Update on Heavy-to-Light ($B \to pi$) form factors from the Fermilab Lattice and MILC collaborations

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Challenges in Semileptonic B meson decays
Barolo, 19-23 April 2022

Fermilab Lattice and MILC collaborations

- Fermilab Lattice and MILC Collaborations (FNAL/MILC):
  Bazavov, Bernard, DeTar, AXK, Gámiz, Gottlieb, Heller, Jay, Jeong, Kronfeld, Lahert, Laiho, Lin, Lynch, Lytle, Mackenzie, Petersson, Neil, Simone, Sugar, Toussaint, Van de Water, Vaquero

- all HISQ SL B, D decay analyses leads:

Will Jay (MIT)

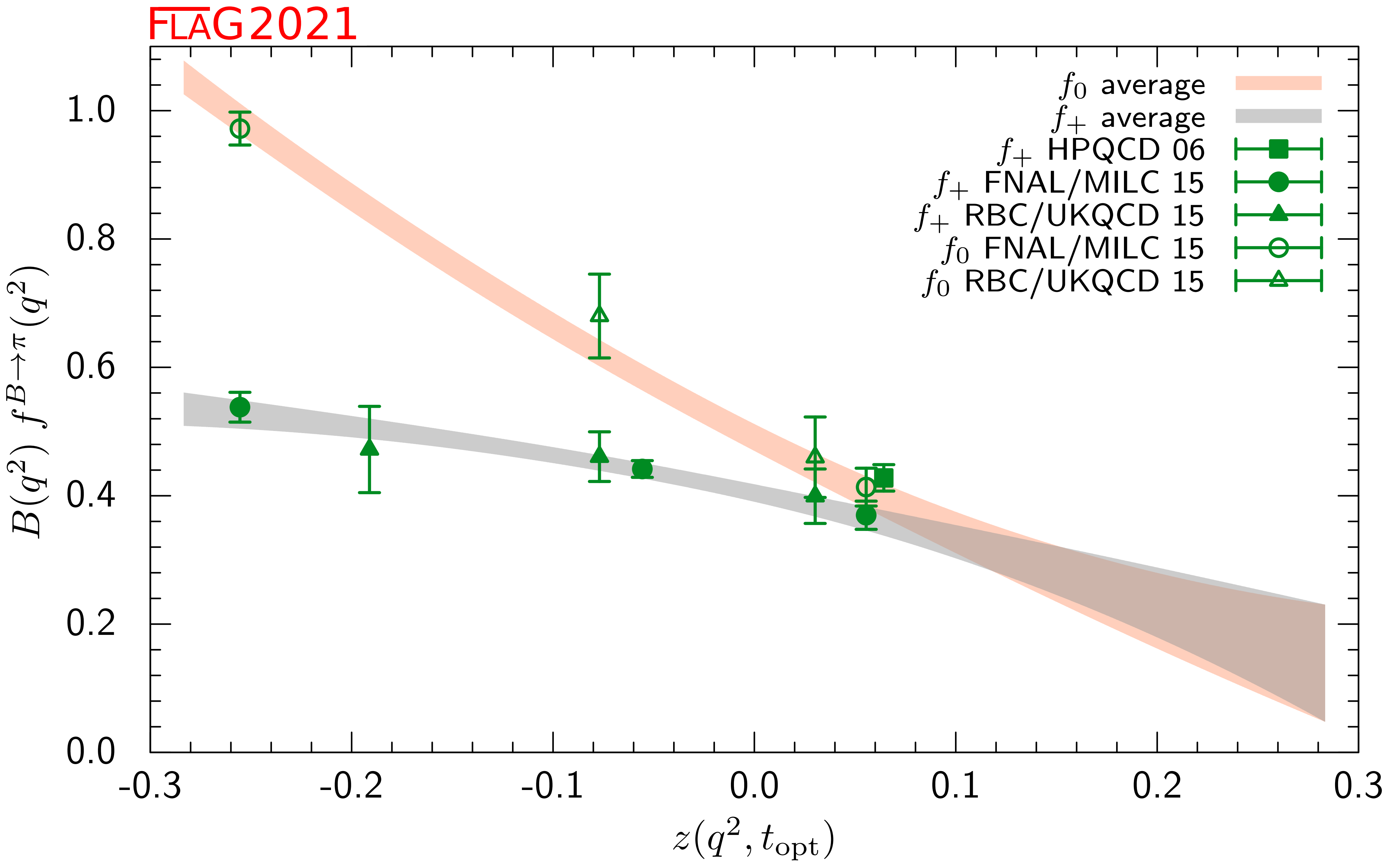
Andrew Lytle (UIUC)

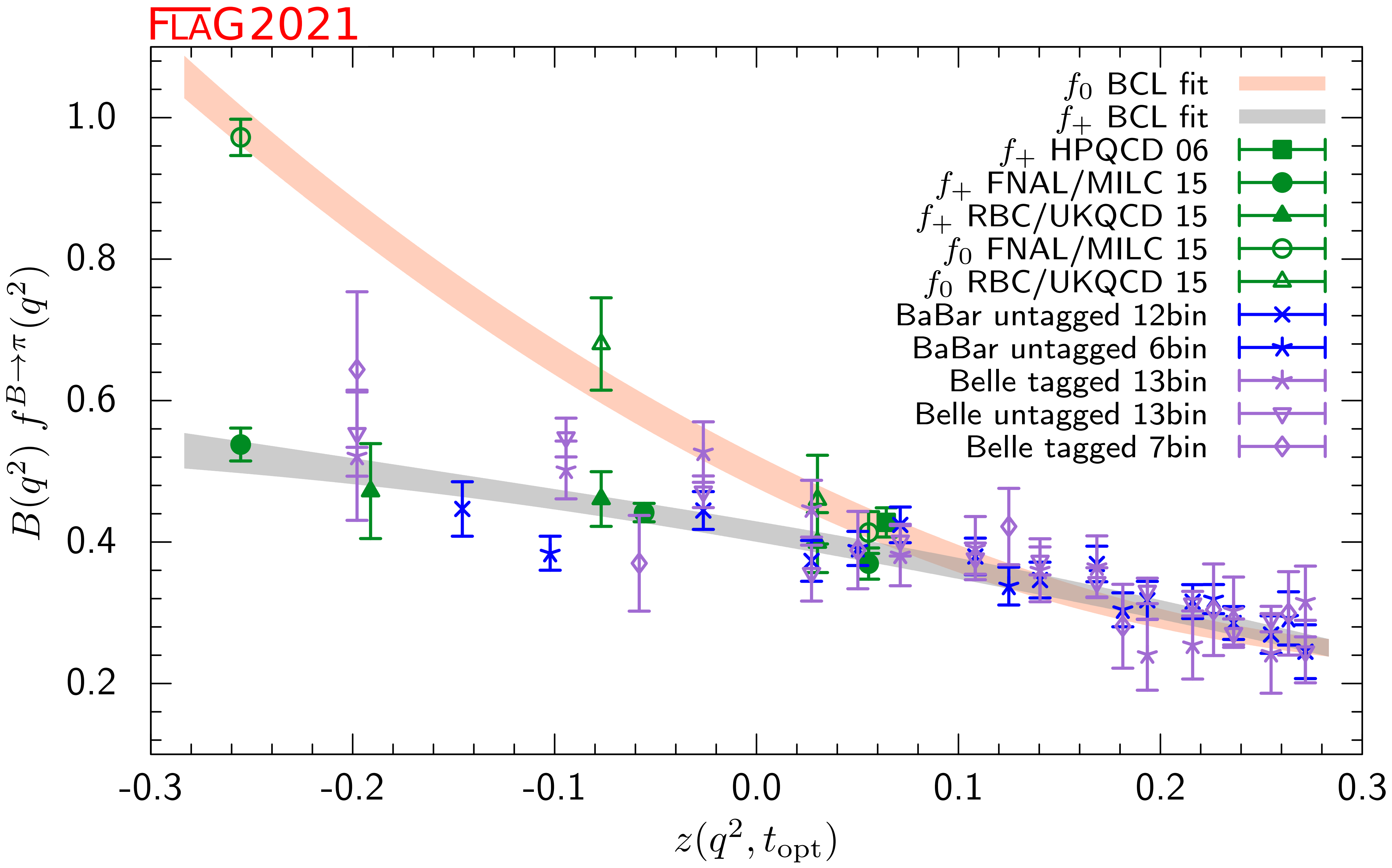
- Fermilab-HISQ SL B decay analyses leads:

Alejandro Vaquero (Utah)

Hwancheol Jeong (IU)

Introduction: $B \to \pi$ form factors



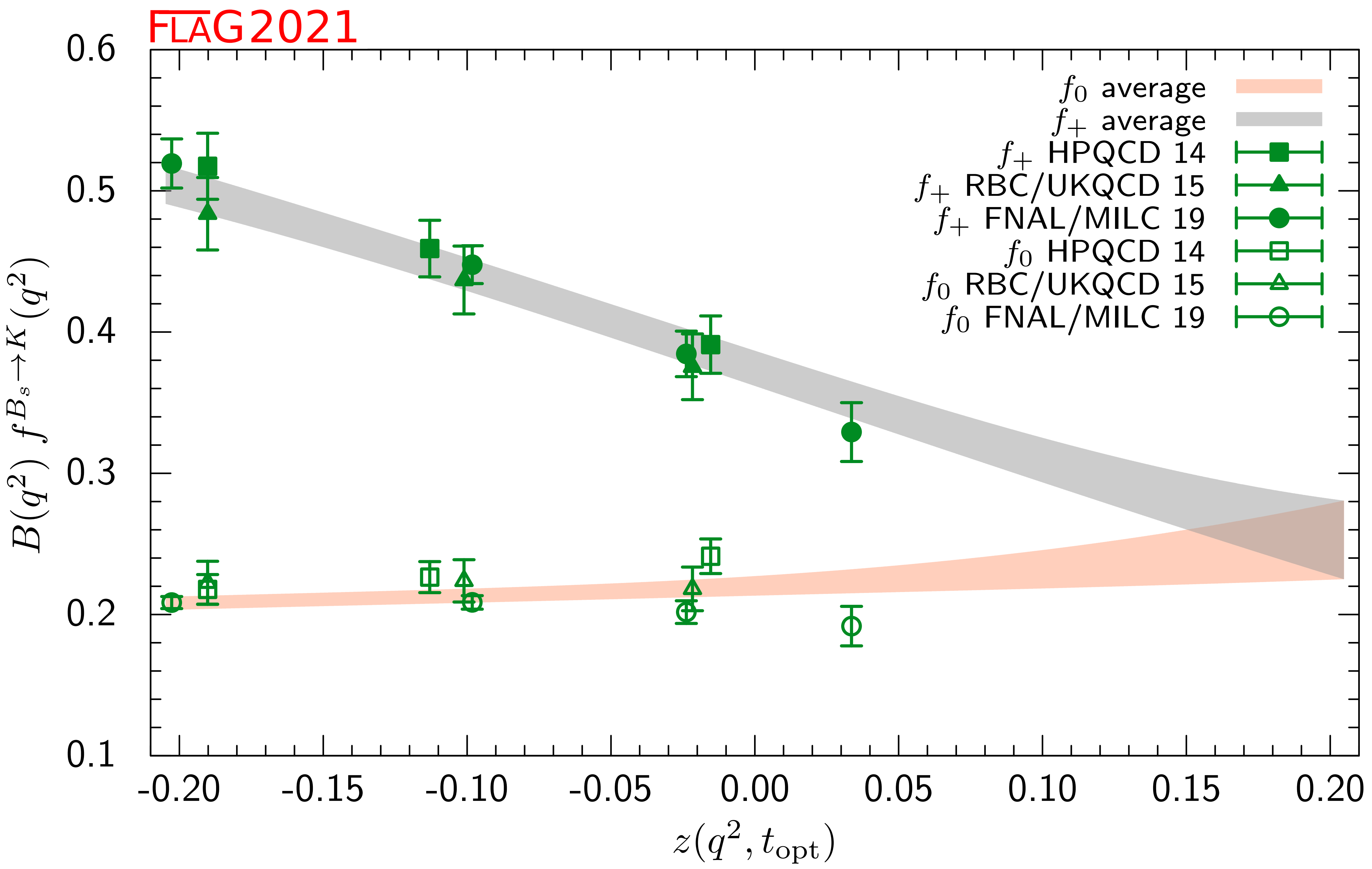


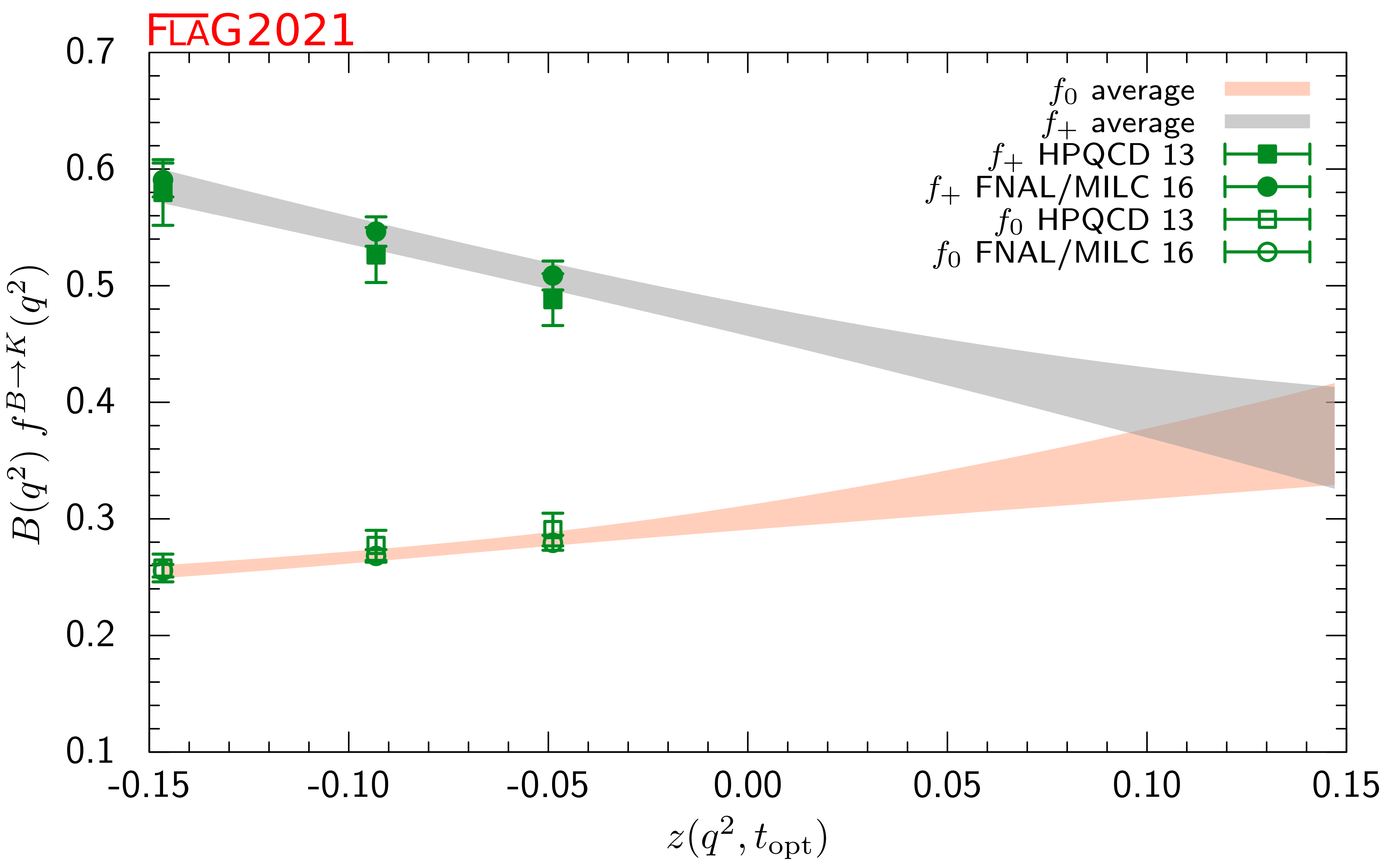
- FNAL/MILC results obtained on asqtad ensembles using Fermilab approach for the b quarks [arXiv:1503.07839, PRD 2015]
- dominant contribution to the FLAG 2021 average

- Enables determinations of $|V_{ub}|$ from exp. measurements of exclusive (differential) decay rates with commensurate contributions to total errors from lattice and experiment

Introduction: $B_s \to K$ and $B \to K$ form factors

- FNAL/MILC results obtained on asqtad ensembles using Fermilab approach for the b quarks

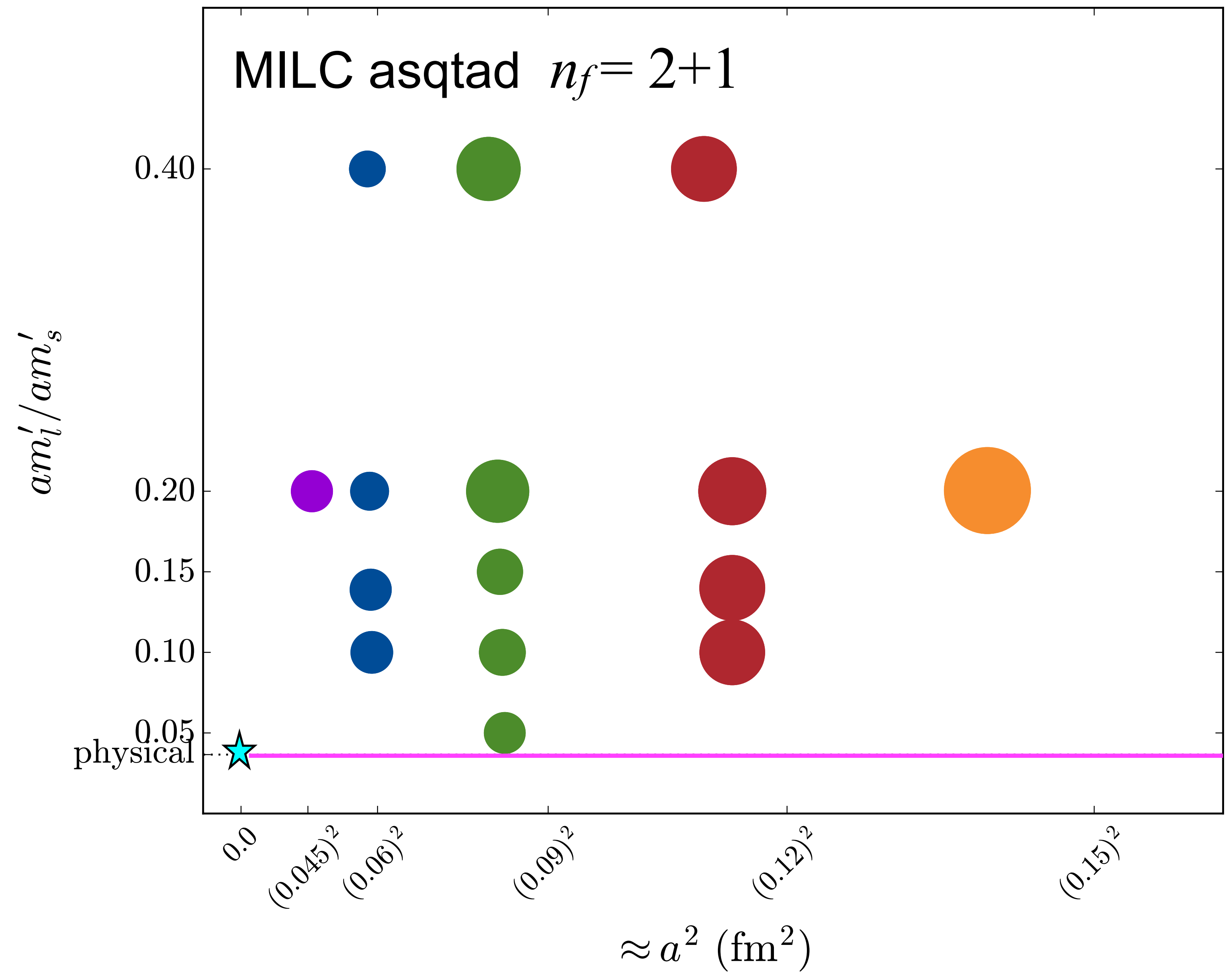


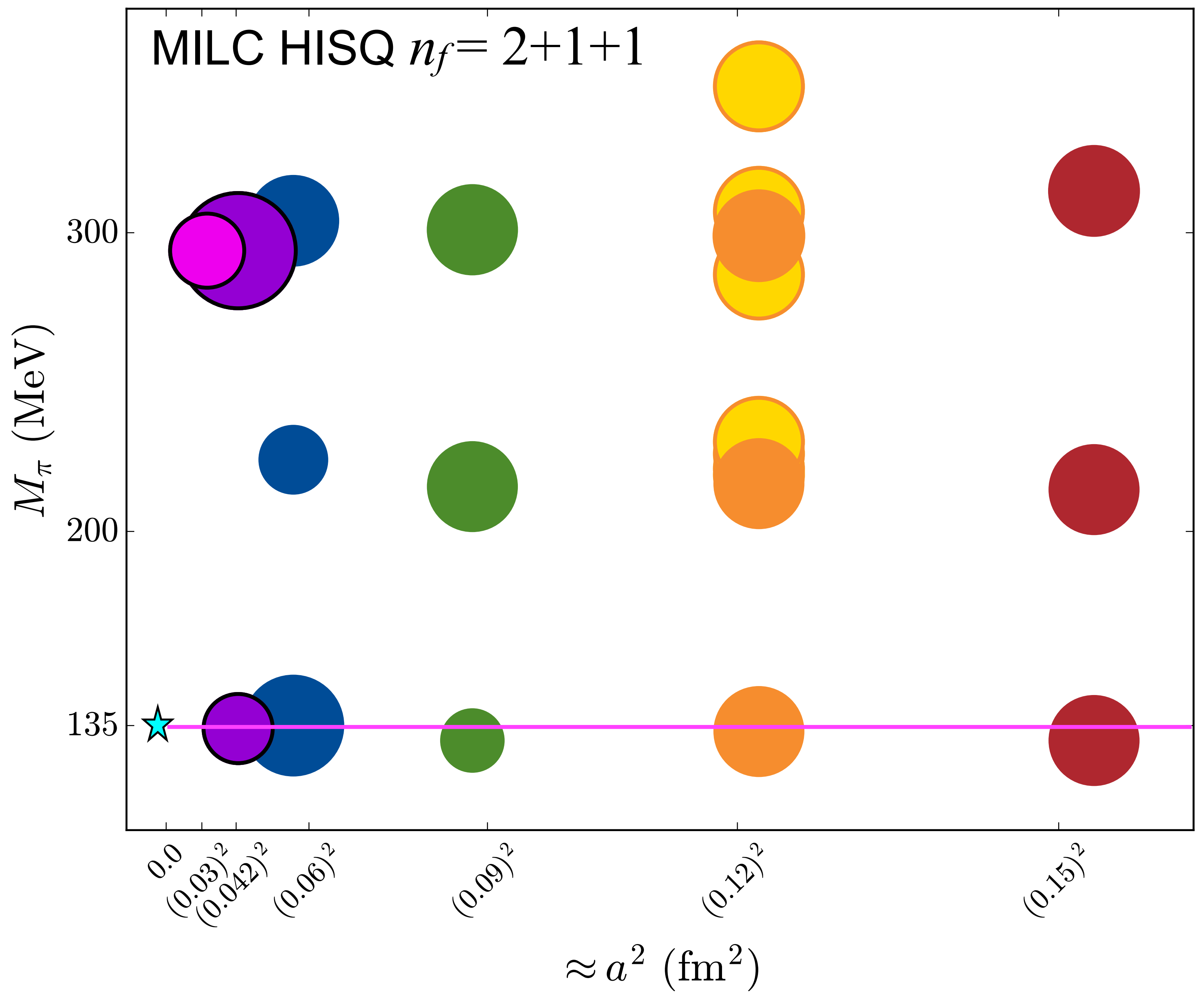


- [arXiv:1901.02561, PRD 2019]
- can be combined with new LHCb measurements

- [arXiv:1509.06235, PRD 2016]
- also results for tensor form factor

From the asqtad to the HISQ ensembles





HISQ ensemble set:

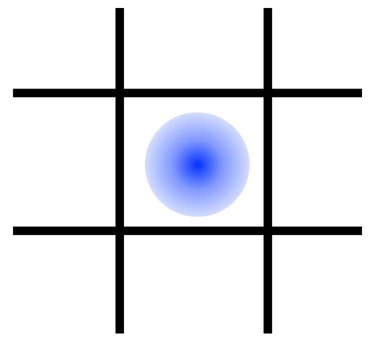
- smaller discretization errors with HISQ action
- physical mass ensembles at every lattice spacing with $L \sim 5.5 - 6\,\mathrm{fm}$
  ➠ chiral interpolation

Form factor projects on HISQ ensembles:

1. Fermilab approach for b quark (same as on asqtad)
   ➠ see Alejandro Vaquero talk
2. HISQ action for heavy and light valence quarks
   ➠ cross check of heavy quark discretization effects

Finding Beauty

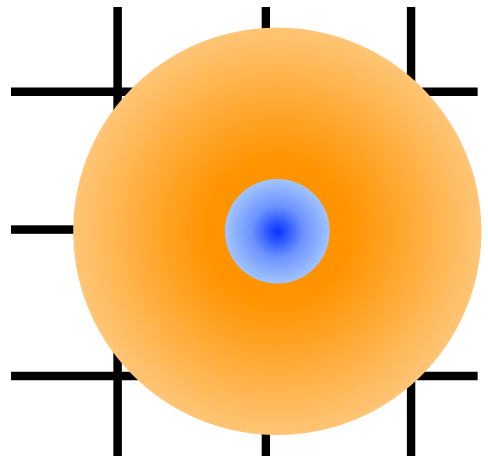
b quark



$m_b \gg \Lambda$ ➠ leading discretization errors $\sim (am_b)^2$
(using same action and matching to cont. QCD as for light quarks)

uncontrolled if $m_b > a^{-1}$

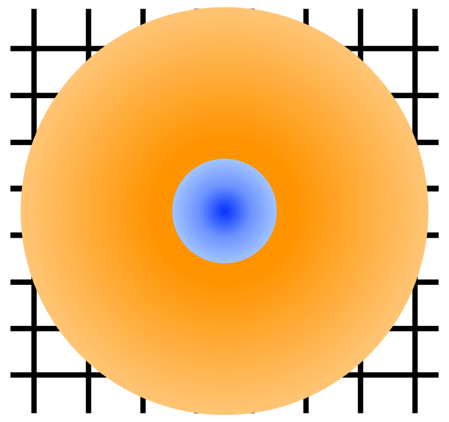
B meson



use EFT

- relativistic heavy quark approach (Fermilab)
  matching relativistic lattice action via HQET to continuum QCD
  nontrivial matching and renormalization

➠ (1-3)% errors



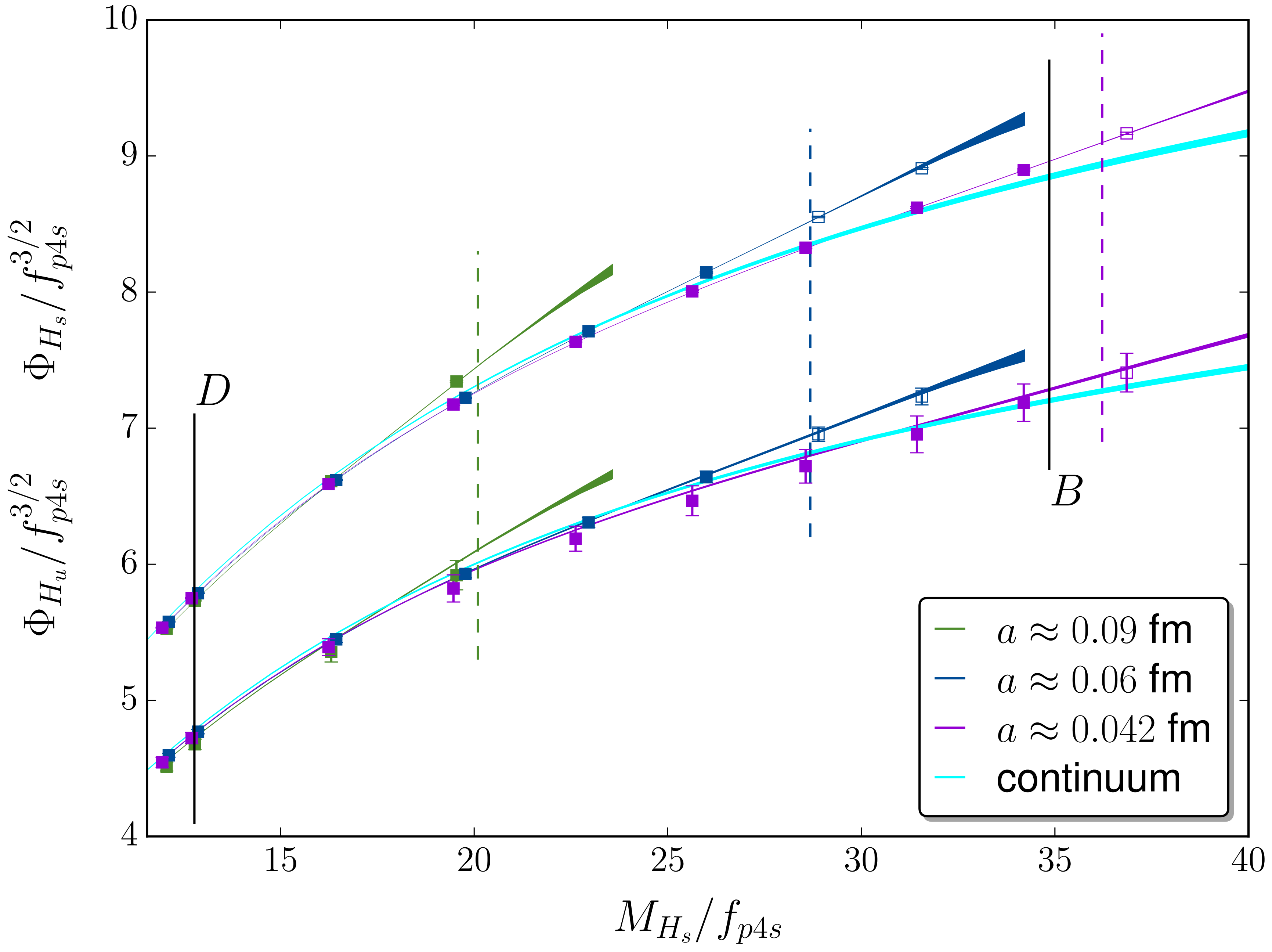
$a^{-1} > m_b \gg \Lambda$ + highly improved staggered quark (HISQ) action

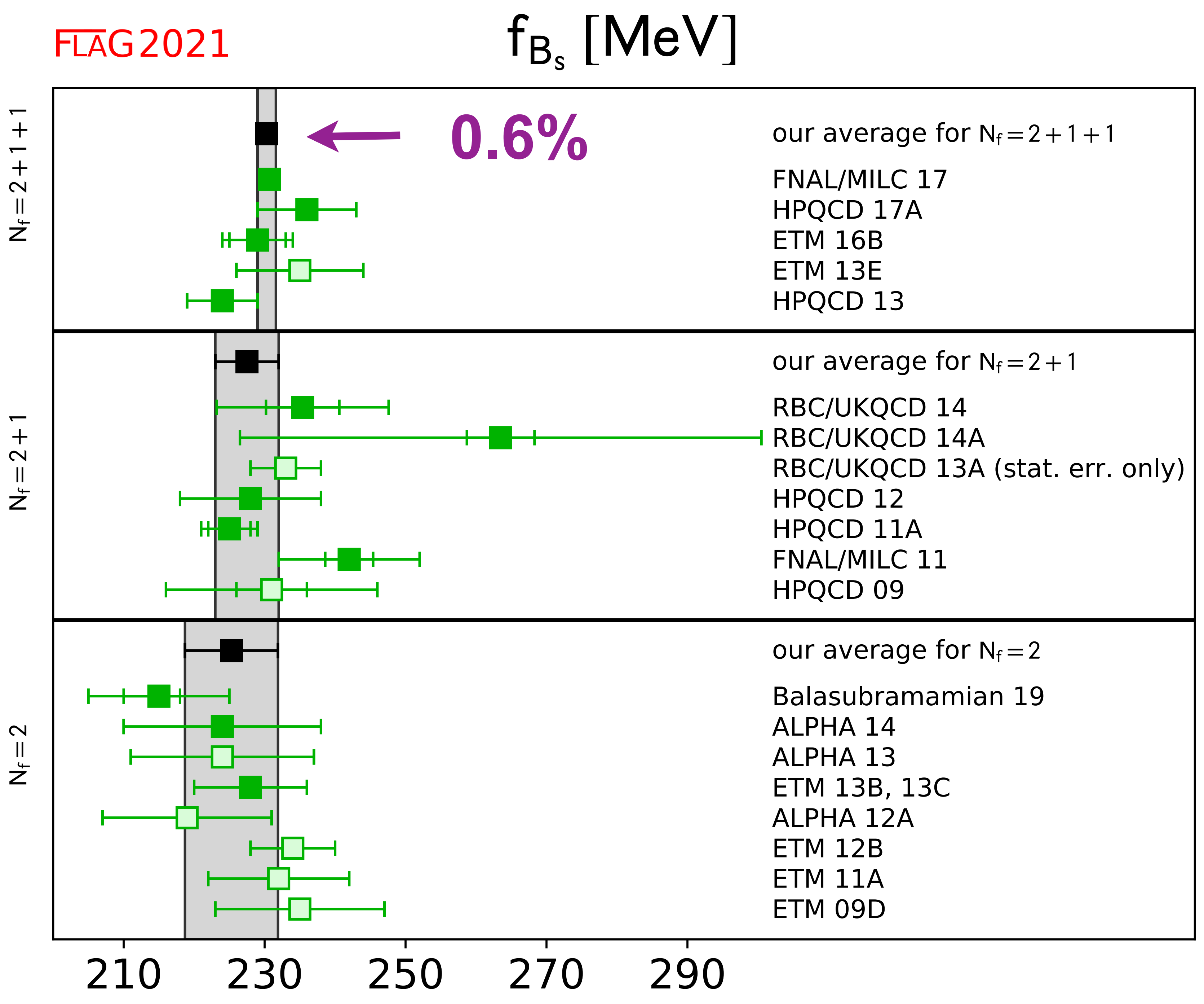
➠ same action for all quarks
➠ simple renormalizations (Ward identities)

➠ < 1% errors are possible

all HISQ B meson decay constants

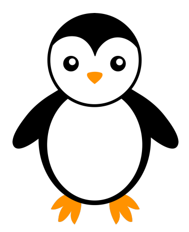
A. Bazavov et al [FNAL/MILC, arXiv:1712.09262, 2018 PRD]

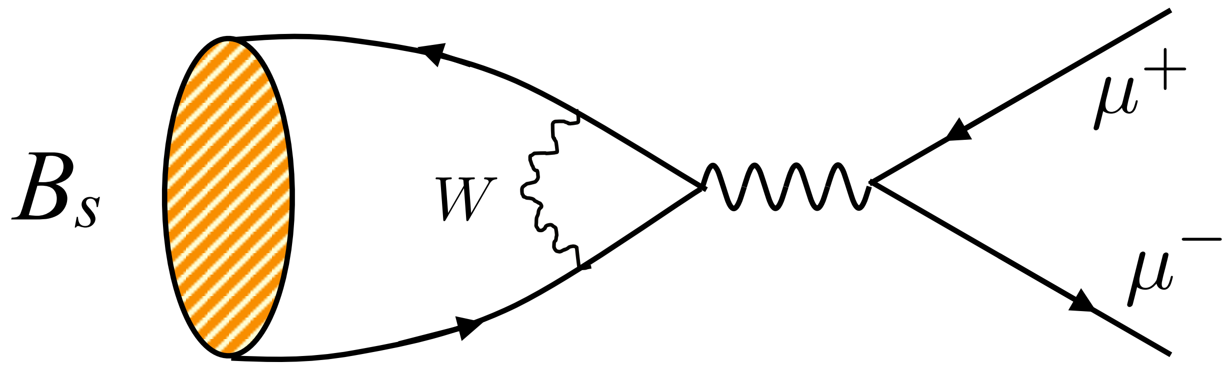




S. Aoki et al [FLAG review, arXiv:2111.09849]

SM prediction for rare leptonic decay rate

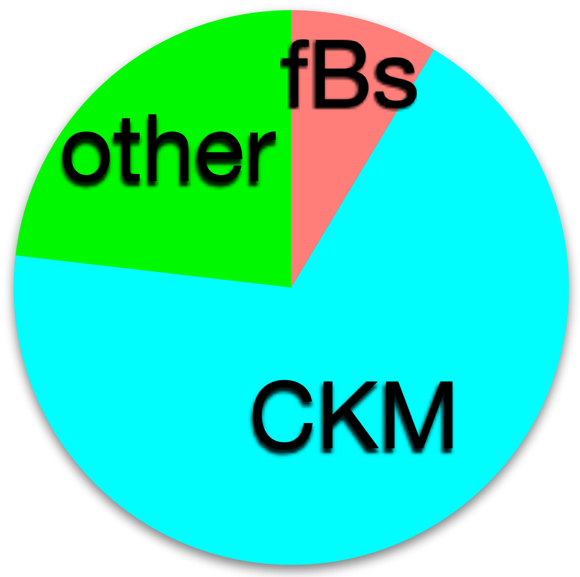




$$\overline{\mathscr{B}}(B_s \to \mu^+\mu^-) = 3.660\,(38) \times 10^{-9}$$

[Beneke et al, arXiv:1908.07011]

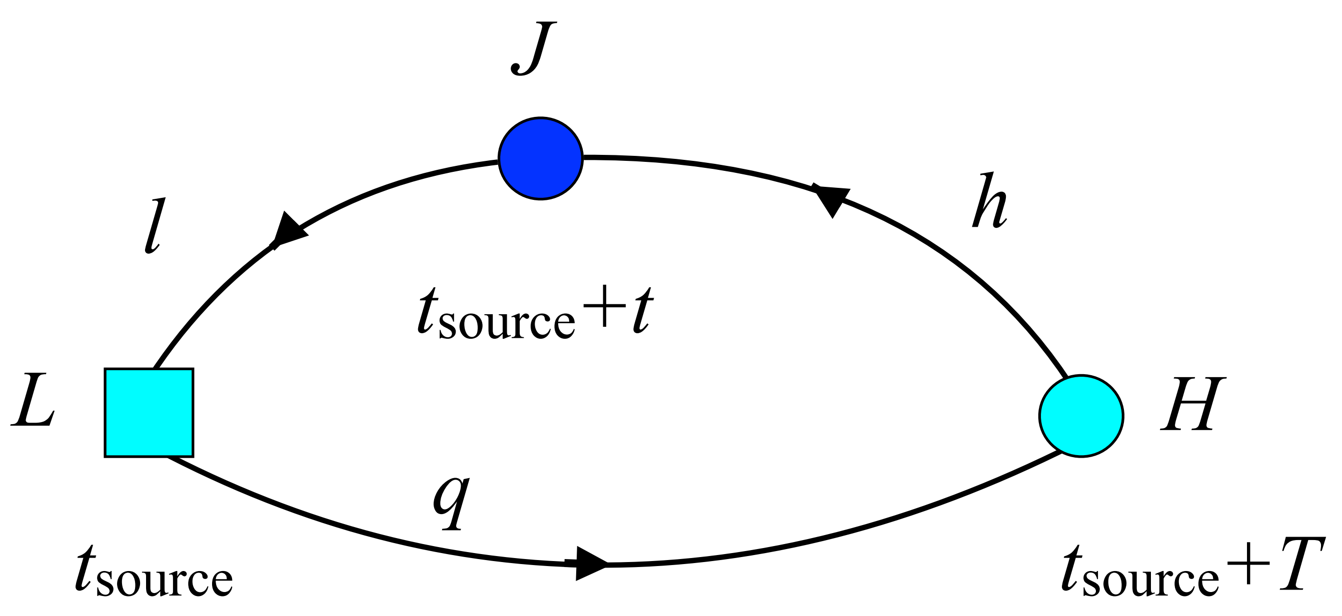
error^2



LQCD decay constant now sub dominant source of uncertainty!

all HISQ semileptonic form factors: setup

- Use HISQ action also for valence heavy and light quarks
- heavy quark masses: $0.9\, m_c \le m_h \lesssim m_b$ light quark masses: $m_l = m_{ud}, 0.1\, m_s, 0.2\, m_s, m_s$
- hadronic matrix elements $\langle L | J_\mu | H \rangle$ from 3-pt functions:



$C_{3\rm pt}(\vec{p}_L, t, T) \sim Z_L Z_H \langle L | J | H \rangle e^{-E_L t} e^{-E_H(T-t)}$
+ excited states

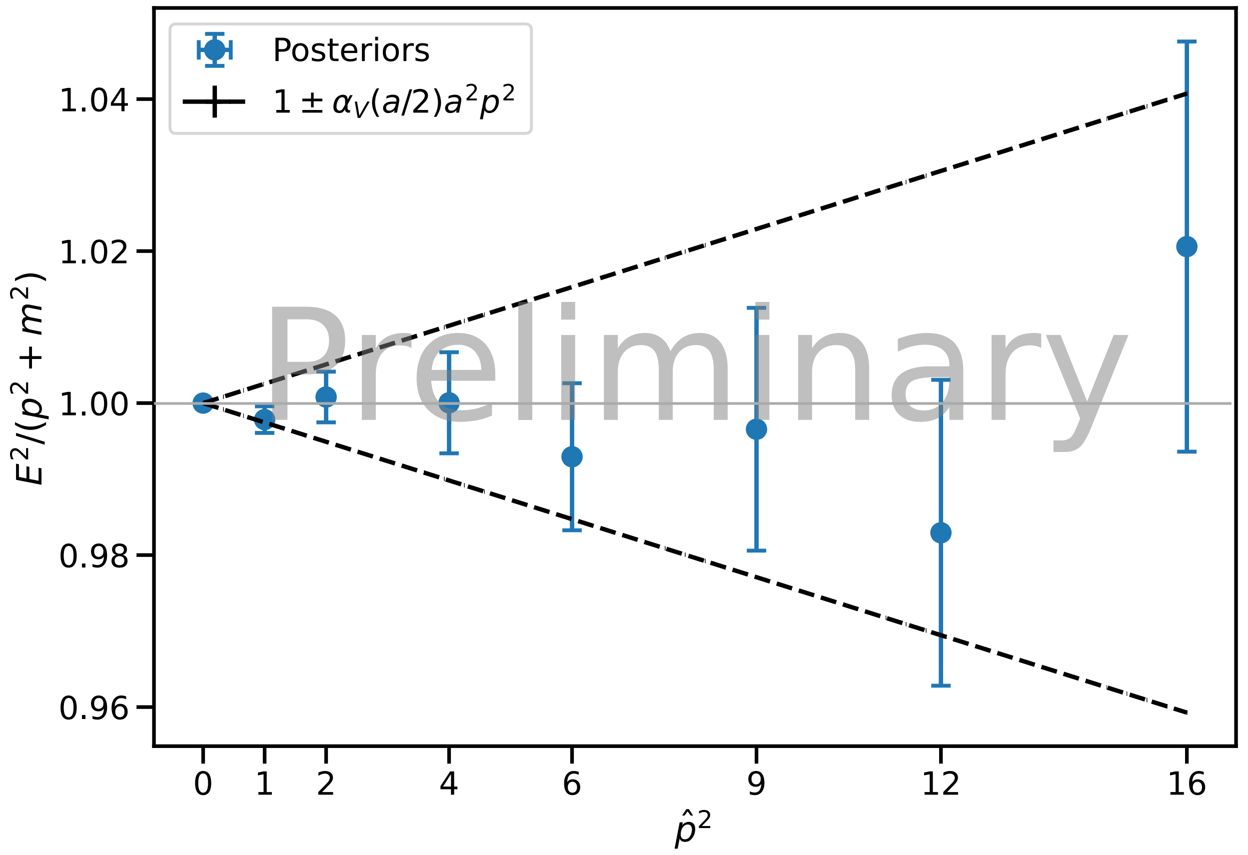
Combine information from 2&3 pt-functions to obtain desired hadronic matrix element

- ✦ H meson at rest, L meson with recoil momentum $|\vec{p}_L| \ge 0$
- ✦ currents: $J = S, V_0, V_i, T_{i4}$
- ✦ use Ward-Takashi identity to obtain renormalized vector current matrix elements $q^\mu \langle V_\mu \rangle = (m_h - m_l) \langle S \rangle$

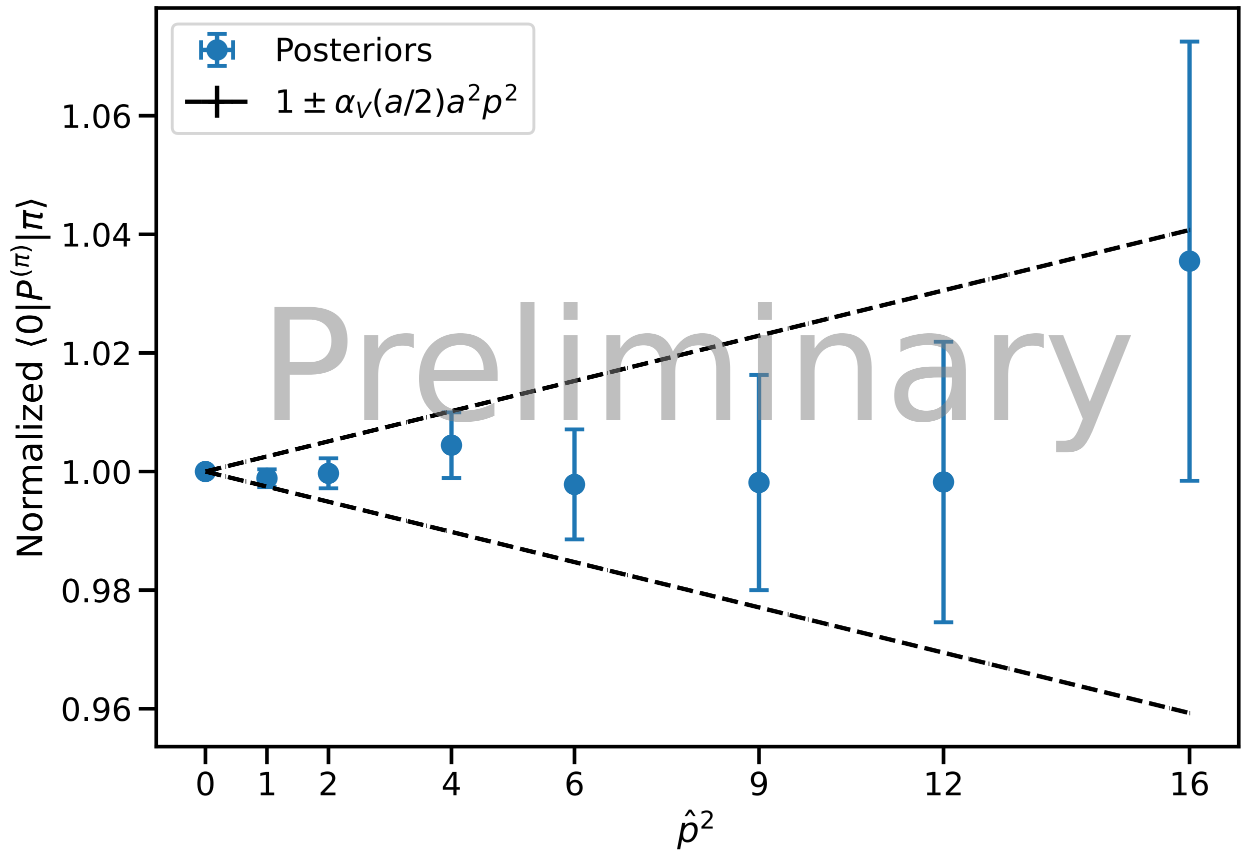
Tests and correlator fits

- Dispersion relation tests from pion two-point functions

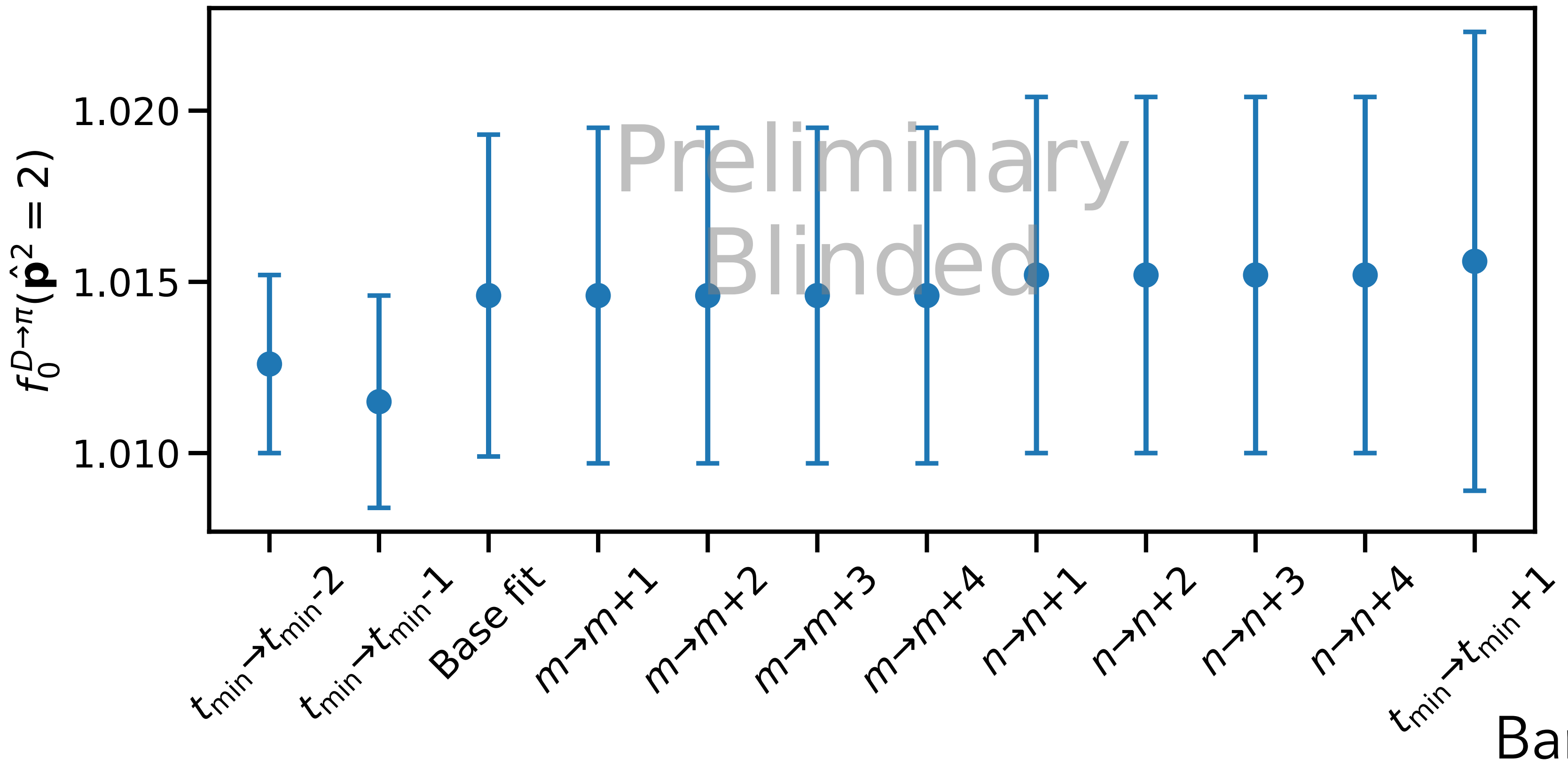
$E^2/(m^2+p^2)$ vs p^2



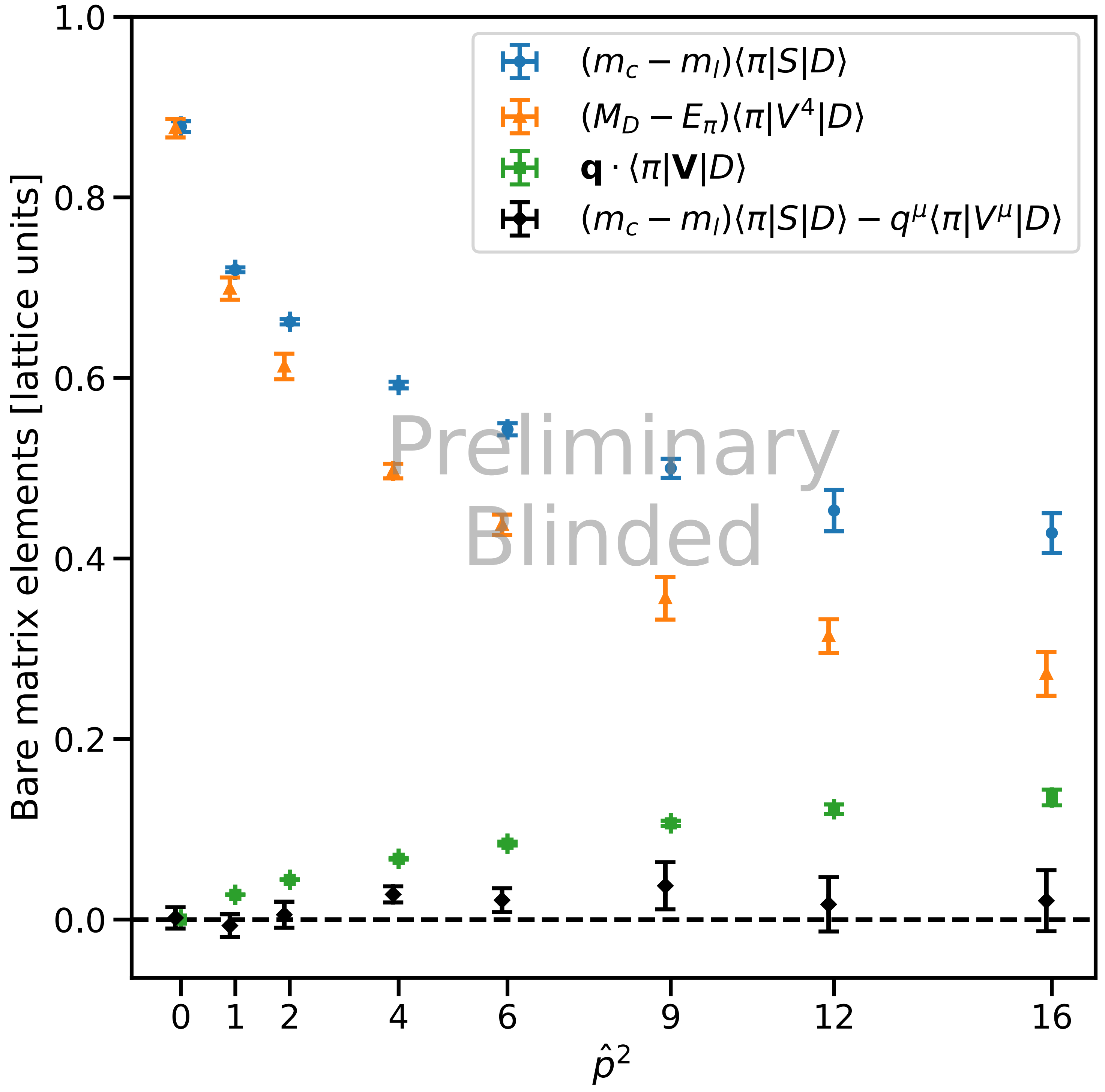
Amplitude vs p^2



- Stability of fit to obtain $f_0^{D\to\pi}$ from 3&2-pt functions under variations of fit parameters (# of states, t_{min})



- Test of Ward identity for $\langle \pi | J | D \rangle$



➠ small statistical uncertainties
➠ small discretization effects
➠ renormalizations $Z_{V_\mu} \simeq 1$

form factors for $B \to \pi \ell \nu$

$$\langle \pi | V^\mu | B \rangle = f_+(q^2) \left[p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right] + f_0(q^2) \frac{M_B^2 - M_\pi^2}{q^2} q^\mu$$

$$\langle \pi | S | B \rangle = \frac{M_B^2 - M_\pi^2}{m_b - m_u} f_0(q^2)$$

$$\langle \pi | T^{\mu\nu} | B \rangle = 2 \frac{p_B^\mu p_\pi^\nu - p_B^\nu p_\pi^\mu}{M_B + M_\pi} f_T(q^2)$$

Convenient for chiral-continuum expansion:

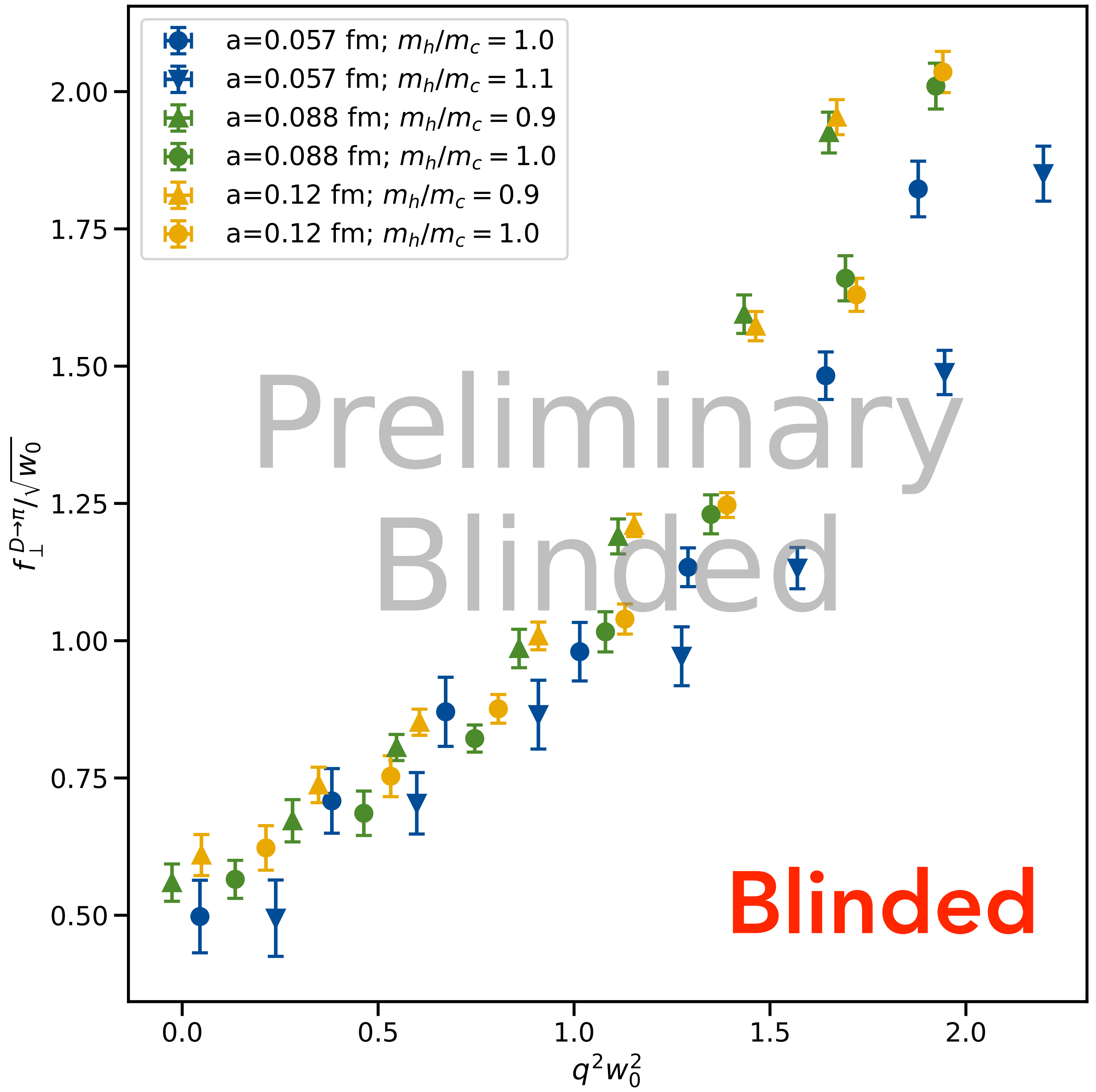
$$\langle \pi | V^\mu | B \rangle = \sqrt{2 M_B} \left[v^\mu f_\parallel(E_\pi) + p_\perp^\mu f_\perp(E_\pi) \right] \quad v = p/M_B$$

$$f_\parallel(E_\pi) = \frac{\langle \pi | V^4 | B \rangle}{\sqrt{2 M_B}}, \qquad p_\perp^\mu = p^\mu - (p \cdot v) v^\mu$$

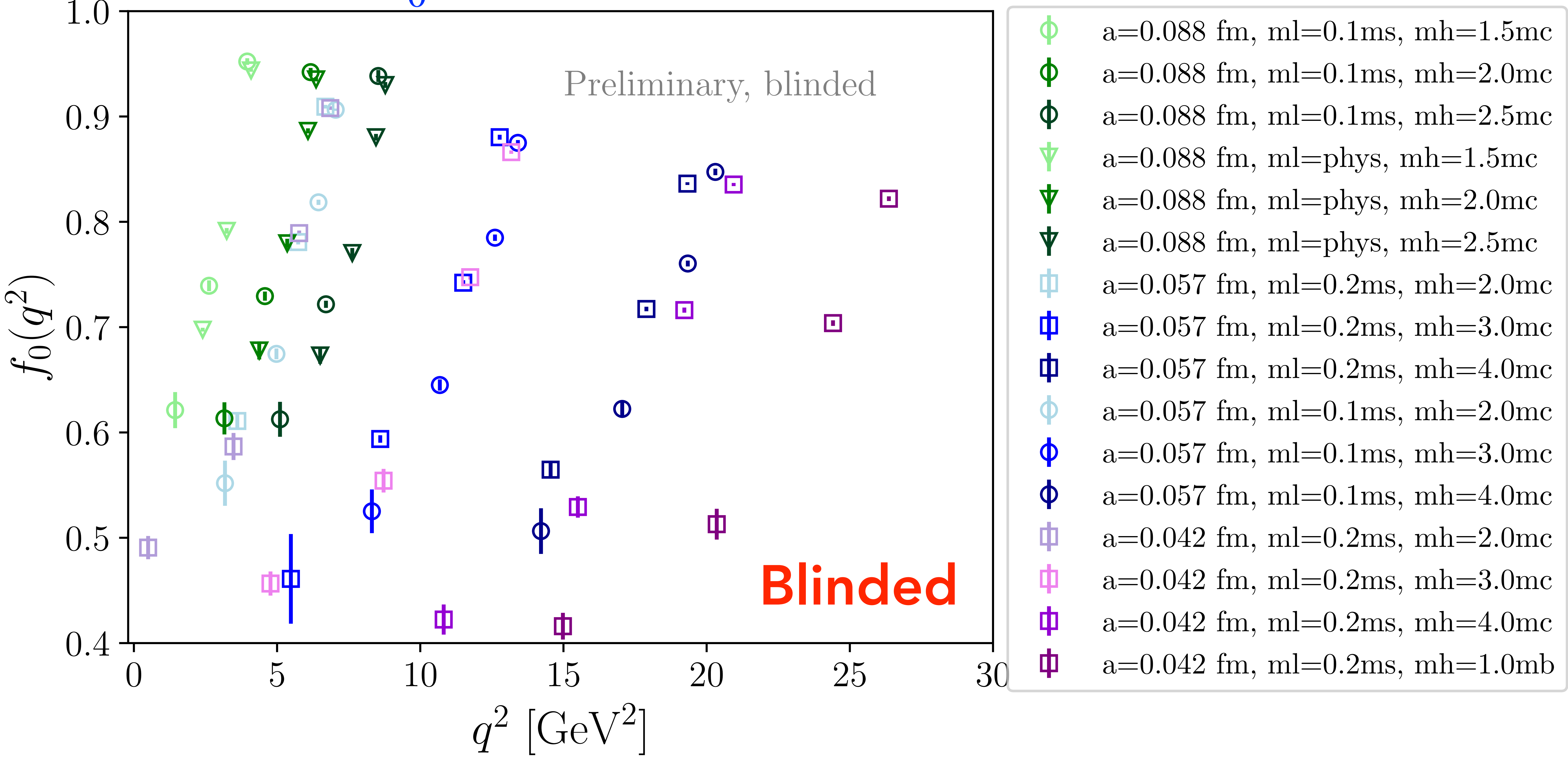
$$f_\perp(E_\pi) = \frac{\langle \pi | V^i | B \rangle}{\sqrt{2 M_B}} \frac{1}{p^i}$$

Form Factors

$f_{\perp}^{D\to\pi}(q^2)$



$f_0^{H_s\to K}(q^2)$



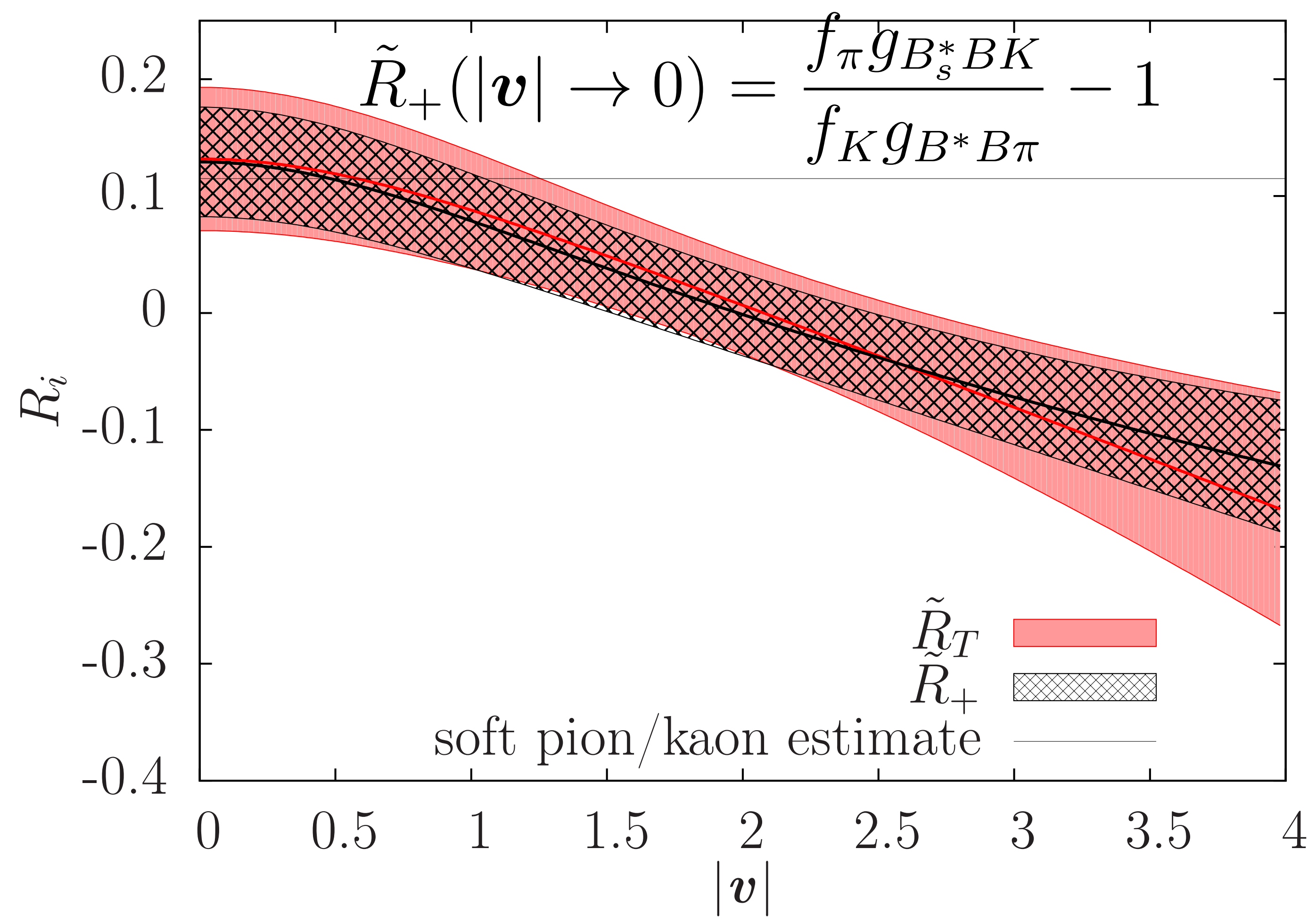
- combined chiral-continuum interpolation/extrapolation in progress

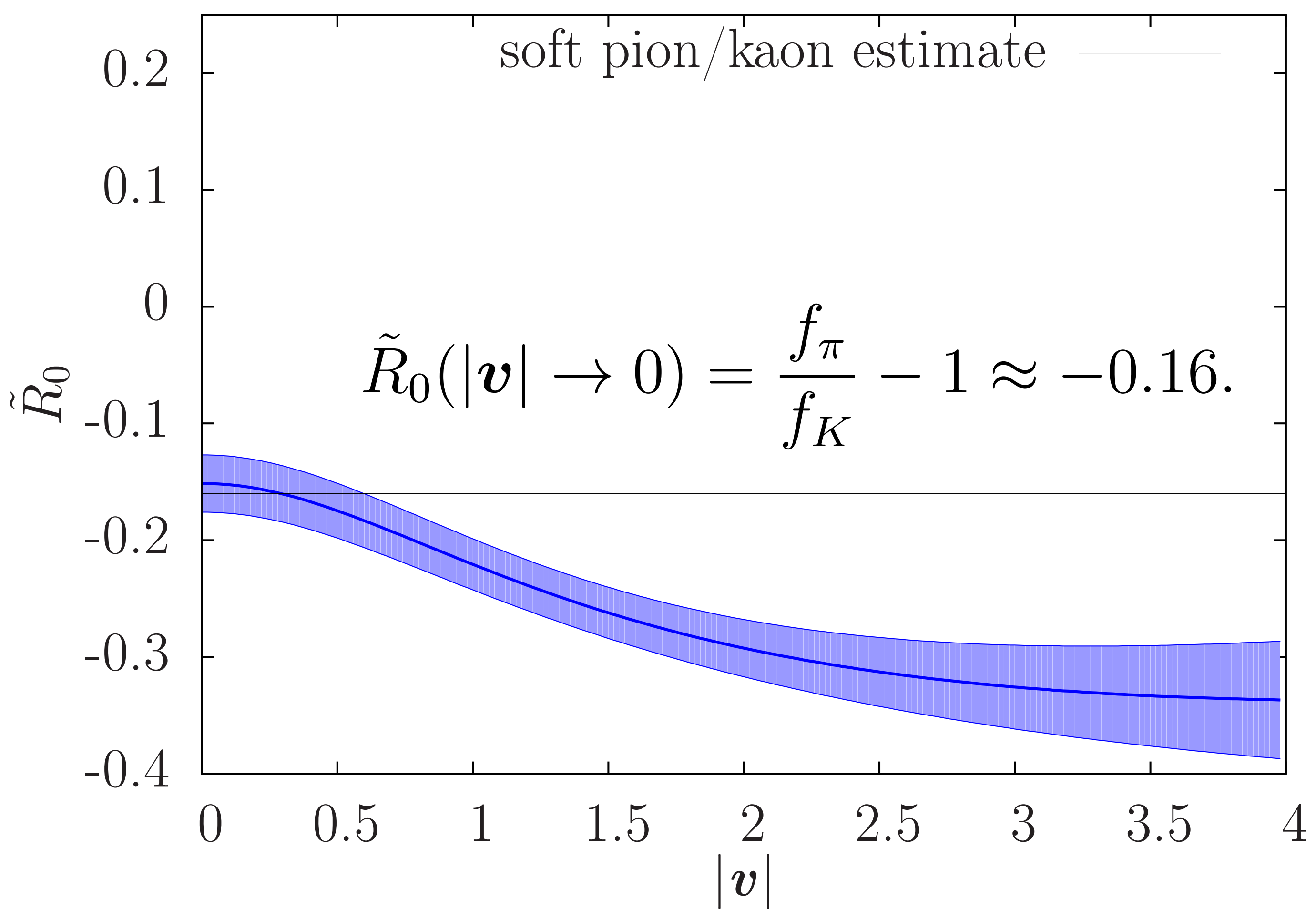
SU(3) ratios

Du et al [arXiv:1510.02349] using FNAL/MILC form factors from [arXiv:1503.07839, 1507.01618, 1509.06235]

$$\tilde{R}_i(|\boldsymbol{v}|) = \frac{f_i^{BK}(|\boldsymbol{v}|)P_i^{BK}}{f_i^{B\pi}(|\boldsymbol{v}|)P_i^{B\pi}} - 1$$

$$P_{+,T}^{B\pi} = 1 - q^2/M_{B^*}^2$$



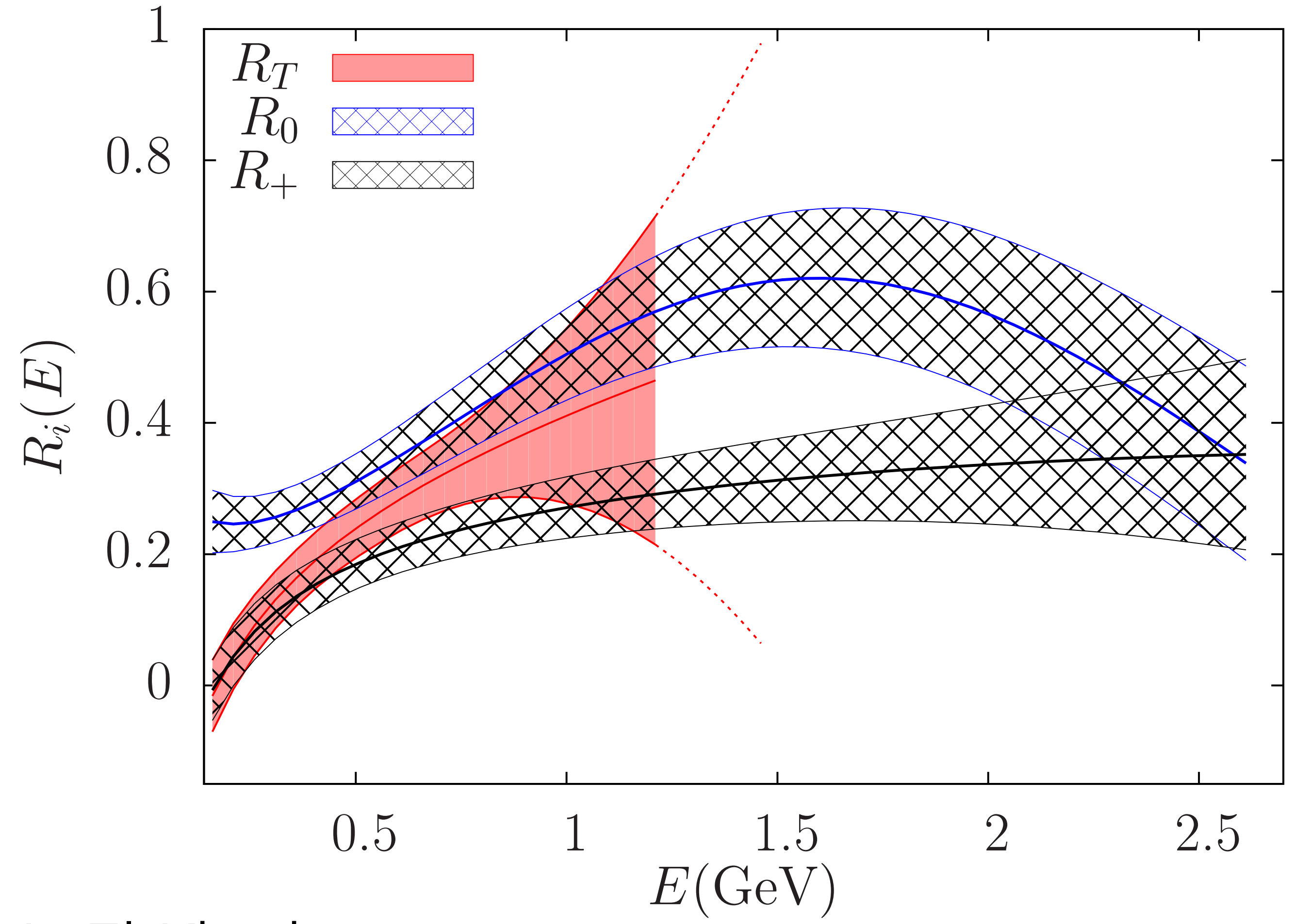


observed effects

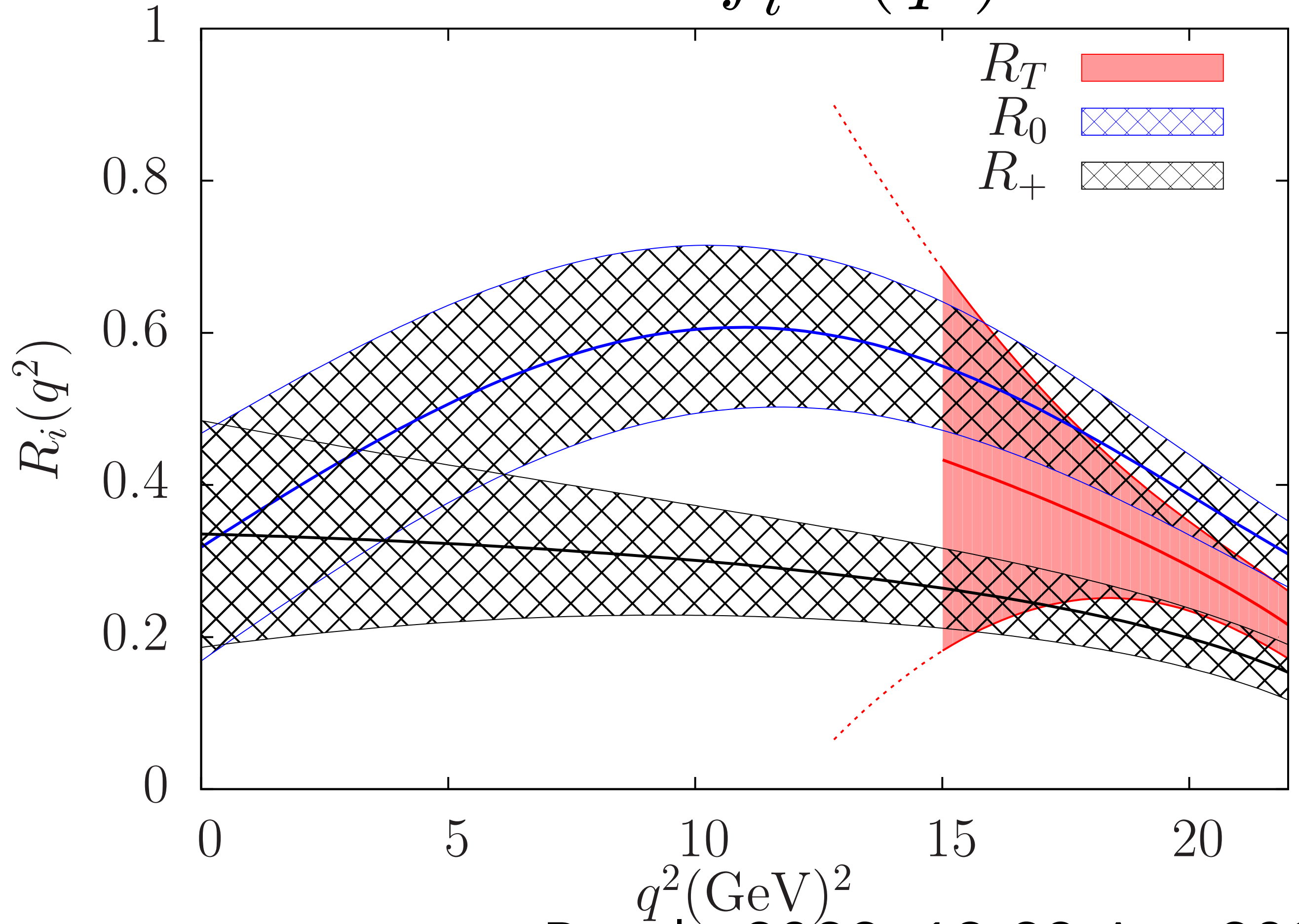
$\tilde{R}_{+,T}(|\mathbf{v}|) \lesssim 20\,\%$

$\tilde{R}_0(|\mathbf{v}|) \lesssim 35\,\%$

$$R_i(E) = \frac{f_i^{BK}(E)}{f_i^{B\pi}(E)} - 1$$



$$R_i(q^2) = \frac{f_i^{BK}(q^2)}{f_i^{B\pi}(q^2)} - 1$$



observed effects

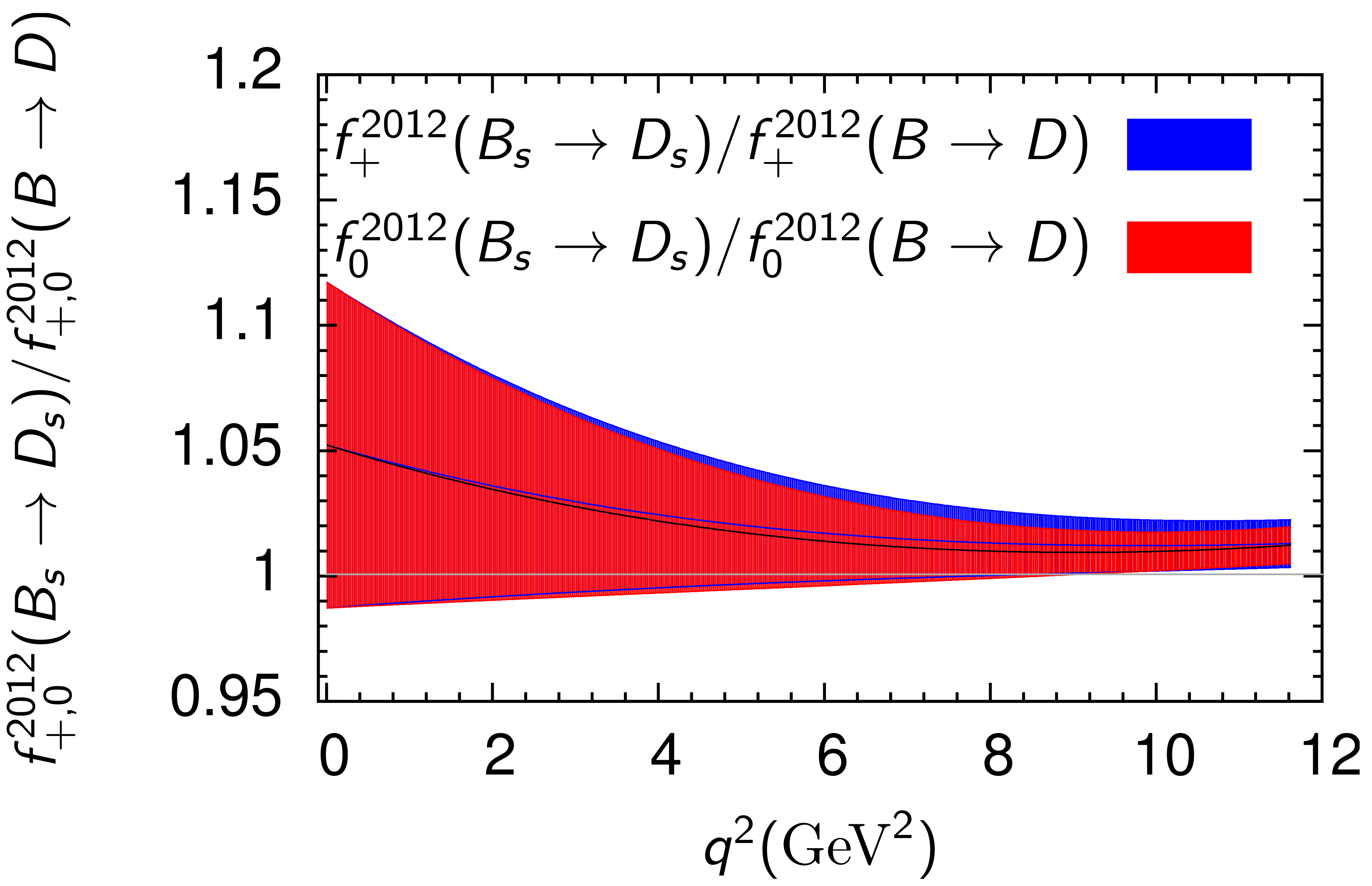
$R_i(q^2), R_i(E)$

$\lesssim 20\% - 60\,\%$

corresponding ratios for $B_s \to K / B \to \pi$ can be obtained from [arXiv:1503.07839, 1901.02561]

SU(3) ratios

Bazavov et al [arXiv:1901.02561] using FNAL/MILC form factors from [arXiv:1202.6346]
(obtained on a subset of the asqtad ensembles)



Conclusions

- work in progress by FNAL/MILC to compute the complete set of semi-leptonic B (and D) meson form factors on the HISQ ensembles using the HISQ action for the valence light and heavy quarks (all HISQ approach) for the following channels: $B \to \pi, B_s \to K, B \to K,$ and $B \to D, B_s \to D_s$ and $B \to D^*, B_s \to D_s^*$

- all analyses are BLINDED until systematic error analysis is finalized
- available range of lattice spacings $a \sim 0.03 - 0.15$ fm:
  ➠ reach $m_h = m_b$ with small discretization errors
- ensembles with physical light-quarks in the sea:
  ➠ chiral interpolation with significantly reduced uncertainties
- analyses set-up to easily obtain correlations and form ratios (SU(3), $B_s \to K/B_s \to D_s$, etc…)
- comparison with form factors calculated in the Fermilab-HISQ project (see Alejandro's talk) will provide cross checks of heavy quark discretization effects.

- goal is to obtain all form factors with percent level precision

Thank you!

Farah Willenbrock

Appendix

z-expansion coefficients: LQCD vs Exp

FNAL/MILC [arXiv:1503.07839]

$$B \to \pi \ell \nu$$

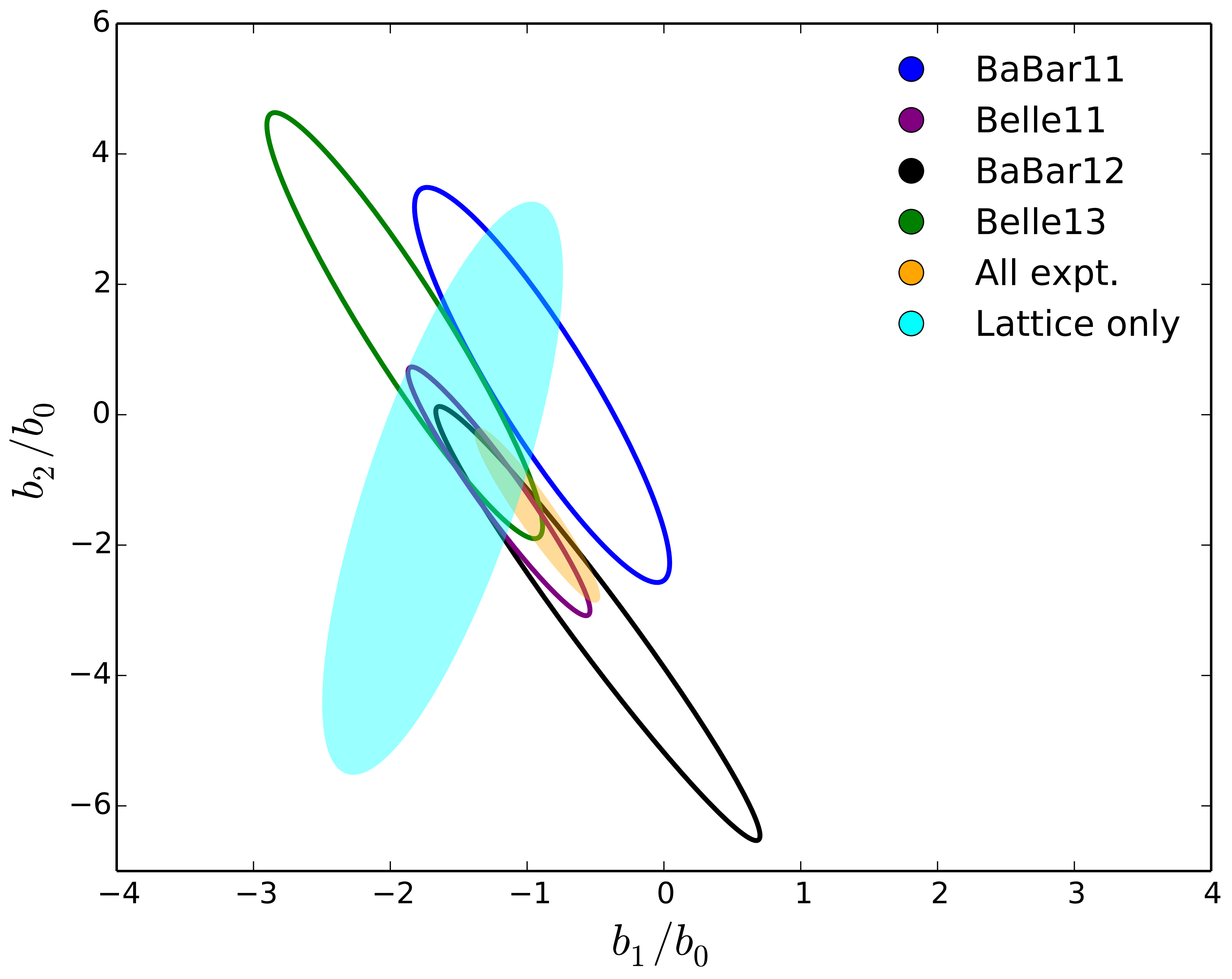


Table XV. The results of fits to experimental data only.

Fit	χ^2/dof	dof	p	b_1/b_0	b_2/b_0	$b_0\|V_{ub}\| \times 10^{-3}$
All exp.	1.5	48	0.02	−0.93(22)	−1.54(65)	1.53(4)
BaBar11 [7]	2	3	0.12	−0.89(47)	0.5(1.5)	1.36(7)
BaBar12 [8]	1.2	9	0.31	−0.48(59)	−3.2(1.7)	1.54(9)
Belle11 [9]	1.1	10	0.36	−1.21(33)	−1.18(95)	1.63(7)
Belle13 [10]	1.2	17	0.23	−1.89(50)	1.4(1.6)	1.56(8)