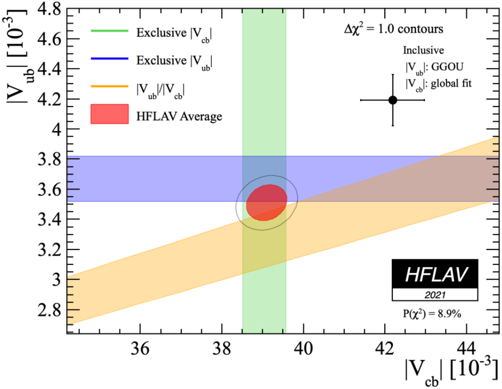
Inclusive V_{cb} determination

Marzia Bordone

Challenges in Semileptonic B Decays

20.04.2022



- Longstanding discrepancy between inclusive and exclusive determinations
- A lot of activity lately
  - $\Rightarrow$ new experimental determinations
  - $\Rightarrow$ new calculations of exclusive form factors

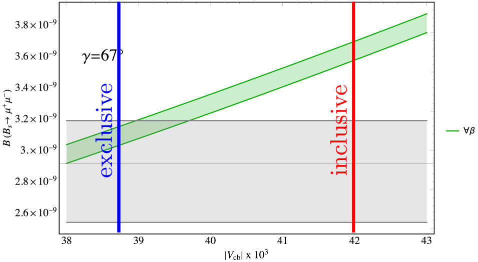
Why is V_{cb} important?

- Key parameter in the prediction of flavour observables

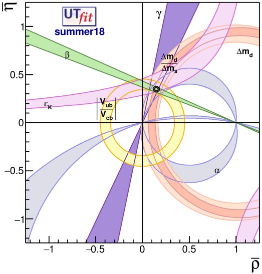
$$\Rightarrow \ \epsilon_K \sim |V_{cb}|^4$$

$$\Rightarrow \ \mathcal{B}(\bar{B}_s \to \mu^+\mu^-) \sim |V_{tb}V_{ts}^*|^2 \sim |V_{cb}|^2[1+\mathcal{O}(\lambda^2)]$$

[Buras, Venturini, '21]



- Tests the SM flavour structure



Theory framework

$$\Gamma = \frac{1}{m_B} \mathrm{Im} \int d^4x \langle B(p)| T \left\{ \mathcal{H}_{\mathrm{eff}}^\dagger(x) \mathcal{H}_{\mathrm{eff}}(0) \right\} |B(p)\rangle$$

Theory framework

$$\Gamma = \frac{1}{m_B} \text{Im} \int d^4x \langle B(p)| T\left\{\mathcal{H}_{\text{eff}}^\dagger(x)\mathcal{H}_{\text{eff}}(0)\right\} |B(p)\rangle$$

$$\uparrow$$

$$\sum_{n,i} \frac{1}{m_b^n} \mathcal{C}_{n,i} \mathcal{O}_{n+3,i}$$

Theory framework

$$\Gamma = \frac{1}{m_B} \mathrm{Im} \int d^4x \langle B(p) | T \left\{ \mathcal{H}_{\mathrm{eff}}^{\dagger}(x) \mathcal{H}_{\mathrm{eff}}(0) \right\} | B(p) \rangle$$

$$\uparrow$$

$$\sum_{n,i} \frac{1}{m_b^n} \mathcal{C}_{n,i} \mathcal{O}_{n+3,i}$$

- The Wilson coefficients are calculated perturbatively
- The matrix elements $\langle B(p) | \mathcal{O}_{n+3,i} | B(p) \rangle$ are non perturbative
  - $\Rightarrow$ They need to be determined with non-perturbative methods **[Hashimoto's talk]**
  - $\Rightarrow$ They can be extracted from data
  - $\Rightarrow$ With large n, large number of operators

Theory framework

$$\Gamma = \frac{1}{m_B} \mathrm{Im} \int d^4x \langle B(p)| T\left\{\mathcal{H}^\dagger_{\mathrm{eff}}(x)\mathcal{H}_{\mathrm{eff}}(0)\right\}|B(p)\rangle$$

$$\uparrow$$

$$\sum_{n,i} \frac{1}{m_b^n} \mathcal{C}_{n,i} \mathcal{O}_{n+3,i}$$

- The Wilson coefficients are calculated perturbatively
- The matrix elements $\langle B(p)|\mathcal{O}_{n+3,i}|B(p)\rangle$ are non perturbative
  - $\Rightarrow$ They need to be determined with non-perturbative methods **[Hashimoto's talk]**
  - $\Rightarrow$ They can be extracted from data
  - $\Rightarrow$ With large n, large number of operators

$\uparrow$

loss of predictivity

Theory framework

$$\Gamma_{sl} = \Gamma_0 f(\rho)\Big[1 + a_1\left(\frac{\alpha_s}{\pi}\right) + a_2\left(\frac{\alpha_s}{\pi}\right)^2 + a_3\left(\frac{\alpha_s}{\pi}\right)^3 - \left(\frac{1}{2} - p_1\left(\frac{\alpha_s}{\pi}\right)\right)\frac{\mu_\pi^2}{m_b^2}$$
$$+ \left(g_0 + g_1\left(\frac{\alpha_s}{\pi}\right)\right)\frac{\mu_G^2(m_b)}{m_b^2} + d_0\frac{\rho_D^3}{m_b^3} - g_0\frac{\rho_{LS}^3}{m_b^3} + \ldots\Big]$$

$$\mu_\pi^2(\mu) = \frac{1}{2m_B}\langle B|\bar{b}_v(i\vec{D})^2 b_v|B\rangle_\mu \qquad \mu_G^2(\mu) = \frac{1}{2m_B}\langle B|\bar{b}_v\frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}b_v|B\rangle_\mu$$

- Coefficients of the expansions are known
- Ellipses stands for higher orders

Theory framework

NEW

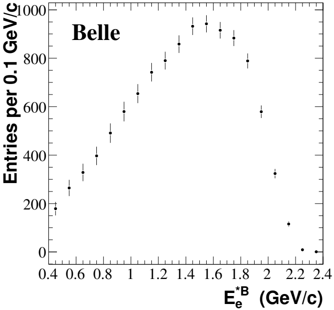
$$\Gamma_{sl} = \Gamma_0 f(\rho)\Big[1 + a_1\left(\frac{\alpha_s}{\pi}\right) + a_2\left(\frac{\alpha_s}{\pi}\right)^2 + a_3\left(\frac{\alpha_s}{\pi}\right)^3 - \left(\frac{1}{2} - p_1\left(\frac{\alpha_s}{\pi}\right)\right)\frac{\mu_\pi^2}{m_b^2}$$

$$+ \left(g_0 + g_1\left(\frac{\alpha_s}{\pi}\right)\right)\frac{\mu_G^2(m_b)}{m_b^2} + d_0\frac{\rho_D^3}{m_b^3} - g_0\frac{\rho_{LS}^3}{m_b^3} + \dots\Big]$$

$$\mu_\pi^2(\mu) = \frac{1}{2m_B}\langle B|\bar{b}_v(i\vec{D})^2 b_v|B\rangle_\mu \qquad \mu_G^2(\mu) = \frac{1}{2m_B}\langle B|\bar{b}_v\frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}b_v|B\rangle_\mu$$

- Coefficients of the expansions are known
- Ellipses stands for higher orders

How do we constrain the OPE parameters?



- Lepton energy and hadronic invariant mass distributions can be used to extract non perturbative information
- The moments admit a Heavy Quark Expansion

$$M_i = M_i^{(0)} + \frac{\alpha_s}{\pi} M_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + M_i^{\mu_\pi^2} \frac{\mu_\pi^2}{m_b^2} + M_i^{\mu_G^2} \frac{\mu_G^2}{m_b^2} + M_i^{\rho_D^3} \frac{\rho_D^3}{m_b^3} + M_i^{\rho_{LS}^3} \frac{\rho_{LS}^3}{m_b^3} + \dots$$

- q^2 moments can also be used

[Fael, Mannel, Vos, '18]

Scheme conventions

- Pole mass scheme

  $\Rightarrow$ Renormalon ambiguity

  $\Rightarrow$ Perturbative series is factorially divergent

$$\Gamma_{sl} \sim \sum_k k! \left(\frac{\beta_0}{2} \frac{\alpha_s}{\pi} \right)^k$$

- We choose to use to b-quark mass and the non perturbative parameters in the kinetic scheme

[Bigi, Shifman,Uraltsev,Vainshtein]

  $\Rightarrow$ Wilsonian cutoff $\mu = 1\,\mathrm{GeV}$

- We express the charm mass in the $\overline{\mathrm{MS}}$ scheme

Inclusion of $\mathcal{O}(\alpha_s^3)$ results

[Fael, Schönwald, Steinhauser, '20]

b-quark mass:

$$m_b^{kin}(1\,\mathrm{GeV}) = [4169 + 259_{\alpha_s} + 78_{\alpha_s^2} + 26_{\alpha_s^3}]\,\mathrm{MeV} = (4526 \pm 15)\,\mathrm{MeV}$$

$\uparrow$ 50% **reduction!**

Semileptonic width

$\Rightarrow$ $\mu = 1\,\mathrm{GeV},\ \mu_b = m_b^{kin},\ \mu_c = 3\,\mathrm{GeV}$

$$\Gamma_{sl} = \Gamma_0 f(\rho)\Big[0.9257 - 0.1163_{\alpha_s} - 0.0349_{\alpha_s^2} - 0.0097_{\alpha_s^3}\Big]$$

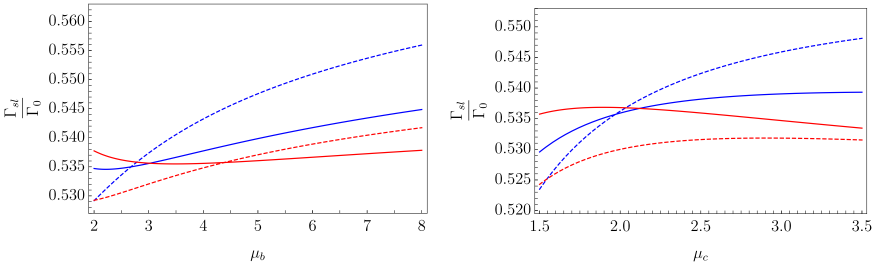
$\Rightarrow$ $\mu = 1\,\mathrm{GeV},\ \mu_b = m_b^{kin}/2,\ \mu_c = 2\,\mathrm{GeV}$

$$\Gamma_{sl} = \Gamma_0 f(\rho)\Big[0.9257 - 0.1138_{\alpha_s} - 0.0011_{\alpha_s^2} + 0.0104_{\alpha_s^3}\Big]$$

$\boxed{\text{residual uncertainty} \sim 0.5\%}$

Residual uncertainty

[MB, Capdevila, Gambino, '21]

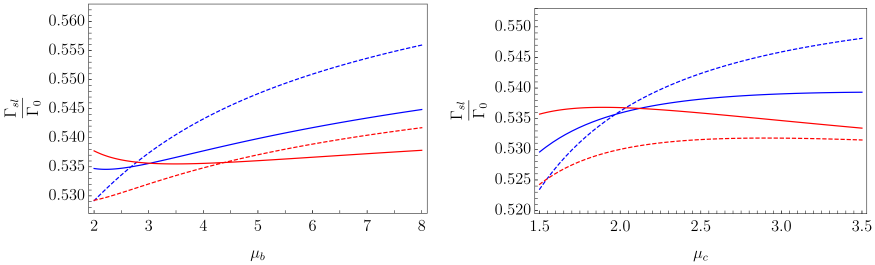


2 loop, $\mu_b = m_b^{kin}$, $\mu_c = 3\,\mathrm{GeV}$
3 loop, $\mu_b = m_b^{kin}$, $\mu_c = 3\,\mathrm{GeV}$
2 loop, $\mu_b = m_b^{kin}/2$, $\mu_c = 2\,\mathrm{GeV}$
3 loop, $\mu_b = m_b^{kin}/2$, $\mu_c = 2\,\mathrm{GeV}$

- Theory uncertainties are essential for a good fit to data [Gambino, Schwanda, '14]
- Residual scale dependence
  - $\Rightarrow$ Milder including $\mathcal{O}(\alpha_s^3)$
  - $\Rightarrow$ We choose $\mu_c = 2\,\mathrm{GeV}$, $\mu_b = m_b^{kin}/2$ and $\mu = 1\,\mathrm{GeV}$ to minimize scale dependence
- Other sources of uncertainties e.g. higher power corrections are slightly smaller

Residual uncertainty

[MB, Capdevila, Gambino, '21]



2 loop, $\mu_b = m_b^{kin}$, $\mu_c = 3\,\mathrm{GeV}$

3 loop, $\mu_b = m_b^{kin}$, $\mu_c = 3\,\mathrm{GeV}$

2 loop, $\mu_b = m_b^{kin}/2$, $\mu_c = 2\,\mathrm{GeV}$

3 loop, $\mu_b = m_b^{kin}/2$, $\mu_c = 2\,\mathrm{GeV}$

- Theory uncertainties are essential for a good fit to data [Gambino, Schwanda, '14]
- Residual scale dependence
  - ⇒ Milder including $\mathcal{O}(\alpha_s^3)$
  - ⇒ We choose $\mu_c = 2\,\mathrm{GeV}$, $\mu_b = m_b^{kin}/2$ and $\mu = 1\,\mathrm{GeV}$ to minimize scale dependence
- Other sources of uncertainties e.g. higher power corrections are slightly smaller

1.2% residual uncertainty

The semileptonic fit

[MB, Capdevila, Gambino, '21]

m_b^{kin}	$\overline{m}_c(2\mathrm{GeV})$	μ_π^2	ρ_D^3	$\mu_g(m_b)$	ρ_{LS}	$\mathrm{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.573	1.092	0.477	0.185	0.306	−0.130	10.66	42.16
0.012	0.008	0.056	0.031	0.050	0.092	0.15	0.51

- Constraints from FLAG $N_f = 2+1+1$: $\overline{m}_b = (\overline{m}_b) = 4.198(12)\,\mathrm{GeV}$ and $\overline{m}_c = (\overline{m}_c) = 0.988(7)\,\mathrm{GeV}$
- No new experimental input wrt to the one in 1411.6560
- The central value of V_{cb} is stable
- Without constraints on m_b, we extract $\bar{m}_b(\bar{m}_b) = 4.210(22)\,\mathrm{GeV}$

The semileptonic fit

[MB, Capdevila, Gambino, '21]

m_b^{kin}	$\overline{m}_c(2\text{GeV})$	μ_π^2	ρ_D^3	$\mu_g(m_b)$	ρ_{LS}	$\text{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.573	1.092	0.477	0.185	0.306	-0.130	10.66	42.16
0.012	0.008	0.056	0.031	0.050	0.092	0.15	0.51

- Constraints from FLAG $N_f = 2+1+1$: $\overline{m}_b = (\overline{m}_b) = 4.198(12)\,\text{GeV}$ and $\overline{m}_c = (\overline{m}_c) = 0.988(7)\,\text{GeV}$
- No new experimental input wrt to the one in 1411.6560
- The central value of V_{cb} is stable
- Without constraints on m_b, we extract $\bar{m}_b(\bar{m}_b) = 4.210(22)\,\text{GeV}$

$$V_{cb} = 42.16(32)_{exp}(30)_{th}(25)_{\Gamma} \cdot 10^{-3}$$

Higher power corrections

- At $\mathcal{O}(1/m^4)$ the number of operators become large
  - $\Rightarrow$ 9 at dim 7
  - $\Rightarrow$ 18 at dim 8

Lowest Lying State Saturation Approximation: [Mannel, Turczyk, Uraltsev, '11]

$$\langle B|\mathcal{O}_1\mathcal{O}_2|B\rangle = \sum_n \langle B|\mathcal{O}_1|n\rangle\langle n|\mathcal{O}_2|B\rangle$$

complete set of states

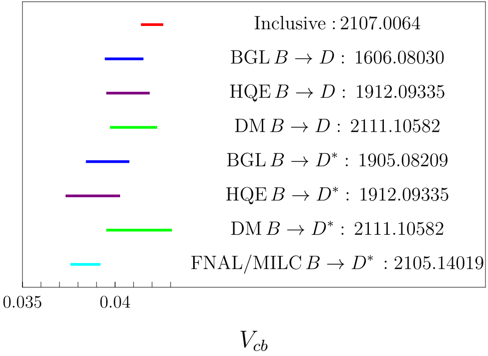
At dimension 6 the LLSA works well:

$$\rho_D^3 = \epsilon\mu_\pi^2 \qquad \rho_{LS}^3 = -\epsilon\mu_G^2 \qquad \epsilon \sim 0.4\,\text{GeV}$$

- Large corrections to the LLSA are possible [Gambino, Mannel, Uraltsev, '12]
- 60% gaussian uncertainty on higher order parameters

$$V_{cb} = 42.00(53) \times 10^{-3}$$

Inclusive vs. Exclusive



- There is a **spread** between inclusive and exclusive determinations of V_{cb}
- The tension between inclusive and FNAL/MILC accounts to almost $4\,\sigma$!
- Determination from q^2 moments **[see Keri's talk]**

The inclusive B_s width

[MB, Gambino, '21]

- B_d and B_s widths are linked through violation of $SU(3)_F$

$$\frac{\delta_{\mu_\pi^2}\Gamma_{\rm sl}(B)}{\Gamma_{\rm sl}(B)} = -0.9(1)\% \qquad \frac{\delta_{\mu_G^2}\Gamma_{\rm sl}(B)}{\Gamma_{\rm sl}(B)} = -3.2(5)\%$$

$$\frac{\delta_{\rho_D^3}\Gamma_{\rm sl}(B)}{\Gamma_{\rm sl}(B)} = -3.2(5)\% \qquad \frac{\delta_{\rho_{LS}^3}\Gamma_{\rm sl}(B)}{\Gamma_{\rm sl}(B)} = -0.3(2)\%$$

- Previous studies used sum rules and HQ relations [Bigi, Mannel, Uraltsev, '11]

- We update those estimates

  $\Rightarrow$ Preliminary lattice estimates [Gambino, Melis, Simula, '17]

  $\Rightarrow$ Most recent semileptonic fit

$$\frac{\Gamma_{\rm sl}(B_s)}{\Gamma_{\rm sl}(B_d)} - 1 = -(1.8 \pm 0.8)\%\,.$$

The inclusive Λ_b width

[MB, Gambino, '21]

- Same arguments as before

- μ_G^2 and ρ_{LS}^3 terms vanish for ground state baryons

$$\frac{\delta_{\mu_G^2}\Gamma_{\rm sl}(B) + \delta_{\rho_{LS}^3}\Gamma_{\rm sl}(B)}{\Gamma_{\rm sl}(B)} = -(3.5 \pm 0.6)\%$$

$\Rightarrow$ Biggest difference comes from these terms

$\Rightarrow$ No big numerical changes from previous determinations

$$\frac{\Gamma_{\rm sl}(\Lambda_b)}{\Gamma_{\rm sl}(B_d)} - 1 = (4.1 \pm 1.6)\%\,,$$

Summary and Prospects

Summary:

- Tension between inclusive and exclusive determination of V_{cb} is not resolved
- New $\mathcal{O}(\alpha_s^3)$ contributions to Γ_{sl} show that

  $\Rightarrow$ perturbative effects are under control

  $\Rightarrow$ reduction of the final uncertainty of 1/3

  $\Rightarrow$ the central value of V_{cb} is stable

Prospects:

- α_s corrections for the hadronic parameters in the moments
- Lattice calculations for the B_s width are ongoing
- Moments measurements for B_s and Λ_b modes