

$|V_{cb}|$ at LHCb (with B_s^0 decays)

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Challenges in SL B decays, 19-23 April 2022

Content

Back in 2020, $|V_{cb}|$ from LHCb [[PRD 101 \(2020\) 072004](#)] just released, was perfect timing for Barolo! No longer “new” today: presented several times, discussed/exploited in a number of phenomenological analyses.

Will refresh key experimental aspects of the LHCb measurement.

Disclaimer: might not be updated with latest @LHCb (I'm in Belle 2 now!)

$|V_{cb}|$ at LHCb, really?

- Could provide new information using other b hadron than $B^{0/+}$. Different system, different uncertainties.
- Had hundred thousands of SL B_s^0 decays in Run I.
- *We just* need a good normalisation to measure a precise branching fraction and do it differential.

Phys. Rev. D 97, 054502 (2018)

Lattice QCD calculation of the $B_{(s)} \rightarrow D_{(s)}^* \ell \nu$ form factors at zero recoil and implications for $|V_{cb}|$

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(HPQCD Collaboration)^b

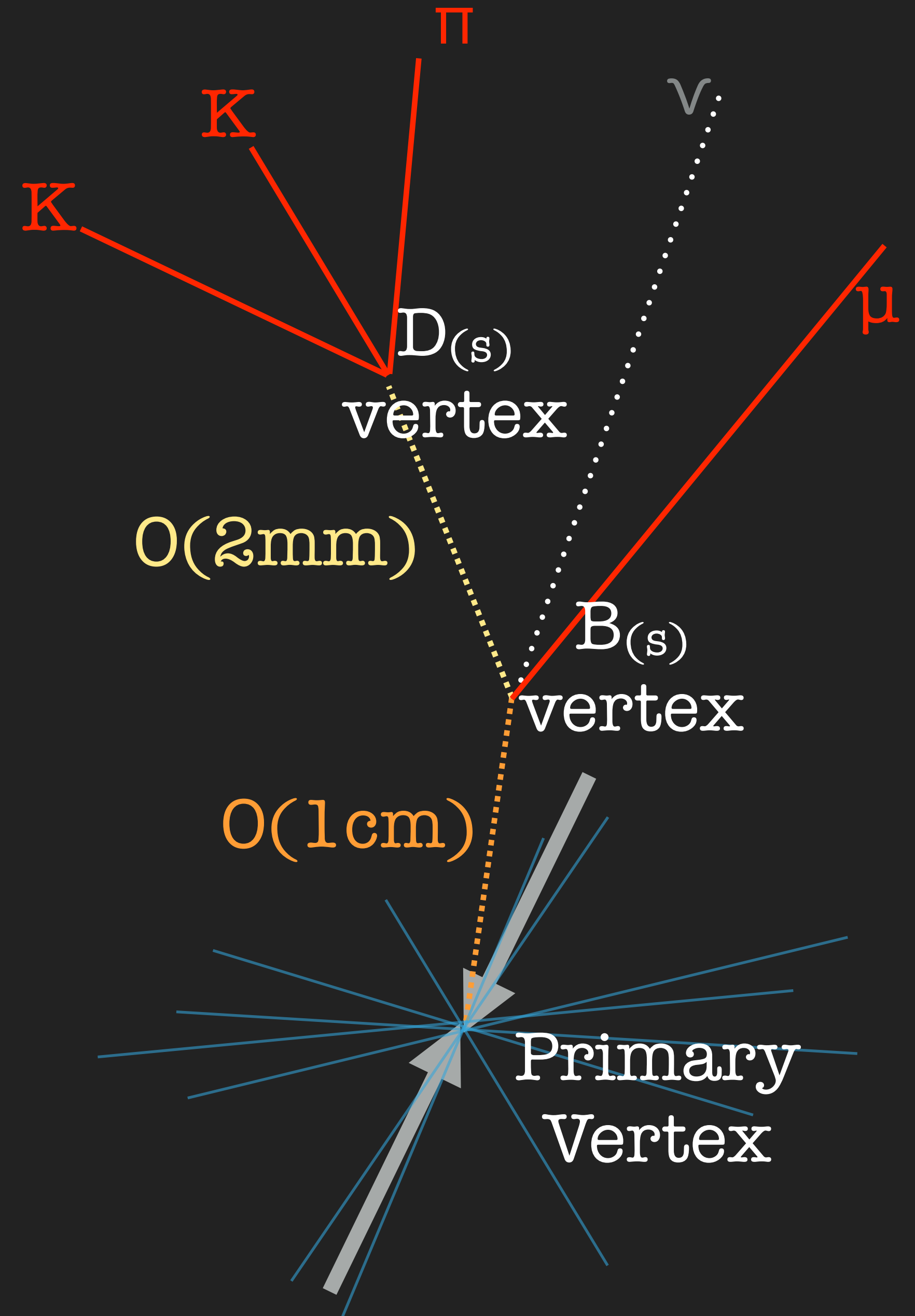
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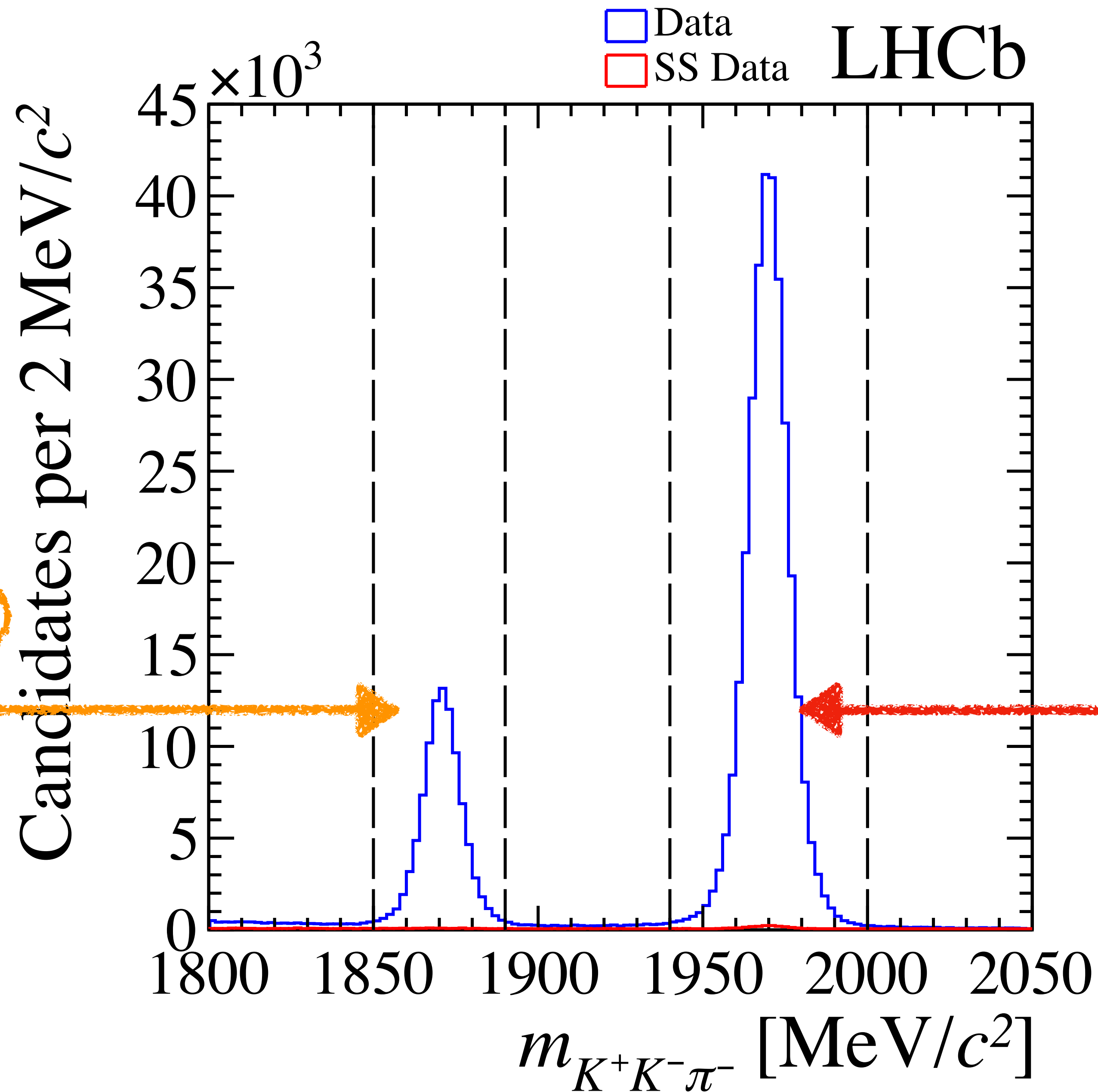
Our result for the $B_s \rightarrow D_s^*$ form factor is the first complete calculation of $h_{A_1}^s(1)$. In the future, measurements of the exclusive decays with a strange spectator, $B_s \rightarrow D_s^{(*)} \ell \nu$, could also provide a constraint on $|V_{cb}|$. LHCb has reconstructed $B_s^0 \rightarrow D_s^{*-} \mu^+ \nu_\mu$ decays [70]. Eventually, with properly normalized branching fractions, these will also provide a method of constraining $|V_{cb}|$.

Signal and normalisation

- Decays $B_{(s)}^0 \rightarrow D_{(s)}^- (\rightarrow K^+ K^- \pi^-) \mu^+ \nu_\mu X$:
 B_s^0 signal and B^0 normalisation.
- Trigger on displaced μ with $p_T > 1.8$ GeV.
- $D_{(s)}^-$ good-quality vertex, displaced from PV.
- Project back the $D_{(s)}^-$ to cross the μ and form a good-quality displaced vertex.
- $m(K^+ K^-)$ 20 MeV around ϕ .



82K $D^- \mu^+$
candidates
(Normalisation)



272K $D_s^- \mu^+$
candidates
(Signal)

Sample composition

Sample components	Efficiency [10^{-3}]	Fraction [%]
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$B_s^0 \rightarrow D_s^- (\rightarrow K^+ K^- \pi^-) \mu^+ \nu_\mu X$

$B_s^0 \rightarrow D_s^- \mu^+ \nu_\mu$ signal

0.481 ± 0.002

30

$B_s^0 \rightarrow D_s^{*-} \mu^+ \nu_\mu$ signal

0.429 ± 0.001

60

B_s^0 feed-down

0.282 ± 0.002

$\mathcal{O}(5)$

B_s^0 semitauonic decays

0.070 ± 0.002

< 1

doubly charmed final states

0.067 ± 0.001

2

B cross-feed

0.130 ± 0.001

2

$B^0 \rightarrow D^- (\rightarrow K^+ K^- \pi^-) \mu^+ \nu_\mu X$

$B^0 \rightarrow D^- \mu^+ \nu_\mu$ signal

0.307 ± 0.001

50

$B^0 \rightarrow D^{*-} \mu^+ \nu_\mu$ signal

0.293 ± 0.001

30

B^0 feed-down

0.104 ± 0.001

9

B^0 semitauonic decays

0.030 ± 0.001

< 1

B^+ decays

0.054 ± 0.001

9

$D_s^- \mu^+$
sample

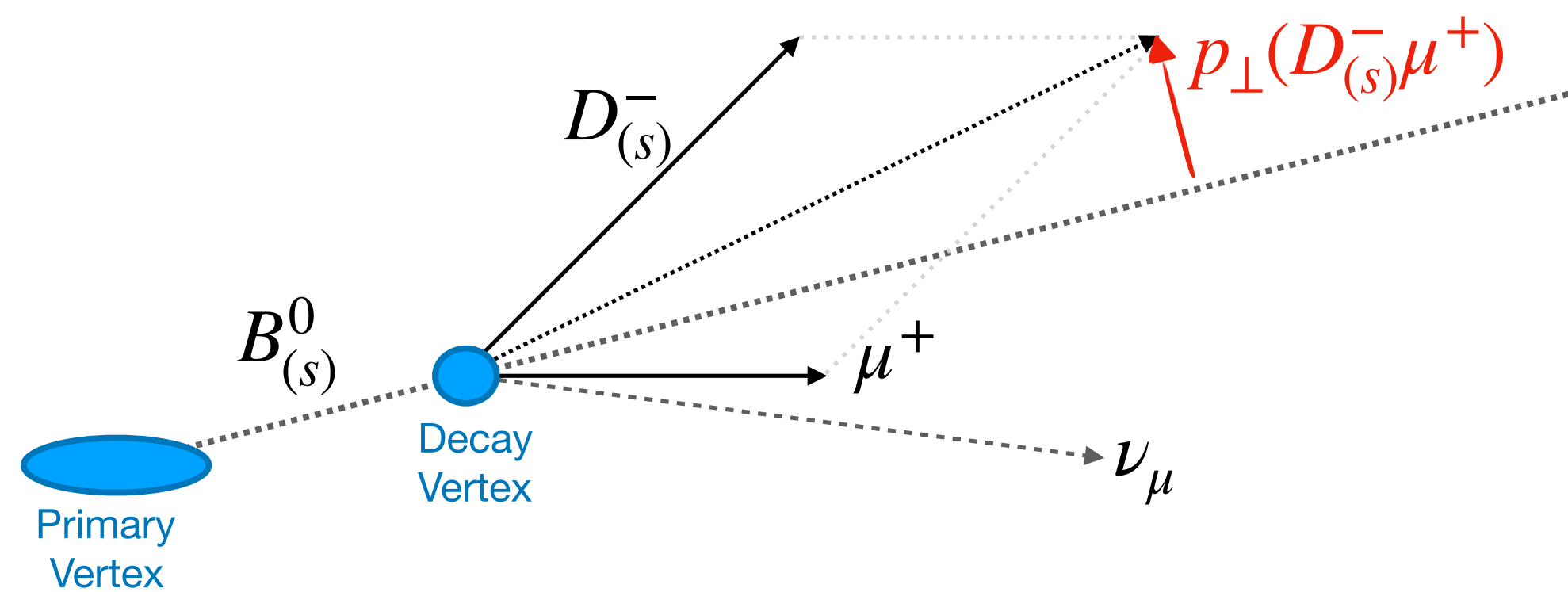
$D^- \mu^+$
sample

90%
signal
decays

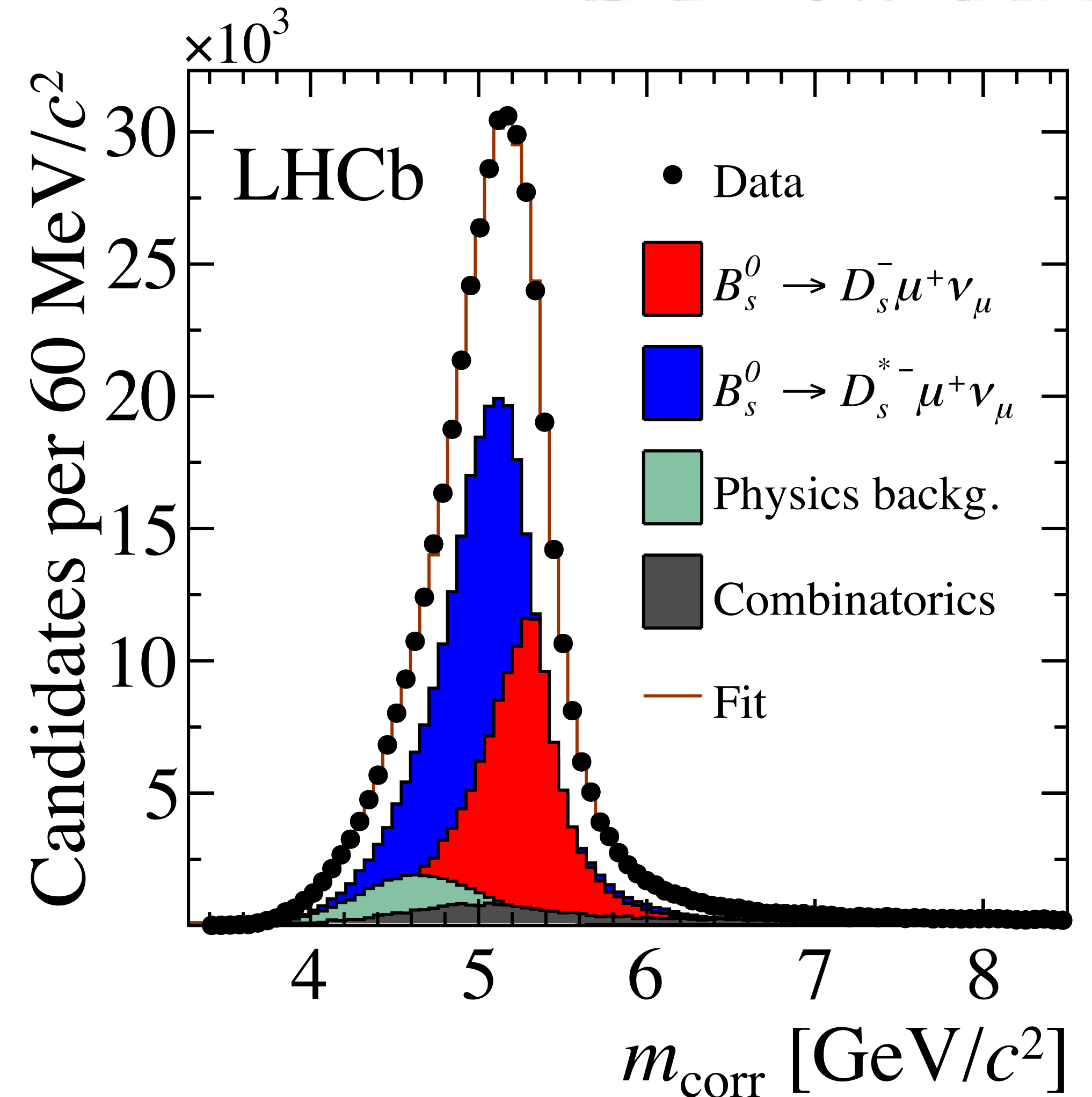
Note: $D_{(s)}^*$ decays not reconstructed

Corrected mass

B direction well measured using primary and decay vertexes. Can recover the missing-mass transverse to the B direction.

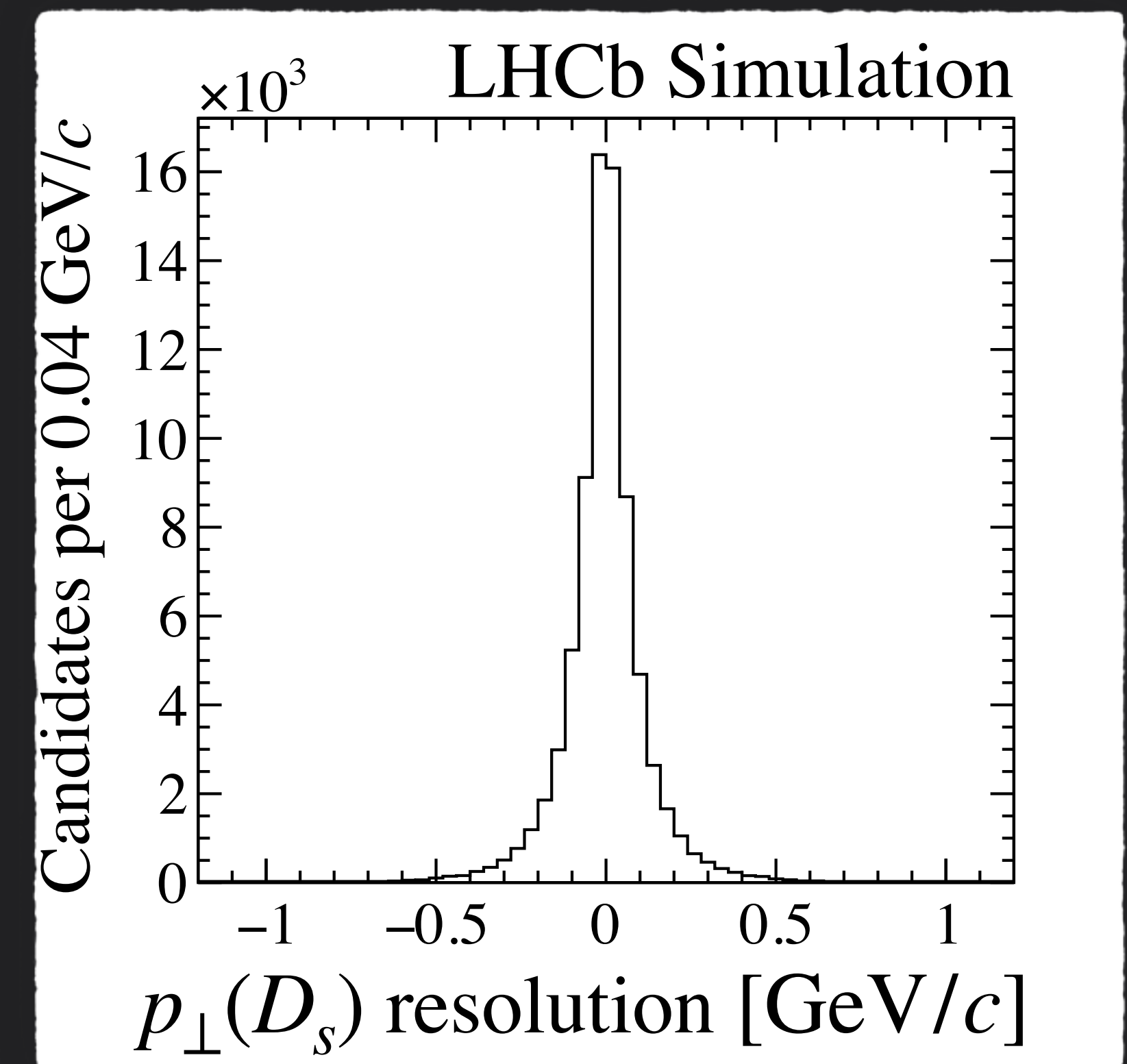
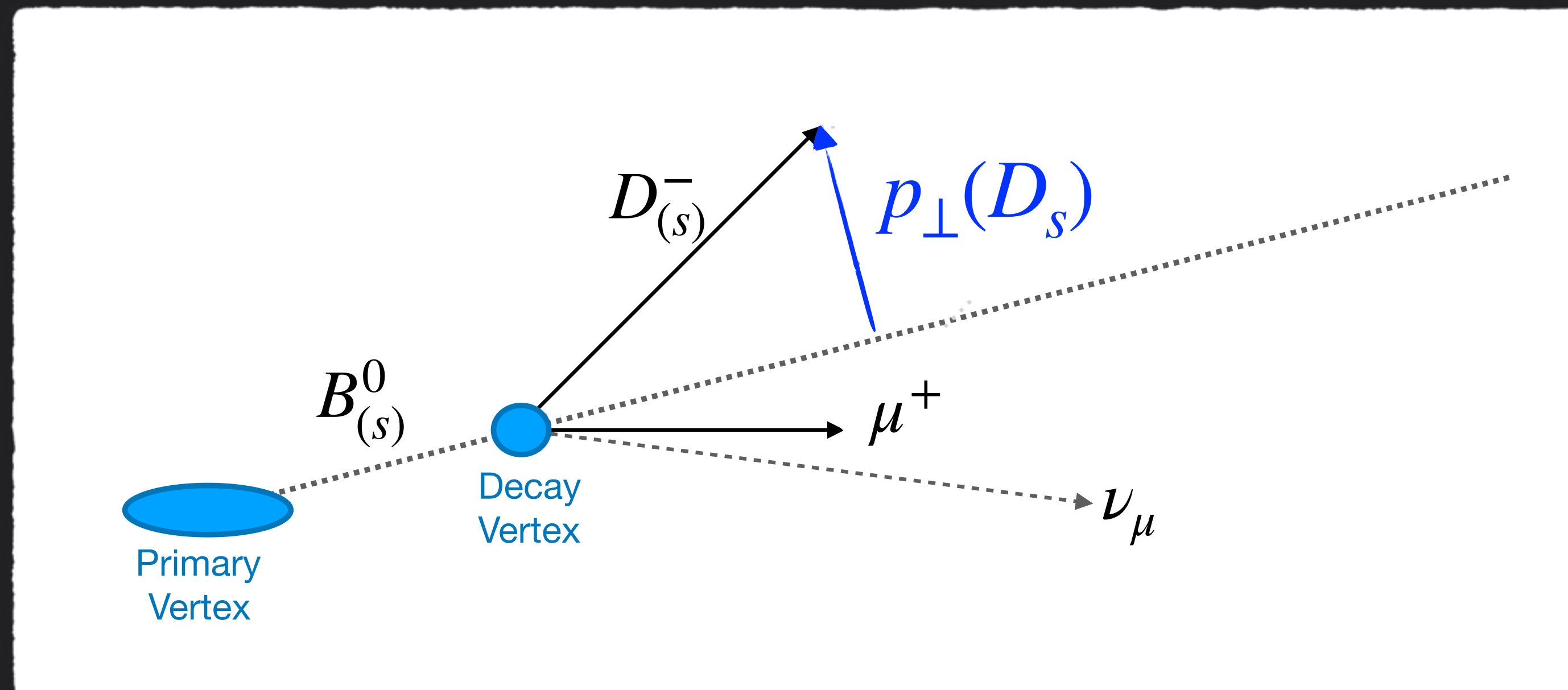


$$m_{corr} \equiv \sqrt{m^2(D_s^- \mu^+) + p_\perp^2(D_s^- \mu^+) + p_\perp(D_s^- \mu^+)}$$

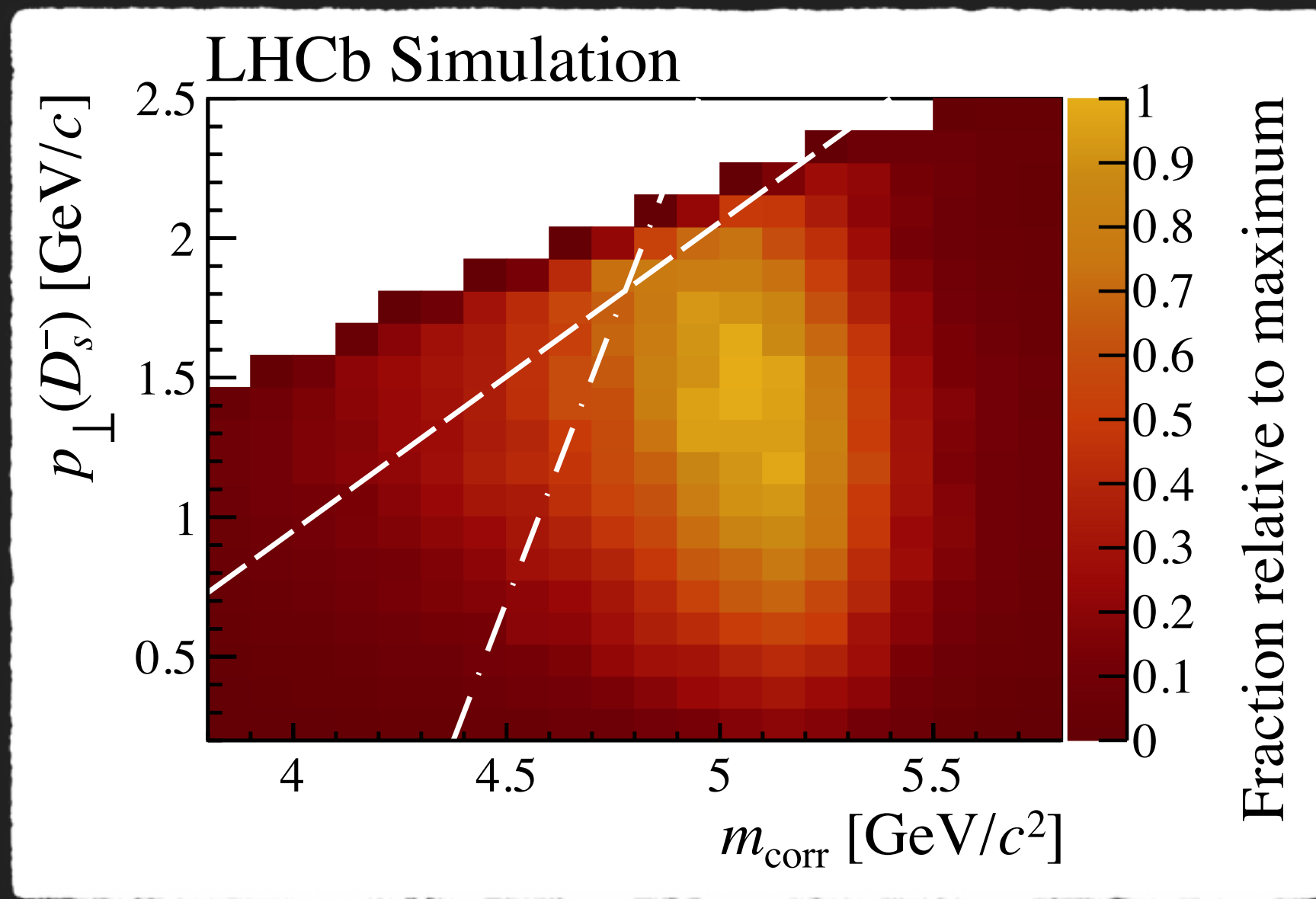


The variable $p_{\perp}(D_s)$

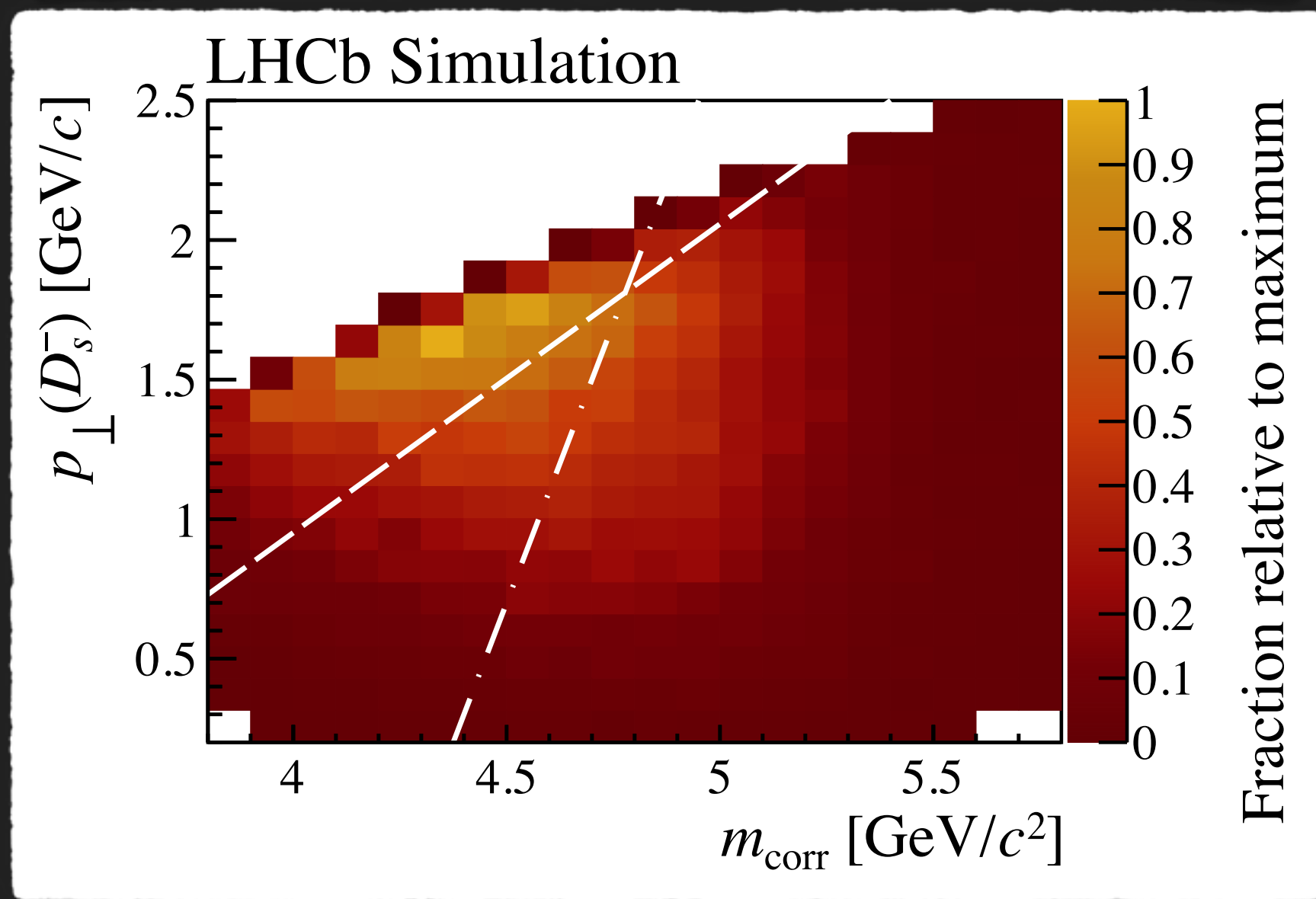
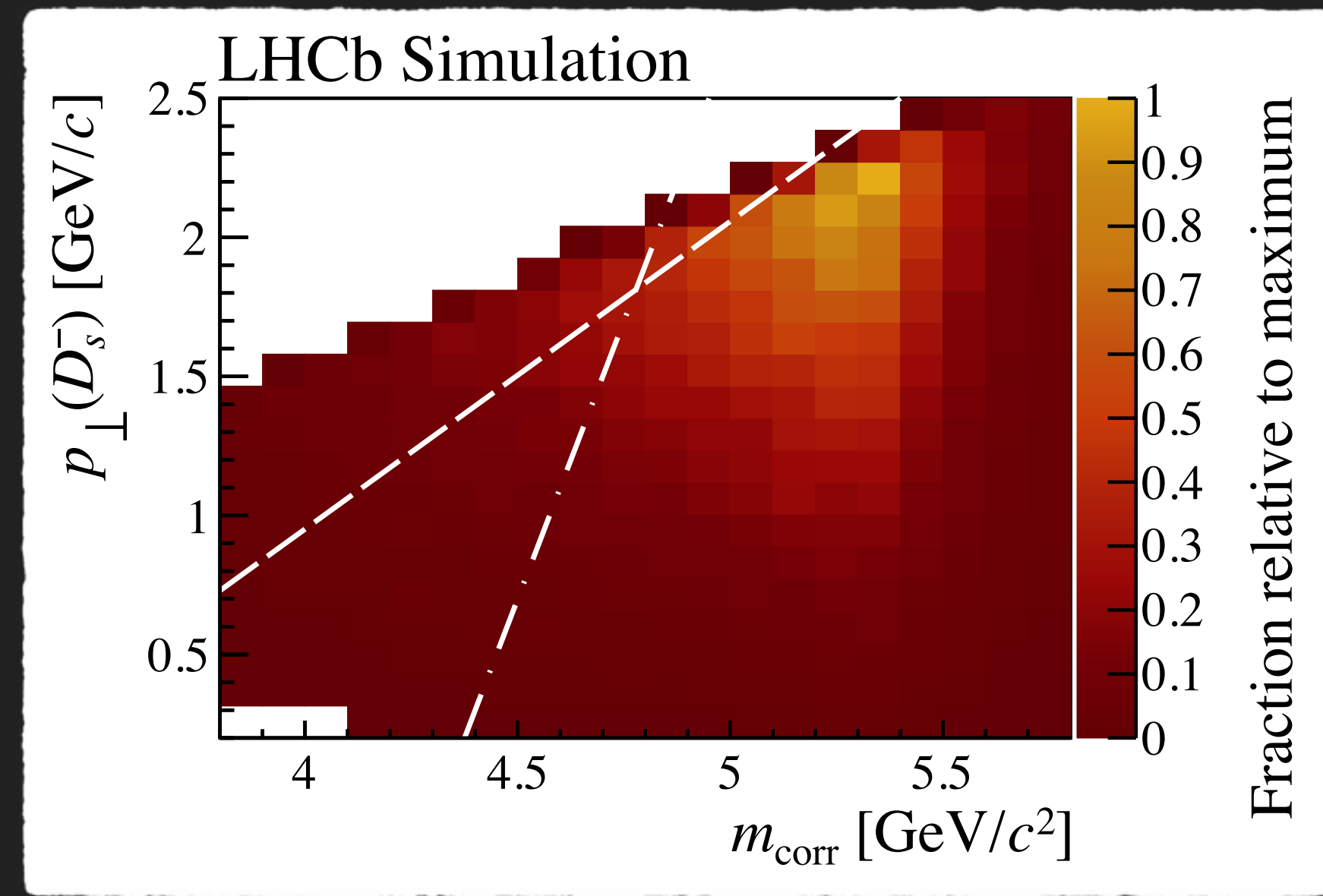
Take the momentum of the D_s transverse to the B_s direction, $p_{\perp}(D_s)$.
Fully reconstructed. Good gaussian resolution (about 120 MeV),
same for $B_s^0 \rightarrow D_s^-$ and $B_s^0 \rightarrow D_s^{*-}$.



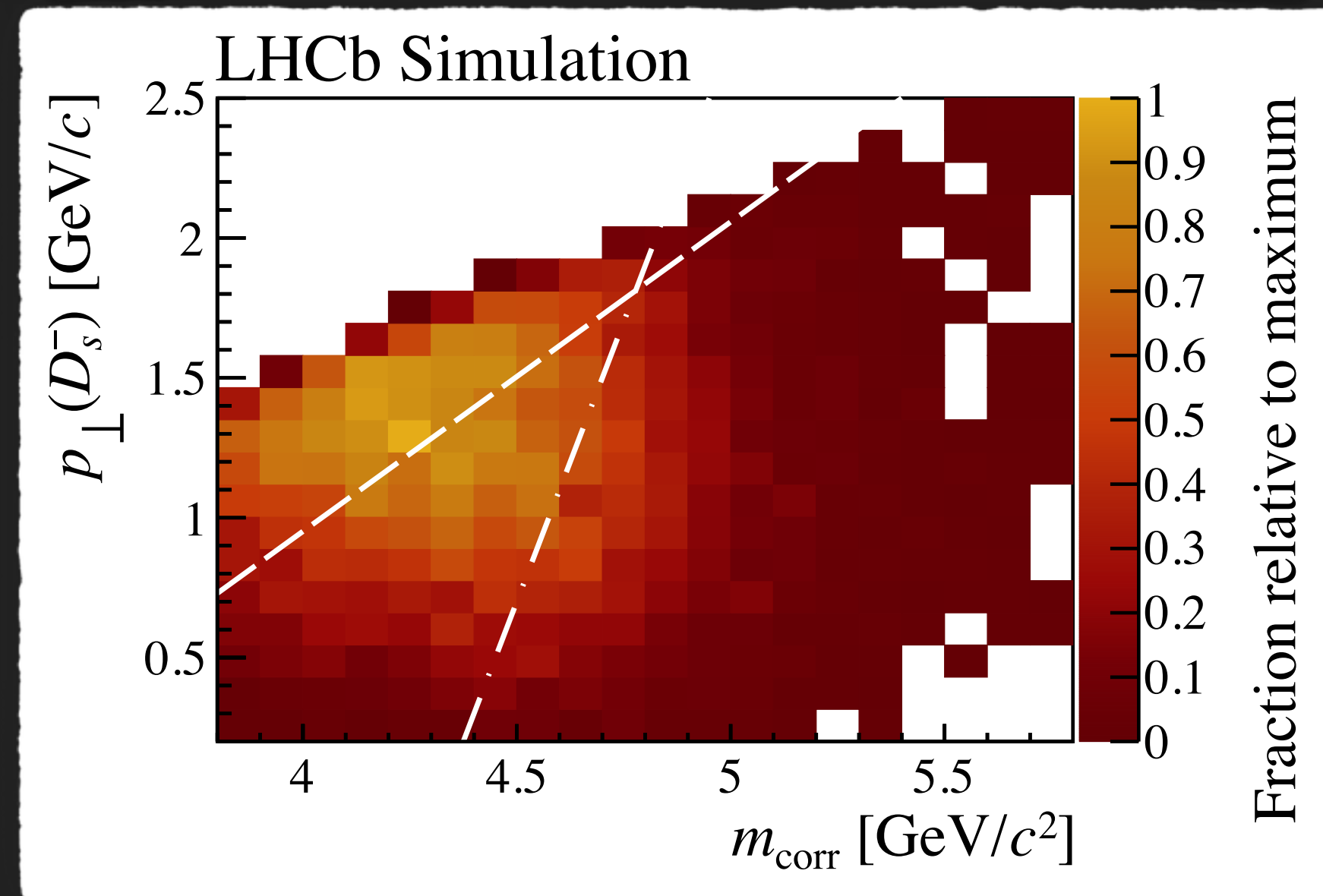
$$B_s \rightarrow D_s^* \mu \nu$$



$$B_s \rightarrow D_s \mu \nu$$



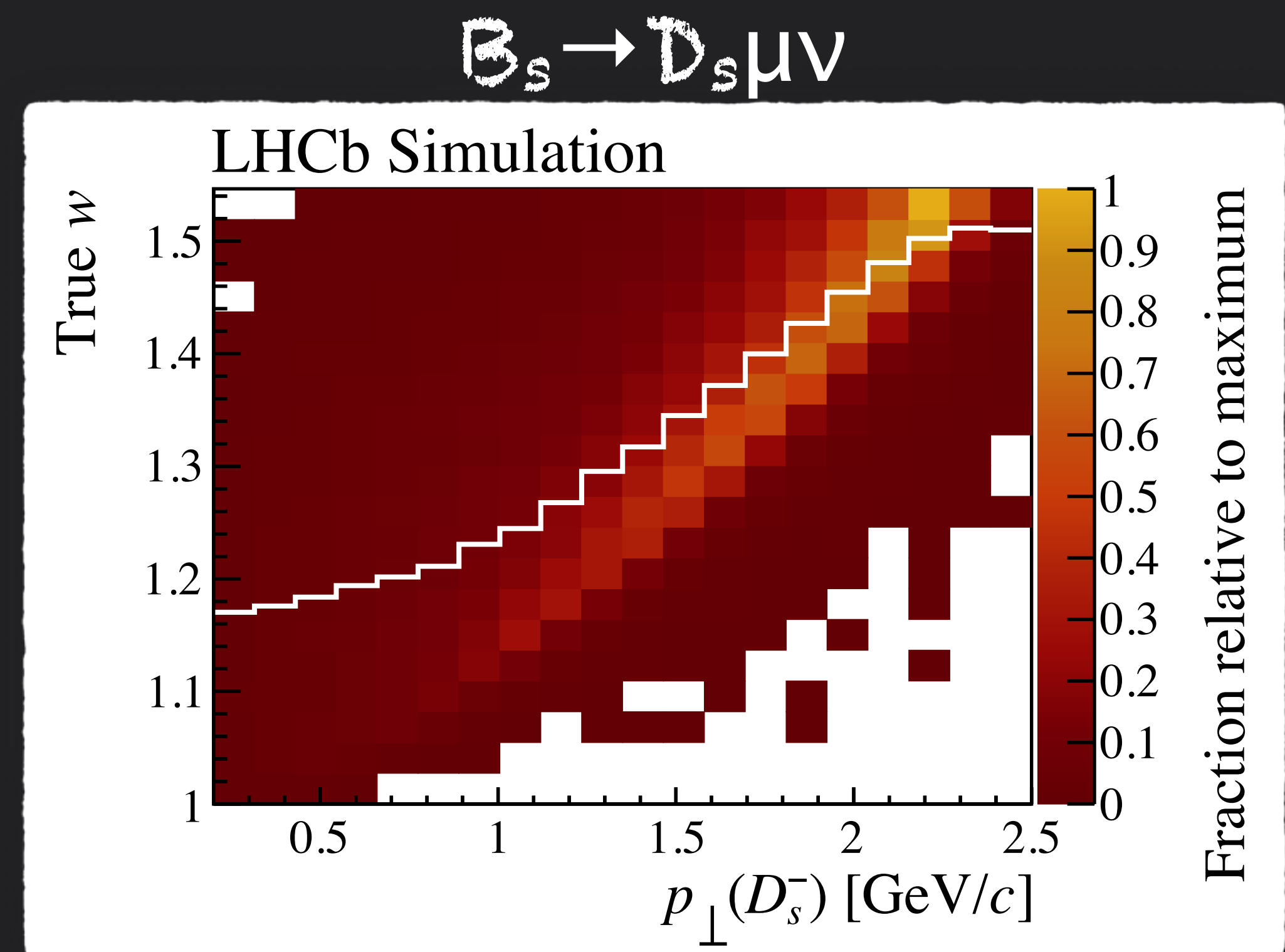
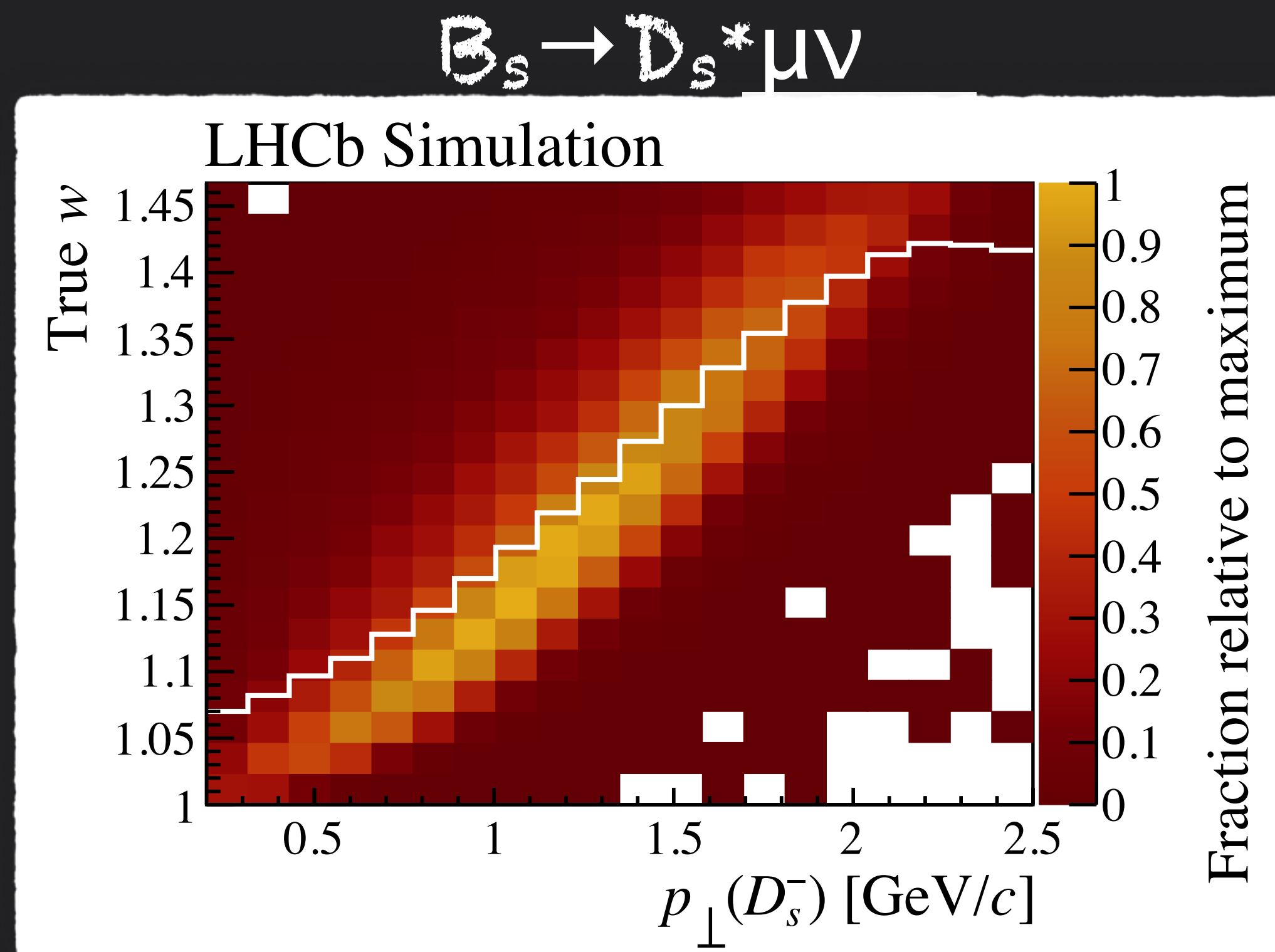
D_s^{**} feed-down +
double charm



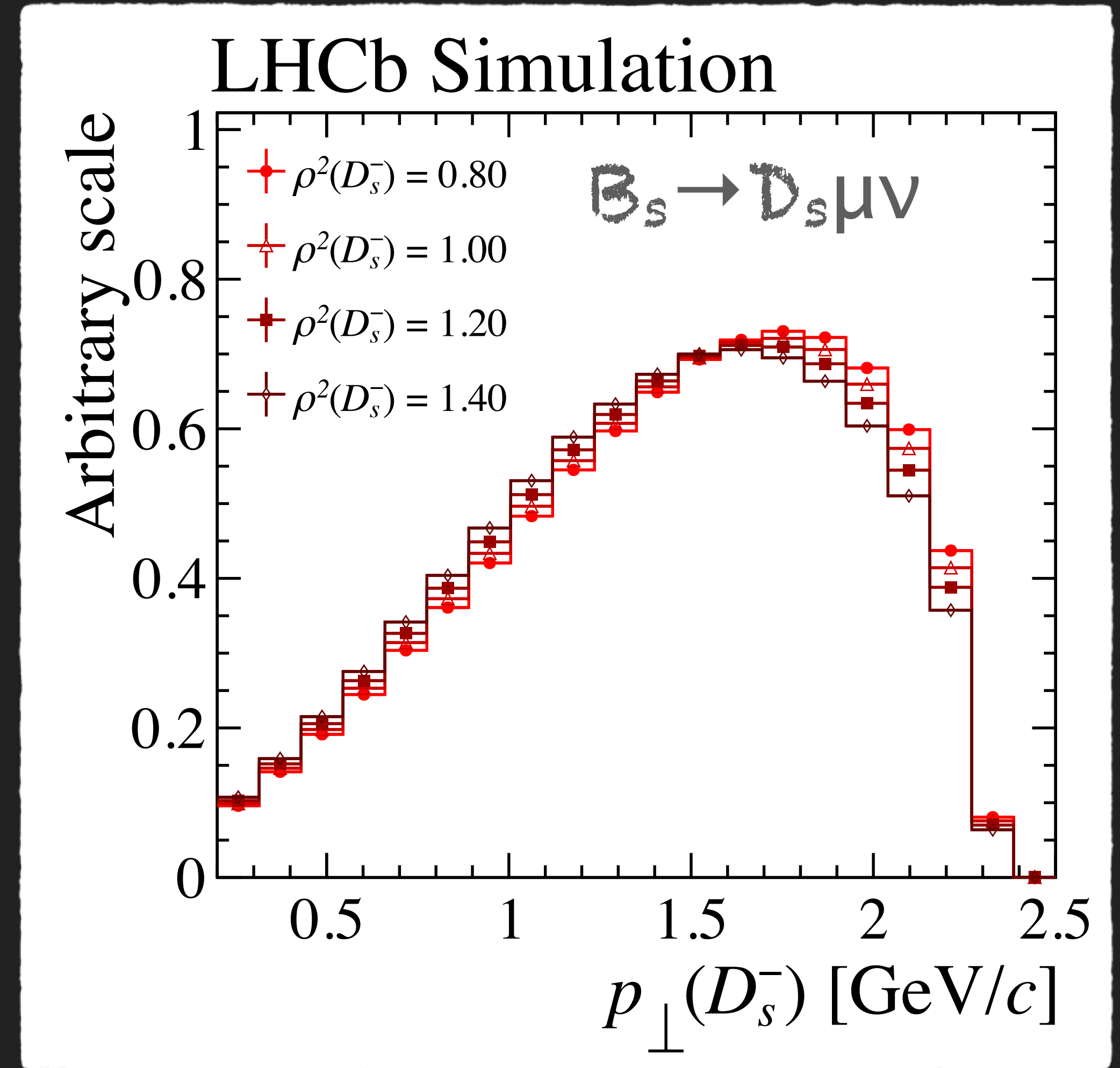
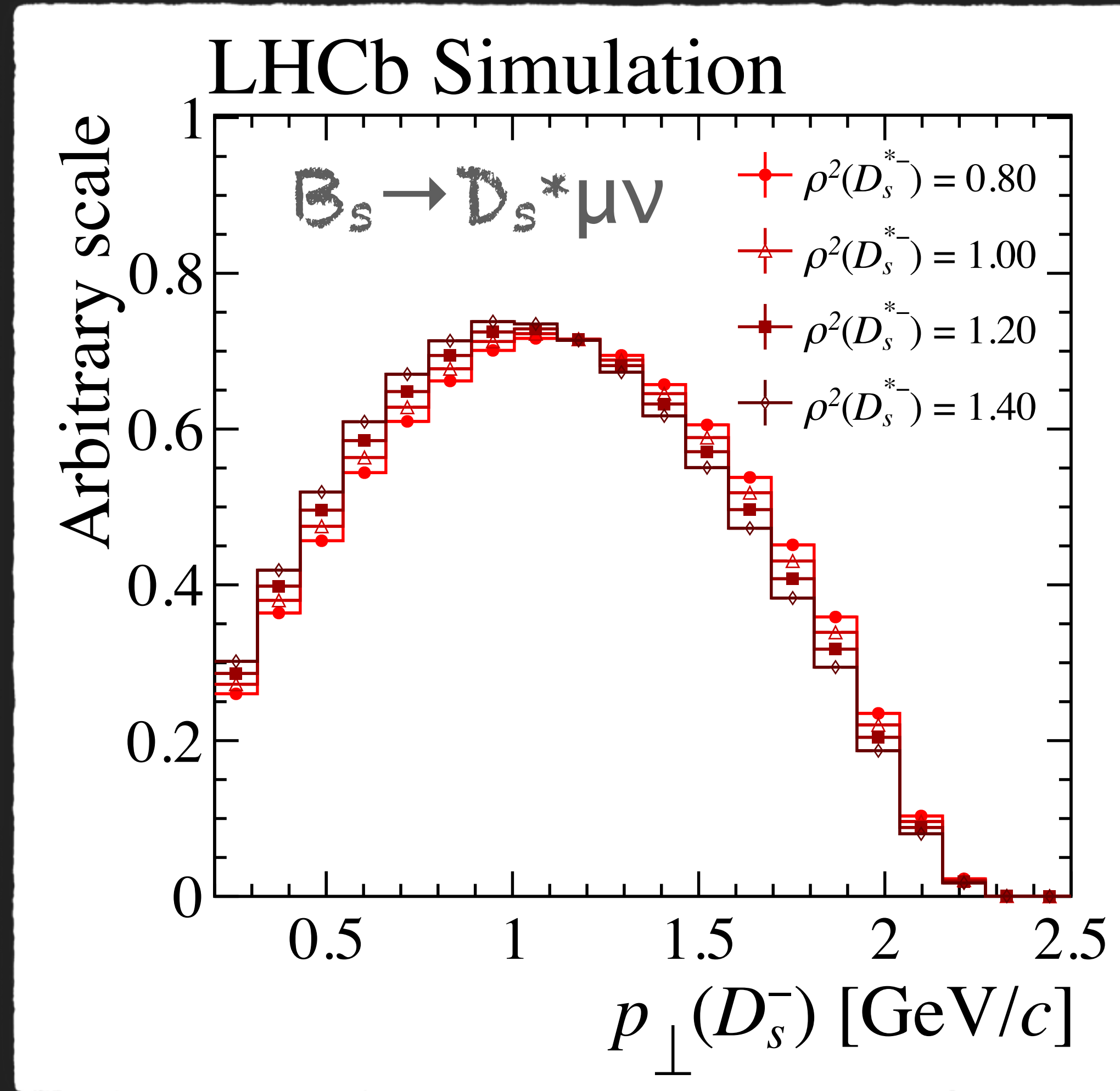
Semitauonic +
Cross-feed

$p_{\perp}(D_s^-)$ correlation with w

Highly correlated with recoil w , it retains the component of the W^* momentum invariant with respect to the B boost. Provides sensitivity to FF.



Sensitivity to form factors



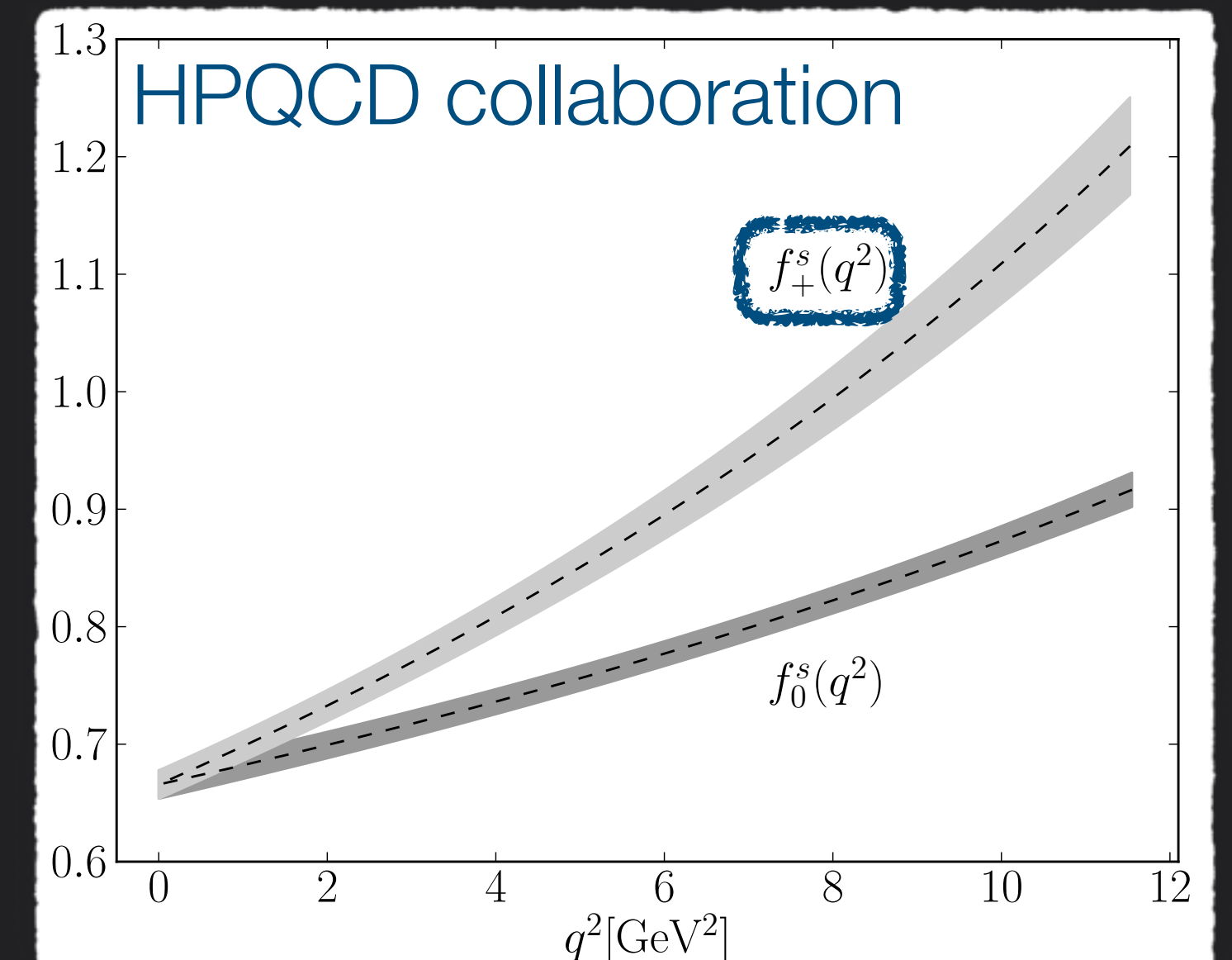
For illustration, dependence on ρ^2 in the CLN parametrisation.

External input (theory)

Use LQCD data for B_s decays to constraint FF

- $B_s \rightarrow D_s^* \mu \nu$ at $w=1$ [PRD 99 (2019) 114512]
- $B_s \rightarrow D_s \mu \nu$ calculations on the full q^2 range [PRD 101 (2020) 074513]
- HPQCD data improve statistical precision on $|V_{cb}|$ by 20% (50%) for CLN (BGL)
- Checked that FF fitted from data w/o constraints are compatible with values from LQCD

Parameter	Value
η_{EW}	1.0066 ± 0.0050
$h_{A_1}(1)$	0.902 ± 0.013
CLN parametrization	
$\mathcal{G}(0)$	1.07 ± 0.04
$\rho^2(D_s^-)$	1.23 ± 0.05
BGL parametrization	
$\mathcal{G}(0)$	1.07 ± 0.04
d_1	-0.012 ± 0.008
d_2	-0.24 ± 0.05



External input (experimental)

Parameter	Value
$f_s/f_d \times \mathcal{B}(D_s^- \rightarrow K^- K^+ \pi^-) \times \tau$ [ps]	0.0191 ± 0.0008
$\mathcal{B}(D^- \rightarrow K^- K^+ \pi^-)$	0.00993 ± 0.00024
$\mathcal{B}(D^{*-} \rightarrow D^- X)$	0.323 ± 0.006
$\mathcal{B}(B^0 \rightarrow D^- \mu^+ \nu_\mu)$	0.0231 ± 0.0010
$\mathcal{B}(B^0 \rightarrow D^{*-} \mu^+ \nu_\mu)$	0.0505 ± 0.0014
B_s^0 mass [GeV/c ²]	5.36688 ± 0.00017
D_s^- mass [GeV/c ²]	1.96834 ± 0.00007
D_s^{*-} mass [GeV/c ²]	2.1122 ± 0.0004

f_s/f_d measured by LHCb,
updated in PRD104 (2021) 032005
with precision improved by ~40%:

$$f_s/f_d \mathcal{B}(D_s^- \rightarrow K^+ K^- \pi^-) \tau = 0.0199 \pm 0.0005 \text{ ps}$$

To obtain B_s^0 branching fractions and get $|V_{cb}|$ from measured signal-to-normalisation ratio of yields (and efficiencies).

Branching-fractions and $|V_{cb}|$

$$\mathcal{B}(B_s^0 \rightarrow D_s^- \mu^+ \nu_\mu) = (2.40 \pm 0.12(\text{stat}) \pm 0.15(\text{syst}) \pm 0.12(\text{ext})) \%$$

$$\mathcal{B}(B_s^0 \rightarrow D_s^{*-} \mu^+ \nu_\mu) = (5.19 \pm 0.24(\text{stat}) \pm 0.47(\text{syst}) \pm 0.19(\text{ext})) \%$$

$$\frac{\mathcal{B}(B_s^0 \rightarrow D_s^- \mu^+ \nu_\mu)}{\mathcal{B}(B_s^0 \rightarrow D_s^{*-} \mu^+ \nu_\mu)} = 0.464 \pm 0.013(\text{stat}) \pm 0.043(\text{syst})$$

$$|V_{cb}|_{\text{CLN}} = (40.8 \pm 0.6(\text{stat}) \pm 0.9(\text{syst}) \pm 1.1(\text{ext})) \times 10^{-3}$$

$$|V_{cb}|_{\text{BGL}} = (41.7 \pm 0.8(\text{stat}) \pm 0.9(\text{syst}) \pm 1.1(\text{ext})) \times 10^{-3}$$

Systematic uncertainty dominated by knowledge of $D_{(s)} \rightarrow KK\pi$ Dalitz structure and background contamination.

Form-factor results

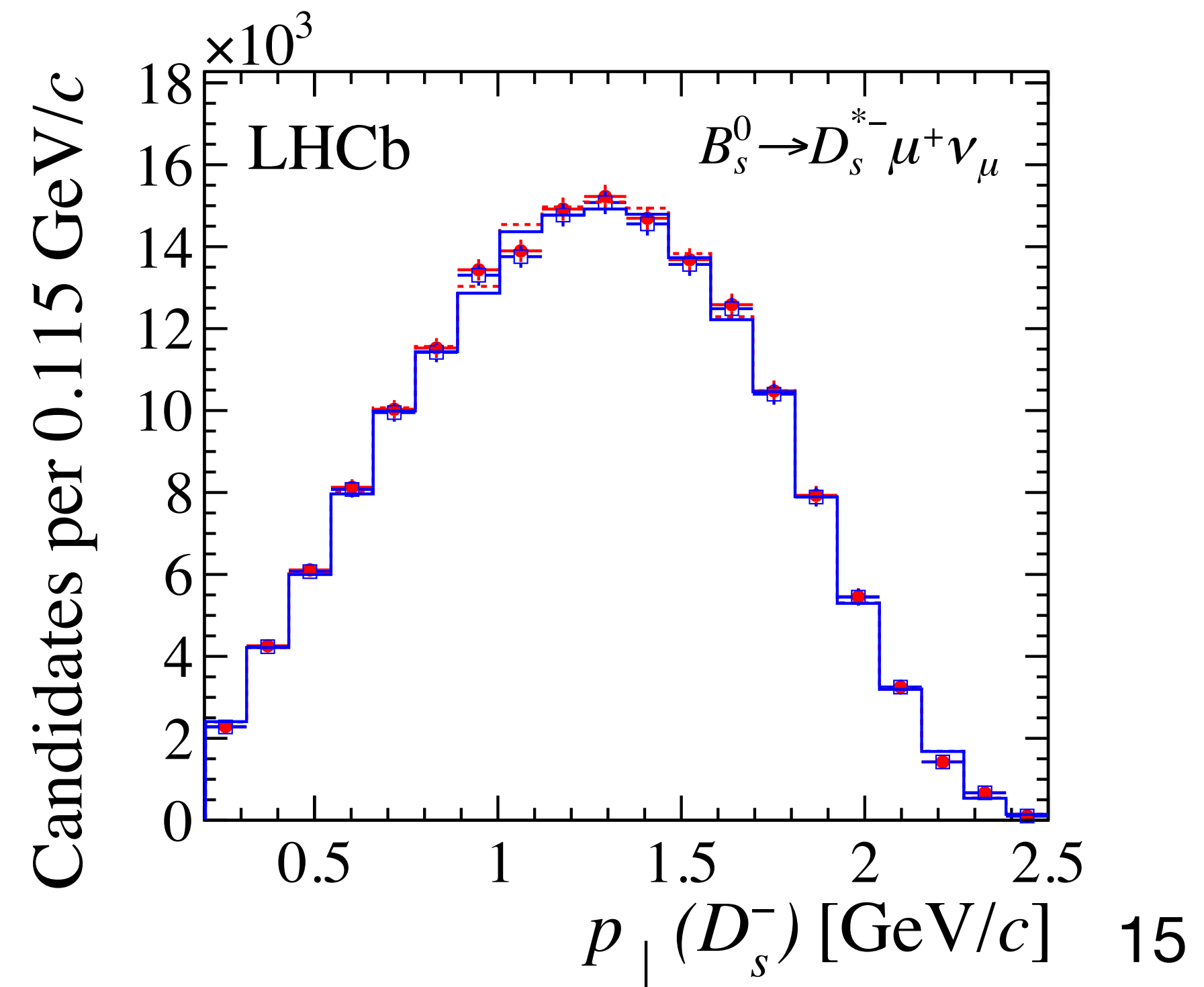
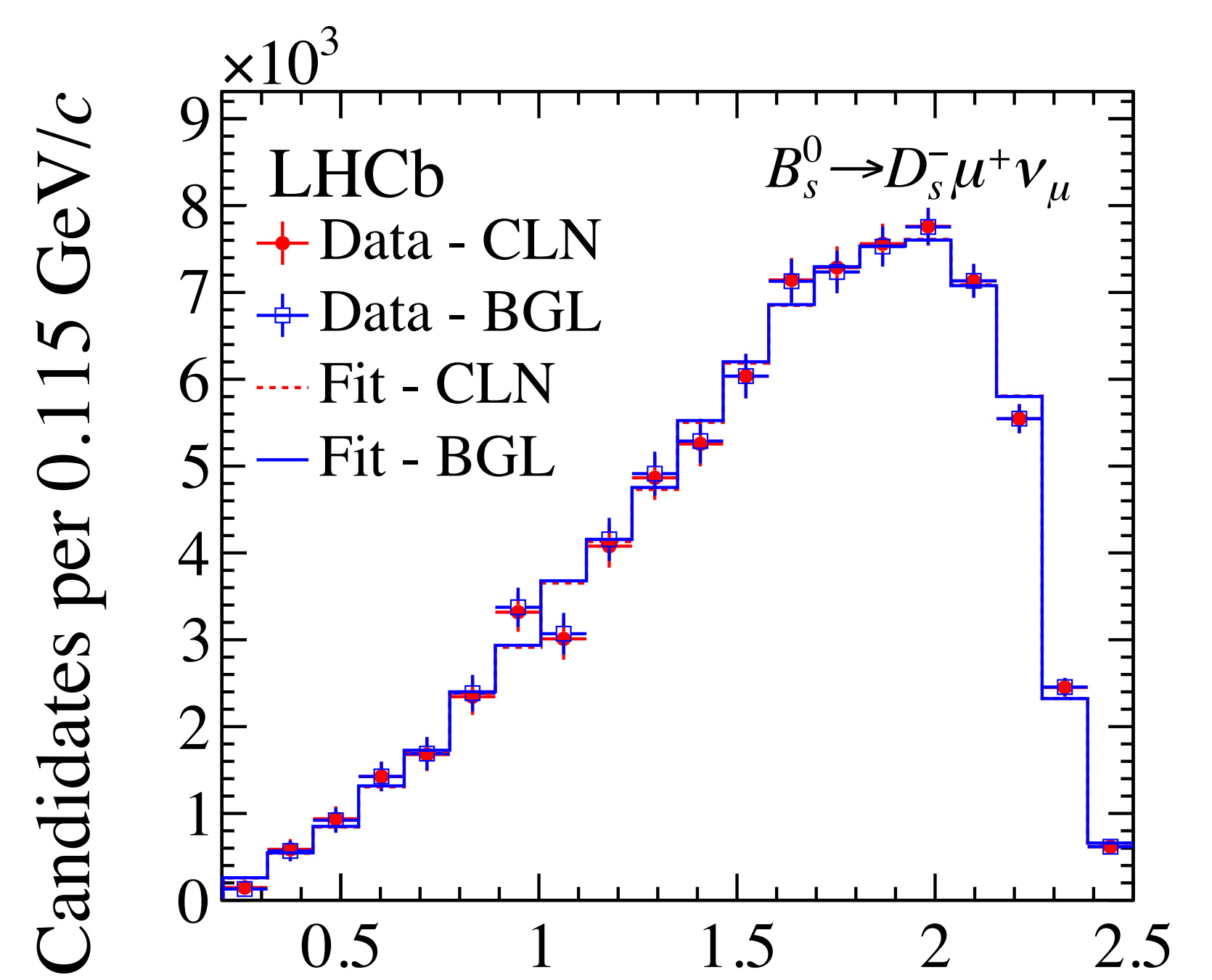
CLN parametrisation

Parameter	Value		
$\mathcal{G}(0)$	1.102 ± 0.034 (stat)	± 0.004 (ext)	
$\rho^2(D_s^-)$	1.27 ± 0.05 (stat)	± 0.00 (ext)	
$\rho^2(D_s^{*-})$	1.23 ± 0.17 (stat)	± 0.01 (ext)	
$R_1(1)$	1.34 ± 0.25 (stat)	± 0.02 (ext)	
$R_2(1)$	0.83 ± 0.16 (stat)	± 0.01 (ext)	

BGL parametrisation (order 2-111)

Parameter	Value		
$\mathcal{G}(0)$	1.097 ± 0.034 (stat)	± 0.001 (ext)	
d_1	-0.017 ± 0.007 (stat)	± 0.001 (ext)	
d_2	-0.26 ± 0.05 (stat)	± 0.00 (ext)	
b_1	-0.06 ± 0.07 (stat)	± 0.01 (ext)	
a_0	0.037 ± 0.009 (stat)	± 0.001 (ext)	
a_1	0.28 ± 0.26 (stat)	± 0.08 (ext)	
c_1	0.0031 ± 0.0022 (stat)	± 0.0006 (ext)	

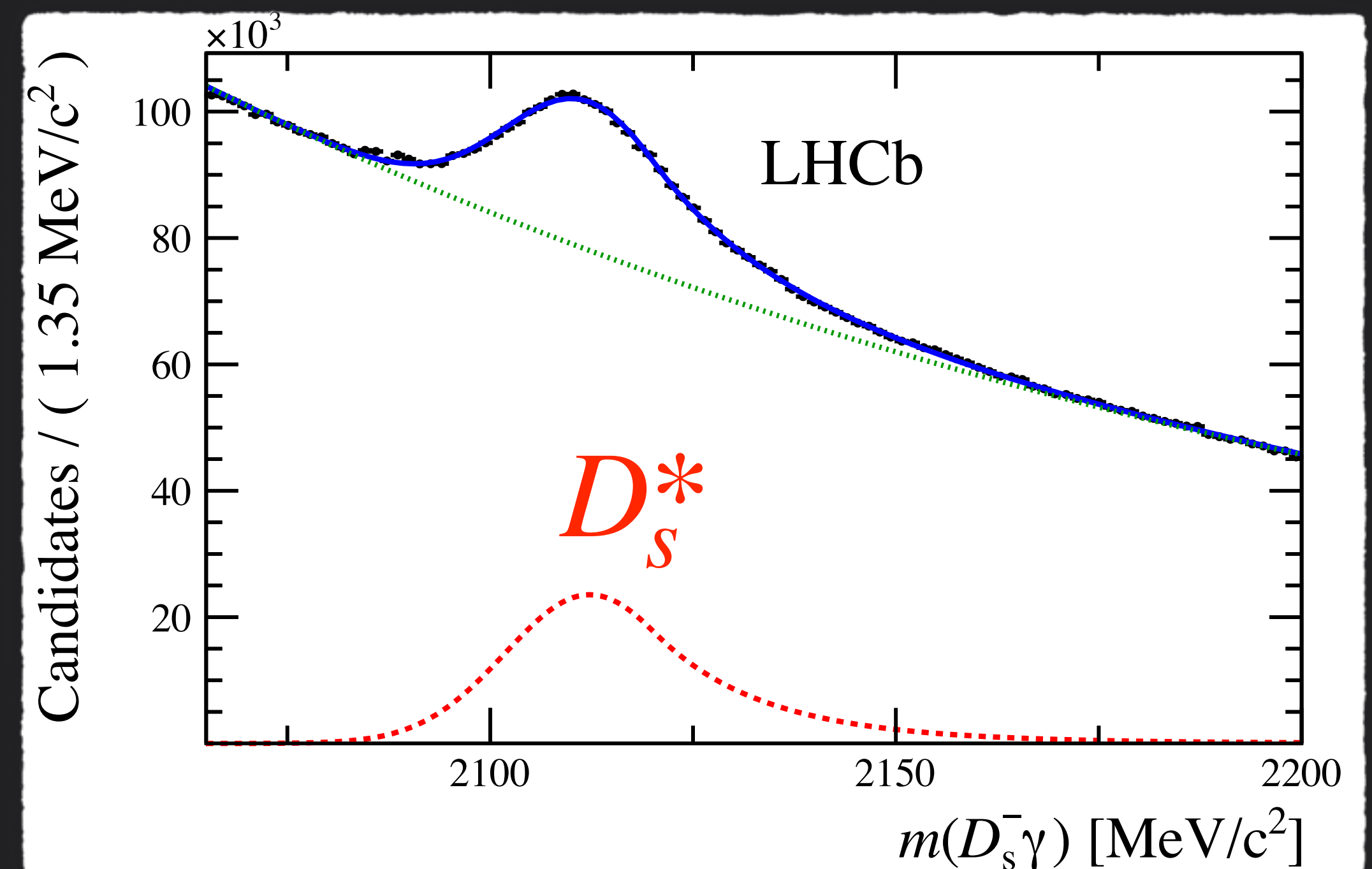
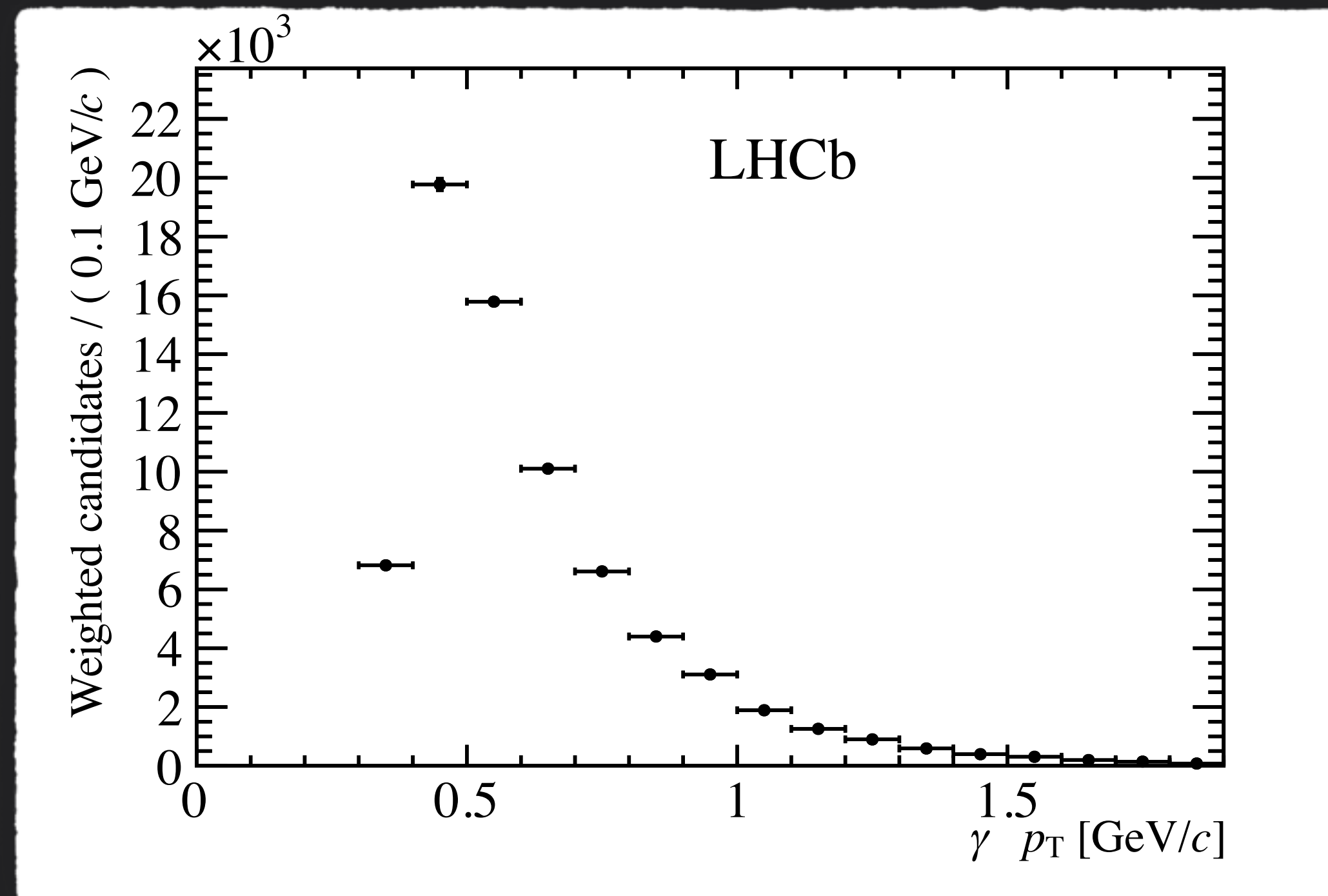
Parameters definition in backup



Supporting the form factors

- Measure the w distribution for $B_s^0 \rightarrow D_s^{*-} \mu^+ \nu_\mu$ decays.
- Independent data set (Run II). Fully reconstruct the $D_s^{*-} \rightarrow D_s^- \gamma$ by selecting the soft photon in a cone around the D_s flight direction.

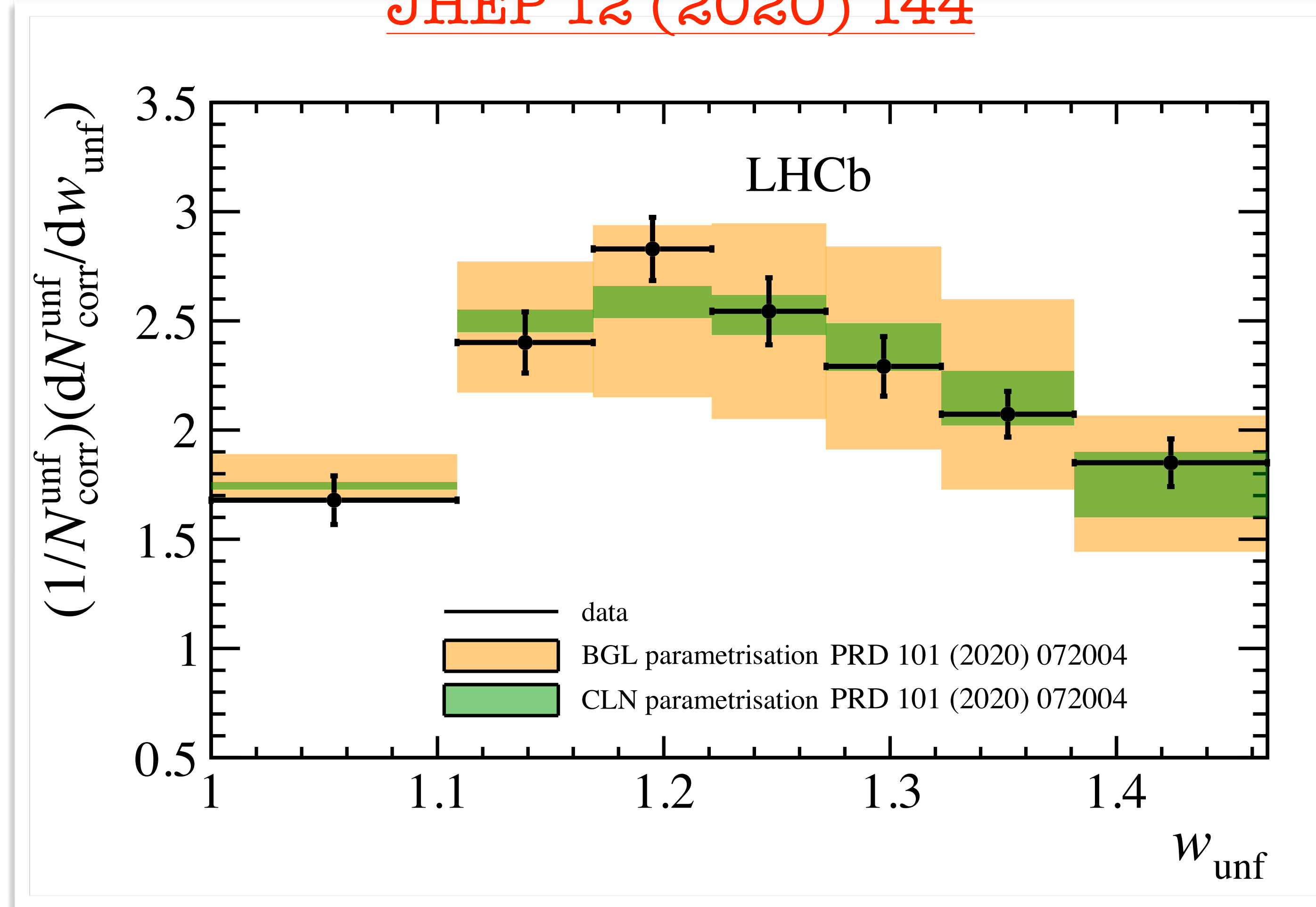
[JHEP 12 \(2020\) 144](#)



w distribution for $B_s^0 \rightarrow D_s^{*-} \mu^+ \nu_\mu$

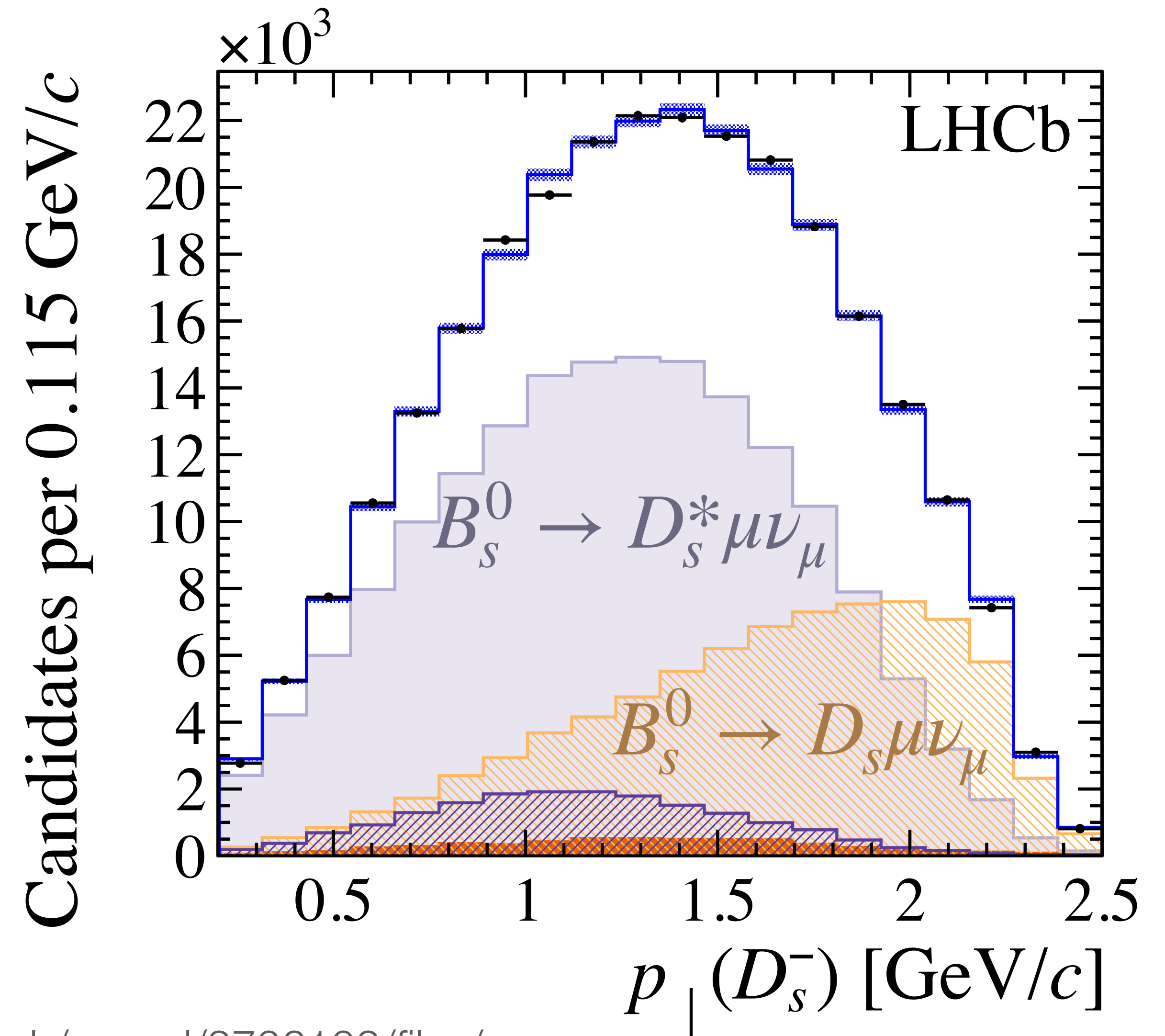
- Use a MVA based algorithm to approximate w [JHEP 02 (2017) 021].
- Fit the corrected mass in bins of the approximate w .
- Unfold efficiency and resolution.
- Good agreement of the measured distribution w.r.t. form factors measured in the $|V_{cb}|$ analysis

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Could $p_{\perp}(D_s)$ data be used?

- Several phenomenological analyses generate $d\Gamma/dw$ from fit results.
- Could $p_{\perp}(D_s)$ data be directly used?
- LHCb provides $p_{\perp}(D_s)$ resolutions and efficiencies. Once a theoretical prediction of $p_{\perp}(D_s)$ is provided, can fold in experimental effects and compare (or fit) to LHCb data.*



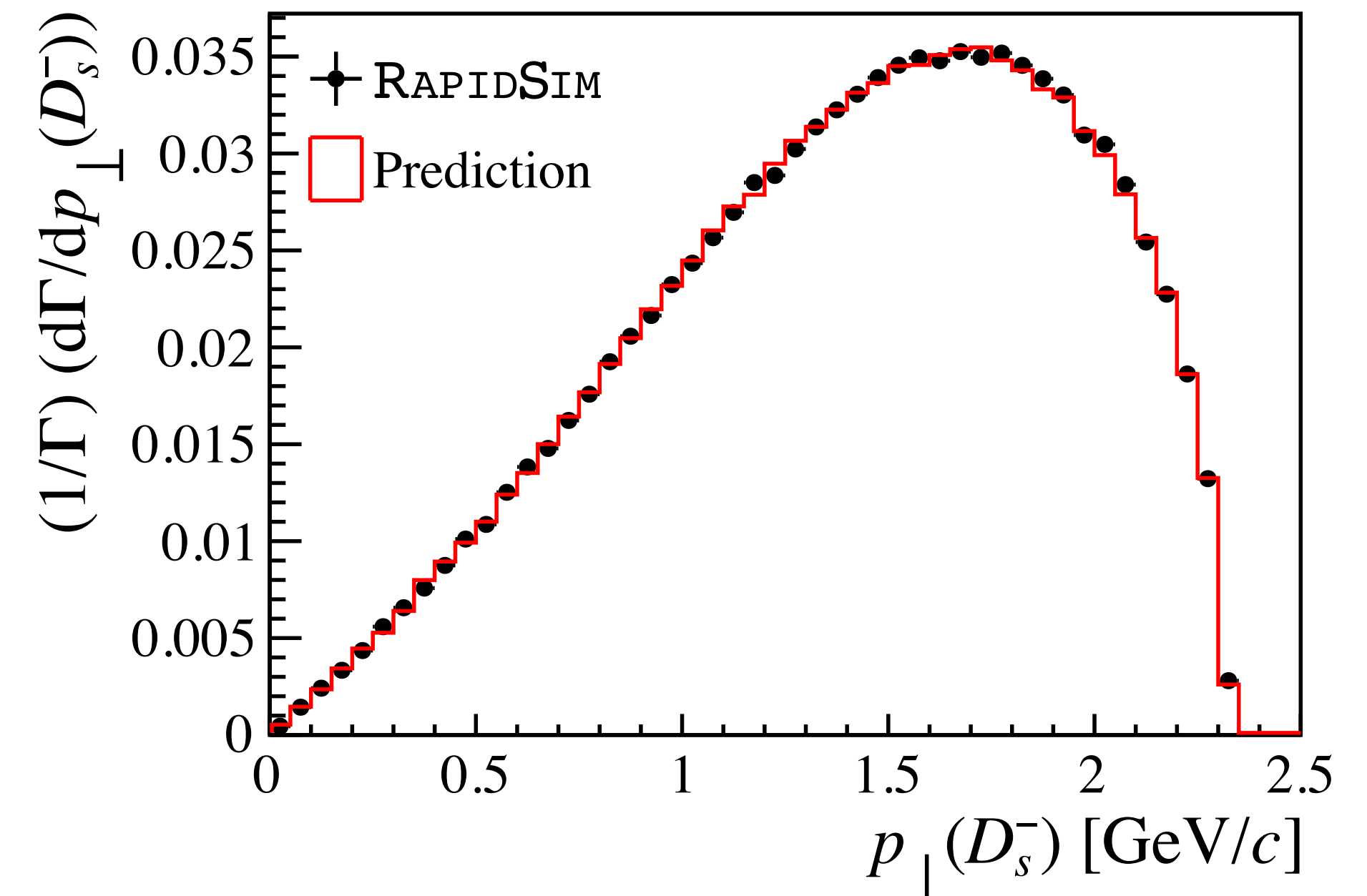
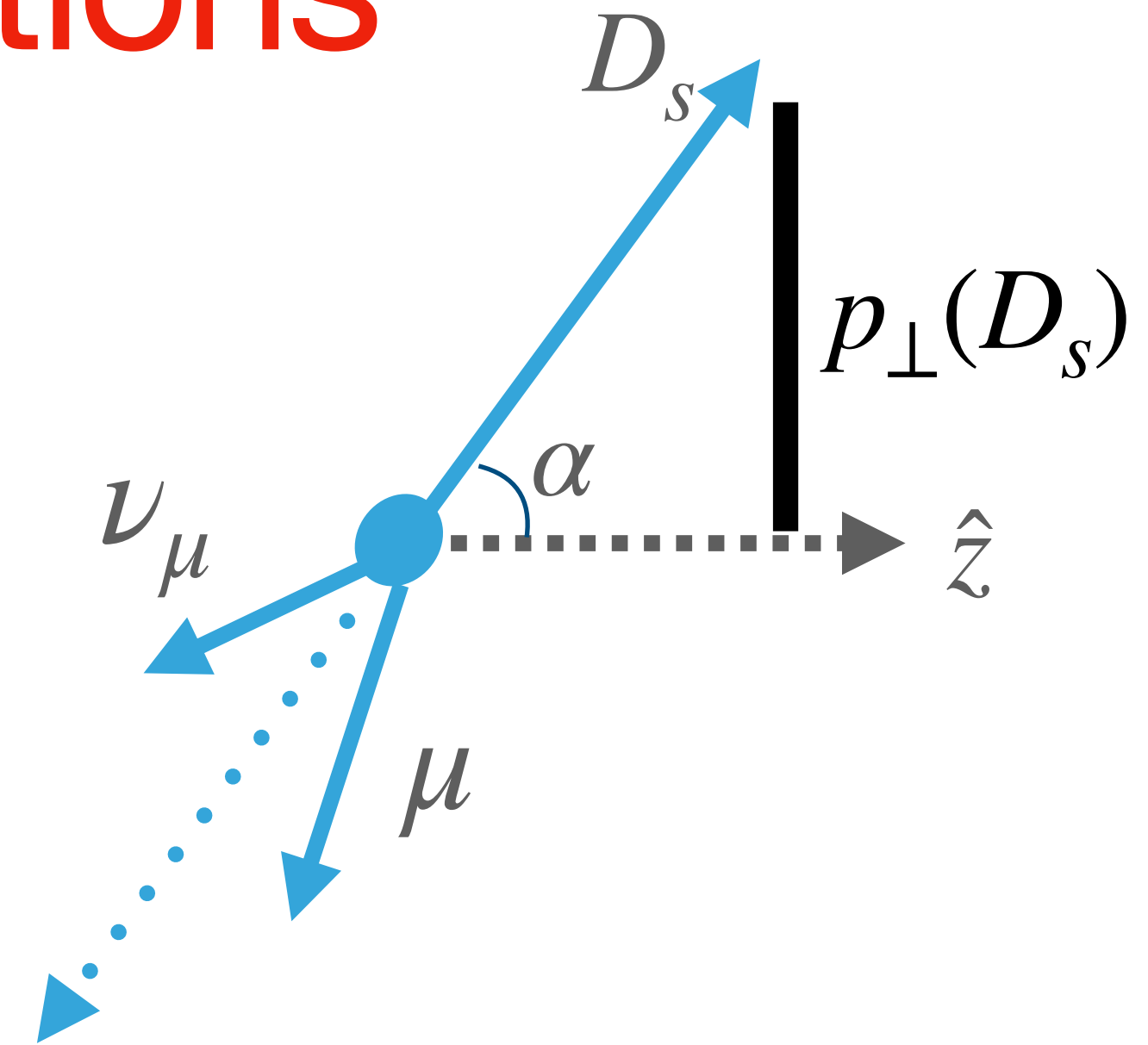
*all provided by LHCb as root macros at <https://cds.cern.ch/record/2706102/files/>

Determining the $p_{\perp}(D_s)$ distributions

- Consider $B_s^0 \rightarrow D_s^- \mu^+ \nu_{\mu}$. In the B_s^0 rest frame, define an arbitrary direction \hat{z}

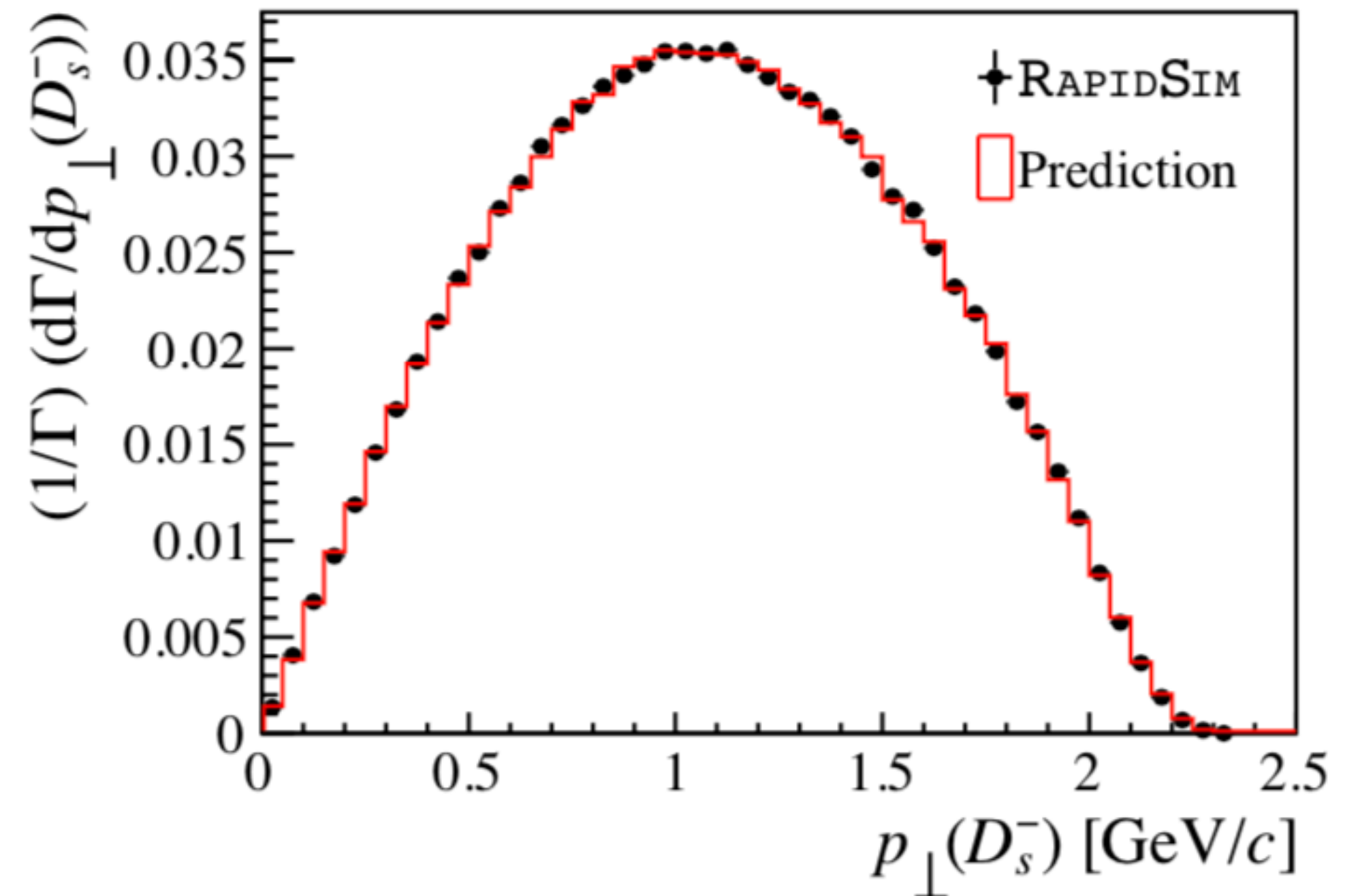
$$p_{\perp}(D_s) = m_{D_s} \sqrt{w^2 - 1} \sin \alpha$$

- Consider \hat{z} along B_s^0 momentum in the lab frame, $p_{\perp}(D_s)$ is invariant.
- Angle α is not measured, integrate over all possible value ($\cos \alpha$ uniform in $[-1, 1]$ since B_s^0 is spin 0)
- Can obtain $p_{\perp}(D_s)$ distribution from $d\Gamma/dw$



In preparation

- Similar calculations for $B_s^0 \rightarrow D_s^{*-} \mu^+ \nu_\mu$ decays, but $p_\perp(D_s)$ depends also on helicity angle $\cos \theta_{D_s}$ when the D_s^* decays is not reconstructed.
- Work in progress (A. Di Canto, M.D., F. Ferrari, S. Jaiswal, N. Soumitra, S. Patra) to reanalyse $p_\perp(D_s)$ data (together with LHCb w measurement and new lattice data).



Conclusion

- Proved that $|V_{cb}|$ could be accessed also in hadron collisions.
Need synergies with B-factories (to get precise normalisation).
Measurement systematically limited: need to improve on f_s/f_d , BR and Dalitz model of $D_{(s)} \rightarrow KK\pi$ decays, knowledge of D_s^{**} ...
- On the other hand, form-factor studies (w distribution, helicity angles?) statistically limited. Could improve with Run 3 data and provide excellent testbeds for calculations (not only for B_s^0 decays).
- Could study baryons too. Λ_b analyses ongoing.

Backup

CLN reminder

- Reminder of the signal parameters, along with $|V_{cb}|$, in the CLN model

$B \rightarrow D^* \mu \nu$

$$h_{A_1}(w) = h_{A_1}(1) [1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3] ,$$
$$R_1(w) = R_1(1) - 0.12(w - 1) + 0.05(w - 1)^2 ,$$
$$R_2(w) = R_2(1) - 0.11(w - 1) - 0.06(w - 1)^2 ,$$

3 floating parameters

$B \rightarrow D \mu \nu$

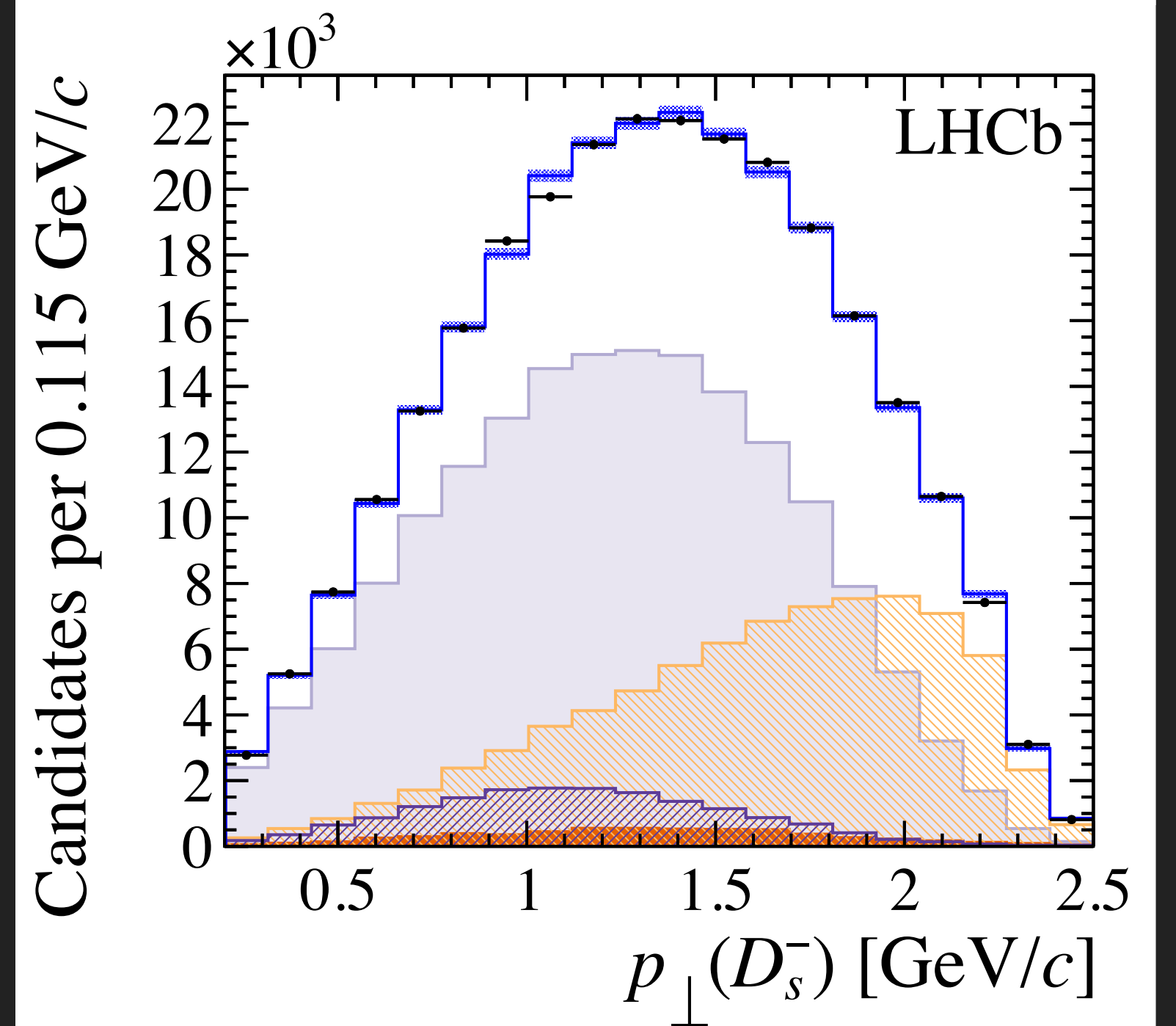
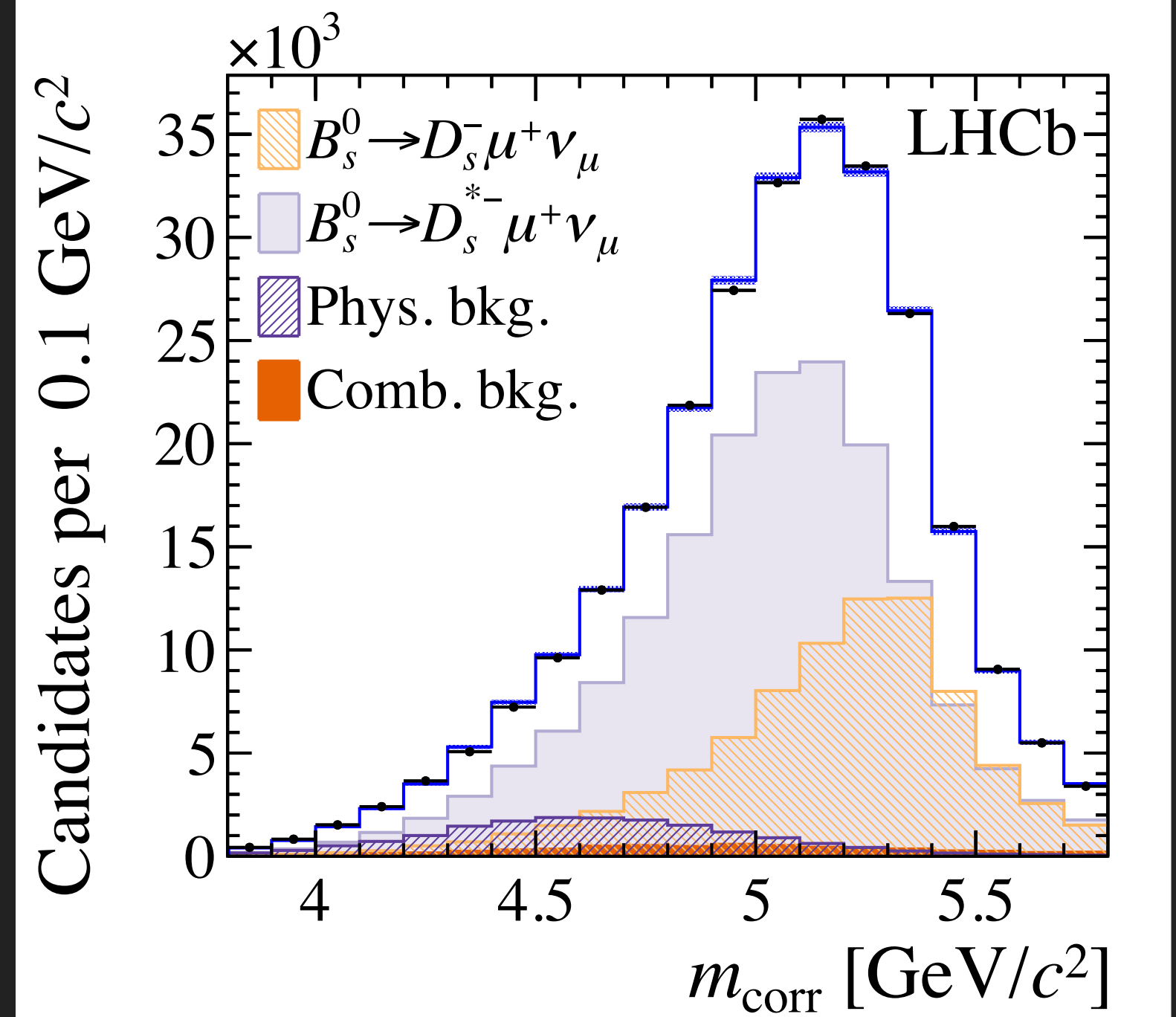
$$\mathcal{G}(z) = \mathcal{G}(0) [1 - 8\rho^2 z + (51\rho^2 - 10)z^2 - (252\rho^2 - 84)z^3]$$

3 constrained
from LQCD

Results — CLN

Parameter	Value			
$ V_{cb} [10^{-3}]$	41.4	± 0.6	(stat) ± 1.2	(ext)
$\mathcal{G}(0)$	1.102	± 0.034	(stat) ± 0.004	(ext)
$\rho^2(D_s^-)$	1.27	± 0.05	(stat) ± 0.00	(ext)
$\rho^2(D_s^{*-})$	1.23	± 0.17	(stat) ± 0.01	(ext)
$R_1(1)$	1.34	± 0.25	(stat) ± 0.02	(ext)
$R_2(1)$	0.83	± 0.16	(stat) ± 0.01	(ext)

- Fit $\chi^2/\text{ndf} = 279/285$, p-value of 58%.
- Statistical uncertainties include those on the templates (MC sample size).
- $|V_{cb}|$ in agreement with both exclusive and inclusive determinations from B decays.
- FF in agreement with those from B decays.



BGL reminder

- Signal parameters, along with $|V_{cb}|$, in the BGL model

$B \rightarrow D\mu\nu$

$$f_+(z) = \frac{1}{P_{1-}(z)\phi(z)} \sum_{n=0}^{\infty} d_n z^n$$

$$d_0 = \frac{1+r}{2\sqrt{r}} \mathcal{G}(0) P_{1-}(0) \phi(0)$$

$\mathcal{G}(0)$, d_1 , d_2 constrained from LQCD

$B \rightarrow D^*\mu\nu$

$$f(z) = \frac{1}{P_{1+}(z)\phi_f(z)} \sum_{n=0}^{\infty} b_n z^n, \quad b_0 = 2\sqrt{m_B m_{D^*}} P_{1+}(0) \phi_f(0) h_{A_1}(1)$$

$$g(z) = \frac{1}{P_{1-}(z)\phi_g(z)} \sum_{n=0}^{\infty} a_n z^n,$$

$$\mathcal{F}_1(z) = \frac{1}{P_{1+}(z)\phi_{\mathcal{F}_1}(z)} \sum_{n=0}^{\infty} c_n z^n. \quad c_0 = (m_B - m_{D^*}) \frac{\phi_{\mathcal{F}_1}(0)}{\phi_f(0)} b_0$$

$h_{A_1}(1)$ constrained from LQCD
 b_1, a_0, a_1, c_1 free parameters

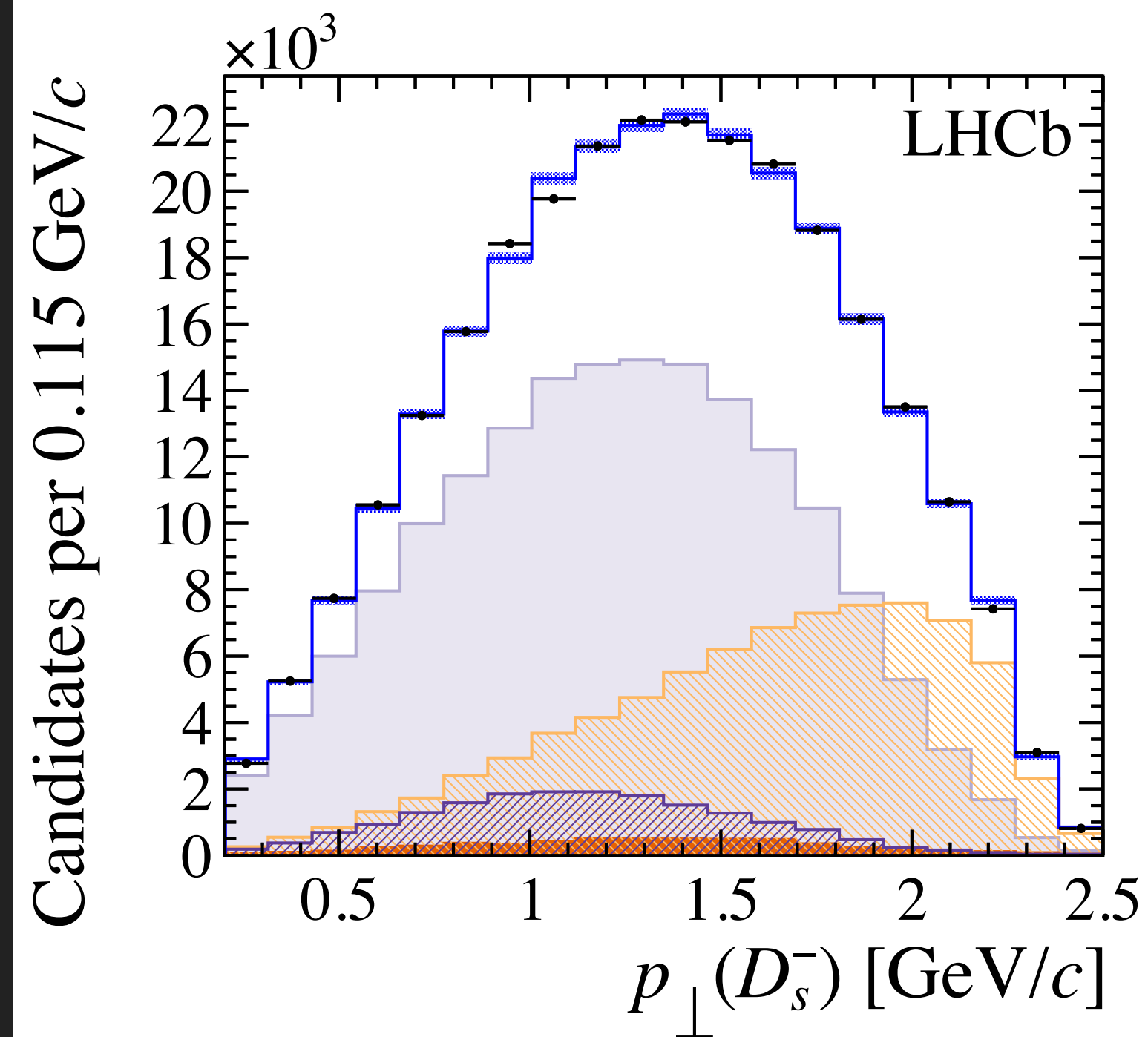
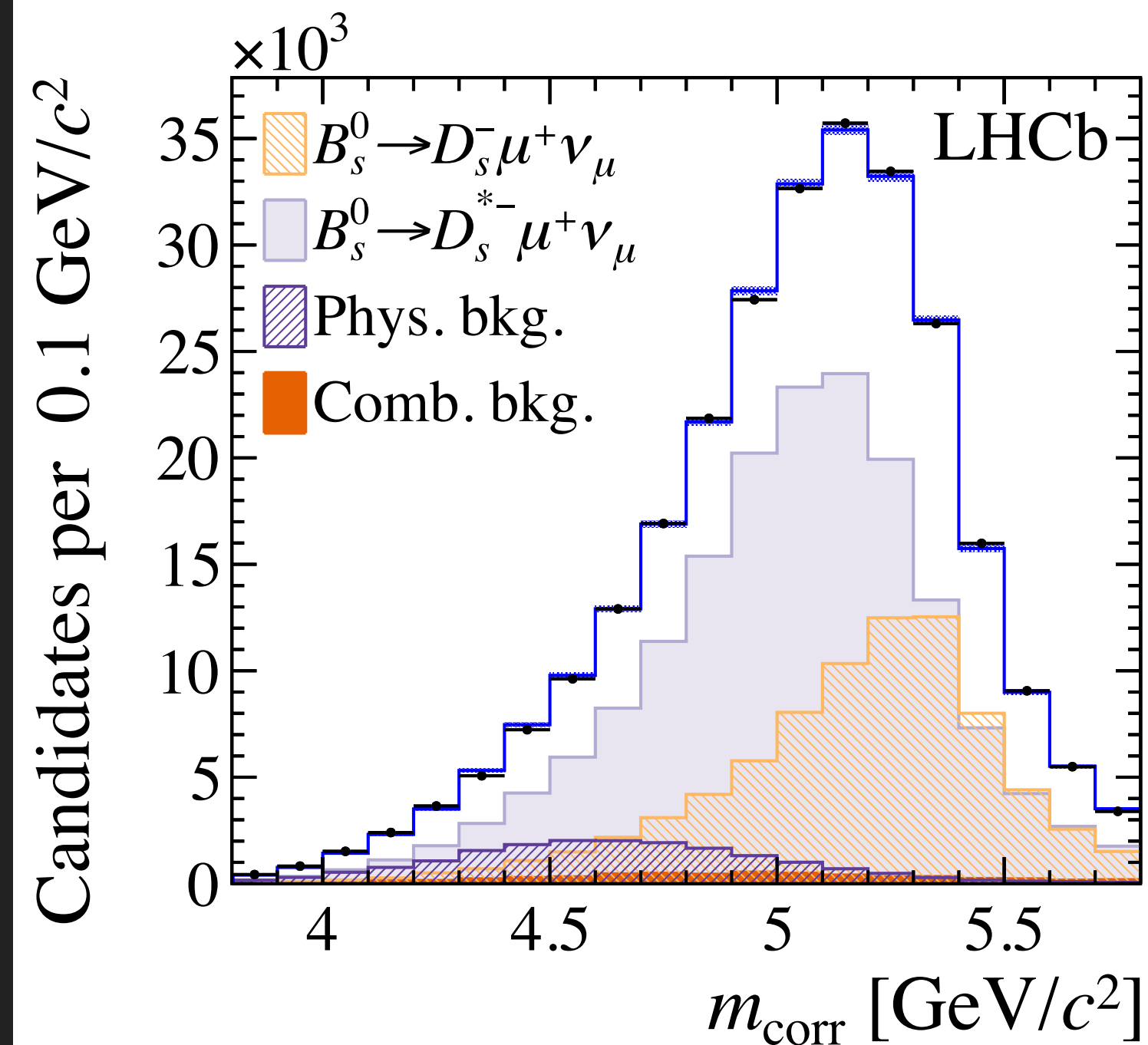
BGL 2-111

- 4 free parameters and 4 parameters constrained from LQCD

Results — BGL

Parameter	Value			
$ V_{cb} [10^{-3}]$	42.3	± 0.8	(stat) ± 1.2	(ext)
$\mathcal{G}(0)$	1.097	± 0.034	(stat) ± 0.001	(ext)
d_1	-0.017	± 0.007	(stat) ± 0.001	(ext)
d_2	-0.26	± 0.05	(stat) ± 0.00	(ext)
b_1	-0.06	± 0.07	(stat) ± 0.01	(ext)
a_0	0.037	± 0.009	(stat) ± 0.001	(ext)
a_1	0.28	± 0.26	(stat) ± 0.08	(ext)
c_1	0.0031	± 0.0022	(stat) ± 0.0006	(ext)

- Configuration BGL 2-111, for D_s up to order z^2 , for D_s^* all 3 serie to order z (b_0 and c_0 set by $h_{A_1}(1)$).
- Fit $\chi^2/\text{ndf} = 276/284$, p-value of 63%.
- $|V_{cb}|$ in agreement with both exclusive and inclusive determinations from B decays.



BGL details

Phase space factors

$$f(z) = \frac{1}{P_{1+}(z)\phi_f(z)} \sum_{n=0}^{\infty} b_n z^n,$$

$$g(z) = \frac{1}{P_{1-}(z)\phi_g(z)} \sum_{n=0}^{\infty} a_n z^n,$$

$$\mathcal{F}_1(z) = \frac{1}{P_{1+}(z)\phi_{\mathcal{F}_1}(z)} \sum_{n=0}^{\infty} c_n z^n.$$

$$f_+(z) = \frac{1}{P_{1-}(z)\phi(z)} \sum_{n=0}^{\infty} d_n z^n.$$

$B \rightarrow D^* \mu \nu$

$B \rightarrow D \mu \nu$

Parameters

$$\phi_f(z) = \frac{4r}{m_B^2} \sqrt{\frac{n_I}{3\pi\tilde{\chi}_{1+}(0)}} \frac{(1+z)\sqrt{(1-z)^3}}{[(1+r)(1-z) + 2\sqrt{r}(1+z)]^4},$$

$$\phi_g(z) = 16r^2 \sqrt{\frac{n_I}{3\pi\tilde{\chi}_{1-}(0)}} \frac{(1+z)^2}{\sqrt{(1-z)}[(1+r)(1-z) + 2\sqrt{r}(1+z)]^4},$$

$$\phi_{\mathcal{F}_1}(z) = \frac{4r}{m_B^3} \sqrt{\frac{n_I}{6\pi\tilde{\chi}_{1+}(0)}} \frac{(1+z)\sqrt{(1-z)^5}}{[(1+r)(1-z) + 2\sqrt{r}(1+z)]^5},$$

$$\phi(z) = \frac{8r^2}{m_B} \sqrt{\frac{8n_I}{3\pi\tilde{\chi}_{1-}(0)}} \frac{(1+z)^2\sqrt{1-z}}{[(1+r)(1-z) + 2\sqrt{r}(1+z)]^5}$$

$$\sum_{n=0}^{\infty} a_n^2 \leq 1,$$

$$\sum_{n=0}^{\infty} (b_n^2 + c_n^2) \leq 1$$

$$\sum_{n=0}^{\infty} d_n^2 < 1$$

Unitarity bounds

Blaschke factors

$$P_{1\pm}(z) = C_{1\pm} \prod_{k=1}^{\text{poles}} \frac{z - z_k}{1 - z z_k}$$

$$z_k = (\sqrt{t_+ - m_k^2} - \sqrt{t_+ - t_-}) / (\sqrt{t_+ - m_k^2} + \sqrt{t_+ - t_-})$$

$$t_{\pm} = (m_B \pm m_{D^*})^2$$

J^P	Pole mass [GeV/c ²]	$\tilde{\chi}_{J^P}(0)$ [10 ⁻⁴ GeV ⁻² c ⁴]	C_{J^P}
1 ⁻	6.329	5.131	2.52733
	6.920		
	7.020		
	7.280		
1 ⁺	6.739	3.894	2.02159
	6.750		
	7.145		
	7.150		
	7.150		

$$b_0 = 2\sqrt{m_B m_{D^*}} P_{1+}(0) \phi_f(0) h_{A_1}(1)$$

$$c_0 = (m_B - m_{D^*}) \frac{\phi_{\mathcal{F}_1}(0)}{\phi_f(0)} b_0$$

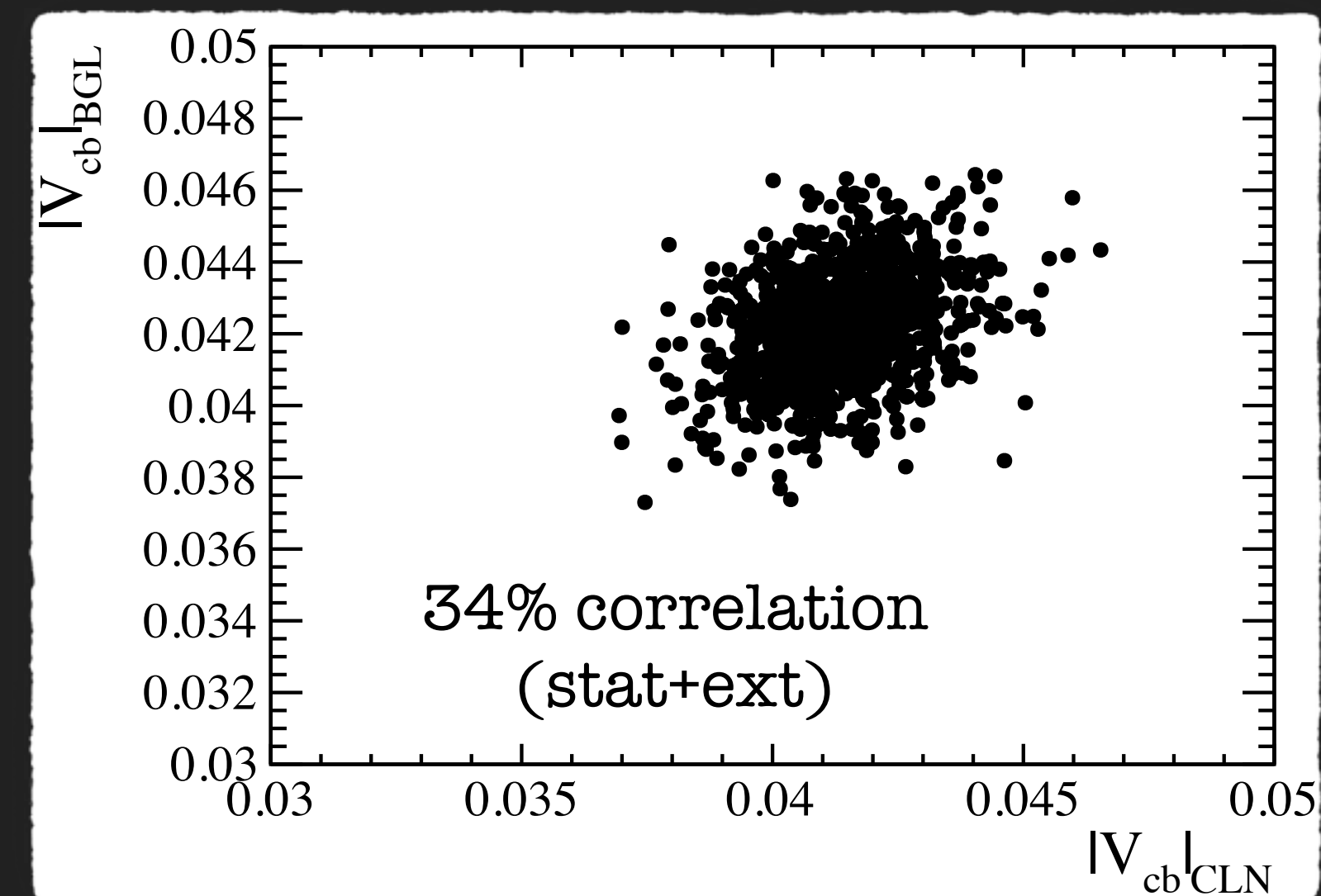
$$d_0 = \frac{1+r}{2\sqrt{r}} \mathcal{G}(0) P_{1-}(0) \phi(0)$$

Only for B_s, 0 for B⁰

Only for B_s, 1 for B⁰

CLN vs BGL

- No significant difference found in the results of $|V_{cb}|$ between CLN and BGL.
- Address correlation by bootstrapping 1K times the data and fitting them with both configurations
- Background-subtracted distributions of p_{\perp} show no significant difference either.



BGL variations

- Different BGL configurations tried to assess stability of $|V_{cb}|$ result.
- For $B_s \rightarrow D_s \mu \nu$ decays, keep always order z^2 (from LQCD constraints).
- Change the order of the series of $B_s \rightarrow D_s^* \mu \nu$ decays. Not shown: discarded configurations which led to poor fit quality (lower orders) or degraded sensitivity (higher orders).

Nominal

Parameter	2-110	2-111	2-210
$ V_{cb} [10^{-3}]$	<u>$41.67 \pm 1.31(0.57)$</u>	$42.26 \pm 1.43(0.80)$	$42.24 \pm 1.41(0.79)$
b_1	$-0.008 \pm 0.039(0.038)$	$-0.060 \pm 0.069(0.068)$	$-0.153 \pm 0.090(0.094)$
b_2	–	–	<u>$1.9 \pm 1.5(1.4)$</u>
a_0	$0.0380 \pm 0.0082(0.0078)$	$0.0374 \pm 0.0086(0.0086)$	$0.046 \pm 0.011(0.011)$
a_1	–	$0.28 \pm 0.27(0.26)$	–
c_1	$0.0046 \pm 0.0016(0.0016)$	$0.0031 \pm 0.0023(0.0022)$	$0.0029 \pm 0.0021(0.0020)$
d_1	-0.0176 ± 0.0074	-0.0172 ± 0.0075	-0.0165 ± 0.0075
d_2	-0.259 ± 0.047	-0.256 ± 0.047	-0.254 ± 0.047
$\mathcal{G}(0)$	1.102 ± 0.034	1.097 ± 0.034	1.094 ± 0.034
$\mathcal{F}(1)$	0.899 ± 0.013	0.901 ± 0.013	0.900 ± 0.013
χ^2/dof	277/285	276/284	275/284
Probability	0.62	0.63	0.64

Shift in $|V_{cb}|$ confirmed with toys

Unitarity bound not imposed

Ratio of BR

- Parametrise the signal yields in terms of ratio of branching fraction between signal and reference decays (and all other inputs).
- FF are shape parameters of the templates (found same values as in the fit for $|V_{cb}|$). CLN used (BGL as a systematic uncertainty).

$$\mathcal{R} \equiv \frac{\mathcal{B}(B_s^0 \rightarrow D_s^- \mu^+ \nu_\mu)}{\mathcal{B}(B^0 \rightarrow D^- \mu^+ \nu_\mu)} = 1.09 \pm 0.05 \text{ (stat)} \pm 0.05 \text{ (ext)}$$

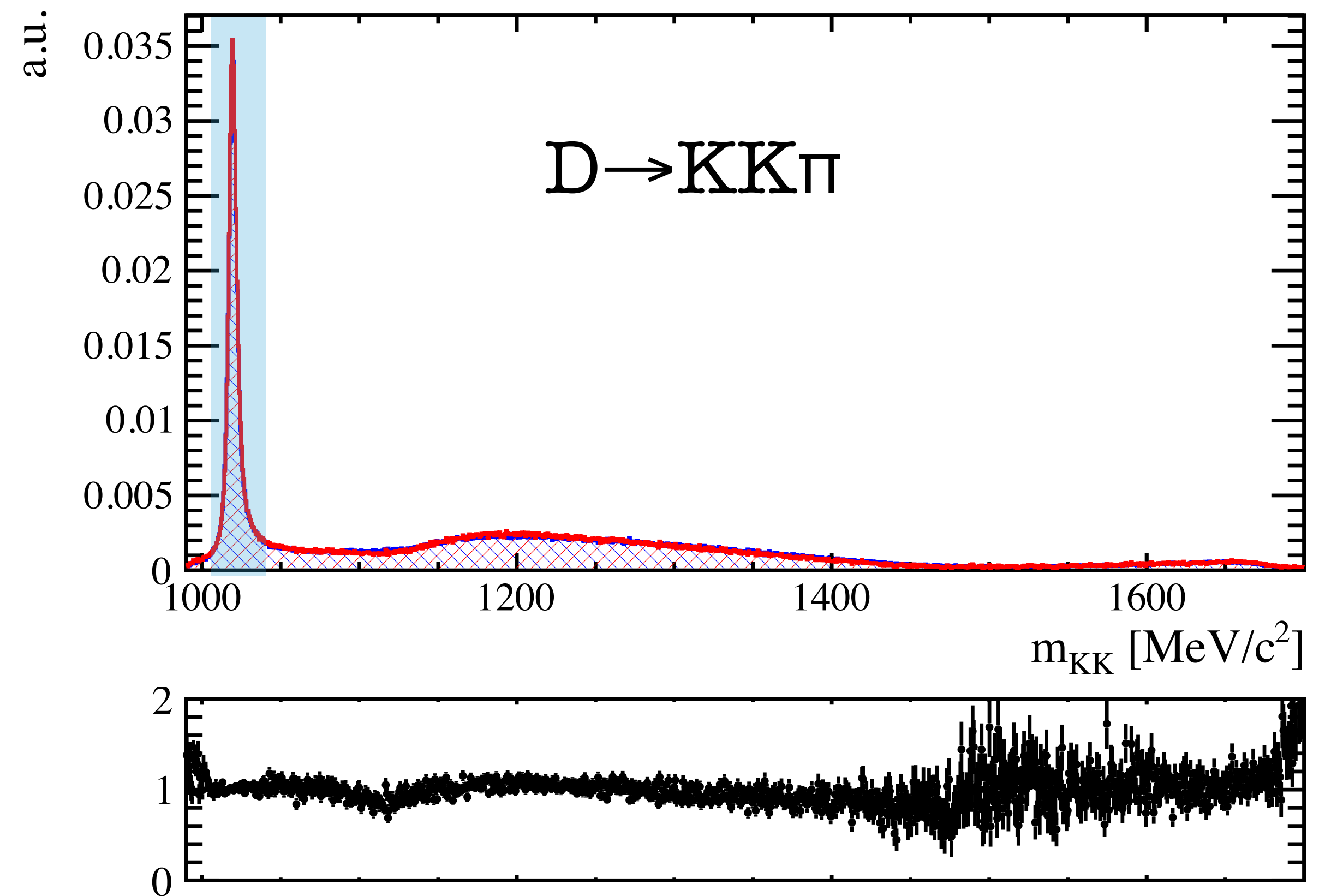
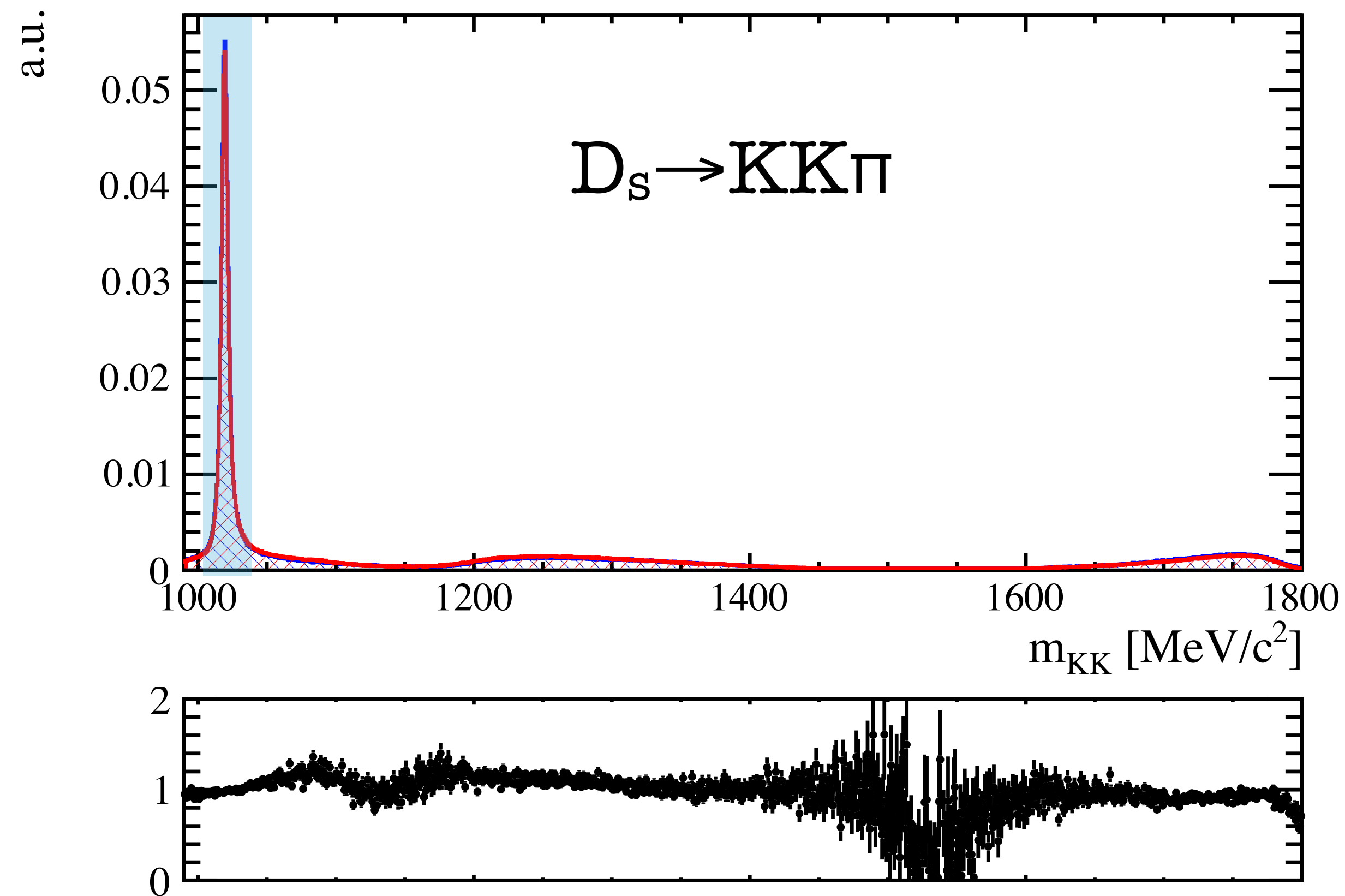
$$\mathcal{R}^* \equiv \frac{\mathcal{B}(B_s^0 \rightarrow D_s^{*-} \mu^+ \nu_\mu)}{\mathcal{B}(B^0 \rightarrow D^{*-} \mu^+ \nu_\mu)} = 1.06 \pm 0.05 \text{ (stat)} \pm 0.05 \text{ (ext)}$$

Systematics (all)

Source	Uncertainty															
	CLN parametrization						BGL parametrization									
	$ V_{cb} $ [10^{-3}]	$\rho^2(D_s^-)$ [10^{-1}]	$\mathcal{G}(0)$ [10^{-2}]	$\rho^2(D_s^{*-})$ [10^{-1}]	$R_1(1)$ [10^{-1}]	$R_2(1)$ [10^{-1}]	$ V_{cb} $ [10^{-3}]	d_1 [10^{-2}]	d_2 [10^{-1}]	$\mathcal{G}(0)$ [10^{-2}]	b_1 [10^{-1}]	c_1 [10^{-3}]	a_0 [10^{-2}]	a_1 [10^{-1}]	\mathcal{R} [10^{-1}]	\mathcal{R}^* [10^{-1}]
$f_s/f_d \times \mathcal{B}(D_s^- \rightarrow K^+ K^- \pi^-) (\times \tau)$	0.8	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.4	0.4
$\mathcal{B}(D^- \rightarrow K^+ K^- \pi^-)$	0.5	0.0	0.0	0.0	0.0	0.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.3
$\mathcal{B}(D^{*-} \rightarrow D^- X)$	0.2	0.0	0.1	0.0	0.1	0.0	0.1	0.0	0.0	0.1	0.0	0.2	0.0	0.3	–	0.2
$\mathcal{B}(B^0 \rightarrow D^- \mu^+ \nu_\mu)$	0.4	0.0	0.3	0.1	0.2	0.1	0.5	0.1	0.0	0.1	0.1	0.4	0.1	0.7	–	–
$\mathcal{B}(B^0 \rightarrow D^{*-} \mu^+ \nu_\mu)$	0.3	0.0	0.2	0.1	0.1	0.1	0.2	0.0	0.0	0.1	0.1	0.3	0.1	0.4	–	–
$m(B_s^0), m(D^{*-})$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	–	–
η_{EW}	0.2	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.1	–	–
$h_{A_1}(1)$	0.3	0.0	0.2	0.1	0.1	0.1	0.3	0.0	0.0	0.1	0.1	0.3	0.1	0.5	–	–
External inputs (ext)	1.2	0.0	0.4	0.1	0.2	0.1	1.2	0.1	0.0	0.1	0.1	0.6	0.1	0.8	0.5	0.5
$D_{(s)}^- \rightarrow K^+ K^- \pi^-$ model	0.8	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.4
Background	0.4	0.3	2.2	0.5	0.9	0.7	0.1	0.5	0.2	2.3	0.7	2.0	0.5	2.0	0.4	0.6
Fit bias	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.2	0.4	0.2	0.4	0.0	0.0
Corrections to simulation	0.0	0.0	0.5	0.0	0.1	0.0	0.0	0.1	0.0	0.1	0.0	0.0	0.0	0.1	0.0	0.0
Form-factor parametrization	–	–	–	–	–	–	–	–	–	–	–	–	–	–	0.0	0.1
Experimental (syst)	0.9	0.3	2.2	0.5	0.9	0.7	0.9	0.5	0.2	2.3	0.7	2.1	0.5	2.0	0.6	0.7
Statistical (stat)	0.6	0.5	3.4	1.7	2.5	1.6	0.8	0.7	0.5	3.4	0.7	2.2	0.9	2.6	0.5	0.5

$m(KK)$ requirement

MC vs bkg-subtracted data for $D_s \rightarrow KK\pi$ and $D \rightarrow KK\pi$ decays



Subleading terms

For $B_s \rightarrow D_s^* \mu \nu$ decays, p_\perp has some (little) sensitivity to $\cos(D, \mu)$, giving possibility to access R_1 and R_2 form factors.

