HPQCD calculations for Heavy $\to$ Heavy Decays

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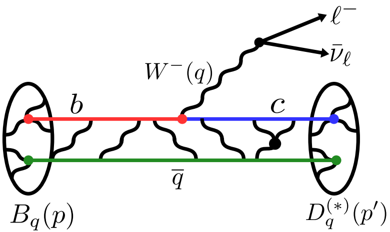
Challenges in semileptonic B decays - April 2022

Background

Many interesting B semileptonic decays currently under active investigation

- Here, focus on three related $b \to c$, pseudoscalar to vector decays: $B_{(s)} \to D^*_{(s)} \ell\nu$ and $B_c \to J/\psi \ell\nu$
  - Complementary determinations of V_{cb},
  - Comparison of observables sensitive to lepton flavor universality violation (LFUV) to experiment



Kinematic variables:

$$q^2 = (p - p')^2$$

$$w = \frac{p' \cdot p}{M_{B_q} M_{D_q^{(*)}}}$$

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$B_{(s)} \to D^*_{(s)}\ell\nu$, $B_c \to J/\psi\ell\nu$

Pseudoscalar to vector decay has the following structure in the SM:

$$\frac{d\Gamma}{dq^2} = \chi(q^2) \times \mathcal{F}^2(q^2)|V_{cb}|^2$$

$$\mathcal{F}^2(q^2) = \left[\left(1 + \frac{m_\ell^2}{2q^2}\right)\left(H_+^2(q^2) + H_-^2(q^2) + H_0^2(q^2)\right) + \frac{3m_\ell^2}{2q^2} H_t^2(q^2)\right]$$

Helicity amplitudes expressed in terms of form factors

$$\left\{H_+(q^2), H_-(q^2), H_0(q^2)\right\} \leftrightarrow \left\{A_1(q^2), A_2(q^2), V(q^2)\right\}$$
$$H_t(q^2) \propto A_0(q^2)$$

- Theoretical predictions for vector meson final state require:
  - 4 form factors within the Standard Model
  - 3 additional tensor form factor for New Physics
- V_{cb} - compare experimental value of $\eta_{\mathrm{EW}}\mathcal{F}(q^2_{\mathrm{max}})|V_{cb}|$ to lattice calculations of $\mathcal{F}(q^2_{\mathrm{max}})$
  - preferred over $B_{(s)} \to D_{(s)}$ due to favorable kinematics near zero-recoil.
- $R(D^*)$
  - Sensitive to LFUV
  - Theory for $R(D^*)$ relies on experimental fits + HQET for A_0
  - On the lattice, typically use unphysically heavy pions and treat $D^* \to D\pi$ resonance using $\chi\mathrm{PT}$
- Lattice calculation of FFs for $B_c \to J/\psi \ < \ B_s \to D_s^* \ < \ B \to D^*$
  - Computational cost of propagators for $c < s << u/d$
  - J/ψ and D_s^* are 'gold-plated'
  - $B \to D^*$ requires careful treatment of chiral effects

Overview of Lattice Results

- SM FFs for $B \to D\ell\nu$ available away from zero recoil[1]
- SM FFs for $B_s \to D_s\ell\nu$ now available across the full kinematic range, tensor FF available close to zero-recoil, with work also ongoing[2]
- SM FFs for $B \to D^*\ell\nu$ recently became available from Fermilab-MILC away from zero-recoil[3], with lattice calculations also underway by JLQCD as well as HPQCD.
- SM FFs for $B_s \to D_s^*\ell\nu$ and $B_c \to J/\psi\ell\nu$ available across full kinematic range from HPQCD[4]
- (Preliminary) SM FFs for $B \to D^*\ell\nu$ across full kinematic range from HPQCD

[1] e.g. 1503.07237,1505.03925
[2] 1906.00701,1310.5238,2110.10061
[3] 2105.14019
[4] 2105.11433, 2007.06957

Current Results

	Lattice only	Lattice+Exp[5]	Experiment	Tension
$R(D)$	$0.293(4)$[6]	$0.299(3)$	$0.340(30)$	1.4σ
$R(D^*)$	$0.265(13)$	$0.2483(13)$	$0.295(14)$	3.3σ
$R(D_s)$	$0.299(5)$	–	–	–
$R(D_s^*)$	$0.249(7)$	–	–	–
$R(J/\psi)$	$0.258(4)$	–	$0.71(25)$[7]	1.8σ

HFLAV average, Fermilab-MILC, HPQCD.

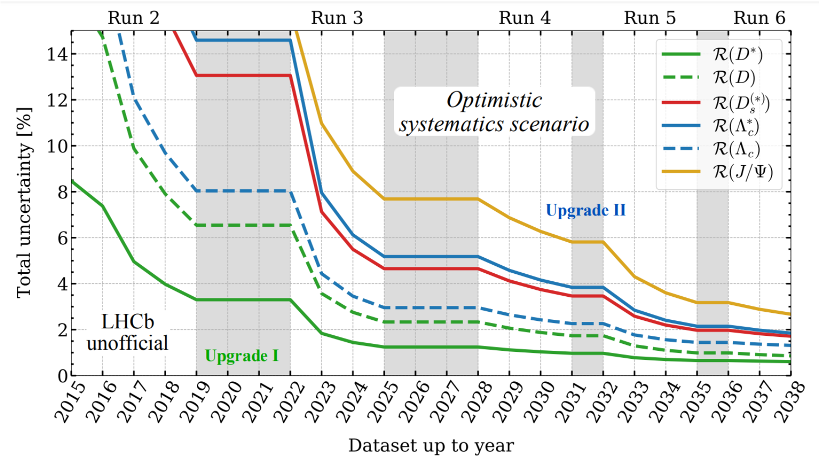
	V_{cb}	
$B \to D$	$39.58(94)_{\mathrm{exp}}(37)_{\mathrm{th}} \times 10^{-3}$	HFLAV
$B \to D^*$	$38.76(42)_{\mathrm{exp}}(55)_{\mathrm{th}} \times 10^{-3}$	
$B_s \to D_s^{(*)}$	$42.3(1.2)_{\mathrm{exp}}(1.2)_{\mathrm{th}} \times 10^{-3}$	LHCb (2001.03225)
$B \to X_c \ell \nu$	$42.16(51) \times 10^{-3}$	Bordone et al.(2107.00604)

[5] Assumes new physics only possible in semitauonic mode

[6] FLAG review

[7] LHCb-1711.05623

Experimental Outlook



- Need precise SM form factors across full kinematic range
  - Resolve discrepancy between inclusive and exclusive determinations of V_{cb}
  - Make first principles predictions for $R(D^*_{(s)})$ independent of experimental measurements
- Need tensor form factors to disentangle possible new physics effects

$b \to c$ Pseudoscalar to Vector Form Factors

In the standard model $\mathcal{F}(q^2)$ is a simple function of the form factors, $A_1(q^2)$, $A_0(q^2)$, $A_2(q^2)$ and $V(q^2)$, defined in terms of matrix elements. For example, for $B_s \to D_s^* \ell\nu$:

$$\langle D_s^*(p',\lambda)|\bar{c}\gamma^\mu b|B_s^0(p)\rangle = \frac{2iV(q^2)}{M_{B_s}+M_{D_s^*}}\varepsilon^{\mu\nu\rho\sigma}\epsilon_\nu^*(p',\lambda)p'_\rho p_\sigma$$

$$\langle D_s^*(p',\lambda)|\bar{c}\gamma^\mu\gamma^5 b|B_s^0(p)\rangle = 2M_{D_s^*}A_0(q^2)\frac{\epsilon^*(p',\lambda)\cdot q}{q^2}q^\mu$$

$$+\,(M_{B_s}+M_{D_s^*})A_1(q^2)\left[\epsilon^{*\mu}(p',\lambda)-\frac{\epsilon^*(p',\lambda)\cdot q}{q^2}q^\mu\right]$$

$$-\,A_2(q^2)\frac{\epsilon^*(p',\lambda)\cdot q}{M_{B_s}+M_{D_s^*}}\left[p^\mu+p'^\mu-\frac{M_{B_s}^2-M_{D_s^*}^2}{q^2}q^\mu\right]$$

Form Factors Across the Full q^2 Range with Lattice QCD[9]

Use "Heavy-HISQ" approach:

- Compute form factors using multiple heavy masses ranging up to close to the physical b-quark mass
- Use **H**ighly **I**mproved **S**taggered **Q**uark action[8] for all quarks - fully relativistic, small discretisation effects
- Nonperturbatively renormalised currents, using PCVC and PCAC relations for vector and axial-vector, RI-SMOM for tensor
- Fit the form factor data including am_h discretisation effects, physical heavy mass dependence, and lattice spacing dependence
  - For $B_s \to D_s^*$ and $B_c \to J/\psi$ first convert to z space, e.g.

$$P(q^2) \times A_1(q^2) = \sum_{n=0}^{3} a_n z^n(q^2)\mathcal{N}_n$$

$$a_n = \sum_{j,k,l=0}^{3} b_n^{jkl} \left(\frac{2\Lambda_{\mathrm{QCD}}}{M_{\eta_h}}\right)^j \left(\frac{am_c^{\mathrm{val}}}{\pi}\right)^{2k} \left(\frac{am_h^{\mathrm{val}}}{\pi}\right)^{2l}$$

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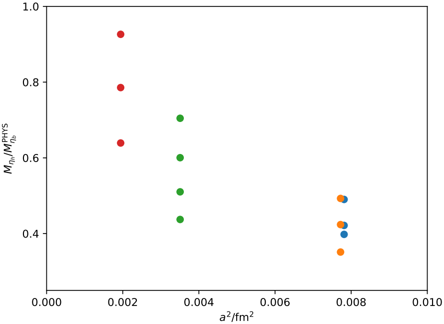
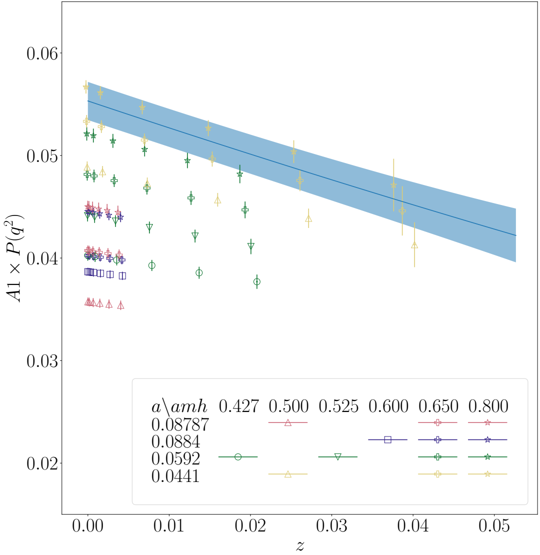
[8] hep-lat/0610092

[9] $B_s \to D_s^*$:2105.11433, $B_c \to J/\psi$:2007.06957

- We use the second generation MILC HISQ gauge configurations with u/d, s and c quarks in the sea.

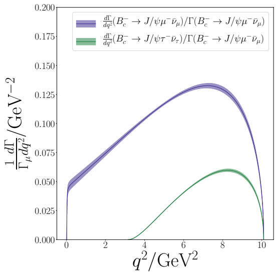


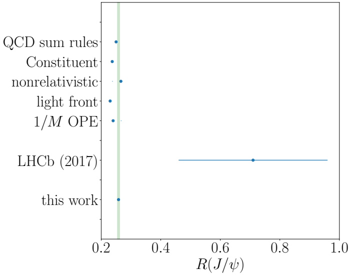
- The subset of configurations we use include physical u/d quark masses, and have small lattice spacings allowing us to come very close to the physical b mass.



$P(q^2) \times A_1$ for $B_c \to J/\psi$, plotted in z space, showing the physical continuuum form factor as a blue band

$B_c \to J/\psi$ Results - 2007.06956, 2007.06957



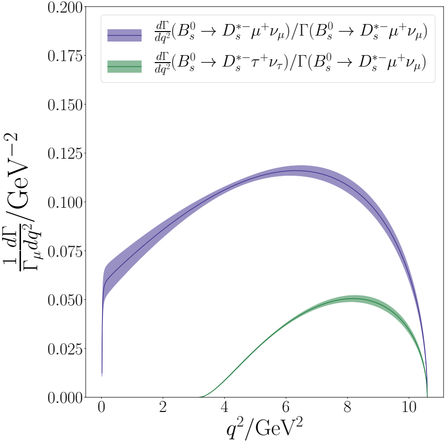


$$R(J/\psi) = 0.2582(38)$$

$$\Gamma(B_c^- \to J/\psi\mu^-\bar{\nu}_\mu)/\eta_{\mathrm{EW}}^2|V_{cb}|^2 = 1.73(12) \times 10^{13}s^{-1}$$

- Experimental results for $B_c \to J/\psi$ are currently much less precise than our lattice results, but expect this to improve in future.
- In addition to $R(J/\psi)$, other observables and ratios may be constructed with high precision from our form factor results
  - Can study the effect of NP couplings - full details in 2007.06956

$B_s \to D_s^*$ Results - 2105.11433



$$R(D_s^*) = 0.249(6)_{\mathrm{latt}}(4)_{\mathrm{EM}}$$

$$\Gamma(B_s^0 \to D_s^{*-}\mu^+\nu_\mu)/\eta_{\mathrm{EW}}^2|V_{cb}|^2 = 2.06(16) \times 10^{13} s^{-1}$$

$R(D_s^*)$, V_{cb}...

Many new lattice predictions for $B_s \to D_s^*$ quantities:

	This work	Exp.[10]	$B \to D^*$ [11]
$\frac{\Gamma(B_s^0 \to D_s^- \mu^+ \nu_\mu)}{\Gamma(B_s^0 \to D_s^{*-} \mu^+ \nu_\mu)}$	0.443(40)	0.464(45)	0.457(23)
$R(D_{(s)}^*)$	0.249(7)	–	0.2483(13)
F_L	0.440(16)	–	0.464(10)
$\mathcal{A}_{\lambda_\tau} = -P_\tau$	0.520(12)	–	0.496(15)

- Can also infer a total experimental rate Γ from LHCb analysis of V_{cb} in 2001.03225, we can use this with our results to give a value of V_{cb}

$$|V_{cb}| = 42.2(2.3) \times 10^{-3}$$

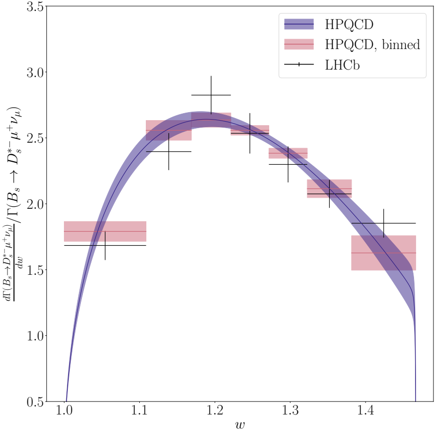
- Consistent with the result using lattice data only at zero-recoil.

[10] LHCb 2001.03225

[11] HFLAV 1909.12524,Bordone et. al 1908.09398

$B_s \to D_s^*$ Shape

We can compare the binned experimental differential rate[12] for the $B_s \to D_s^*$ shape to our results



$\chi^2/\mathrm{dof} = 1.8$ (0.62 excluding third bin)

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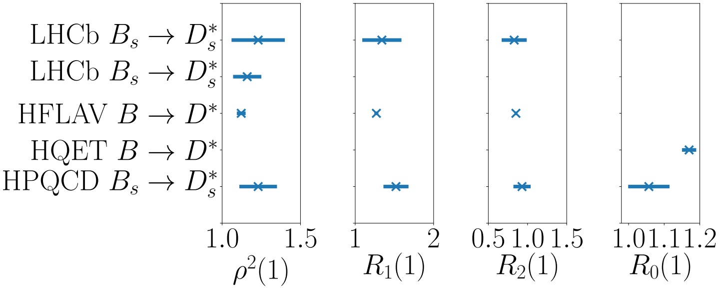
[12] LHCb:2003.08453

$B_s \to D_s^*$ Shape Parameters

In the CLN parameterisation, the shape of the decay for massive leptons in the SM is fully described by the four parameters ρ^2, $R_1(1)$, $R_2(1)$ and $R_0(1)$, with ρ^2, $R_1(1)$, $R_2(1)$ determined from experiment and $R_0(1)$ known to NLO in HQET[13]



- Our results are broadly consistent with the measured values of ρ^2, $R_1(1)$ and $R_2(1)$ for $B_s \to D_s^*$, and with the NLO HQET value of $R_0(1)$.

[13] LHCb:2001.03225+2003.08453, HFLAV:1909.12524, HQET:1703.05330

Preliminary results for $B \to D^*$

For $B \to D^*$, use HQET form factors:

$$\frac{\langle D^*(p',\lambda)|\bar{c}\gamma^\mu b|B(p)\rangle}{\sqrt{M_B M_{D^*}}} = h_V(w)\varepsilon^{\mu\nu}{}_{\rho\sigma}\epsilon^*_\nu(v_{D^*},\lambda)v^\rho_{D^*}v^\sigma_B$$

$$\frac{\langle D^*(p',\lambda)|\bar{c}\gamma^\mu\gamma^5 b|B(p)\rangle}{\sqrt{M_B M_{D^*}}}$$

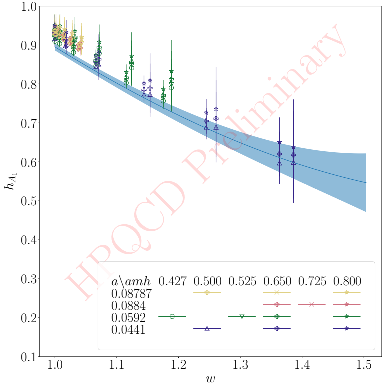
$$= i\epsilon^*_\nu\left[g^{\mu\nu}(w+1)h_{A_1}(w) - v^\nu_B\left(v^\mu_B h_{A_2}(w) + v^\mu_{D^*}h_{A_3}(w)\right)\right]$$

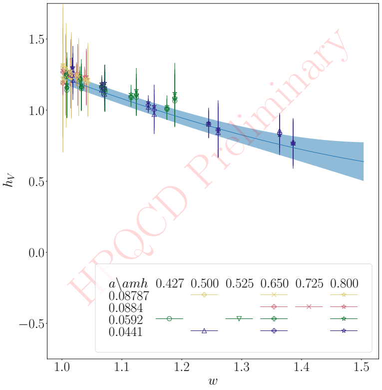
Computation completed on $a = 0.045\mathrm{fm}$, $a = 0.06\mathrm{fm}$ and $a = 0.09\mathrm{fm}$ $m_l = m_s/5$ lattices and $a = 0.09\mathrm{fm}$ physical m_l lattices. In the process of generating correlation functions on $a = 0.06\mathrm{fm}$ physical m_l lattices. Fit form factors to HQET inspired form, including chiral terms:

$$F = \sum_{nijk} a^n_{ijk}(w-1)^n\left(\frac{am_c}{\pi}\right)^i\left(\frac{am_h}{\pi}\right)^j\left(\frac{\Lambda_{\mathrm{QCD}}}{M_B}\right)^k\mathcal{N}_n$$

$$+ X_{\log}(M_\pi/\Lambda_\chi) + A\left(\frac{M_\pi}{\Lambda_\chi}\right)^2$$

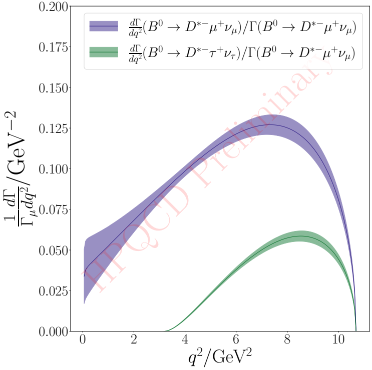
Preliminary results for $B \to D^*$

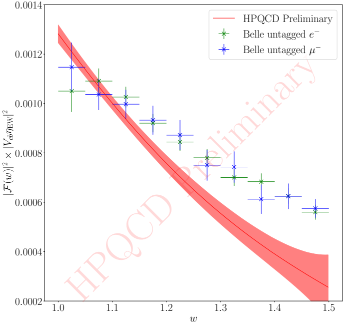




We include data from $B_s \to D_s^*$ in our chiral extrapolation.

Preliminary results for $B \to D^*$

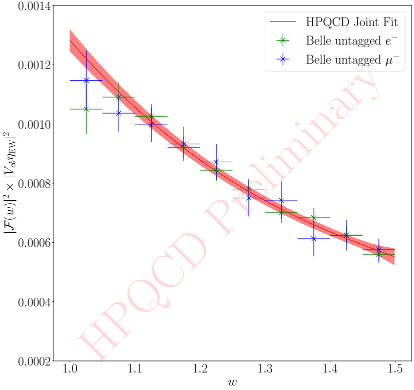




$$\frac{d\mathcal{F}}{dw}\Big|_{w=1} = -0.96(9)$$

(PRELIMINARY)

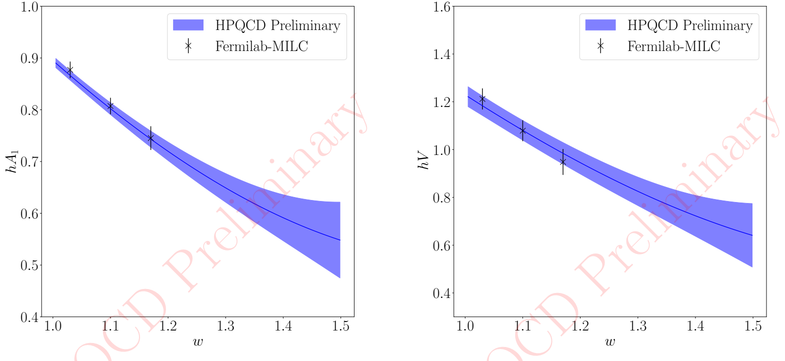
Preliminary results for $B \to D^*$



Joint fit to HPQCD lattice and Belle untagged data - $\chi^2/\mathrm{dof} = 1.6$

$V_{cb} = 39.7(0.5)_{\mathrm{latt}}(0.5)_{\mathrm{exp}} \times 10^{-3}$ (PRELIMINARY)

Comparison to Fermilab-MILC (2105.14019)



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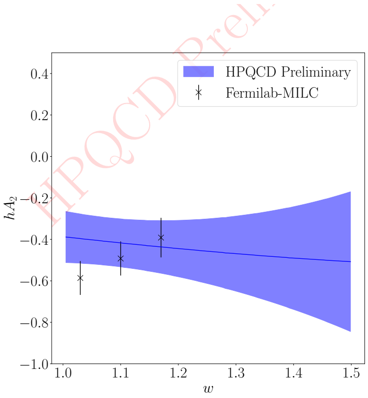

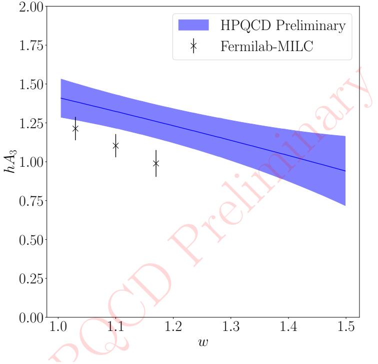

Summary

- Published lattice results for $B_c \to J/\psi$ form factors, corresponding experimental measurements are currently imprecise.
  - Experimental results for $B_c \to J/\psi$ decays are expected to become more precise
- Results for the $B_s \to D_s^*$ form factors on arXiv
  - Model independent determinations of $R(D_s^*)$ and other observables
  - Model independent determination of $|V_{cb}|$, though ideally would use experimental results directly
- Work on $B \to D^*$ form factors, including Tensor form factors, almost complete

Thanks for listening!