Lattice calculations of exclusive $b \to u\ell\nu$

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Challenges in Semileptonic B Decays
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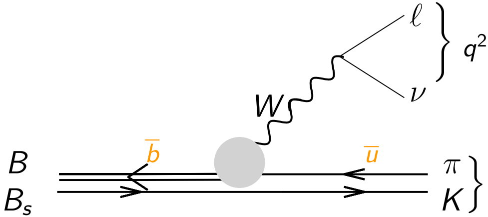
Exclusive semileptonic $B_{(s)}$ decays



$$q^2 = M_{B_{(s)}}^2 + M_P^2 - 2M_{B_{(s)}} E_P$$

$\pi, K = P$

pseudoscalar initial state — charged vector current — pseudoscalar final state

$B \to \pi\ell\nu$

$B_s \to K\ell\nu$

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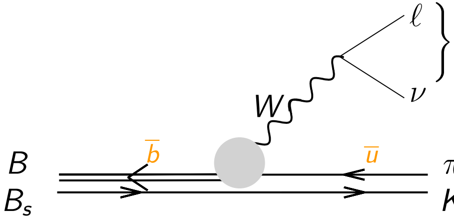
baryonic decays (work by Meinel et al.)

vector final state

$B_s \to K^*\ell\nu$ (ongoing challenge)

$B \to \rho\ell\nu$ (future challenge)

Exclusive semileptonic $B_{(s)}$ decays



$$q^2 = M^2_{B_{(s)}} + M^2_P - 2M_{B_{(s)}}E_P$$

$$\pi, K = P$$

- Conventionally parametrized ($B_{(s)}$ meson at rest)

$$\frac{d\Gamma(B_{(s)} \to P\ell\nu)}{dq^2} = \frac{\eta_{EW} G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_P^2 - M_P^2}}{q^4 M^2_{B_{(s)}}}$$

experiment CKM known

$$\times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) M^2_{B_{(s)}}(E_P^2 - M_P^2)|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(M^2_{B_{(s)}} - M_P^2)^2 |f_0(q^2)|^2 \right]$$

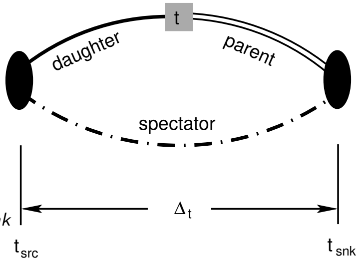
nonperturbative input

Lattice determination of form factors

- Parametrize hadronic matrix element for flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle P|V^\mu|B_{(s)}\rangle = f_+(q^2)\left(p^\mu_{B_{(s)}} + p^\mu_P - \frac{M^2_{B_{(s)}} - M^2_P}{q^2}q^\mu\right) + f_0(q^2)\frac{M^2_{B_{(s)}} - M^2_P}{q^2}q^\mu$$

- Calculate the corresponding 3-point function
  - → Inserting source for spectator quark at t_{src}
  - → Allow it to propagate to t_{sink}
  - → Turn it into a sequential source for b quark
  - → Propagate light daughter quark from t_{src}
  - → Contract with b quark at t with $t_{src} \le t \le t_{sink}$



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- Prefer to compute

$$f_\parallel(E_P) = \langle P|V^0|B_{(s)}\rangle/\sqrt{2M_{B_{(s)}}} \quad \text{and} \quad f_\perp(E_P)p^i_P = \langle P|V^i|B_{(s)}\rangle/\sqrt{2M_{B_{(s)}}}$$

which are directly proportional to 3-point functions

- Both are related by

$$f_0(q^2) = \frac{\sqrt{2M_{B_{(s)}}}}{M^2_{B_{(s)}} - M^2_P}\left[(M_{B_{(s)}} - E_P)f_\parallel(E_P) + (E^2_P - M^2_P)f_\perp(E_P)\right]$$

$$f_+(q^2) = \frac{1}{\sqrt{2M_{B_{(s)}}}}\left[f_\parallel(E_P) + (M_{B_{(s)}} - E_P)f_\perp(E_P)\right]$$

Lattice determination of form factors

- Parametrize hadronic matrix element for flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle P|V^\mu|B_{(s)}\rangle = f_+(q^2)\left(p^\mu_{B_{(s)}} + p^\mu_P - \frac{M^2_{B_{(s)}} - M^2_P}{q^2}q^\mu\right) + f_0(q^2)\frac{M^2_{B_{(s)}} - M^2_P}{q^2}q^\mu$$

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$$f_\parallel(E_P) = \langle P|V^0|B_{(s)}\rangle/\sqrt{2M_{B_{(s)}}} \quad \text{and} \quad f_\perp(E_P)p^i_P = \langle P|V^i|B_{(s)}\rangle/\sqrt{2M_{B_{(s)}}}$$

which are directly proportional to 3-point functions

- Alternatively use f_1 and f_2 with $v^\mu = p^\mu_B/M_B$ motivated by HQET

$$f_1(v\cdot p_\pi) + f_2(v\cdot p_\pi) = f_\parallel(E_\pi)/\sqrt{2}$$
$$f_2(v\cdot p_\pi) = f_\perp(E_\pi)\cdot\left(v\cdot p_\pi/\sqrt{2}\right)$$

Challenges for $B_s \to K\ell\nu$ and $B \to \pi\ell\nu$ on the lattice

- Kinematics (range in q^2 to cover)

  $B_s \to K\ell\nu$: $q^2 \approx [m_\ell^2, 24]$ GeV^2 $B \to \pi\ell\nu$: $q^2 \approx [m_\ell^2, 27]$ GeV^2

  → Kinematical z-expansion

  → Dispersive bound method

- Ratio of quark masses

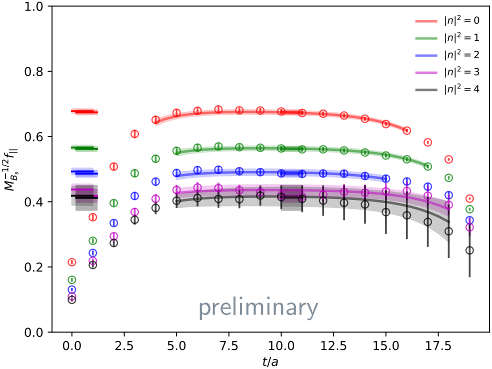
  → Pions need a large box to not feel finite volume effects

  ⇝ Simulate heavier than physical pions

  → b quarks need fine lattice spacings to control discretization effects

  ⇝ Use effective b-quark action or perform additional extrapolation

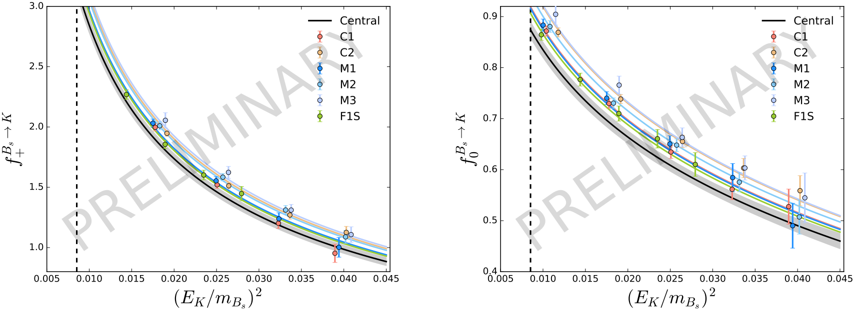
$B_s \to K\ell\nu$: form factor



- Example: form factor $f_{||}$ on coarse ensemble C1 (RBC-UKQCD)

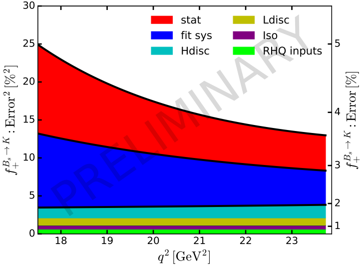
- Individually fit ratios of 3-point over 2-point functions
  - → Each fit has a χ^2/dof or p-value
  - → Smaller correlation matrix
- Combined correlated fit to all states using several 3- and 2-point functions
  - → One "global" χ^2/dof or p-value
  - → Much larger correlation matrix
    - ⇝ Use SVD cut or alike
- Vary range of fitted time slices, source-sink separation

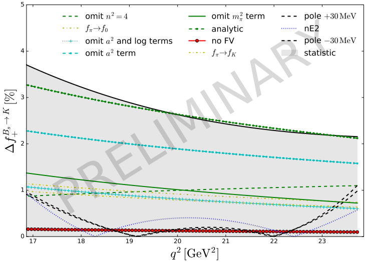
$B_s \to K\ell\nu$: chiral-continuum extrapolation (RBC-UKQCD)

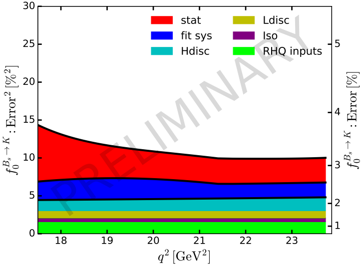


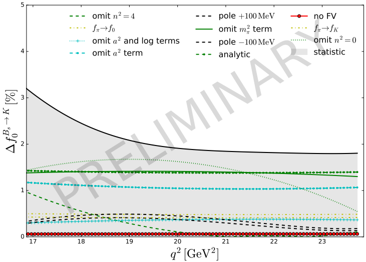
- Chiral-continuum extrapolation using SU(2) hard-kaon χPT
  - $\rightarrow$ $f_{pole}(M_K, E_K, a^2) = \frac{c_0\Lambda}{E_K+\Delta} \times \left[1 + \frac{\delta f}{(4\pi f)^2} + c_1\frac{M_\pi^2}{\Lambda^2} + c_2\frac{E_K}{\Lambda} + c_3\frac{E_K^2}{\Lambda^2} + c_4(a\Lambda)^2\right]$
  - $\rightarrow$ δf non-analytic logs of the kaon mass and hard-kaon limit is taken by M_K/E_K

$B_s \to K\ell\nu$: error budget (RBC-UKQCD)









Kinematical extrapolation (z-expansion)

[Boyd, Grinstein, Lebed, PRL 74 (1995) 4603] [Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]

- Map complex q^2 plane with cut $q^2 > t_*$ onto the unit disk in z

$$z(q^2, t_*, t_0) = \frac{\sqrt{t_* - q^2} - \sqrt{t_* - t_0}}{\sqrt{t_* - q^2} + \sqrt{t_* - t_0}}$$

with

$$t_* = (M_B + M_\pi)^2 \qquad \text{(two-particle production threshold)}$$
$$t_\pm = (M_{B_s} \pm M_K)^2 \qquad (\text{with } t_- = q^2_{max})$$
$$t_0 \equiv t_{\text{opt}} = t_* - \sqrt{t_*(t_* - t_-)} \qquad (\text{symmetrize range of } z)$$

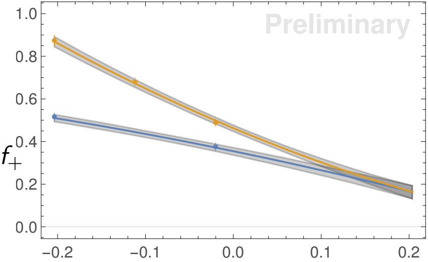
- BCL express form factor f_+ for $B \to \pi\ell\nu$

$$f_+(q^2) = \frac{1}{1 - q^2/M^2_{pole}} \sum_{k=0}^{K-1} b_k^+(t_0) z^k$$

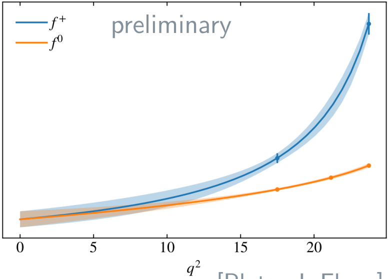
- For other decays use product of factors for subthreshold poles for both f_+ and f_0 paralleling the Blaschke factors for a BGL fit to the same decay

Kinematical extrapolation (RBC-UKQCD)

- BCL z-expansion
  - → Perform fit in z-space with K parameters
  - → Then convert back to physical q^2
  - → BCL with pole $M_+ = B^* = 5.33$ GeV for f_+
  - → Exploit kinematic constraint $f_+ = f_0\big|_{q^2=0}$
  - → Include HQ power counting to constrain size of f_+ coefficients (work in progress)



- Alternative: Dispersive Bound Method
  [Martinelli et al. PRD 104(2021)094512]
  [PRD 105(2022)034503][arXiv:2202.10285]
  - ⇝ See talk by Silvano



- Compare to other results/combine with LHCb

[Plots: J. Flynn]

Ratios testing lepton flavor universality

Traditional R-ratio

$$R_P = \frac{\int_{m_\tau^2}^{q^2_{max}} dq^2 \, \frac{d\Gamma(B_{(s)} \to P\tau\bar{\nu}_\tau)}{dq^2}}{\int_{m_\ell^2}^{q^2_{max}} dq^2 \, \frac{d\Gamma(B_{(s)} \to P\ell\bar{\nu}_\ell)}{dq^2}}$$

Alternative R-ratio

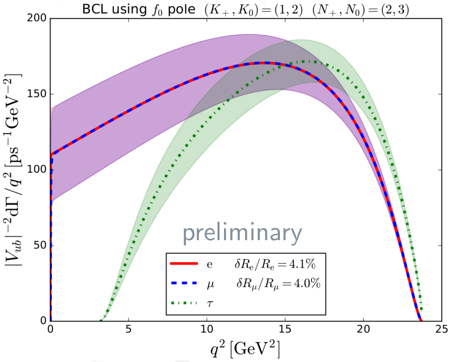
$$R_P^{\text{imp}} = \frac{\int_{q^2_{min}}^{q^2_{max}} dq^2 \, \frac{d\Gamma(B_{(s)} \to P\tau\bar{\nu}_\tau)}{dq^2}}{\int_{q^2_{min}}^{q^2_{max}} dq^2 \, \frac{\omega_\tau(q^2)}{\omega_\ell(q^2)} \frac{d\Gamma(B_{(s)} \to P\ell\bar{\nu}_\ell)}{dq^2}}$$

- Follow idea proposed for $B \to V$ [Isidori and Sumensari EPJC80 (2020)1078]
  - → Common integration range: $q^2_{min} \geq m_\tau^2$ [Freytsis et al. PRD92(2015)054018] [Bernlochner and Ligeti PRD95(2017)014022] [Flynn et al. PoS ICHEP2020 436]
  - → Same weights in integrands for τ and ℓ

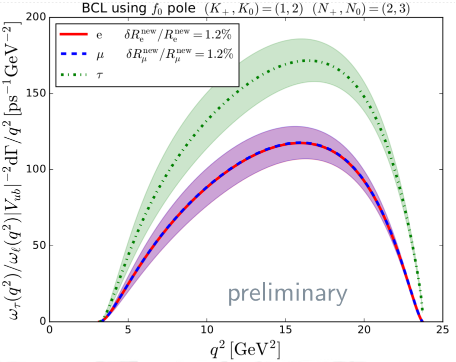
$$\omega_l(q^2) = \left(1 - \frac{m_l^2}{q^2}\right)^2 \left(1 + \frac{m_\ell^2}{2q^2}\right) \quad \text{for } l = e, \mu, \tau$$

Ratios testing lepton flavor universality

Traditional *R*-ratio

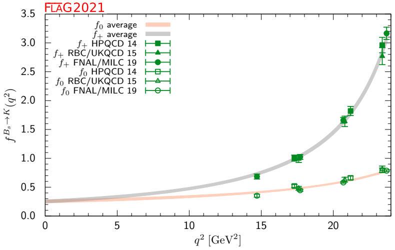


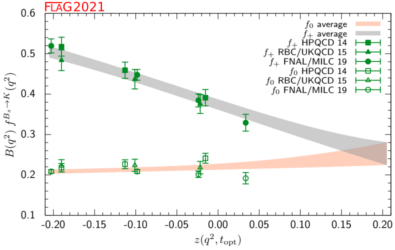
Alternative *R*-ratio



- Preliminary RBC-UKQCD data for $B_s \to K\ell\nu$ [Plots: J. Flynn]

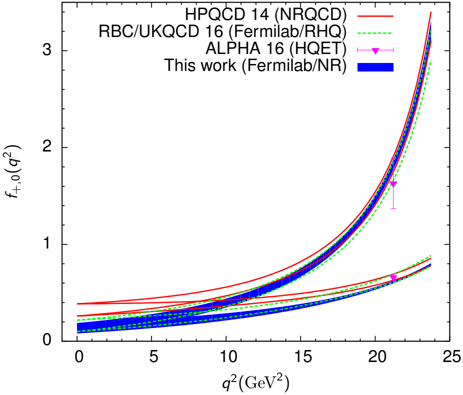
Overview lattice results $B_s \to K\ell\nu$ [FLAG 2021]

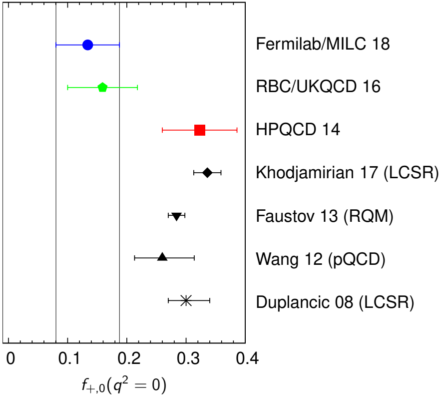




- HPQCD 14 [Bouchard et al. PRD90(2014)054506]
  - → HISQ light/strange, NRQCD b quarks
  - → $a \approx 0.12$ fm, 0.09 fm (2+1)
  - → $M_\pi \gtrsim 260$ MeV
- RBC/UKQCD 15 [Flynn et al. PRD91(2015)074510]
  - → Domain wall light/strange, RHQ b quarks
  - → $a = 0.11$ GeV, 0.08 GeV (2+1)
  - → $M_\pi \gtrsim 290$ MeV
- FNAL/MILC 19 [Bazavov et al. PRD100(2019)034501]
  - → Asqtad light/strange, Fermilab b quarks
  - → $a \approx 0.12$ fm, 0.09 fm, 0.06 fm (2+1)
  - → $M_\pi \gtrsim 177$ MeV

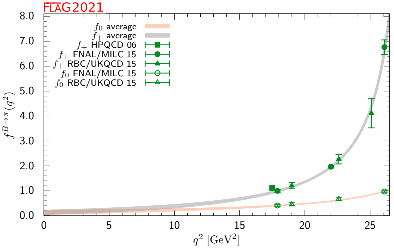
Overview lattice results $B_s \to K\ell\nu$ [Plots: Bazavov et al. PRD100(2019)034501]

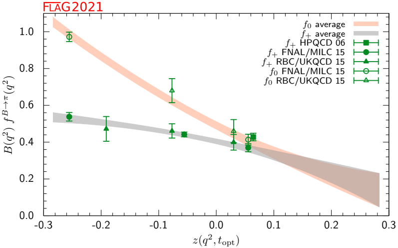




- ALPHA [Bahr et al. PLB757(2016)473]
  - → Wilson light/strange, HQET b quarks
  - → $a \approx 0.048$ fm, 0.065 fm, 0.075 fm (2+0)
  - → $M_\pi \gtrsim 310$ MeV

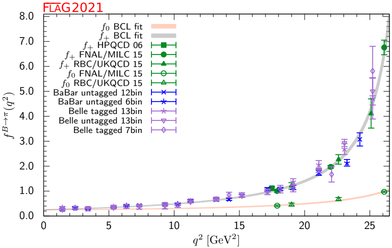
Overview lattice results $B \to \pi\ell\nu$ [FLAG 2021]

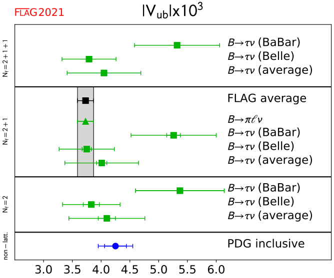


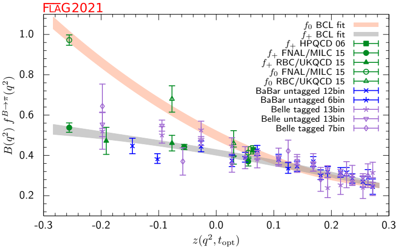


- HPQCD 06 [Dalgic et al. PRD73(2006)074502][PRD75(20
  - → HISQ light/strange, NRQCD b quark
  - → $a \approx 0.12$ fm, 0.09 fm
  - → $M_\pi \gtrsim 400$ MeV
- RBC/UKQCD 15 [Flynn et al. PRD91(2015)074510]
  - → SDWF light/strange, RHQ b quark
  - → $a \approx 0.11$ fm, 0.08 fm
  - → $M_\pi \gtrsim 290$ MeV
- FNAL/MILC 15 [Bailey et al. PRD92(2015)014024]
  - → Asqtad light/strange, Fermilab b quark
  - → $a \approx 0.12$ fm, 0.09 fm, 0.06 fm, 0.045 fm
  - → $M_\pi \gtrsim 177$ MeV

Extracting $|V_{ub}|$ [FLAG 2021]





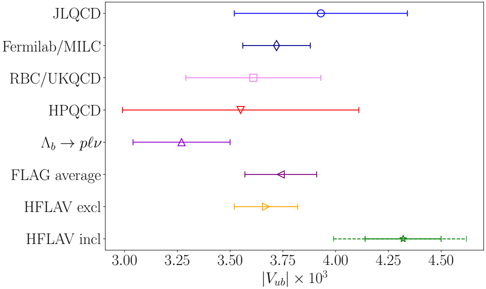


- $|V_{ub}| = 3.74(17) \cdot 10^{-3}$

New: JLQCD [Colquhoun et al. arXiv:2203.04938]

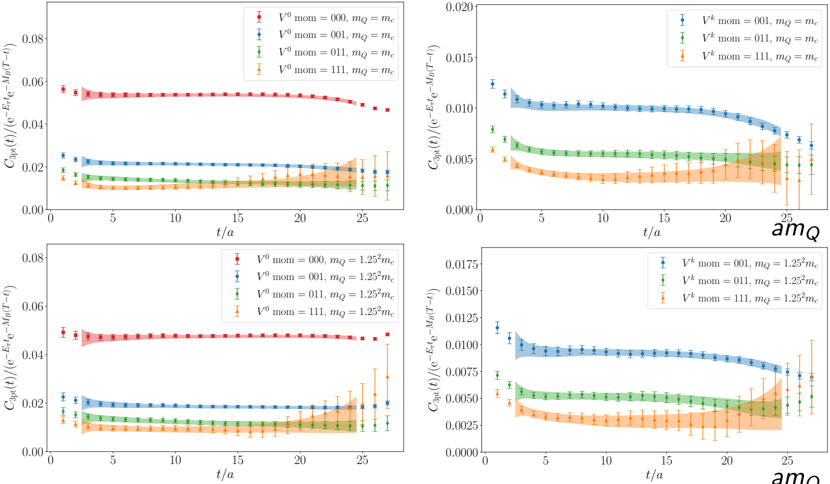
- Unitary setup: MDWF light/strange and heavy quarks with $am_c \leq am_Q \leq 2.44 \cdot am_c$
  → Additional extrapolation in the heavy quark mass to reach m_b
- $a \approx 0.044$ fm, 0.055 fm, 0.080 fm
- $M_\pi \gtrsim 230$ MeV

$\Rightarrow |V_{ub}| = (3.93 \pm 0.41) \cdot 10^{-3}$



New: JLQCD [Colquhoun et al. arXiv:2203.04938]

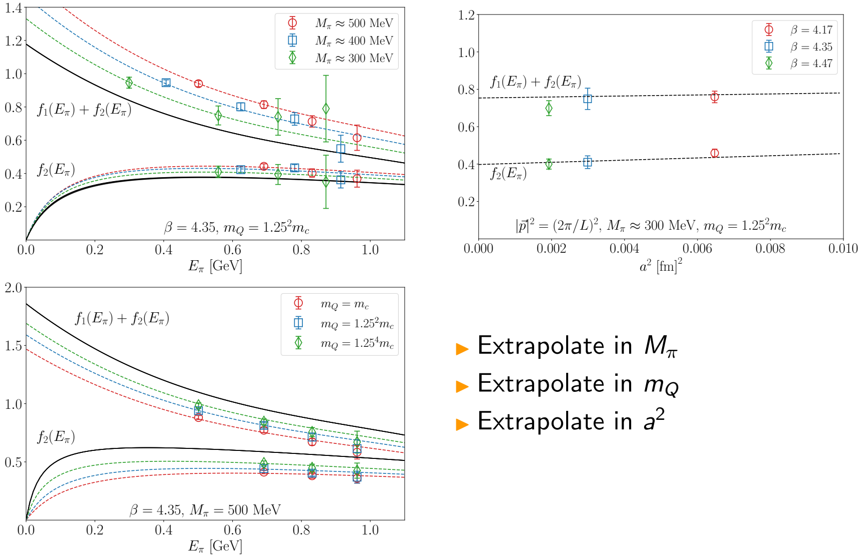
- $a \approx 0.080$ fm, $am_{u/d} = 0.007$



$am_Q = 0.44037$

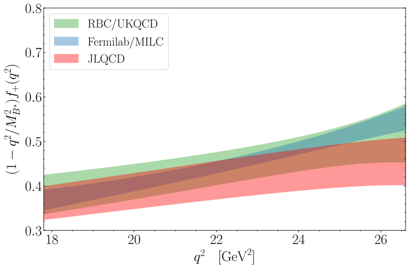
$am_Q = 0.68808$

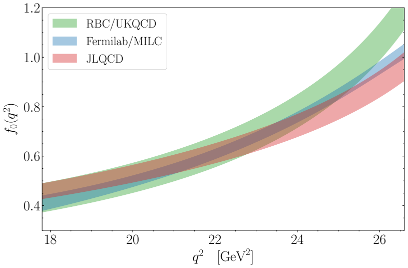
New: JLQCD [Colquhoun et al. arXiv:2203.04938]



- Extrapolate in M_π
- Extrapolate in m_Q
- Extrapolate in a^2

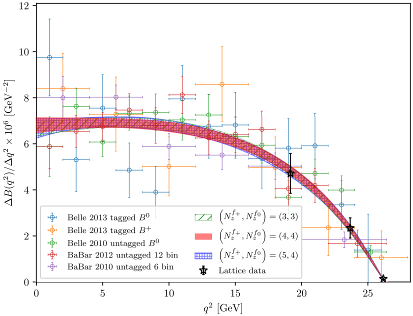
New: JLQCD [Colquhoun et al. arXiv:2203.04938]

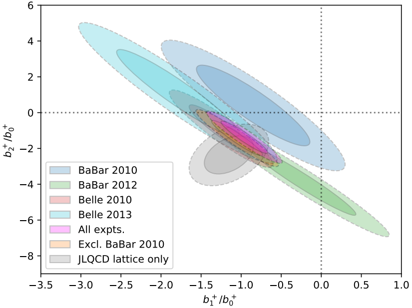




- Comparison to results by Fermilab/MILC and RBC/UKQCD (simulated q^2 range only)

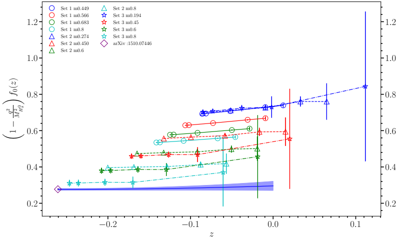
New: JLQCD [Colquhoun et al. arXiv:2203.04938]

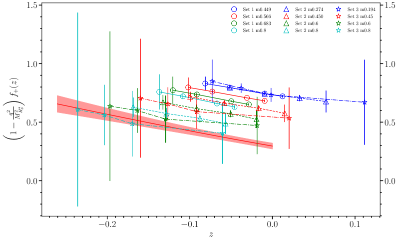




- Determination of $|V_{ub}|$
  → Joint fit of experimental and lattice data
- Shape parameters of BCL z-fit
  → Tension with BaBar 2010 data set

HPQCD: toward $B \to$ light [Parrott et al. PRD 103(2021)094506]

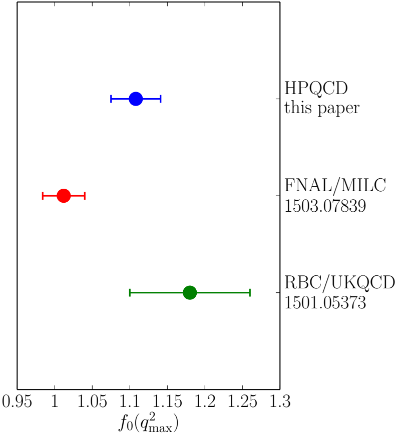




- "$B_s \to \eta_s$"
  - $\to$ HISQ light/strange/charm quarks, HISQ b quarks with $0.2 \lesssim am_H \lesssim 0.8$
  - $\to$ $a \approx 0.044$ fm, 0.060 fm, 0.090 fm (2+1+1)

HPQCD: $B \to \pi\ell\nu$ [Colquhoun et al. PRD93(2016)034502]

- Calculation at zero recoil only
  - → HISQ light/strange/charm quark, improved NRQCD b quarks
  - → $a \approx 0.09$ fm, 0.12 fm, 0.15 fm (2+1+1)
  - → $M_\pi = M_\pi^{\text{phys}}$



Challenges (easy and not so easy)

- Include physical light quarks in the calculations (improve chiral limit)
- Include finer lattices for improved control on discretization effects and better continuum limit
- Increase range in q^2 directly covered in lattice simulations
- More groups studying baryonic decays
- Semileptonic $b \to u\ell\nu$ decays with vector final states