First determination of inclusive V_{cb} from q^2 moments

Keri Vos

in collaboration with M. Fael, Th. Mannel, K. Olschewsky and M. Rahimi

in collaboration with F. Bernlochner, M. Welsch, R. van Tonder, E. Persson

JHEP 1902 (2019) 177 and work in progress
arXiv:1812.07472 and arXiv:2105.02163

Why V_{cb}?

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Inclusive V_{cb} is like a bottle of Barolo

- An excellent example of the strength of Heavy Flavour
- True connoisseurs sample it over and over again in the search for correlations
- Already pretty impressive, but there is always room for improvement

State-of-the-art in inclusive $b \to c$

Jezabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290; Fael, Schonwald, Steinhauser, Phys Rev. D 104 (2021) 016003; Fael, Schonwald, Steinhauser, Phys Rev. Lett. 125 (2020) 052003; Fael, Schonwald, Steinhauser, Phys Rev. D 103 (2021) 014005,

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \Gamma_0^{(3)} \left(\frac{\alpha_s}{\pi}\right)^3 + \frac{\mu_\pi^2}{m_b^2}\left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi}\Gamma^{(\pi,1)}\right)\right.$$
$$\left. + \frac{\mu_G^2}{m_b^2}\left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi}\Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3}\left(\Gamma^{(D,0)} + \Gamma_0^{(1)}\left(\frac{\alpha_s}{\pi}\right)\right) + \mathcal{O}\left(\frac{1}{m_b^4}\right) + \cdots\right)$$

- Include terms up to $1/m_b^3$* see also Gambino, Healey, Turczyk [2016]
- Using lepton energy and hadronic mass moments
- Recent progress: α_s^3 to total rate and kinetic mass Fael, Schonwald, Steinhauser [2020, 2021]
- Recent progress: $\alpha_s \rho_D^3$ for total rate Mannel, Pivovarov [2020]

Recent update:

$$|V_{cb}|_{\rm incl} = (42.16 \pm 0.51) \times 10^{-3}$$

Gambino, Schwanda, PRD 89 (2014) 014022;
Alberti, Gambino et al, PRL 114 (2015) 061802;
Bordone, Capdevila, Gambino, Phys.Lett.B 822 (2021) 136679

Towards the ultimate precision in inclusive V_{cb}

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \Gamma_0^{(3)} \left(\frac{\alpha_s}{\pi}\right)^3 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi}\Gamma^{(\pi,1)}\right) \right.$$
$$\left. + \frac{\mu_G^2}{m_b^2} \left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi}\Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3} (\Gamma^{(D,0)} + \Gamma_0^{(1)} \left(\frac{\alpha_s}{\pi}\right)) + \mathcal{O}\left(\frac{1}{m_b^4}\right) + \cdots \right)$$

- Proliferation of non-perturbative matrix elements
  - 4 up to $1/m_b^3$
  - 13 up to $1/m_b^4$ Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087
  - 31 up to $1/m_b^5$ Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109

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$$\left. + \frac{\mu_G^2}{m_b^2} \left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi} \Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3} (\Gamma^{(D,0)} + \Gamma_0^{(1)} \left(\frac{\alpha_s}{\pi}\right)) + \mathcal{O}\left(\frac{1}{m_b^4}\right) + \cdots \right)$$

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The story of $|V_{cb}|$ continued: [in progress]

- Include α_s corrections for ρ_D^3 for moments Mannel, Pivovarov, Moreno [2022]
- Alternative method using q^2 moments up to $1/m_b^4$ Fael, KKV, Bernlochner et al. [in progress]
- Reconsider how to deal with backgrounds Mannel, Rahimi, KKV [2105.02163]

Reparametrization Invariance

Reparametrization invariance

Dugan, Golden, Grinstein, Chen, Luke, Manohar, Hill, Solon, Heinonen, Mannel
Mannel, KKV, JHEP 1806 (2018) 115

- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Choice of v not unique
- Reparametrization Invariant (RPI) under an infinitesimal change

$$v_\mu \to v_\mu + \delta v_\mu$$

$$\delta_{RP}\ v_\mu = \delta v_\mu \ \text{ and } \ \delta_{RP}\ iD_\mu = -m_b \delta v_\mu$$

- Reparametrization invariance links different orders in $1/m_b$
  - Remember: HQE parameters contain chains of covariant derivatives
  - Gives exact relations between different orders
  - Resums towers of operators
  - Reduces the number of independent parameters

Non-perturbative matrix elements

Mannel, KKV, JHEP 1806 (2018) 115

- 1:
  - $2M_B\mu_3 = \langle B|\bar{b}_v b_v|B\rangle = 2M_B\left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b}\right)$
- $1/m_b^2$:
  - $2M_B\mu_G^2 = \langle B|\bar{b}_v(-i\sigma^{\mu\nu})iD_\mu iD_\nu b_v|B\rangle$
- $1/m_b^3$:
  - $2M_B\tilde{\rho}_D^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left[iD_\mu,\left[\left(ivD + \frac{(iD)^2}{m_b}\right), iD^\mu\right]\right] b_v|B\right\rangle$
- $1/m_b^4$:
  - $2M_B r_G^4 \equiv \frac{1}{2}\langle B|\bar{b}_v[iD_\mu, iD_\nu][iD^\mu, iD^\nu] b_v|B\rangle \propto \langle \vec{E}^2 - \vec{B}^2\rangle$
  - $2M_B r_E^4 \equiv \frac{1}{2}\langle B|\bar{b}_v[ivD, iD_\mu][ivD, iD^\mu] b_v|B\rangle \propto \langle \vec{E}^2\rangle$
  - $2M_B s_B^4 \equiv \frac{1}{2}\langle B|\bar{b}_v[iD_\mu, iD_\alpha][iD^\mu, iD_\beta]\ (-i\sigma^{\alpha\beta})b_v|B\rangle \propto \langle \vec{\sigma}\cdot\vec{B}\times\vec{B}\rangle$
  - $2M_B s_E^4 \equiv \frac{1}{2}\langle B|\bar{b}_v[ivD, iD_\alpha][ivD, iD_\beta]\ (-i\sigma^{\alpha\beta})b_v|B\rangle \propto \langle \vec{\sigma}\cdot\vec{E}\times\vec{E}\rangle$
  - $2M_B s_{qB}^4 \equiv \frac{1}{2}\langle B|\bar{b}_v[iD_\mu,[iD^\mu,[iD_\alpha, iD_\beta]]]\ (-i\sigma^{\alpha\beta})b_v|B\rangle \propto \langle \Box\,\vec{\sigma}\cdot\vec{B}\rangle$.

Up to $1/m_b^4$: 8 parameters   versus previous 13

RPI Observables

Mannel, KKV, JHEP 1806 (2018) 115; Fael, Mannel, KKV, JHEP 02 (2019) 177

- Ratio between the rate with and without a cut

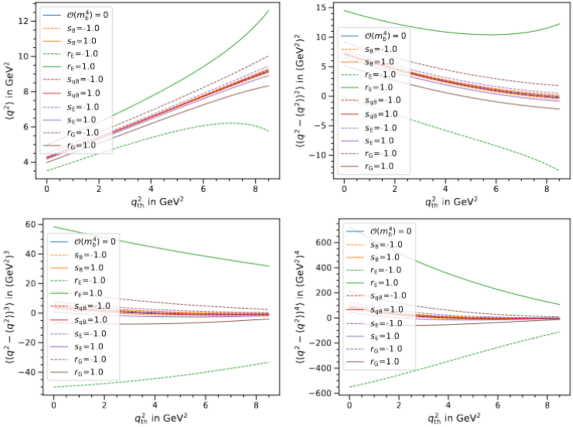
$$R^*(q^2_{\rm cut}) = \int_{q^2>q^2_{\rm cut}} dq^2 \, \frac{d\Gamma}{dq^2} \bigg/ \int_0 dq^2 \, \frac{d\Gamma}{dq^2}$$

- q^2 moments

$$\langle (q^2)^n \rangle_{\rm cut} = \int_{q^2>q^2_{\rm cut}} dq^2 \, (q^2)^n \, \frac{d\Gamma}{dq^2} \bigg/ \int_{q^2>q^2_{\rm cut}} dq^2 \, \frac{d\Gamma}{dq^2}$$

- Hadronic mass and lepton energy moments are NOT RPI
- A cut on lepton energy is not RPI, but $q^2_{\rm cut}$ is RPI

Sensitivity to higher orders



Most sensitivity to r_E and r_G

Alternative V_{cb} determination

Alternative V_{cb} Method

$$R^*(q^2_{\rm cut}) \quad \langle (q^2)^n \rangle_{\rm cut}$$

$$\downarrow$$

$$\mu_3, \mu_G, \tilde{\rho}_D, r_E, r_G, s_E, s_B, s_{qB}, m_b, m_c$$

$$\downarrow$$

$$\mathrm{Br}(\bar{B} \to X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B}\left[\Gamma_{\mu_3}\mu_3 + \Gamma_{\mu_G}\frac{\mu_G^2}{m_b^2} + \Gamma_{\tilde{\rho}_D}\frac{\tilde{\rho}_D^3}{m_b^3} + \Gamma_{r_E}\frac{r_E^4}{m_b^4} + \Gamma_{r_G}\frac{r_G^4}{m_b^4} + \Gamma_{s_B}\frac{s_B^4}{m_b^4} + \Gamma_{s_E}\frac{s_E^4}{m_b^4} + \Gamma_{s_{qB}}\frac{s_{qB}^4}{m_b^4}\right]$$

$$\downarrow$$

$$V_{cb} = ?$$

Fael, Mannel, KKV, JHEP 02 (2019) 177;

Alternative V_{cb} Method

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$$\downarrow$$

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$$\downarrow$$

$V_{cb} =$ Stay Tuned!

Fael, Mannel, KKV, JHEP 02 (2019) 177;

V_{cb} from q^2 moments

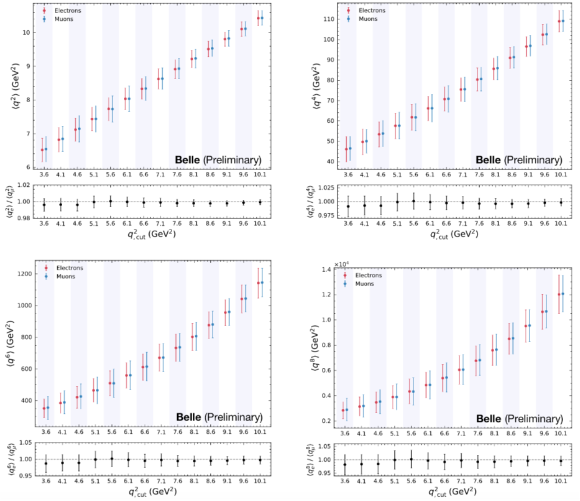
in collaboration with

F. Bernlochner, M. Welsch, M. Fael, K. Olschewsky, R. van Tonder and R. Persson

in progress

New Belle q^2 measurements!

See talk by Ray [2109.01685,2105.08001]



Belle II results to appear soon!

Towards a new extraction of V_{cb}

In progress

Γ	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓	✓	✓
μ_G^2	✓	✓		
ρ_D^3	✓	✓		
$1/m_b^4$	✓			
$m_b^{\rm kin}/m_c^{\overline{\rm MS}}$		✓	✓	✓

$\langle (q^2)^n \rangle$	tree	α_s	α_s^2	α_s^3
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In progress: Software package

- Moments and centralized moments
- α_s^3 to partonic rate included Fael, Schoenwald, Steinhauser [2020, 2021]
- Kinetic scheme for bottom
- Both kinetic scheme and $\overline{\rm MS}$ for charm
- Flexible theoretical covariance matrix

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First extraction:

- Sensitive to r_E^4 and r_G^4
- Goal: Extracting both $|V_{cb}|$ and ρ_D^3, r_E^4 and r_G^4

The fit

$$\chi^2 = \frac{\left(\mathcal{B} - \Gamma(|V_{cb}|, \vec{\theta})\,\frac{\tau_B}{\hbar}\right)^2}{\sigma_{\mathcal{B}}^2 + \sigma_{\Gamma}^2} + \left(\vec{q}(\vec{\theta}) - \vec{q}_{\rm meas}\right) C^{-1} \left(\vec{q}(\vec{\theta}) - \vec{q}_{\rm meas}\right)^T + \sum_i \frac{(\theta_i - \theta_i^{\rm cons})^2}{\sigma_{\theta_i}^2}\,,$$

- $\vec{\theta} = \left\{m_b^{\rm kin}, m_c, \rho_D^3, \mu_G^2, \mu_\pi^2, r_E^4, r_G^4\right\}$
- External constraints on $m_b^{\rm kin}, m_c, \mu_G^2$ and μ_π^2
- Covariance matrix $C = C_{\rm stat} + C_{\rm syst} + C_{\rm theo}$

Theoretical Uncertainties and Correlations

To account for missing higher-order

- perturbative corrections: variation of $\alpha_s(\mu_s)$ between $m_b^{\mathrm{kin}}/2 < \mu_s < 2m_b^{\mathrm{kin}}$.
- $1/m_b$ corrections: variation of ρ_D^3 by 30%.
- $\alpha_s/m_b^{2,3}$ corrections: variation of μ_G^2 by 20%.

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Flexible theory covariance matrix

- Correlation between different q^2 cuts

$$\rho(q_n(q^2_{\rm cut,A}), q_n(q^2_{\rm cut,B}) = \rho_{\rm cut}^x \quad \text{and} \quad x = \frac{|q^2_{\rm cut,B} - q^2_{\rm cut,A}|}{0.5\,\text{GeV}^2},$$

- Correlation between different moments

$$\rho(q_m(q^2_{\rm cut,A}), q_n(q^2_{\rm cut,B}) = \text{sign}(\rho_{\rm mom}) \cdot |\rho_{\rm mom}|^{|m-n|}\, \rho_{\rm cut}^x$$

Theoretical Uncertainties and Correlations

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Default fit: add $\rho_{\rm cut}$ and $\rho_{\rm mom}$ as nuisance parameters to χ^2

Branching ratio input

$$\mathcal{B}(B \to X_c \ell \bar{\nu}_\ell) = \mathcal{B}(B \to X \ell \bar{\nu}_\ell) - \Delta\mathcal{B}(B \to X_u \ell \bar{\nu}_\ell)/\epsilon_{\Delta\mathcal{B}} \,,$$

- $\Delta\mathcal{B}(B \to X_u \ell \bar{\nu}_\ell)$ is the branching ratio with a lepton-energy cut
- Correction factor $\epsilon_{\Delta\mathcal{B}} = 0.858 \pm 0.008$ Belle [2021]
- $\Delta\mathcal{B}(B \to X_u \ell \bar{\nu}_\ell) = 0.00159 \pm 0.00017$ Belle [2021]

$$\mathcal{B}(B \to X_c \ell \bar{\nu}_\ell) = \mathcal{B} \pm \sigma_{\mathcal{B}} = (10.30 \pm 0.16) \cdot 10^{-2} \,.$$

- Differs from $10.66 \pm 0.15 \times 10^{-2}$ obtained by Bordone, Capdevila, Gambino [2021]

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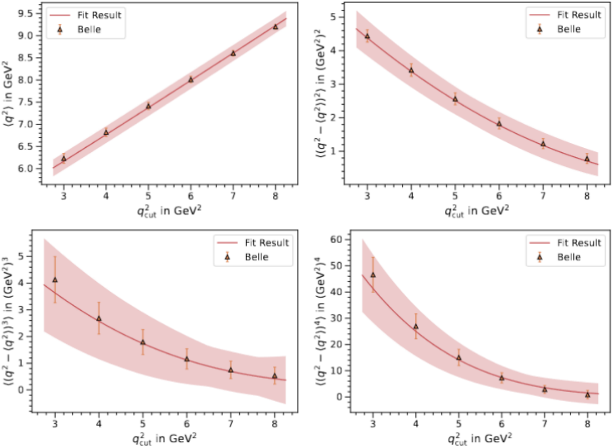
- Differs from $10.66 \pm 0.15 \times 10^{-2}$ obtained by Bordone, Capdevila, Gambino [2021]
- Important to get an update!

Fit projections



First determination of $|V_{cb}|$

First determination of $|V_{cb}|$ from Belle data:

$$
\begin{aligned}
|V_{cb}| &= (41.20 \pm 0.33|_{\mathcal{B}} \pm 0.30|_{\Gamma} \pm 0.19|_{\rm Exp.} \pm 0.18|_{\rm Theo.} \pm 0.43|_{\rm Constr.}) \times 10^{-3} \\
&= (41.20 \pm 0.67) \times 10^{-3} \ .
\end{aligned}
$$

Uncertainties coming from

- $\mathcal{B}$: experimental branching ratio
- Γ: theoretical branching ratio
- Exp. (Theo.): experimental (theoretical) q^2 moments
- Constr.: external constraints

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$$= (41.20 \pm 0.67) \times 10^{-3} \, .$$

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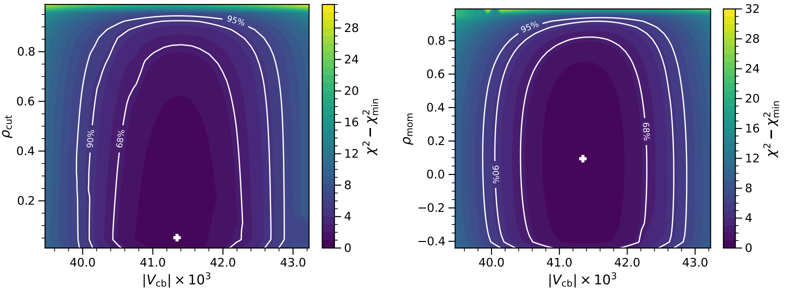
- Additional conservative uncertainty: vary s_E, s_B and s_{qB} by ± 1 GeV^4:

$$|V_{cb}| = (41.20 \pm 0.67 \pm 0.23) \times 10^{-3} = (41.20 \pm 0.71) \times 10^{-3} \, .$$

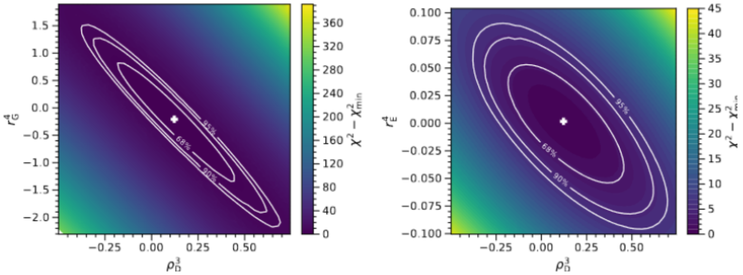
- Note: adding s_E, s_B and s_{qB} as Gaussian constraint gives same uncertainty

Covariance parameters versus V_{cb}



First extraction:

- Profiled over a large range of correlations
- Included in the uncertainty of $|V_{cb}|$



First extraction:

- HQE parameters highly correlated
- Interesting to extract, but large uncertainties
- $\rho_D^3 = 0.12 \pm 0.28$, $r_G^4 = -0.27 \pm 0.98$ and $r_E^4 = (0.01 \pm 0.48) \cdot 10^{-1}$
- Still under investigation

Outlook

Summary & Outlook

The story of V_{cb} continues:

- RPI reduces number of non-perturbative matrix elements
- Total rate and q^2 moments are RPI: 8 instead of 13 up to $1/m_b^4$
- NEW! q^2 moments available!

Coming soon:

- V_{cb} from q^2 moments combined Belle and Belle II
- Study of quark-hadron duality violation
- New Physics analysis on moments of $B \to X_c \ell \nu$ spectrum

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Close collaboration between theory and experiment necessary!

Backup

	$\lvert V_{cb}\rvert$	m_b	m_c	μ_G^2	μ_π^2	ρ_D^3	r_G^4	r_E^4	ρ_{cut}	ρ_{mom}
$\lvert V_{cb}\rvert$	1.00	-0.52	0.14	0.04	0.17	0.24	-0.27	-0.25	-0.11	0.11
m_b	-0.52	1.00	0.00	0.00	-0.00	0.34	-0.26	-0.08	-0.00	0.03
m_c	0.14	0.00	1.00	-0.00	0.00	-0.16	0.12	0.03	0.00	-0.01
μ_G^2	0.04	0.00	-0.00	1.00	-0.00	-0.21	0.11	0.00	-0.00	-0.02
μ_π^2	0.17	-0.00	0.00	-0.00	1.00	-0.01	0.00	0.00	0.00	-0.00
ρ_D^3	0.24	0.34	-0.16	-0.21	-0.01	1.00	-0.97	-0.75	-0.19	0.20
r_G^4	-0.27	-0.26	0.12	0.11	0.00	-0.97	1.00	0.88	0.20	-0.18
r_E^4	-0.25	-0.08	0.03	0.00	0.00	-0.75	0.88	1.00	0.17	-0.09
ρ_{cut}	-0.11	-0.00	0.00	-0.00	0.00	-0.19	0.20	0.17	1.00	0.06
ρ_{mom}	0.11	0.03	-0.01	-0.02	-0.00	0.20	-0.18	-0.09	0.06	1.00

Reduced sensitivity to μ_π^2

- The μ_3 RPI combination drops out in the normalized moments

$$\frac{\mathcal{Q}_n(\hat{q}^2_{\rm cut})}{\mathcal{Q}_0(\hat{q}^2_{\rm cut})} = \frac{\mu_3 X_0^{(n)} + \frac{\mu_G^2}{m_b^2} g_0^{(n)} + \mathcal{O}(\frac{1}{m_b^3})}{\mu_3 X_0^{(0)} + \frac{\mu_G^2}{m_b^2} g_0^{(0)} + \mathcal{O}(\frac{1}{m_b^3})}$$

$$= \frac{X_0^{(n)}}{X_0^{(0)}} \left[1 + \frac{\mu_G^2}{m_b^2} \left(\frac{g_0^{(n)}}{X_0^{(n)}} - \frac{g_0^{(0)}}{X_0^{(0)}} \right) - \frac{(\mu_G^2)^2}{m_b^4} \left(1 + \frac{g_0^{(0)}}{X_0^{(0)}} \right) \left(\frac{g_0^{(n)}}{X_0^{(n)}} - \frac{g_0^{(0)}}{X_0^{(0)}} \right) + \frac{\mu_\pi^2 \mu_G^2}{m_b^4} \left(\frac{g_0^{(n)}}{X_0^{(n)}} - \frac{g_0^{(0)}}{X_0^{(0)}} \right) \right].$$

- Requires (at the moment) external constraint on μ_π^2

Theoretical Uncertainties and Correlations

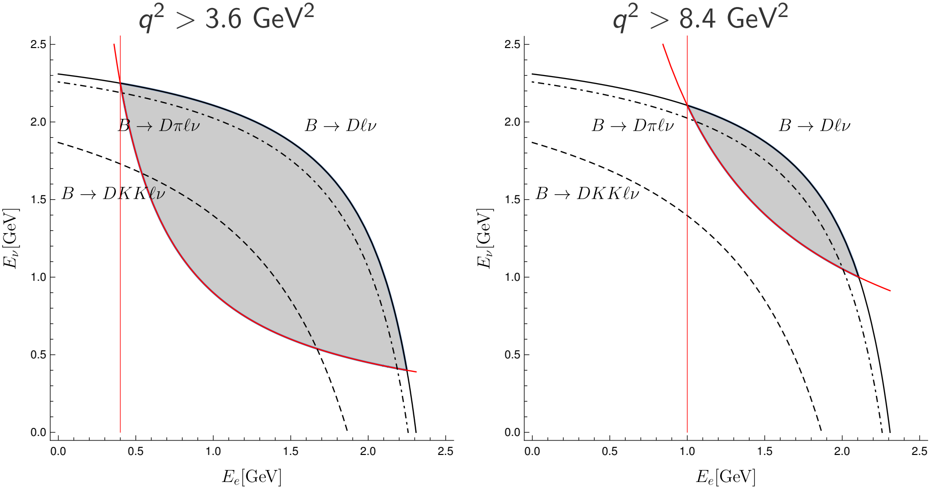
To account for missing higher-order

- perturbative corrections: variation of $\alpha_s(\mu_s)$ between $m_b^{\rm kin}/2 < \mu_s < 2m_b^{\rm kin}$.
- $1/m_b$ corrections: variation of ρ_D^3 by 30%.
- $\alpha_s/m_b^{2,3}$ corrections: variation of μ_G^2 by 20%.

Default fit: external inputs

- $m_b^{\rm kin}(1 \text{ GeV}) = 4.565 \pm 0.020 \text{ GeV}$,
- $\overline{m}_c(2 \text{ GeV}) = 1.093 \pm 0.008 \text{ GeV}$.
- μ_G^2 and μ_π^2

q^2 versus energy cut



Details on the theory covariance

- Correlation between different q^2 cuts

$$\rho(q_n(q^2_{\mathrm{cut,A}}), q_n(q^2_{\mathrm{cut,B}}) = \rho^x_{\mathrm{cut}} \quad \text{and} \quad x = \frac{\left|q^2_{\mathrm{cut,B}} - q^2_{\mathrm{cut,A}}\right|}{0.5\,\mathrm{GeV}^2},$$

- Correlation between different moments

$$\rho(q_m(q^2_{\mathrm{cut,A}}), q_n(q^2_{\mathrm{cut,B}}) = \mathrm{sign}(\rho_{\mathrm{mom}}) \cdot |\rho_{\mathrm{mom}}|^{|m-n|} \cdot \rho(q_n(q^2_{\mathrm{cut,A}}), q_n(q^2_{\mathrm{cut,B}})).$$

- Include correlation parameters as nuisance parameters:

$$f_{\mathrm{DFD}}(\rho, a, b) = \frac{1}{2(1+e^{w(\rho-b)})(1+e^{-w(\rho-a)})} \quad (b > a)\,, \tag{1}$$

with $w = 50$.

- Included in the fit as:

$$\chi^2 \to \chi^2 + \chi^2_{\mathrm{DF}} = \chi^2 - 2\ln f_{\mathrm{DFD}}(\rho_c, 0, 1) - 2\ln f_{\mathrm{DFD}}(\rho_m, -0.45, 1)$$

Reparametrization invariance

Total rate at tree level

Mannel, KKV, JHEP 1806 (2018) 115

$$R = \sum_{n=0}^{\infty} C^{(n)}_{\mu_1 .. \mu_n}(v) \otimes \bar{b}_v (iD_{\mu_1} \cdots iD_{\mu_n}) b_v$$

$$\delta_{\rm RP} R = 0 \quad = \quad \sum_{n=0}^{\infty} \left[\delta_{\rm RP} C^{(n)}_{\mu_1 \cdots \mu_n} \right] \bar{b}_v (iD^{\mu_1} \cdots iD^{\mu_n}) b_v + \sum_{n=0}^{\infty} C^{(n)}_{\mu_1 \cdots \mu_n} \left[\delta_{\rm RP} \bar{b}_v (iD^{\mu_1} \cdots iD^{\mu_n}) b_v \right]$$

The RPI relation:

$$\delta_{\rm RP} C^{(n)}_{\mu_1 \cdots \mu_n} = m_b \delta v^{\alpha} \left[C^{(n+1)}_{\alpha \mu_1 \cdots \mu_n} + C^{(n+1)}_{\mu_1 \alpha \mu_2 \cdots \mu_n} + \cdots + C^{(n+1)}_{\mu_1 \cdots \mu_n \alpha} \right]$$

Parameter reduction: an example ρ_{LS}

- $1/m_b^2$: $\mu_G^2 \to \underbrace{\eta\,(-i\sigma_{\mu\nu})}_{c^{(2)}_{\mu\nu}} \otimes \bar{b}_v\,(iD^\mu\, iD^\nu)\, b_v$
- $1/m_b^3$: $\rho_{LS}^3 \to \underbrace{\xi\, v_\alpha (-i\sigma_{\mu\nu})}_{c^{(3)}_{\mu\alpha\nu}} \otimes \bar{b}_v\,(iD^\mu\, iD^\alpha\, iD^\nu)\, b_v$

The RPI relation:

$$
\begin{aligned}
\delta_{\mathrm{RP}}\, C^{(2)}_{\mu\nu} &= 0 \\
&= m_b\, \delta v^\alpha \left(C^{(3)}_{\mu\nu\alpha} + C^{(3)}_{\mu\alpha\nu} + C^{(3)}_{\alpha\mu\nu} \right) \\
&= -i m_b\, \xi\, \delta v^\alpha \,(\sigma_{\mu\alpha} v_\nu + \sigma_{\alpha\nu} v_\mu) \\
&\leftrightarrow \xi = 0
\end{aligned}
$$

RPI Observables

- RPI reduced set to directly fit the higher order terms
- Requires RPI observables

$$O = \int w(v, p_e, p_\nu) \langle \mathrm{Im}\, T(S) \rangle L(p_e, p_\nu) d\Phi_3$$

- The observable O is RPI if $\delta_{\mathrm{RP}} w(v, p_e, p_\nu) = 0$

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O	$w(v, p_e, p_\nu)$	RPI
Total Rate	1	✓
Moments charged lepton energy	$(v \cdot p_e)^n$	✗
Moments hadronic invariant mass	$(M_B v - q)^{2n}$	✗
Moments leptonic invariant mass	$(q^2)^n$	✓

Fael, Mannel, KKV, JHEP 02 (2019) 177

Theory guidance to include power corrections

Lowest State Saturation Approximation (LSSA)

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

$$\rho_D^3 = \varepsilon\mu_\pi^2, \qquad \rho_{LS}^3 = -\varepsilon\mu_G^2, \qquad \varepsilon \sim 0.4 \text{ GeV}$$

Mannel, Turczyk, Uraltsev JHEP 1011 (2010) 109; Heinonen, Mannel, NPB 889 (2014) 46

- LSSA estimated as priors (60% gaussian uncertainty)
- $\mathcal{O}(1/m_b^4, 1/m_b^5)$ can then be included in fit Healey, Turczyk, Gambino, PLB 763 (2016) 60
- -0.25% shift in V_{cb}

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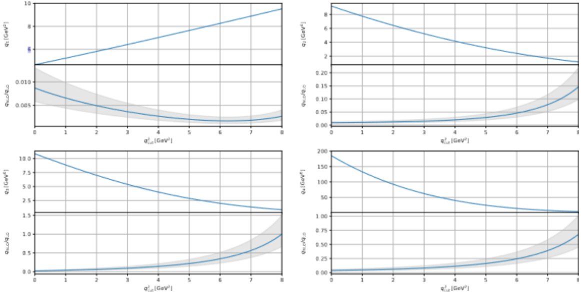
- -0.25% shift in V_{cb}
- Full determination up to $1/m_b^4$ from data possible?



α_s corrections to moments