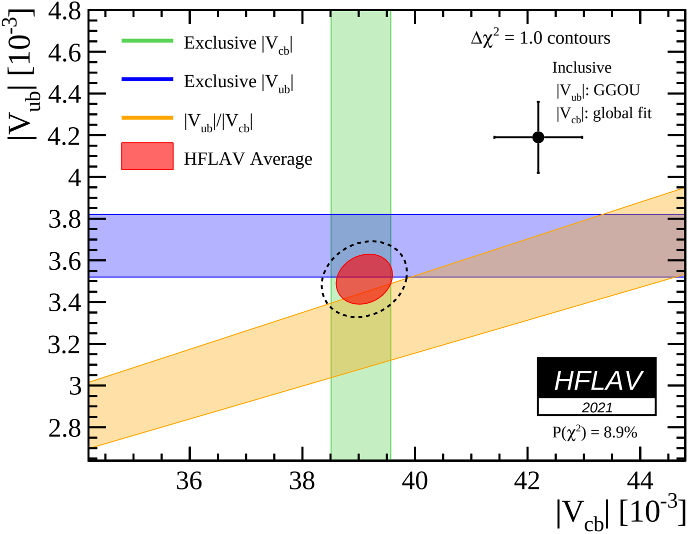
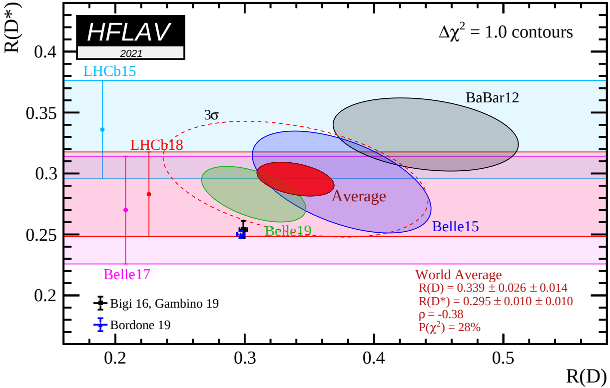
# The $V_{cb}$ matrix element: Tensions




Current status of $|V_{ub}|$ vs $|V_{cb}|$ (HFLAV 2021)

# The $V_{cb}$ matrix element: Tensions in lepton universality

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$




- Current $\approx 3\sigma$ tension with the SM

# The $V_{cb}$ matrix element: Measurement from exclusive processes

$$\underbrace{\frac{d\Gamma}{dw}\left(\bar{B}\to D^*\ell\bar{\nu}_\ell\right)}_{\text{Experiment}} = \left[\underbrace{K_1(w,m_\ell)}_{\text{Known factors}}\underbrace{|\mathcal{F}(w)|^2}_{\text{Theory}} + \underbrace{K_2(w,m_\ell)}_{\text{Known factors}}\underbrace{|\mathcal{F}_2(w)|^2}_{\text{Theory}}\right] \times |V_{cb}|^2$$

- The amplitude $\mathcal{F}$ must be calculated in the theory
  - Can use effective theories (HQET) to say something about $\mathcal{F}(1)$
  - $K_i(w,m_\ell) \propto (w^2-1)^{\frac{1}{2}}$ factor requires extrapolation of experimental data
- $R(D^*)$ requires an extra term that only contributes with the $\tau$

$$R(D^*) = \frac{\int_1^{w_{\mathrm{Max},\tau}} dw \left[K_1(w,m_\tau)\,|\mathcal{F}(w)|^2 + K_2(w,m_\tau)\,|\mathcal{F}_2(w)|^2\right] \times \cancel{|V_{cb}|^2}}{\int_1^{w_{\mathrm{Max}}} dw \left[K_1(w,0)\,|\mathcal{F}(w)|^2\right] \times \cancel{|V_{cb}|^2}}$$

- It is possible to extract $R(D^*)$ without experimental data!

# Calculating $V_{cb}$ on the lattice: Formalism

- Form factors

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{V}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^*}}} = \frac{1}{2}\epsilon^{\nu *}\varepsilon^{\mu\nu}_{\rho\sigma}v_B^\rho v_{D^*}^\sigma \boldsymbol{h_V}(w)$$

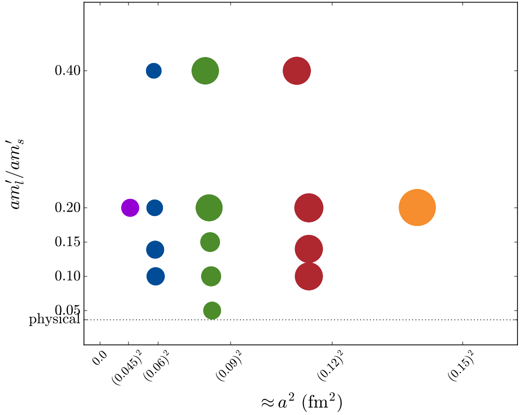
$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{A}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^*}}} =$$

$$\frac{i}{2}\epsilon^{\nu *}\left[g^{\mu\nu}\left(1+w\right)\boldsymbol{h_{A_1}}(w) - v_B^\nu\left(v_B^\mu \boldsymbol{h_{A_2}}(w) + v_{D^*}^\mu \boldsymbol{h_{A_3}}(w)\right)\right]$$

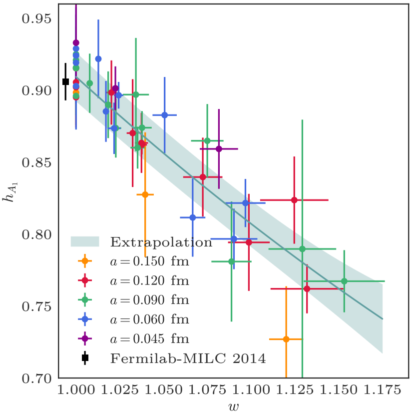
- $\mathcal{V}$ and $\mathcal{A}$ are the vector/axial currents in the continuum
- The $h_X$ enter in the definition of $\mathcal{F}$
- We can calculate $h_{A_{1,2,3},V}$ directly from the lattice

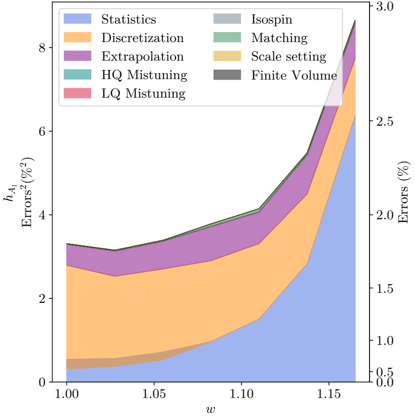
# Available data: ASQTAD analysis

- Using 15 $N_f = 2+1$ MILC ensembles of asqtad sea quarks
- The heavy quarks are treated using the Fermilab action



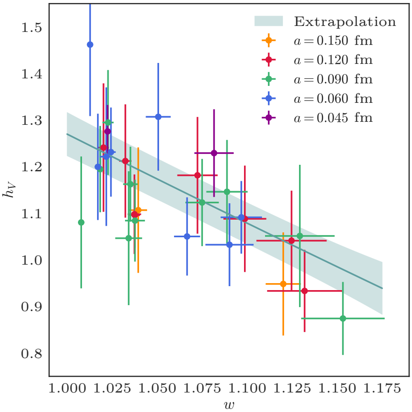
# Analysis: Chiral-continuum fits

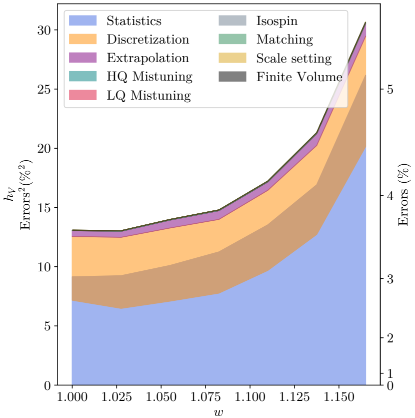



- Combined fit $p - \text{value} = 0.75$
- $h_{A_1}(1) = 0.909(17)$

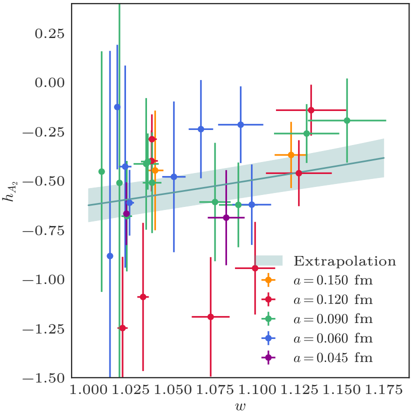
# Analysis: Chiral-continuum fits

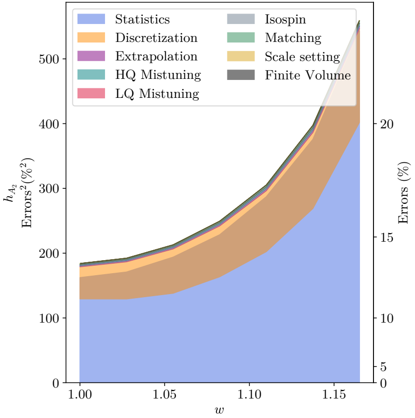



- Combined fit $p -$ value $= 0.75$
- $h_V(1) = 1.270(46)$

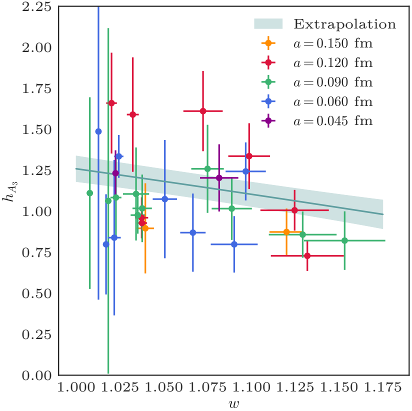
# Analysis: Chiral-continuum fits

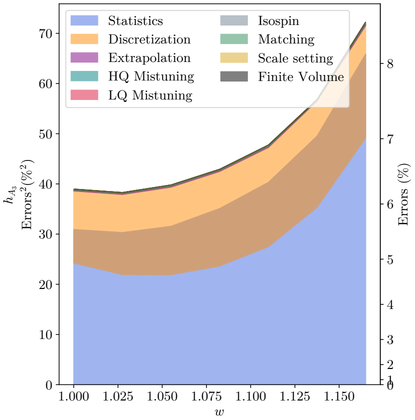



- Combined fit $p - \text{value} = 0.75$
- $h_{A_2}(1) = -0.624(85)$

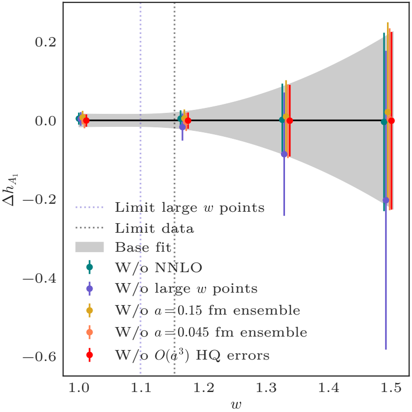
# Analysis: Chiral-continuum fits

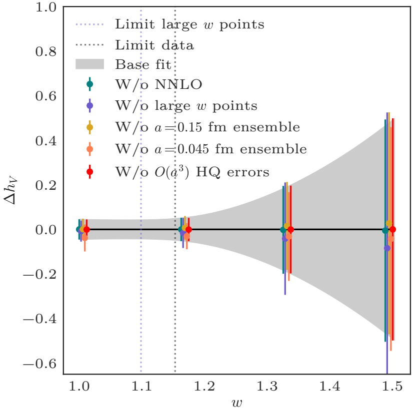



- Combined fit $p - \text{value} = 0.75$
- $h_{A_3}(1) = 1.259(79)$

# Results: Stability of chiral-continuum fits




| | **Base** | W/o NNLO | W/o large $w$ | W/o $a = 0.15$ fm |
|---|---|---|---|---|
| $\chi^2$/dof | **85.5/95** | 86.0/107 | 71.1/75 | 79.4/86 |
| | | W/o $a = 0.045$ fm | W/o HQ $O(a^3)$ | |
| $\chi^2$/dof | | 81.6/86 | 85.3/99 | |

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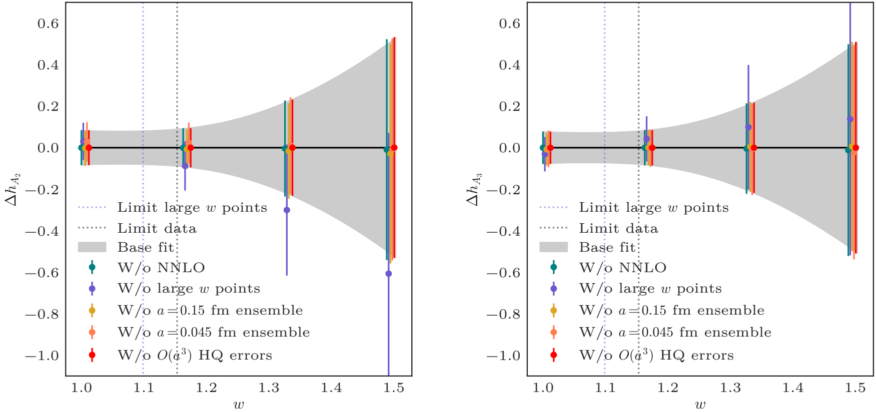


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# Analysis: z-Expansion

- The BGL expansion is performed on different (more convenient) form factors

Phys.Lett. **B**769, 441 (2017), Phys.Lett. **B**771, 359 (2017)

$$g = \frac{h_V(w)}{\sqrt{m_B\, m_{D^*}}} = \frac{1}{\phi_g(z) B_g(z)} \sum_j a_j z^j$$

$$f = \sqrt{m_B\, m_{D^*}}(1+w) h_{A_1}(w) = \frac{1}{\phi_f(z) B_f(z)} \sum_j b_j z^j$$

$$\mathcal{F}_1 = \sqrt{q^2} H_0 = \frac{1}{\phi_{\mathcal{F}_1}(z) B_{\mathcal{F}_1}(z)} \sum_j c_j z^j$$

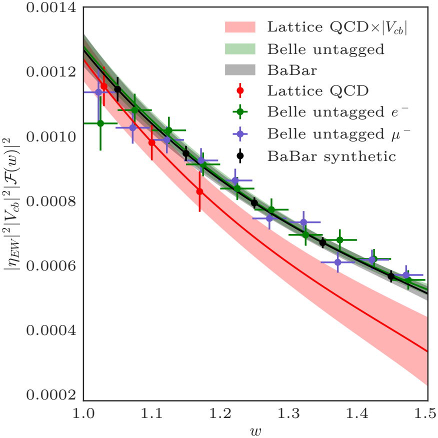
$$\mathcal{F}_2 = \frac{\sqrt{q^2}}{m_{D^*}\sqrt{w^2-1}} H_S = \frac{1}{\phi_{\mathcal{F}_2}(z) B_{\mathcal{F}_2}(z)} \sum_j d_j z^j$$

- Constraint $\mathcal{F}_1(z=0) = (m_B - m_{D^*}) f(z=0)$
- Constraint $(1+w) m_B^2 (1-r) \mathcal{F}_1(z = z_{\mathrm{Max}}) = (1+r) \mathcal{F}_2(z = z_{\mathrm{Max}})$
- BGL (weak) unitarity constraints

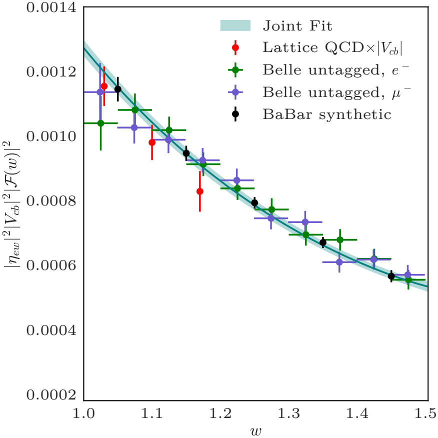
$$\sum_j a_j^2 \leq 1, \qquad \sum_j b_j^2 + c_j^2 \leq 1, \qquad \sum_j d_j^2 \leq 1$$

# Results: Separate fits and joint fit

## Separate fits

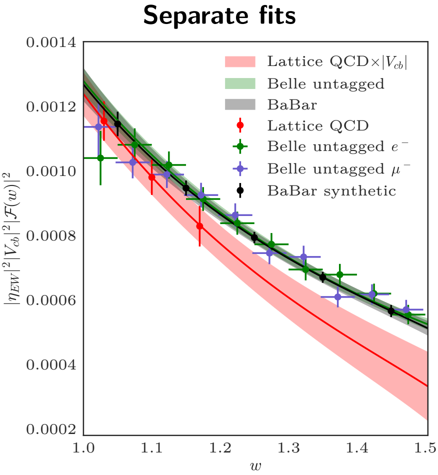


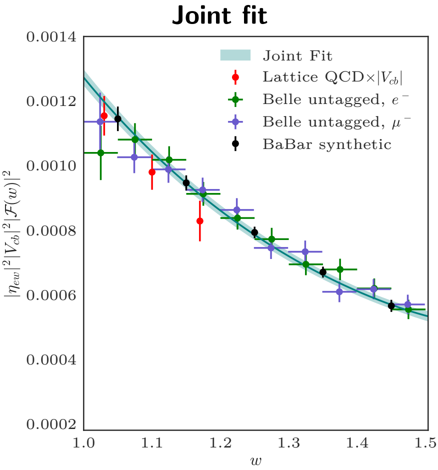

## Joint fit




| Fit | Lattice | Exp | Lat + Belle | Lat + BaBar | Lat + Exp |
|---|---|---|---|---|---|
| $p$-Value | 0.43 | 0.015 | 0.010 | 0.075 | 0.002 |

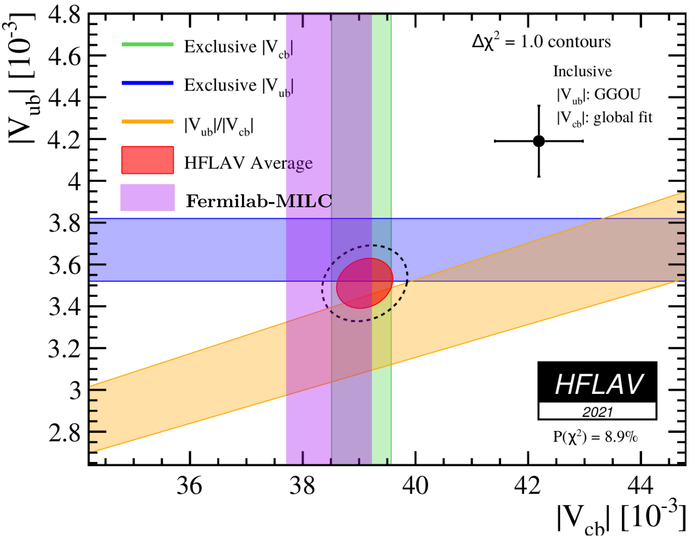
# Results: Separate fits and joint fit




**Unblinded, final result** $|V_{cb}| = 38.47(75) \times 10^{-3}$

# Results: Update of $|V_{ub}|$ vs $|V_{cb}|$

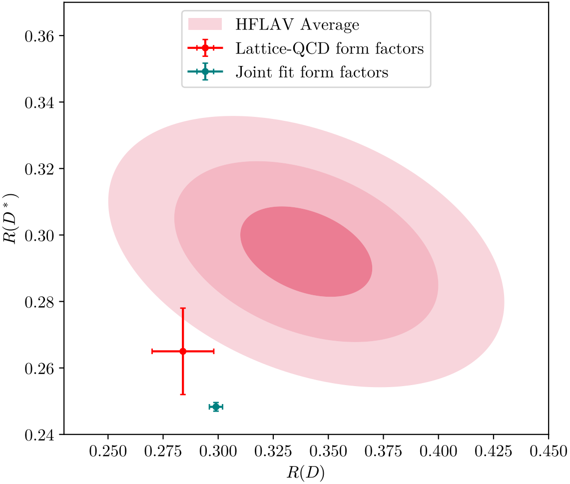



**The $|V_{cb}|$ puzzle remains**

# Results: $R(D^*)$ in context

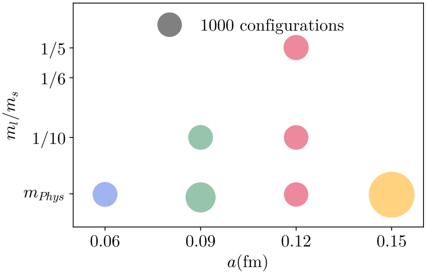
$$R(D^*)_{\mathbf{Lat}} = 0.265(13) \quad R(D^*)_{\mathbf{Lat+Exp}} = 0.2485(13)$$

Phys.Rev.D**92** (2015), 034506; Phys.Rev.D**100** (2019), 052007; Phys.Rev.D**103** (2021), 079901; Phys.Rev.Lett. **123** (2019), 091801



# New analysis: HISQ + Fermilab heavy quarks

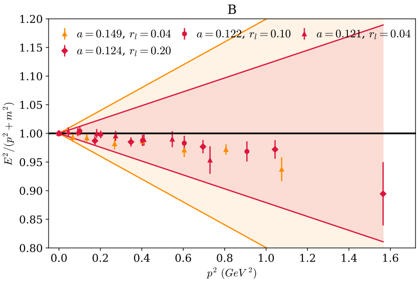
- 7 $N_f = 2 + 1 + 1$ MILC ensembles of HISQ sea quarks + Fermilab heavy quarks
- Same or better statistics than in the asqtad analysis
- Correlated $H \to H$ and $H \to l$ analysis

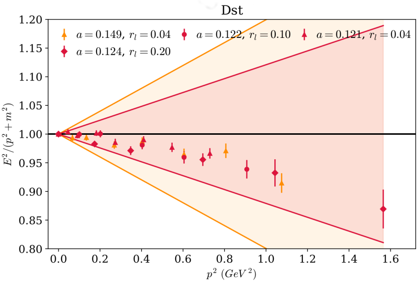



- More channels $B_{(s)} \to D_{(s)}^{(*)} \ell \nu$
- HISQ fermions behave better than asqtad $\to$ smaller light discretization errors
- Lower pion masses (4 ensembles with physical pion masses vs 0)
- Similar heavy-quark and renormalization errors

# New analysis: a glimpse of discretization errors

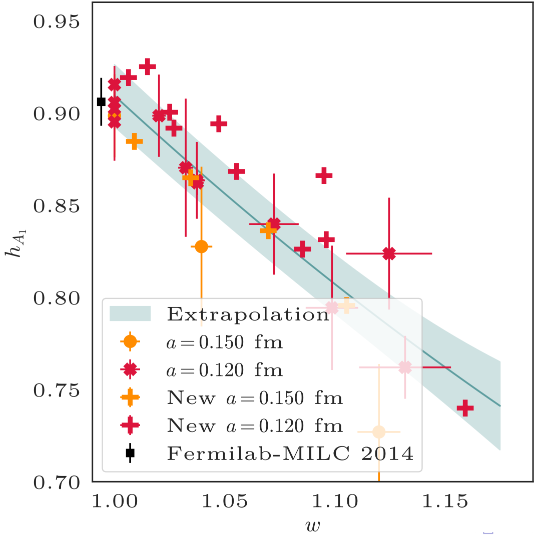
- **Very preliminary**
- The discretization errors seem to be under control



# New analysis: form factors

- **Very preliminary**
- First peek at three-point correlators



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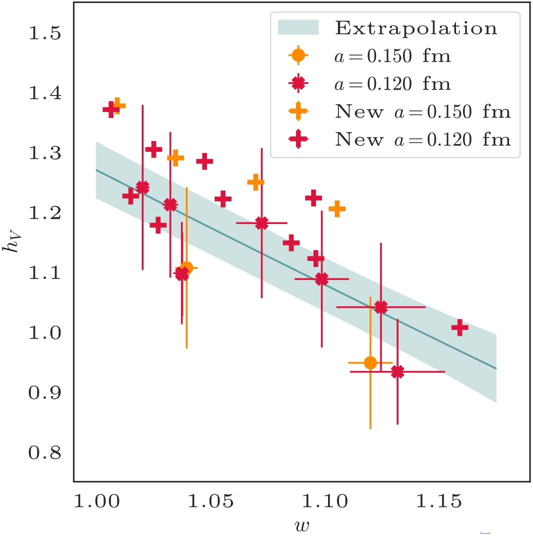

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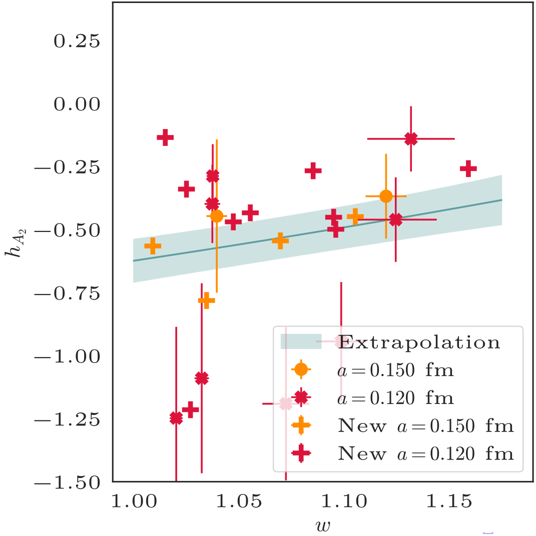

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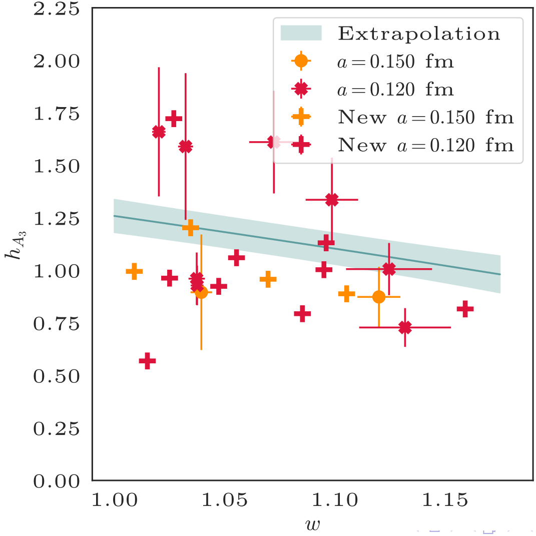

# Conclusions

- Complete lattice QCD calculation of the $B \to D^* \ell \nu$ form factors at non-zero recoil
- Please note related work by other groups
  - HPQCD is exploring different channels (see J. Harrison talk)
  - JLQCD working on a $B \to D^{(*)} \ell \nu$ at non-zero recoil with very different systematics
- New analysis will include several improvements
  - Fewer ensembles, but better statistics (especially the finest ones)
  - Light quarks at physical masses
  - Improved light quark discretization
- HISQ analysis includes several channels $B_{(s)} \to D_{(s)}^{(*)} \ell \nu$
  - Correlated plot $R(D)$ vs $R(D^*)|$
  - Cross-symmetry tests
  - Strange channels very interesting
- HISQ analysis coordinated with $B \to \pi, K$ and $B_s \to K$
  - Correlated plot $|V_{ub}|$ vs $|V_{cb}|$

# THANK YOU