Light-cone sum rules for $b \to u$ exclusive channels

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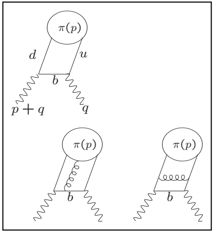
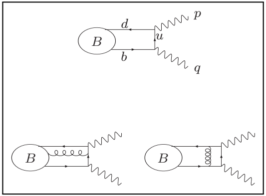
□ Exclusive $b \to u$ channels

- optimal for $|V_{ub}|$ determination
  - $B \to \pi \ell \nu_\ell$, $B_s \to K \ell \nu_\ell$
  - $B \to 2\pi(\rho, f_0, f_2) \ell \nu_\ell$,
  - $B \to \eta \ell \nu_\ell$, $B \to \omega \ell \nu_\ell$
  - $B_c \to D \ell \nu_\ell$

- form factors at large recoil (small q^2) accessible with LCSRs:



- Method 1:
with light-meson DAs



- Method 2
with B-meson DAs

☐ Which LCSR method is better?

LCSR	method 1 (light meson DAs)	method 2 (B DAs)
Correlator	finite m_b	HQET
OPE twist expansion α_s expansion	$\leq$ tw 6 (N)NLO tw 2, NLO tw3	$\leq$ tw 6 NLO tw 2
parameters of DAs exp. data	sufficiently accurate for π,ρ pion FFs	λ_B,$\lambda_{H,E}$ uncertainty $B \to \ell\nu_\ell\gamma$
duality threshold	s_0^B from B-mass fixing	s_0^π from 2point SR
validity check with $D \to h$	yes	no
flexibility $B \to h$	needs a set of DAs for each h	change the inteprol.current
$B \to 2\pi$ FFs	dipion DAs poorly known	accessible via disp. relation

- Uncertainty estimates:
  - “traditional” way: parametric uncertainties in quadrature
  - more advanced: statistical (Bayesian) analysis,

    [I.S.Imsong, A.K., T. Mannel and D. van Dyk, (2015)]

    [N.Gubernari, A.Kokulu, D. van Dyk, (2018)]
  - channel-dependent “systematic” error due to quark-hadron duality approximation

☐ Most recent LCSR analysis for $B \to \pi$ form factors with pion DAs

[D.Leljak, B. Melic, D.van Dyk, 2102.0723]

▶ Comparison with earlier LCSRS results

	$f^{+}_{B\pi}(0)$	$f^{T}_{B\pi}(0)$
Light-cone sum rules		
Duplancic et al. [15]	$0.26^{+0.04}_{-0.03}$	0.255 ± 0.035
Imsong et al. [20]	0.31 ± 0.02	—
Khodjamirian/Rusov [29]	0.301 ± 0.023	0.273 ± 0.021
Gubernari et al. (B LCDA) [21]	0.21 ± 0.07	0.19 ± 0.06
this work	0.283 ± 0.027	0.282 ± 0.026
Light-cone sum rules + Lattice QCD combination		
this work	0.235 ± 0.019	0.235 ± 0.017

Table 6. Comparison of our results for the form factor normalizations with other QCD-bas results in the literature. The result of ref. [21] is included for completeness, although the auth caution that the threshold setting procedure employed in that work fails for the $\bar{B} \to \pi$ form facto

□ Recent results and comparison for $B \to \rho$ form factors

from [N.Gubernari, A.Kokulu, D. van Dyk, ArXiv[1811.00983] (2018)]

form factor at $q^2 = 0$	our result	literature	DAs	[Ref.]
$A_1^{B\to\rho}$	0.22 ± 0.10	0.24 ± 0.08	B	[AK,Mannel,Offen 06']
		0.262 ± 0.026	ρ	[Bharucha,Straub,Zwicky 15
$A_2^{B\to\rho}$	0.19 ± 0.11	0.21 ± 0.09	B	[AK,Mannel,Offen 06']
$V^{B\to\rho}$	0.27 ± 0.14	0.32 ± 0.10	B	[AK,Mannel,Offen 06']
		0.327 ± 0.031	ρ	[Bharucha,Straub,Zwicky 15
$T_1^{B\to\rho}$	0.24 ± 0.12	0.28 ± 0.09	B	[AK,Mannel,Offen 06']
		0.272 ± 0.026	ρ	[Bharucha,Straub,Zwicky 15
$T_{23}^{B\to\rho}$	0.56 ± 0.15	0.747 ± 0.076	ρ	[Bharucha,Straub,Zwicky 15

ρ DAs, neglecting Γ_ρ

☐ Comparison with lattice QCD results

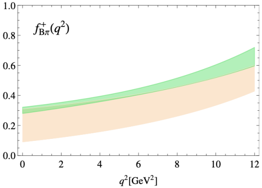
- LCSRs (the method with pion DAs) predict somewhat larger central values of $f_{B\pi}$ and f_{B_sK}

green [A.K., A.Rusov, (2017)]

orange [Fermilab-MILC 1503.07839]

using in LCSR the f_B FLAG aver. instead of 2point SRs

slightly reduces the difference



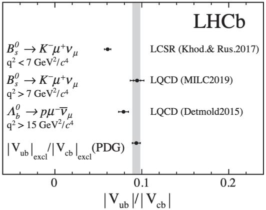
- a "pure LCSR" observable:

$$\Delta\zeta_{B\pi}\,(0, 12\text{GeV}^2) = \frac{1}{|V_{ub}|^2}\int\limits_0^{12\,\text{GeV}^2} dq^2\,\frac{d\Gamma}{dq^2}(B\to\pi\ell\nu_\ell) \equiv \frac{G_F^2}{24\pi^3}\int\limits_0^{12\,\text{GeV}^2} dq^2\,p_\pi^3\,|f^+_{B\pi}(q^2)|^2 = (5.25^{+0.68}_{-0.54})\,\text{ps}^{-1}$$

[I. S.Imsong, A.K., T. Mannel and D. van Dyk, (2015)]

$$\Delta\zeta_{B_sK}\,(0, 12\,\text{GeV}^2) = 6.92^{+1.09}_{-0.90}\ \text{ps}^{-1}$$

[A.K., A.Rusov, (2017)]



□ $b \to u$ transitions to dipion and resonance states

- Measured are $B \to 2\pi\ell\nu_\ell$ decays
- a practical problem: to isolate contributions of resonances from "nonresonant" background in $B \to \pi\pi\ell\nu_\ell$
- the recent Belle results (full data sample) $B \to \pi^+\pi^-\ell\nu_\ell$ $(\ell = e, \mu)$
  modeled with four resonances : $\rho, \rho', f_0(500), f_2(1270)$ [Belle Collab. hep-ex 2005.07766]
  see the talk by Christoph Schwanda
- in the theory language:
  see e.g. [S. Faller, T. Feldmann, A. Khodjamirian, T. Mannel and D. van Dyk, Phy1310.6660]
  - define $B \to \pi\pi$ form factors, e.g.,:

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma^\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle = -F_\perp(q^2,k^2,\zeta)\,\frac{4}{\sqrt{k^2\lambda_B}}\,i\epsilon^{\mu\alpha\beta\gamma}\,q_\alpha\,k_{1\beta}\,k_{2\gamma} + ...$$

$$(2\zeta - 1) = (1 - 4m_\pi^2/k^2)^{1/2} cos\theta_\pi, \text{ in dipion c.m.}$$

  - expand in partial waves, isolate dipion $S, P, D, ...$-waves

$$F_\perp(q^2,k^2,\zeta) \Rightarrow F_\perp^{(\ell=1)}(q^2,k^2)$$

  - for each partial wave apply hadronic dispersion relation in the dipion invariant mass

□ Dispersion relation for the $B \to \pi^+\pi^0$ vector FF

- assuming only *P*-wave; three-resonance ansatz:

$$\frac{\sqrt{3}F_{\perp}^{(\ell=1)}(q^2,k^2)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{g_{\rho\pi\pi}}{m_\rho^2 - k^2 - im_\rho\Gamma_\rho(k^2)}\frac{V^{B\to\rho}(q^2)}{m_B+m_\rho}$$
$$+\frac{g_{\rho'\pi\pi}}{m_{\rho'}^2 - k^2 - im_{\rho'}\Gamma_{\rho'}(k^2)}\frac{V^{B\to\rho'}(q^2)}{m_B+m_{\rho'}} +$$
$$+\frac{g_{\rho''\pi\pi}}{m_{\rho''}^2 - k^2 - im_{\rho''}\Gamma_{\rho''}(k^2)}\frac{V^{B\to\rho''}(q^2)}{m_B+m_{\rho''}} + ...$$

- a more refined dispersion analysis possible

[e.g., G. Colangelo, M. Hoferichter and P. Stoffer, 1810.00007].

- calculate $B \to \pi\pi$ form factors from LCSRs

(not yet accessible in the lattice QCD)

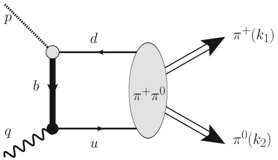
- $\rho, \rho', ...$ have to be "embedded" in this calculation
- model-dependence of the input is unavoidable

□ LCSRs for $B \to \pi\pi$ form factors: Method 1

[Ch. Hambrock, AK, Nucl. Phys. B (2016); 1511.02509 [hep-ph]]

- applicable for dipion with a small invariant mass and large recoil:
  $k^2 \lesssim 1$ GeV^2, $0 \leq q^2 \leq$ 12-14 GeV^2.
- nonperturbative input: dipion distribution amplitudes (DAs)
- vacuum $\to$ on-shell dipion hadronic matrix elements of nonlocal $\bar{u}(x)d(0)$ operators

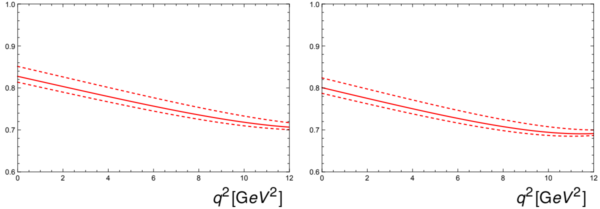
  FSI including the ρ-meson "embedded" in DAs



- considered only $\bar{B}^0 \to \pi^+\pi^0\ell^-\nu_\ell$,
  isospin 1, $L = 1, 3, , ..$
- only LO, twist-2 approximation for dipion DAs available
- quark-hadron duality in the B-channel, $\Rightarrow$ effective threshold s_0^B,

□ How dominant is ρ ?

$$\frac{[F_{\perp}^{(\ell=1)}(q^2,k^2_{min})]^{(\rho)}}{[F_{\perp}^{(\ell=1)}(q^2,k^2_{min})]^{(LCSR)}} \qquad \frac{[F_{||}^{(\ell=1)}(q^2,k^2_{min})]^{(\rho)}}{[F_{||}^{(\ell=1)}(q^2,k^2_{min})]^{(LCSR)}}$$



Relative contribution of ρ-meson to the $B \to \pi^+\pi^0$ P-wave form factors

$F_{\perp}^{(\ell=1)}(q^2, k^2_{min})$ (left panel) and $F_{||}^{(\ell=1)}(q^2, k^2_{min})$ (right panel) from LCSRs.

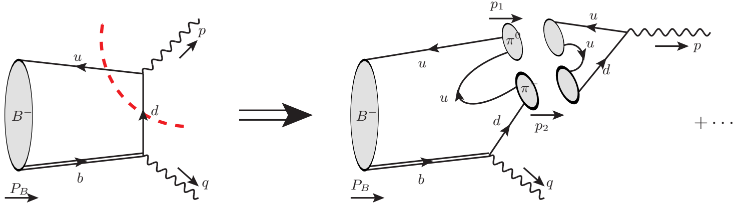
Dashed lines - the uncertainty due to the variation of the Borel parameter.

- main problem: the Gegenbauer coefficents of dipion DAs
  (complex functions of dipion invariant mass)
- to model/extract these coefficients switching the LCSRs to $D \to \pi^-\pi^0\ell^+\nu_\ell$ form factors and fitting to the decay distribution measured by BESS III [R.Kellermann, AK, G.Tetlalmatzi-Xolocotzi, in progress]

□ LCSRs for $B \to \pi\pi$ FFs: Method 2

[S.Cheng, AK, J.Virto, 1701.01633 [hep-ph]]

- LCSRs with B-meson DA and $\bar{u}\gamma_\mu d$ interpolating current
- The correlation function:



- insert a dispersion.relation for $B \to 2\pi$ form factors
  and a (dispersion rel. $\oplus$ experiment) parametrization for F_π
- not a direct calculation, given the ansatz of the $B \to 2\pi$ form factors, these sum rules provide normalization parameters

Probing ρ-resonance models

- ansatz for the $B \to \pi\pi$ FF:

$$\frac{\sqrt{3}F_\perp^{(\ell=1)}(q^2,k^2)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{g_{\rho\pi\pi}}{m_\rho^2 - k^2 - im_\rho\Gamma_\rho(k^2)}\frac{V^{B\to\rho}(q^2)}{m_B+m_\rho}$$
$$+\frac{g_{\rho'\pi\pi}}{m_{\rho'}^2 - k^2 - im_{\rho'}\Gamma_{\rho'}(k^2)}\frac{V^{B\to\rho'}(q^2)}{m_B+m_{\rho'}} + \frac{g_{\rho''\pi\pi}}{m_{\rho''}^2 - k^2 - im_{\rho''}\Gamma_{\rho''}(k^2)}\frac{V^{B\to\rho''}(q^2)}{m_B+m_{\rho''}}$$

inspired by experimental fit of timelike $F_\pi(s)$

- Model 1:
  - $V^{B\to\rho}(q^2)$ from LCSR with ρ-meson DAs (in which $\Gamma_\rho = 0$)

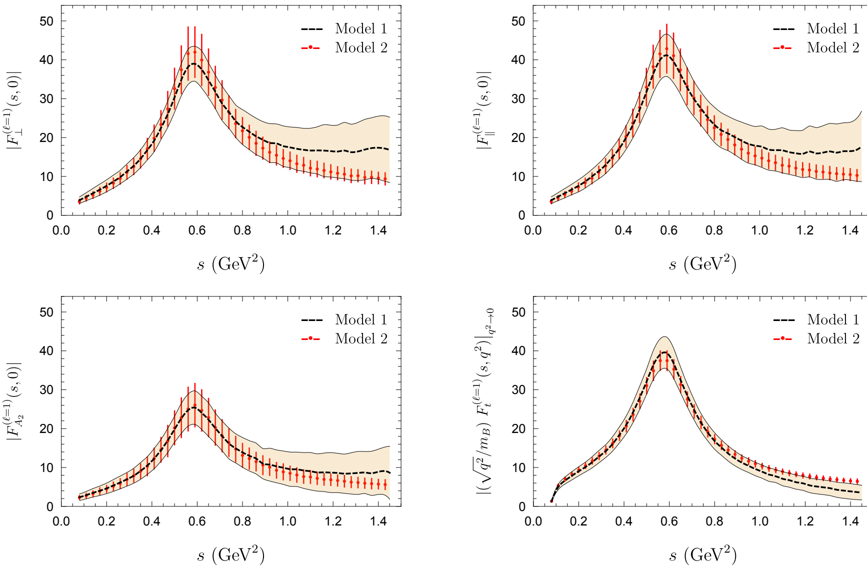
    taken from A.Bharucha, D.Straub and R.Zwicky,1503.05534

  - neglect ρ''

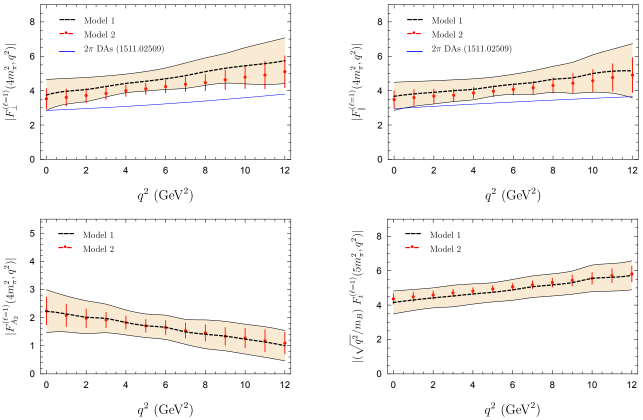
  $\Rightarrow$ contribution of ρ' up to 20% of ρ in residue consistent with the fit

- Model 2 :
  - all three resonances taken into account

□$B \to 2\pi$ ($\ell = 1$) FFs: dipion mass dependence



□ $B \to 2\pi$ $(\ell = 1)$ FFs: q^2-dependence at small k^2



□ Concluding:

- wish list (experiment):
  - accurate q^2-slope measurement in $B \to \pi \ell \nu_\ell$, $B_s \to K \ell \nu_\ell$
  - $B \to \gamma \ell \nu_\ell$ partial width and photon energy distribution
  - partial wave expansion in $B \to \pi\pi \ell \nu_\ell$

- wish list (theory):
  - to assess "intrinsic" $1/m_b$-corrections to the B-DA correlator
  - further improvements in λ_B, $\lambda_{E,H}$, λ_{B_s} determination
  - Gegenbauer functions for dipon DAs
  - $B \to \pi\pi$ FFs with $J^P = 0^+, 2^+$ from LCSRs