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$|V_{ub}|$: Exclusive semileptonic decays (B-> π)

Based on JHEP 07 (2021) 082, in collaboration with I. Ray, A. Biswas and S. Patra

Prologue

- Exclusive $B \to \pi l v$ decays: $|\mathrm{V_{ub}}|$ extraction. $l = e, \mu$.
- **Inclusive measurement:** Considerable background from $b \to c l v$ decays. Experimental cuts required to separate $b \to u$ from $b \to c$ decays…Restricts phase space….. OPE breaks down and sensitivity to non-perturbative aspects increases.
- **Exclusive measurement** $B \to \pi l v$ decays: Babar(2011) (Phys. Rev. D **83**(2011) 032007), Belle(2011) (Phys. Rev. D **83** (2011) 071101), Babar (2012) (Phys. Rev. D **86** (2012) 092004), Belle(2013) (Phys. Rev. D **88** (2013) 032005). Averaging by HFLAV (p-value~ 6%) followed by fitting $|\mathrm{V_{ub}}|$ to averaged dataset…………………………Statistically significance?
- Disagreement of $\geq 2.2\,\sigma$. $\mathrm{V_{ub}^{incl}}$:$(3.70 \pm 0.16) \times 10^{-3}$, $\mathrm{V_{ub}^{excl}}$:$(4.25 \pm 0.12^{+0.15}_{-0.14}) \times 10^{-3}$.

Form Factors

- Precise extractions of $|V_{ub}|$ requires information on form factors at different q^2 regions.
- Non-perturbative techniques: **Lattice-QCD** (High q^2) **LCSR**: (Low q^2).
- Shape of decay distribution ($d\Gamma/dq^2$): Shape of the form factors over full q^2.
- **z expansion:** model independent technique based on analyticity arguments. Conformally maps the allowed (real) q^2 range within a (complex) disc of radius $|z|< 1$.

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}},$$

$$t_\pm \equiv (m_B \pm m_\pi)^2 \text{ and } t_0 \equiv t_+(1 - \sqrt{1 - t_-/t_+})$$

BCL & BSZ: appropriate for B-> π

- **BCL:**

$$f_+(z) = \frac{1}{1-q^2/m_{B^*}^2}\sum_{n=0}^{N_z-1} b_n^+\left[z^n - (-1)^{n-N_z}\frac{n}{N_z}z^{N_z}\right],$$

$$f_0(z) = \sum_{n=0}^{N_z-1} b_n^0 z^n .$$

$$\Sigma(b^{0/+}, N_z) \equiv \sum_{m,n=0}^{N_z} B_{mn}\, b_m^{0/+} b_n^{0/+} \leq 1\,,$$

B_{mn} satisfies $B_{mn} = B_{nm} = B_{0|m-n|}$

- **BSZ:**

$$f_i(q^2) = \frac{1}{1-q^2/m_{R,i}^2}\sum_{k=0}^{N} a_k^i\,[z(q^2)-z(0)]^k$$

where $m_{R,i}$ denotes the mass of sub-threshold resonances

$m_{B^*} = 5.54\ GeV$

BCL obeys asymtptotic behaviour near Bπ threshold: $Im(f_+(q^2)\sim(q^2-t_+)^{3/2})$. Hence at $q^2 = t_+\ (z=-1)$, $\frac{df_+}{dz}|_{z=+1} = 0$, removes extra d.o.f for f_+. For BSZ $f_i(0) = \mathrm{a}_0^i$, also the QCD relation $f_+(0) = f_0(0)$ boils down to $\mathrm{a}_+^0 = \mathrm{a}_0^0$. However, for BCL:

$$b_3^0 = 45.70(b_0^+ - b_0^0) - 12.78b_1^0 - 3.58b_2^0 + 12.85b_1^+ + 3.44b_2^+ + 1.21b_3^+$$

Inputs

- $\boldsymbol{B \to \pi l v}$ **:** Babar(2011), Combined analysis (B-> π, B-> ρ) (Phys. Rev. D **83**(2011) 032007), Belle(2011) (Phys. Rev. D **83** (2011) 071101), Babar (2012), combined + individual modes (Phys. Rev. D **86** (2012) 092004), Belle(2013) (Phys. Rev. D **88** (2013) 032005).
- $\boldsymbol{B \to \pi}$ **Form Factors:** Lattice (high q^2): MILC(*Phys. Rev. D* **92** (2015) 014024, *PoS* **LATTICE 2019** (2019) 236), UKQCD(*Phys. Rev. D* **91** (2015) 074510). **Tensor:** NP analysis. , Lattice: MILC *(Phys. Rev. Lett.* **115** (2015) 152002). LCSR(low q^2): GKD (*JHEP* **01** (2019) 150) , LMD (*JHEP* **07** (2021) 036). LMD use two-particle twist-two pion LCDA. More precise than GKD which is an LO calculation with the ill-known B-meson LCDA.

Comparison with MILC

- Apart from HFLAV, MILC have also combined expt. data.
- No averaging. Fit direct to data w/wo lattice.
- Reproduce their analysis where just expt. data from each measurement are fitted with (n=3) BCL parameterization.
- Great agreement except for Babar (2012). We have a much better p-value.
- Cross-check with analysis done by Babar (2012) with n=4 BGL parametrization.

Scenario	Fit	χ^2/dof	dof	p-value (%)	b_1/b_0	b_2/b_0	$b_0\lvert V_{ub}\rvert \times 10^3$
This Work	All exp	1.4	48	3	−1.05(19)	−1.08(60)	1.52(3)
	BaBar(11) [6]	2	3	10	−0.95(43)	0.7(1.4)	1.36(7)
	BaBar(12) [8]	0.4	9	91	−0.32(44)	−3.6(1.3)	1.50(6)
	Belle(11) [7]	1.2	10	30	−1.32(27)	−0.69(91)	1.60(6)
	Belle(13) [9]	1.3	17	19	−1.89(50)	1.4(1.6)	1.56(8)
MILC [12]	All exp	1.5	48	2	−0.93(22)	−1.54(65)	1.53(4)
	BaBar(11)	2	3	12	−0.89(47)	0.5(1.5)	1.36(7)
	BaBar(12)	1.2	9	31	−0.48(59)	−3.2(1.7)	1.54(9)
	Belle(11)	1.1	10	36	−1.21(33)	−1.18(95)	1.63(7)
	Belle(13)	1.2	17	23	−1.89(50)	1.4(1.6)	1.56(8)

Scenario	a_1/a_0	a_2/a_0	χ^2/dof	p-value (%)	$\lvert V_{ub}\, f_+(0)\rvert \times 10^4$
BaBar(12)	−0.93 (19)	−5.4 (1.0)	4.07/9	90.7	8.7 (3)
This Work	−0.91(27)	−5.54(149)	3.93/9	91.6	8.6(4)

The BaBar dataset: closer look

- For the analysis, one has to carefully inspect all the datasets.
- A closer look reveals Babar 2012 significantly better than Babar 2011.

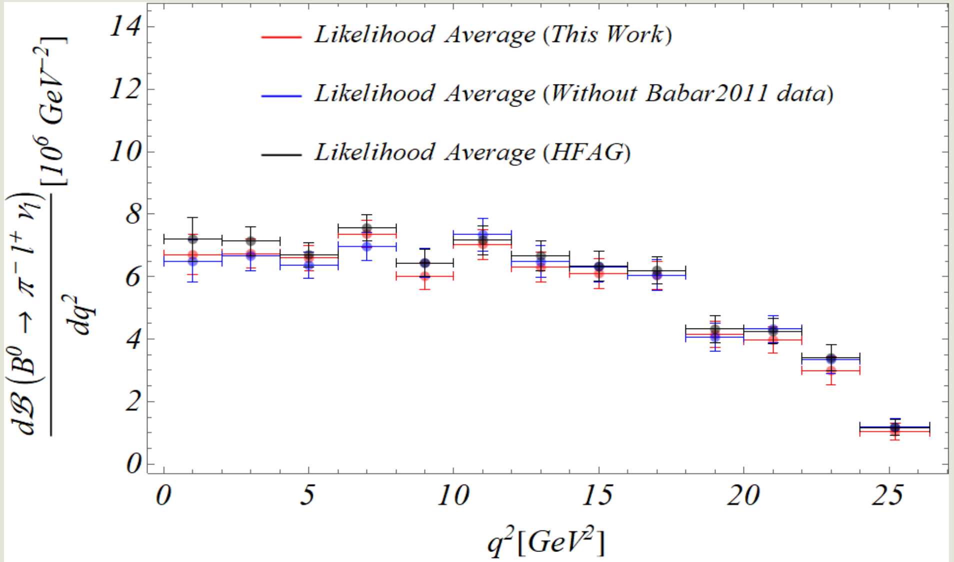
- The event selection has been optimized over the entire fit region instead of the signal-enhanced region, as was done previously.
- The tighter selections produce a data-set with a better signal to background ratio and higher purity in the $B^0 \to \pi^+ \ell \nu$ decays.
- This analysis uses the full BaBar data-set compared to only a subset in the analysis of 2011.

- Babar (2011) present their results from an exclusive analysis of $B^0 \to \pi^+ l\nu, B^+ \to \pi^0 l\nu, B^0 \to \rho^+ l\nu$ and $B^0 \to \rho^+ l\nu$: This might reduce some of the background contributions. The Babar (2011) analysis is also markedly different from the analyses by Belle along with that by Babar itself in 2012.
- MILC also pointed out that Babar (2011) is at odds with the rest.
- Problematic?

Comparison with HFLAV

HFLAV follows a two step process:

1. Binned maximum likelihood fit to determine avg. partial B.F. in each q^2 interval.
2. This avg. spectrum is then used in order to fit V_{ub}



Δq^2 $[GeV^2]$	$(\Delta\mathcal{B}(B^0 \to \pi^- \ell^+ \nu_\ell)/\Delta q^2)$ $[10^{-7}]$		
	HFLAV Average	Our Average	Average (Dropping BaBar (11) [6])
0–2	72.0±7.0	67.0±6.4	64.9 ± 6.7
2–4	71.4±4.6	67.4±4.7	66.6 ± 4.8
4–6	67.0±3.9	65.9±3.9	63.7 ± 4.2
6–8	75.6±4.3	73.7±4.2	69.6 ± 4.6
8–10	64.4±4.3	60.2±4.4	64.3 ± 4.7
10–12	71.7±4.6	70.4±4.8	73.4 ± 5.2
12–14	66.7±4.7	63.0±4.8	64.8 ± 5.1
14–16	63.3±4.8	61.0±4.8	63.2 ± 5.1
16–18	62.0±4.4	60.5±4.5	60.5 ± 4.8
18–20	43.2±4.3	41.5±4.2	40.6 ± 4.4
20–22	42.5±4.1	39.7±4.1	43.2 ± 4.3
22–24	34.0±4.2	29.9±4.4	33.5 ± 4.7
24–26.4	11.7±2.6	10.4±2.7	11.8 ± 2.7
p-value	6%	≈ 1%	24.8%

Lattice and LCSR (new): $|V_{ub}|$

- LCSR: GKD provide synthetic data points (central values, uncertainties and the corresponding correlation matrix) for values of ff's at $q^2 = [-15, -10, -5, 0, 5]\ GeV^2$.

- Lattice:
  - UKQCD: synthetic data points with (systematic and statistical) covariance matrices at $q^2 = [19, 22.6, 25.1]\ GeV^2$.
  - MILC: fit results for the coefficients, we have generated synthetic data at the same points as UKQCD..

Parameters	Our Avg. q^2 spec. w/o BaBar (11) + New Lattice & LCSR + BaBar (11) re-introduced (p value = 0.75%)	Our Avg. q^2 spec. + New Lattice & LCSR (p value = 20.9%)	Our Avg. q^2 spec. w/o BaBar (11) + New Lattice & LCSR (p value = 31%)
$V_{ub} \times 10^3$	3.78(13)	3.78(13)	3.89(14)
b_0^+	0.410(12)	0.410(12)	0.408(12)
b_1^+	−0.526(44)	−0.526(44)	−0.561(46)
b_2^+	−0.39(13)	−0.39(13)	−0.40(13)
b_3^+	0.59(24)	0.59(24)	0.59(25)
b_0^0	0.540(16)	0.540(16)	0.536(16)
b_1^0	−1.617(66)	−1.617(66)	−1.647(66)
b_2^0	1.294(146)	1.294(146)	1.257(146)

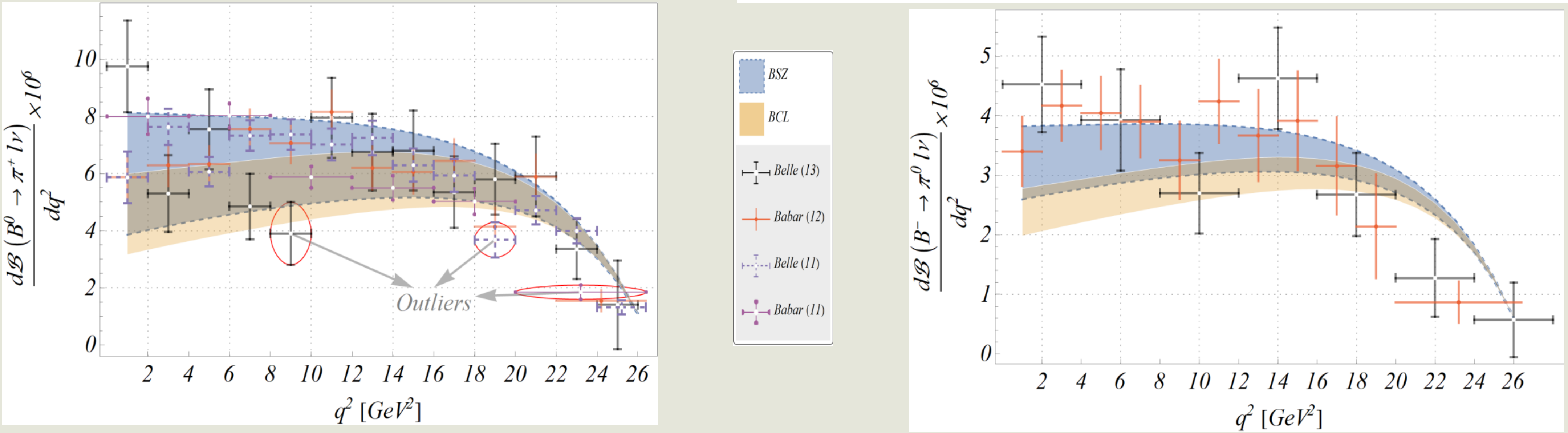
More checks: strategy

- DO NOT AVERAGE!. Look for outliers in the fit to exclusive data, also check the spectrum using inclusive $|V_{ub}|$:

  Are their any common outliers?

- Plots using $|V_{ub}|$ inclusive $(4.10 \pm 0.22 \pm 0.22 \pm 0.22) \times 10^{-3}$ (Belle).

BSZ				BCL			
$\chi^2_{\rm min}/$DOF	p-value (%)	Parameters	Values	$\chi^2_{\rm min}/$DOF	p-value (%)	Parameters	Values
4.48/15	99.6	a_0^+	0.213(22)	12.88/15	61	b_0^+	0.396(13)
		a_1^+	−0.65(14)			b_1^+	−0.707(70)
		a_2^+	0.263(425)			b_2^+	−0.36(18)
		a_3^+	0.67(31)			b_3^+	0.77(32)
		a_1^0	0.41(17)			b_0^0	0.521(17)
		a_2^0	1.46(51)			b_1^0	−1.756(78)
		a_3^0	1.78(49)			b_2^0	1.15(16)



Outliers: data with pull > 2

Form-Factors	Inclusive V_{ub} used	$[B^0 \to \pi^-]$ $q^2:\ 18-20$ Belle (11)	$[B^0 \to \pi^-]$ $q^2:\ 20-26.4$ BaBar (11)	$[B^0 \to \pi^-]$ $q^2:\ 18-20$ BaBar (12)	$[B^+ \to \pi^0]$ $q^2:\ 20-26.4$ BaBar (12)	$[B^0 \to \pi^+]$ $q^2:\ 0.0111637-2$ Belle (13)	$[B^0 \to \pi^+]$ $q^2:\ 8-10$ Belle (13)
BSZ	HFLAV (GGOU)	−2.55	−3.54	−2.13	−2.35	—	−2.04
	HFLAV (BLNP)	−2.50	−3.27	−2.14	−2.41	—	−2.09
	Belle (New)	—	−2.32	—	—	2.22	—
BCL	HFLAV (GGOU)	−2.32	−3.49	—	−2.28	2.30	—
	HFLAV (BLNP)	−2.32	−3.23	—	−2.35	2.07	—
	Belle (New)	—	−2.29	—	—	2.54	—

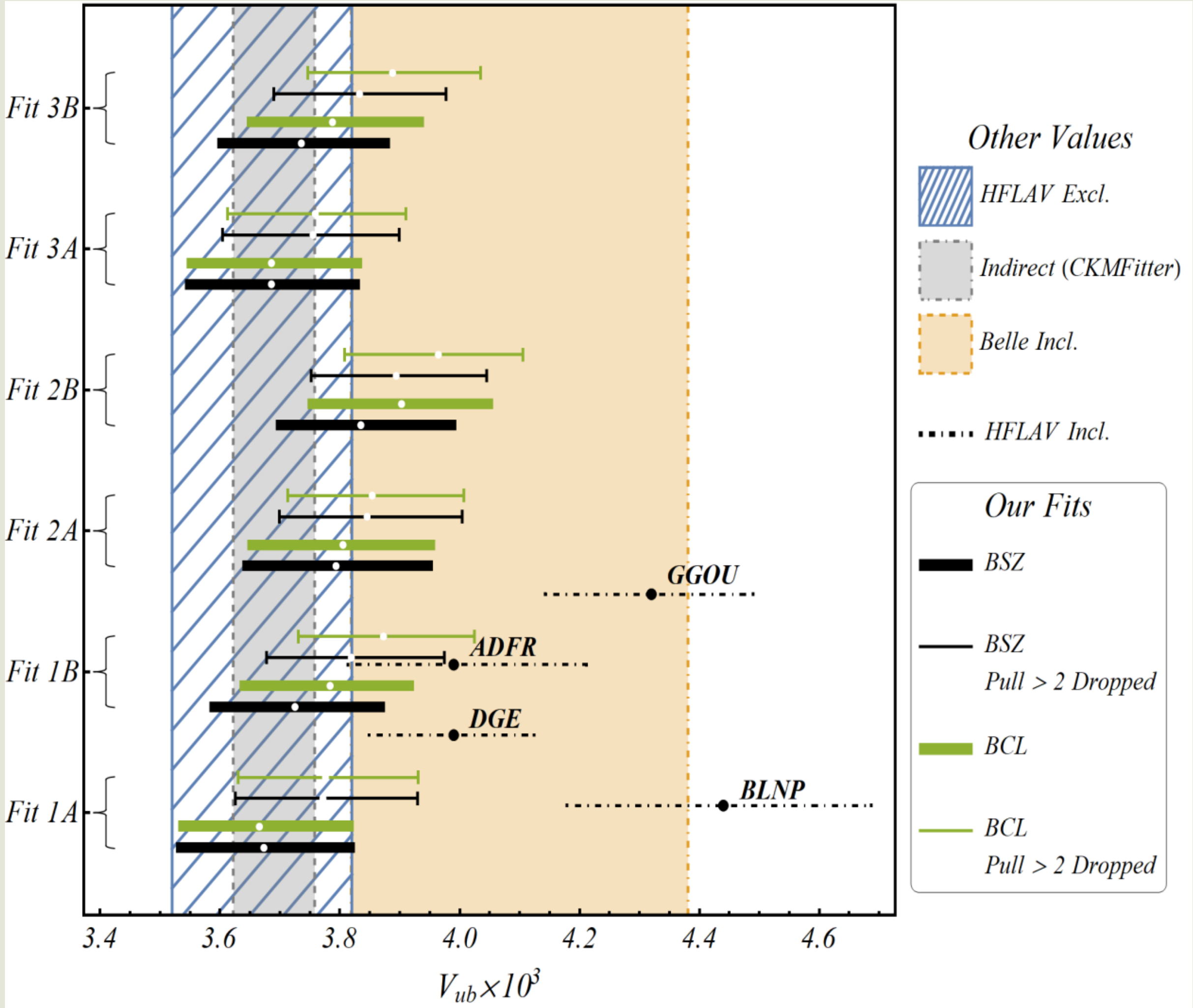
Form-Factors	Fit Index	$[B^0 \to \pi^-]$ $q^2:\ 4-8$ BaBar (11)	$[B^0 \to \pi^-]$ $q^2:\ 20-26.4$ BaBar (11)	$[B^0 \to \pi^-]$ $q^2:\ 10-12$ BaBar (12)	$[B^0 \to \pi^-]$ $q^2:\ 20-22$ BaBar (12)	$[B^0 \to \pi^-]$ $q^2:\ 18-20$ Belle (11)	$[B^0 \to \pi^+]$ $q^2:\ 0.0111637-2$ Belle (13)	$[B^0 \to \pi^+]$ $q^2:\ 8-10$ Belle (13)
BSZ	Fit 1A	2.46	−2.30	2.08	—	—	—	−2.42
	Fit 1B	2.52	−2.42	2.07	—	—	—	−2.41
	Fit 2A	—	—	—	—	−2.02	—	−2.43
	Fit 2B	—	—	—	—	−2.07	—	−2.42
	Fit 3A	2.40	−2.35	2.00	2.01	—	—	−2.44
	Fit 3B	2.45	−2.46	—	—	—	—	−2.43
BCL	Fit 1A	2.45	−2.30	2.07	—	—	—	−2.42
	Fit 1B	2.59	−2.56	2.07	—	—	—	−2.40
	Fit 2A	—	—	—	—	−2.03	—	−2.45
	Fit 2B	—	—	—	—	−2.18	2.00	−2.42
	Fit 3A	2.36	−2.36	—	2.00	—	—	−2.45
	Fit 3B	2.48	−2.61	—	—	—	—	−2.44

- **Fit 1**: B^0 decays from Belle (2011) and Belle (2013); B^- decays from Belle (2013); the combined modes from BaBar (2011) and BaBar (2012). We have subdivided this set into two sets depending on whether or not LCSR inputs are taken into account, like the following:
    1. *Fit 1A*: experimental data (Fit 1) + synthetic Lattice data points,
    2. *Fit 1B*: experimental data (Fit 1) + synthetic Lattice data points + LCSR.
- **Fit 2**: B^0 decays from Belle (2011), BaBar (2012), and Belle (2013); B^- decays from BaBar (2012) and Belle (2013). As above we have defined the following sets:
    - *Fit 2A*: experimental data (Fit 2) + synthetic Lattice data points,
    - *Fit 2B*: experimental data (Fit 2) + synthetic Lattice data points + LCSR.

Fit 3: the combined modes from BaBar (2011) along with the *Fit 2* dataset, with and without LCSR as follows:

- *Fit 3A*: experimental data (Fit 3) + synthetic Lattice data points,
- *Fit 3B*: experimental data (Fit 3) + synthetic Lattice data points + LCSR.

$|V_{ub}|$ Extraction



$|V_{ub}|$ Extraction: Results

Form-Factors	Inclusive V_{ub} used	$[B^0 \to \pi^-]$ q^2: 18 – 20 Belle (11)	$[B^0 \to \pi^-]$ q^2: 20 – 26.4 BaBar (11)	$[B^0 \to \pi^-]$ q^2: 18 – 20 BaBar (12)	$[B^+ \to \pi^0]$ q^2: 20 – 26.4 BaBar (12)	$[B^0 \to \pi^+]$ q^2: 0.0111637 – 2 Belle (13)	$[B^0 \to \pi^+]$ q^2: 8 – 10 Belle (13)
BSZ	HFLAV (GGOU)	−2.55	−3.54	−2.13	−2.35	—	−2.04
	HFLAV (BLNP)	−2.50	−3.27	−2.14	−2.41	—	−2.09
	Belle (New)	—	−2.32	—	—	2.22	—
BCL	HFLAV (GGOU)	−2.32	−3.49	—	−2.28	2.30	—
	HFLAV (BLNP)	−2.32	−3.23	—	−2.35	2.07	—
	Belle (New)	—	−2.29	—	—	2.54	—

- **Fit 2B-I**: input used in *Fit 2B* without the data on $\mathcal{B}(B^0 \to \pi^-)^{[18,20]}$ (Belle 2011).
- **Fit 3B-I**: input used in *Fit 3B* without the data on $\mathcal{B}(B^0 \to \pi^-)^{[20,26.4]}$ (BaBar 2011).
- **Fit 3B-II**: input used in *Fit 3B* without the data on $\mathcal{B}(B^0 \to \pi^-)^{[18,20]}$ (Belle 2011) and $\mathcal{B}(B^0 \to \pi^-)^{[20,26.4]}$ (BaBar 2011).

Form-Factors	Fit Index	$[B^0 \to \pi^-]$ q^2: 4 – 8 BaBar (11)	$[B^0 \to \pi^-]$ q^2: 20 – 26.4 BaBar (11)	$[B^0 \to \pi^-]$ q^2: 10 – 12 BaBar (12)	$[B^0 \to \pi^-]$ q^2: 20 – 22 BaBar (12)	$[B^0 \to \pi^-]$ q^2: 18 – 20 Belle (11)	$[B^0 \to \pi^+]$ q^2: 0.0111637 – 2 Belle (13)	$[B^0 \to \pi^+]$ q^2: 8 – 10 Belle (13)
BSZ	Fit 1A	2.46	−2.30	2.08	—	—	—	−2.42
	Fit 1B	2.52	−2.42	2.07	—	—	—	−2.41
	Fit 2A	—	—	—	—	−2.02	—	−2.43
	Fit 2B	—	—	—	—	−2.07	—	−2.42
	Fit 3A	2.40	−2.35	2.00	2.01	—	—	−2.44
	Fit 3B	2.45	−2.46	—	—	—	—	−2.43
BCL	Fit 1A	2.45	−2.30	2.07	—	—	—	−2.42
	Fit 1B	2.59	−2.56	2.07	—	—	—	−2.40
	Fit 2A	—	—	—	—	−2.03	—	−2.45
	Fit 2B	—	—	—	—	−2.18	2.00	−2.42
	Fit 3A	2.36	−2.36	—	2.00	—	—	−2.45
	Fit 3B	2.48	−2.61	—	—	—	—	−2.44

Fit Scenario	BSZ				BCL			
	χ^2/DOF	p-value (%)	$V_{ub} \times 10^3$		χ^2/DOF	p-value (%)	$V_{ub} \times 10^3$	
			Frequentist	Bayesian			Frequentist	Bayesian
F2B-I	55.4/69	88.14	3.90(14)	$3.89^{+0.14}_{-0.15}$	68.85/69	48.25	3.96(14)	$3.95^{+0.14}_{-0.15}$
F3B-I	78.86/75	35.8	3.83(14)	3.83(13)	93.6/75	7.19	3.89(14)	3.89(14)
F3B-II	72.96/74	51.25	3.88(14)	$3.87^{+0.14}_{-0.15}$	87.2/74	13.99	3.94(14)	$3.93^{+0.14}_{-0.15}$

Update: Including LMD, value :

$|V_{ub}| = (3.91 \pm 0.13) \times 10^{-3}$

Deviation < 1σ

THANK YOU!