JLQCD form factors for $B \to D^{(*)} \ell \nu$

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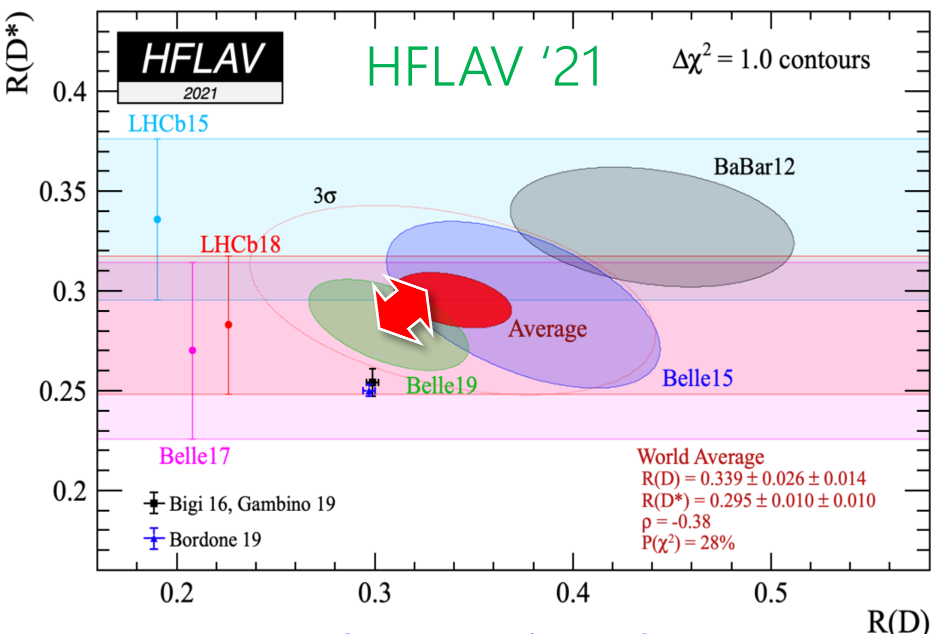
Challenges in Semileptonic B decays

April 21st, 2022, Barolo / Zoom

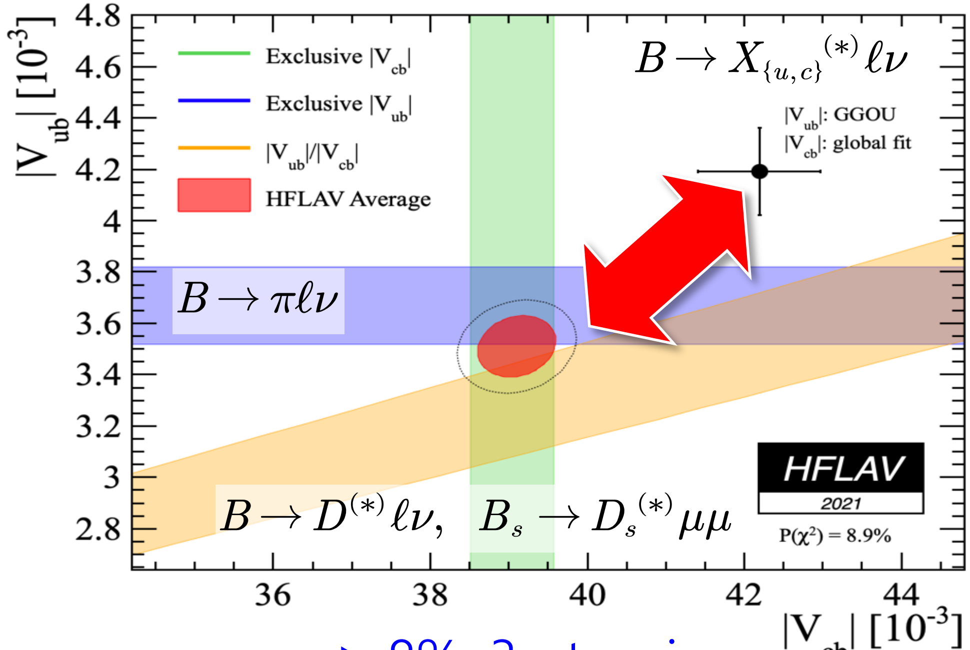
introduction

hint of new physics ?

$|V_{cb}|$ tension



3 σ tension (?)



≥ 8%, 3σ tension

new physics? ⇔ Crivellin-Pokorski '18 : $d_L^{qb}\partial^\nu\left(\bar{q}\sigma_{\mu\nu}P_L b\right)$ ⇔ $\Gamma\left(Z \to b\bar{b}\right)$

need deeper understanding of th. and/or exp't uncertainties

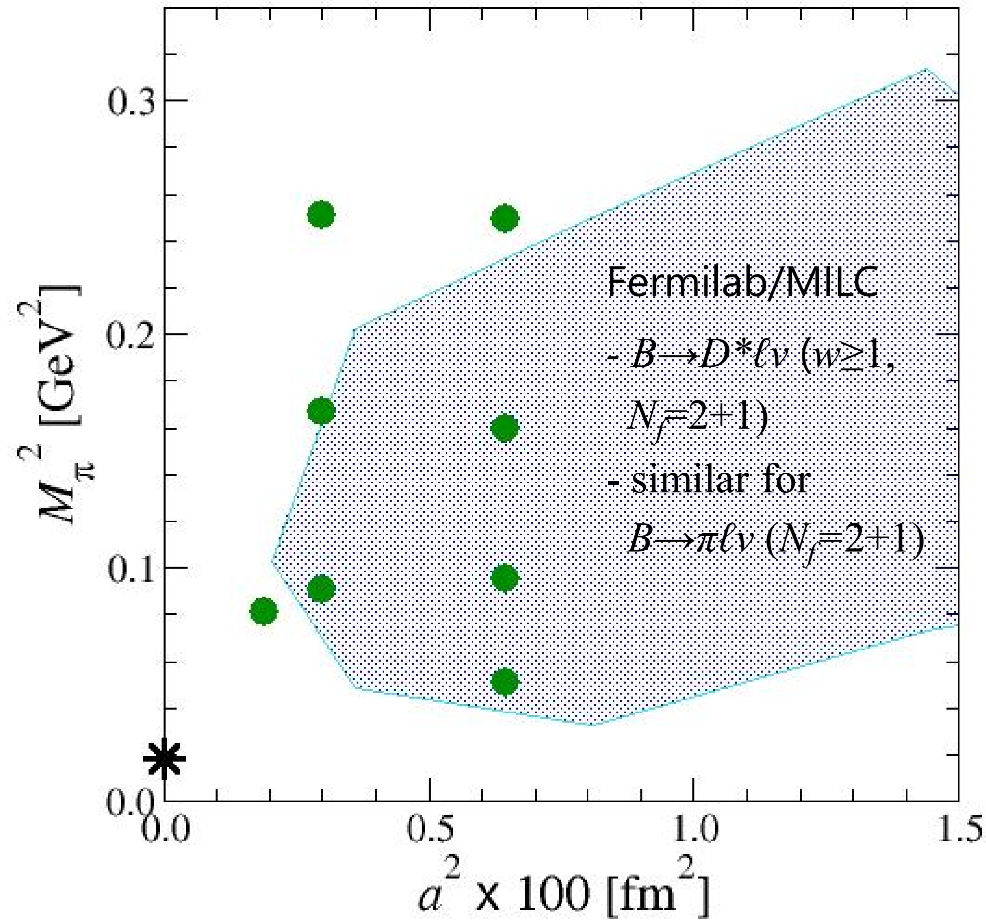
- theory side : form factors (FFs) describing non-perturbative QCD effects
- this talk: JLQCD's study on $B\to D^*\ell\nu$ @ non-zero recoils ⇔ Fermilab/MILC '21

gauge ensembles

domain-wall quarks to preserve chiral symmetry

simulation parameters

- $N_f = 2+1$
- $a^{-1} \lesssim 2.5, 3.6,$ 4.5 GeV $\sim m_b$
  + $m_{\rm res} \lesssim$ a few MeV
  + no $O(a^{2n+1})$ errors
- $M_\pi \gtrsim$ 230 MeV
  + $D^* \not\to D\pi$
  + chiral log from a=0 HMChPT
- $M_\pi L \gtrsim 4$
  + $M_\pi L$ = 3 to directly check FVEs
- statistics < staggered-type



$a^{-1} \lesssim m_b \Rightarrow$ need a careful treatment of heavy quarks

“relativistic approach”

QCD action for all flavors

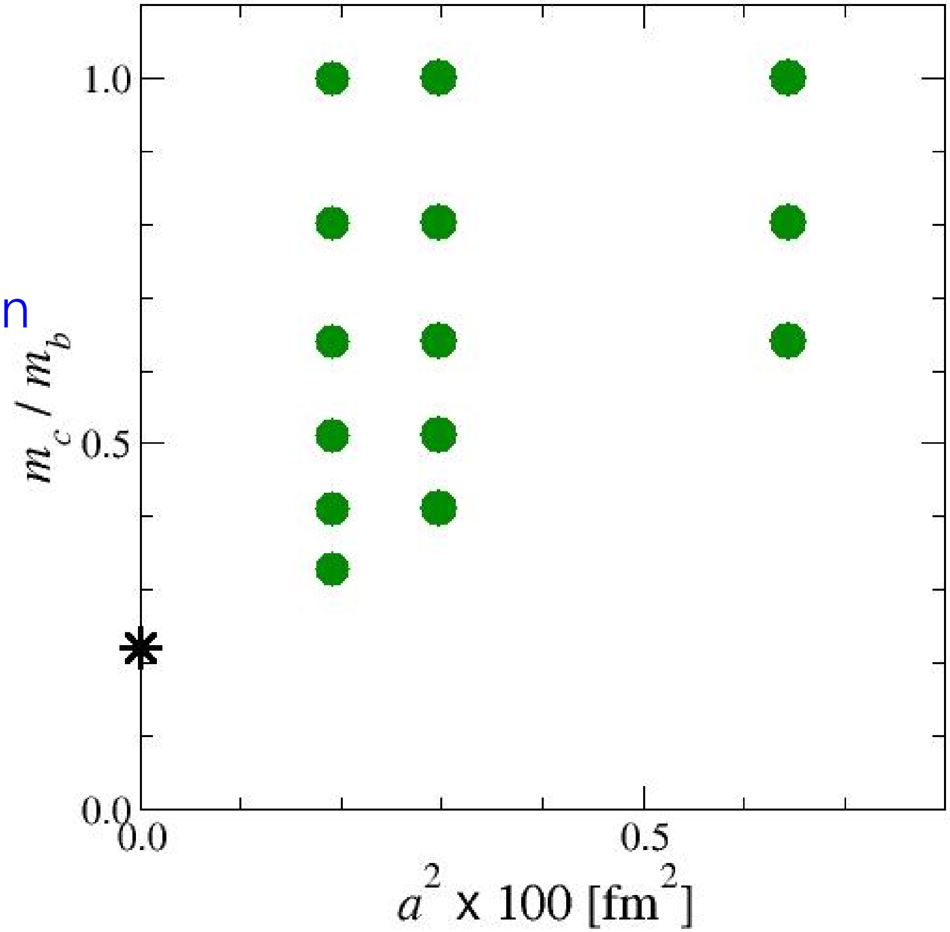
- do not need matching of EFT
- + chiral symmetry

  ⇒ not need explicit renormalization for SM FFs
- charm is OK : $m_c < a^{-1}$'s

  fix m_c to physical value
- simulate 3 – 6 $m_b \leq 0.7a^{-1}$

  ⇒ extrapolate to $m_{b,\mathrm{phys}}$

  ⇔ m_b dependence not large



very different systematics from other studies

$B\to\pi\ell\nu$ (2203.04938), inclusive (talk by Hashimoto), B mixing w/ RBC/UKQCD, ...

$B \to D^* \ell \nu$ FFs

"relativistic" convention

$$\langle D^*(p',\epsilon)|V^\mu|\bar{B}(p)\rangle = ig\epsilon^{\mu\alpha\beta\gamma}\epsilon^*_\alpha p'_\beta p_\gamma,$$

$$\langle D^*(p',\epsilon)|A^\mu|\bar{B}(p)\rangle = f\epsilon^{*\mu} + (\epsilon^*\cdot p)\left[a_+(p+p')^\mu + a_-(p-p')^\mu\right]$$

$$\mathcal{F}_1 = \frac{1}{M_D^*}\left\{2k^2q^2a_+ - \frac{1}{2}\left(q^2 - M_B^2 + M_{D^*}^2\right)f\right\}, \quad \mathcal{F}_2 = \frac{1}{M_D^*}\left\{f + (M_B^2 - M_{D^*})a_+ + q^2a_-\right\}$$

"HQET" convention $|\mathrm{HQET}\rangle = |\mathrm{rel}\rangle/\sqrt{M}$

$$\langle D^*(v',\varepsilon')|V^\mu|\bar{B}(v)\rangle = ih_V(w)\,\epsilon^{\mu\nu\alpha\beta}\,\varepsilon'^*_\nu v'_\alpha v_\beta$$

$$\langle D^*(v',\varepsilon')|A^\mu|\bar{B}(v)\rangle = h_{A_1}(w)\,(w+1)\,\varepsilon'^{*\mu} - \left[h_{A_2}(w)\,v^\mu + h_{A_3}(w)\,v'^\mu\right]\varepsilon^*\cdot v$$

setup for correlation functions on the lattice

- B at rest
- $|\mathbf{p}_{D^*}|^2$ = 0, 1, 2, 3 in units of $(2\pi/L)^2$ $\Rightarrow$ w = 1.0 – 1.1

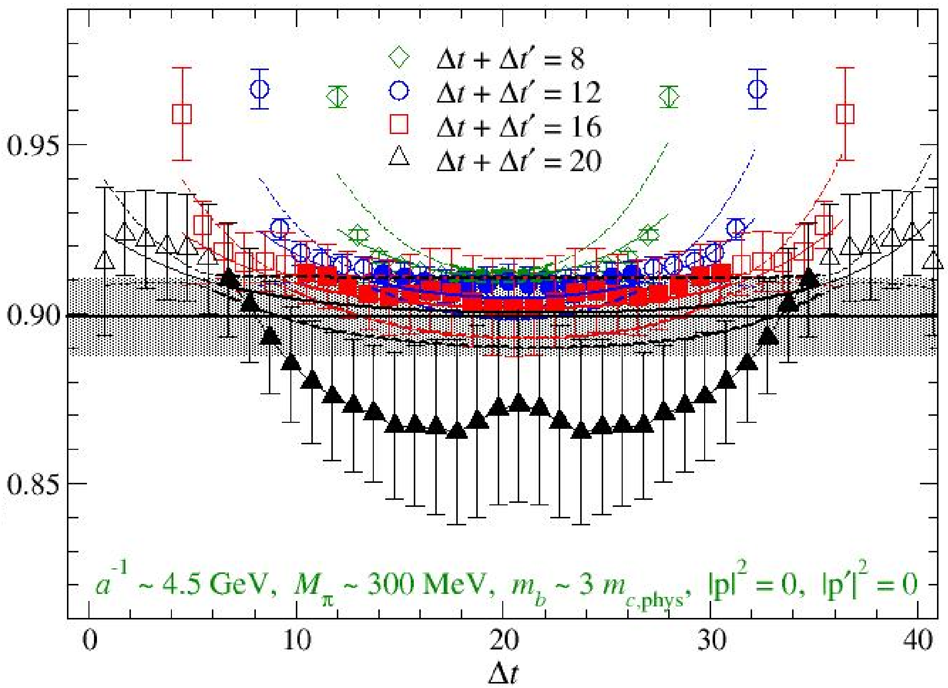
extracting FFs

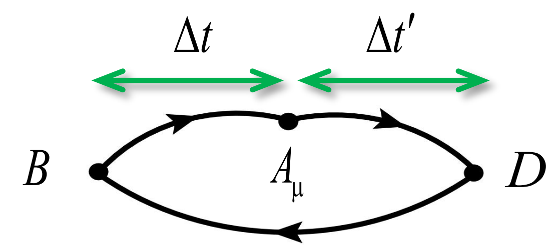
ratio method (Hashimoto et al. '99)

$$e.g.\quad C_{A_\mu}^{BD^*}(\mathbf{p},\mathbf{p}';\Delta t,\Delta t') = \frac{Z_B Z_{D^*}^*}{4E_B E_{D^*}} \langle D^*(p)|A_\mu|B(p)\rangle e^{-E_B\Delta t} e^{-E_{D^*}\Delta t'}$$

$$\frac{C_{A_4}^{BD^*}(\mathbf{0},\mathbf{0}';\Delta t,\Delta t')C_{A_4}^{D^*B}(\mathbf{0},\mathbf{0}';\Delta t,\Delta t')}{C_{V_4}^{BB}(\mathbf{0},\mathbf{0}';\Delta t,\Delta t')C_{V_4}^{D^*D^*}(\mathbf{0},\mathbf{0}';\Delta t,\Delta t')} = \frac{\langle D^*(0)|A_4|B(0)\rangle\langle B(0)|A_4|D^*(0)\rangle}{\langle B(0)|V_4|B(0)\rangle\langle D^*(0)|V_4|D^*(0)\rangle} = h_{A_1}(1)^2$$

multiple source-sink separations





- large $\Delta t+\Delta t'$
  to eliminate excited state contribu.
- small $\Delta t+\Delta t'$
  to achieve good statistical accuracy

continuum + chiral extrapolation

NLO HMChPT (Randall-Wise '92, Savage '01) + polynomial corrections

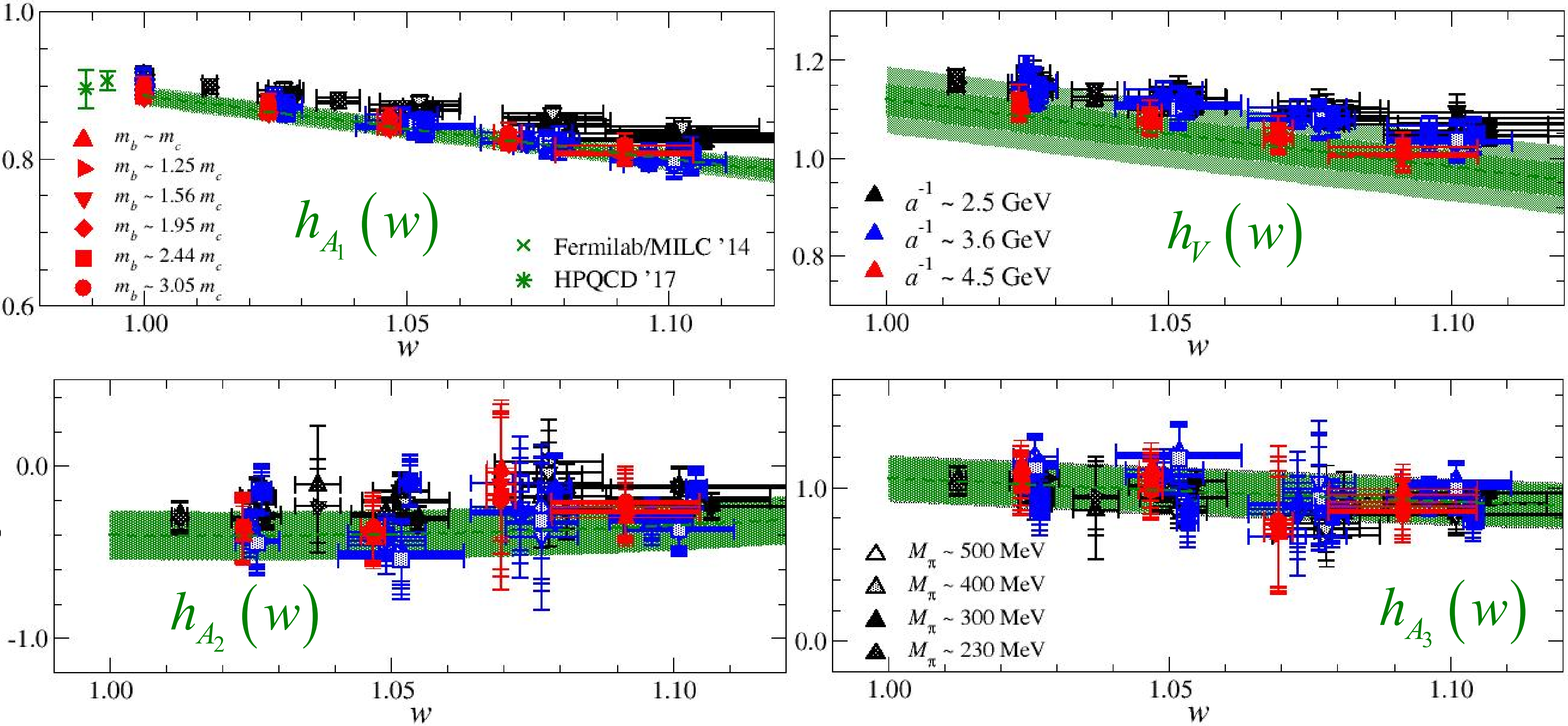
$$\frac{h_{A_1}(w)}{\eta_{A_1}} = c + \frac{g_{D^*D\pi}^2}{16\pi^2 f_\pi^2}\Delta_c^2 b_{\log}\bar{F}_{\log}\left(M_\pi, \Delta_c, \Lambda_\chi\right)$$

$$+c_w\left(w-1\right)+d_w\left(w-1\right)^2+c_b\left(w-1\right)\varepsilon_b+c_\pi\xi_\pi+c_{\eta s}\xi_{\eta s}+c_a\xi_a+c_{am_b}\xi_{amb}$$

$$\varepsilon_b = \frac{\bar{\Lambda}}{2m_b}, \quad \xi_\pi = \frac{M_\pi^2}{\left(4\pi f_\pi\right)^2}, \quad \xi_{\eta s} = \frac{M_{\eta s}^2}{\left(4\pi f_\pi\right)^2}, \quad \xi_a = \left(a\Lambda_{\rm QCD}\right)^2, \quad \xi_a = \left(am_b\right)^2$$

- singular correlation matrix ⇒ SVD cut, shrinkage ⇔ Fermilab/MILC
- η_X : one-loop radiative correction (Neubert '92)
- $g_{D^*D\pi}$ = 0.53(8) (Fermilab/MILC '14) ⇒ small systematic error
- ξ - expansion : better convergence for light quark obs. (JLQCD '08)
- $O((w\text{-}1)/m_b)$ for h_{A1}, h_+ ⇔ Luke's theorem '90 ; include $O(1/m_b^2)$

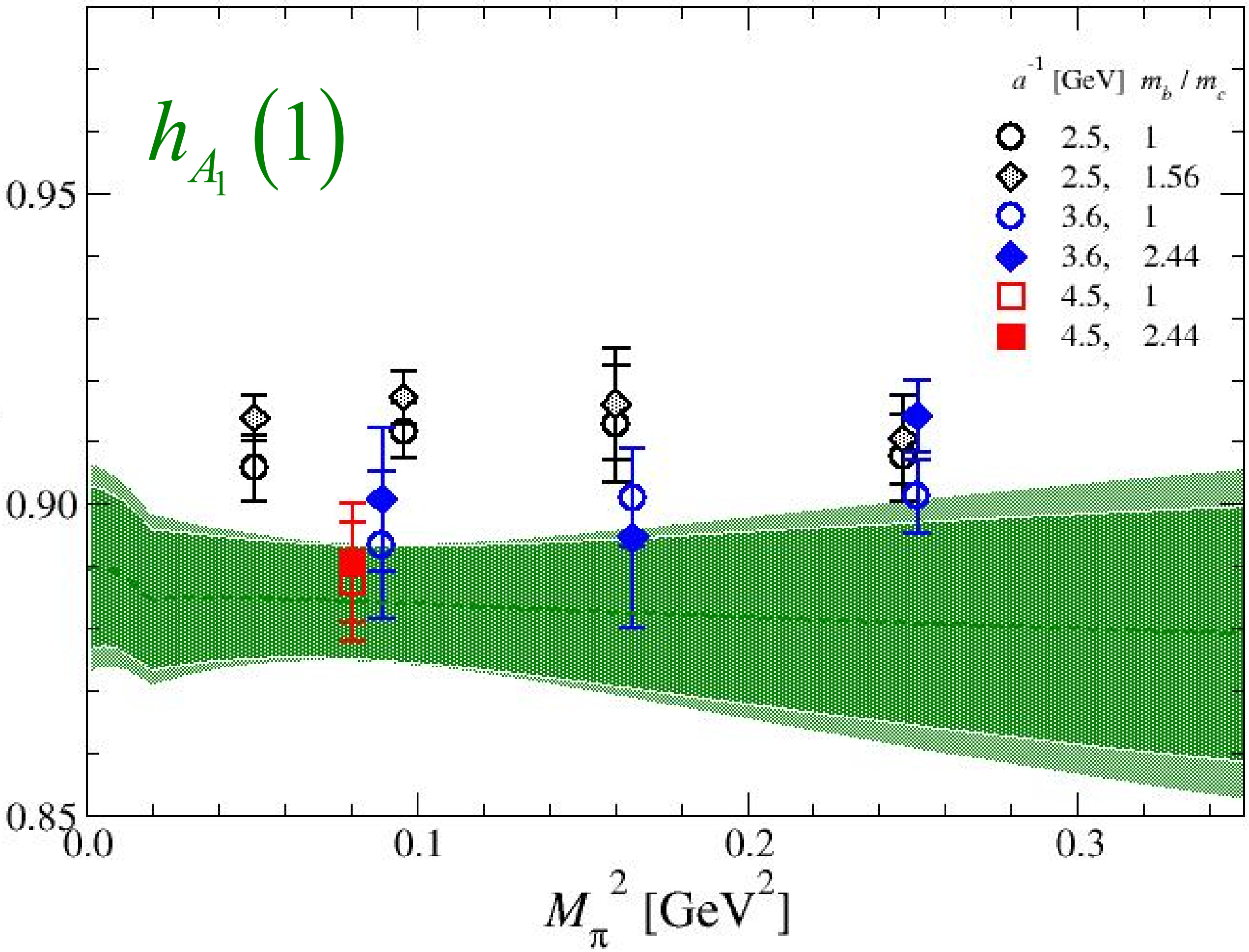
$B \to D^* \ell \nu$ form factors



- mild dependence on a, M_π, m_s, m_b

  $\Rightarrow$ all coefficients $c_X \leq O(1)$; $\geq$ 50% error for c_π, $c_{\eta s}$, c_a [except h_{A1}]
- extrapolation : reasonably controlled w/ χ^2/d.o.f. ~ 0.5

M_π dependence



- mild dependence
  - suppressed log
  - no valence π
- similar for other w , FFs

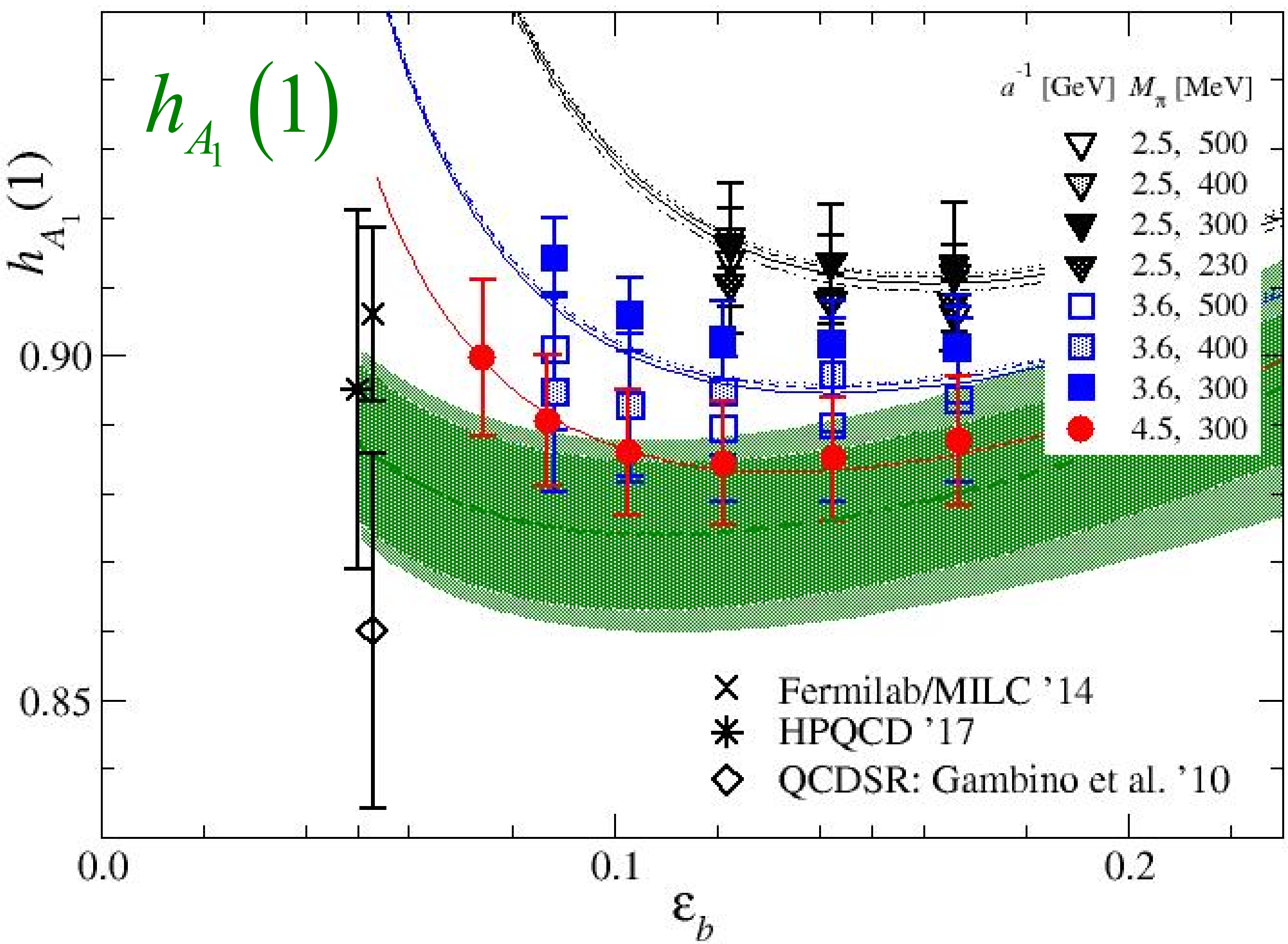
w/ $H^{(Q)}$, π

$$\sqrt{\Delta^2 - M_\pi^2}\ \ln\left[\frac{\Delta + \sqrt{\Delta^2 - M_\pi^2}}{\Delta - \sqrt{\Delta^2 - M_\pi^2}}\right]$$

$$\Delta = M_{D*} - M_D$$

- $D^* \to D\pi$ ⇒ concave structure < statistical accuracy
- mild dependence ⇒ controlled extrapolation ⇔ $B \to \pi \ell \nu$

m_b and a dependences



$$\varepsilon_b = \frac{\bar{\Lambda}}{2m_b} = \frac{\bar{\Lambda}}{M_{\eta_b}}$$

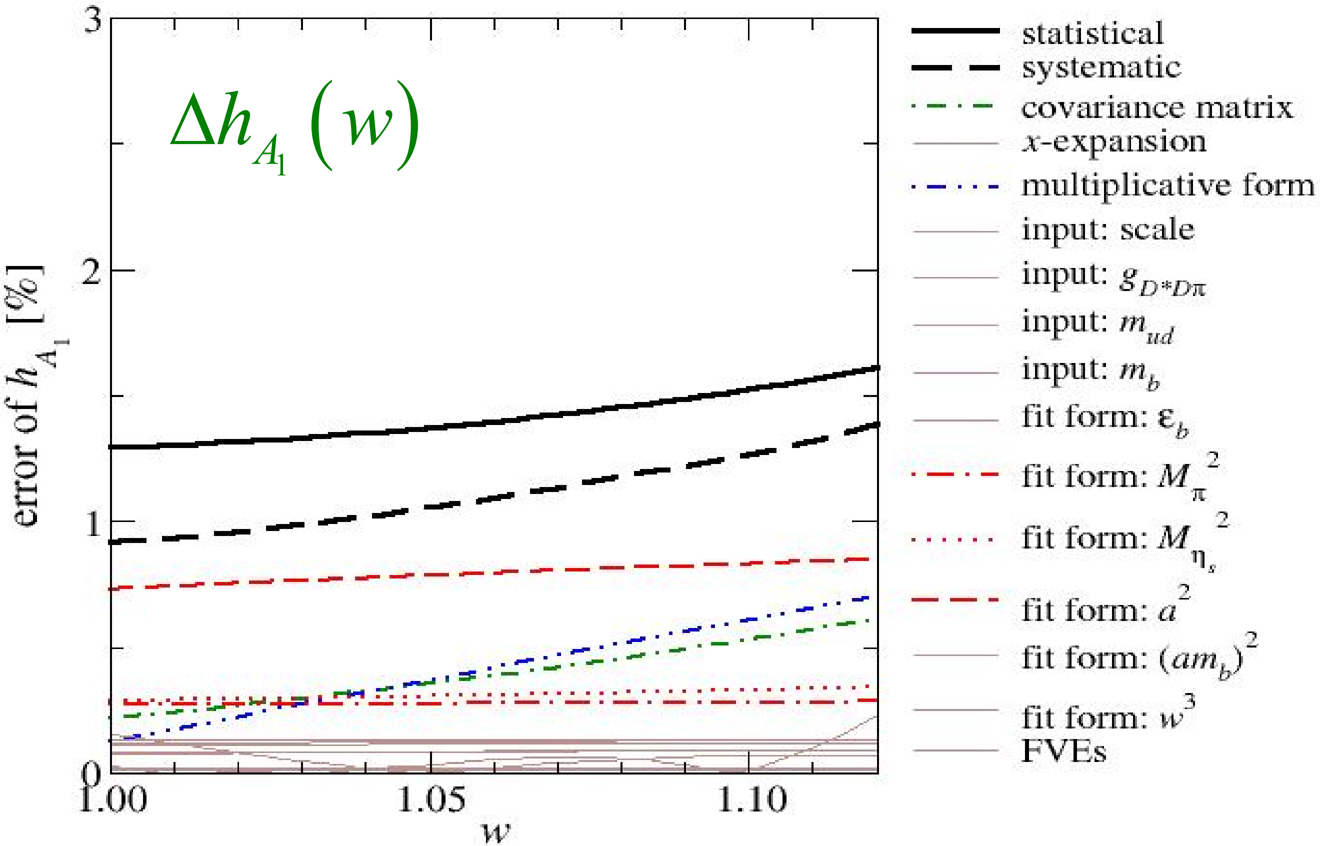
$\bar{\Lambda} = 0.5\text{GeV}$

- two $a \neq 0$ effects
  - $(a\Lambda)^{2n}$, $(am_c)^{2n}$
  - $(am_b)^{2n}$
- consistency w/ QCDSR (?)

  Gambino-Mannel-Uraltsev '10

- turn out to be a few % effects
- reasonably controlled extrapolation in ε_b and a $\Leftrightarrow$ smaller a ?

uncertainties



correlation matrix
extrapolation form

inputs
a, m_{q}, $g_{D^*D\pi}$

polynomial corr.
add higher order
remove poorly determined term

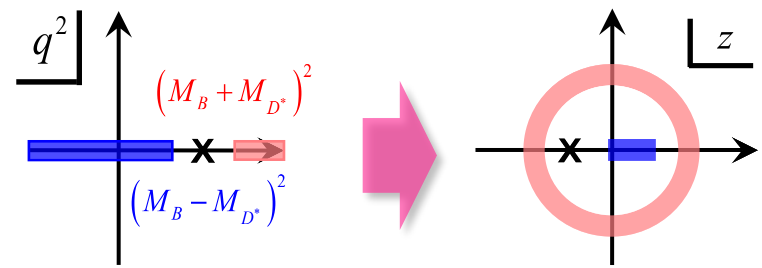
\- small by $2L$'s

- h_{A1} : largest uncertainties – statistics and discretization – but 1-2 %
- other FFs : larger and more dominant statistical error

BGL parameterization

Boyd-Grinstein-Lebed parameterization '97

$$F(w) = \frac{1}{P_F(z)\phi_F(z)} \sum_{n=1}^{N_F} a_n^F z^n, \quad z = \frac{\sqrt{w+1}-\sqrt{2}}{\sqrt{w+1}+\sqrt{2}}$$



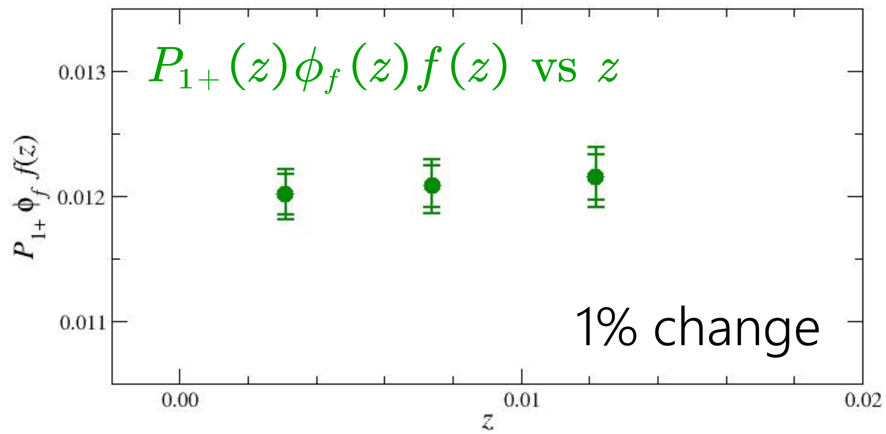
- w = [1.0, 1.1], q^2 = [13.0, 10.7] $\longrightarrow$ z =[0.000, 0.012]

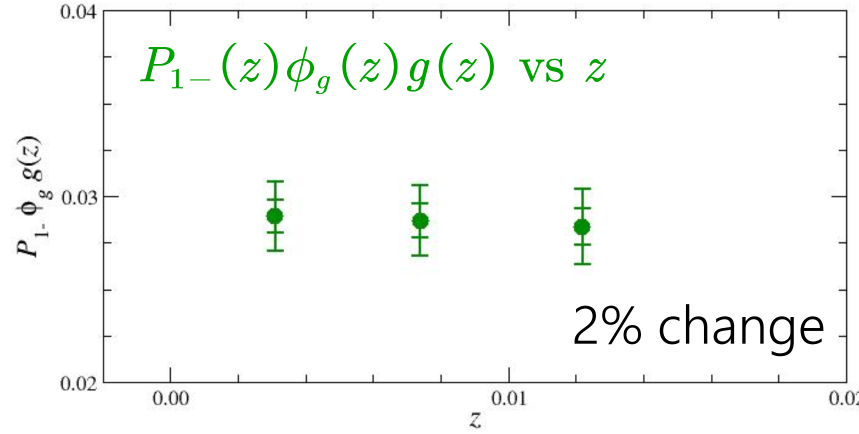
  [1.0, 1.5] → z =[0.000,0.056]

- Baschke factors P_F to factor out singularity
- outer functions ϕ_F leading to unitarity constraint $\Sigma\, |a^F{}_n|^2 \leq 1$

  resonance masses, derivatives of VPFs same as Bigi-Gambino-Schacht '17
- w/o unitarity constraint, but satisfied within errors

BGL parameterization





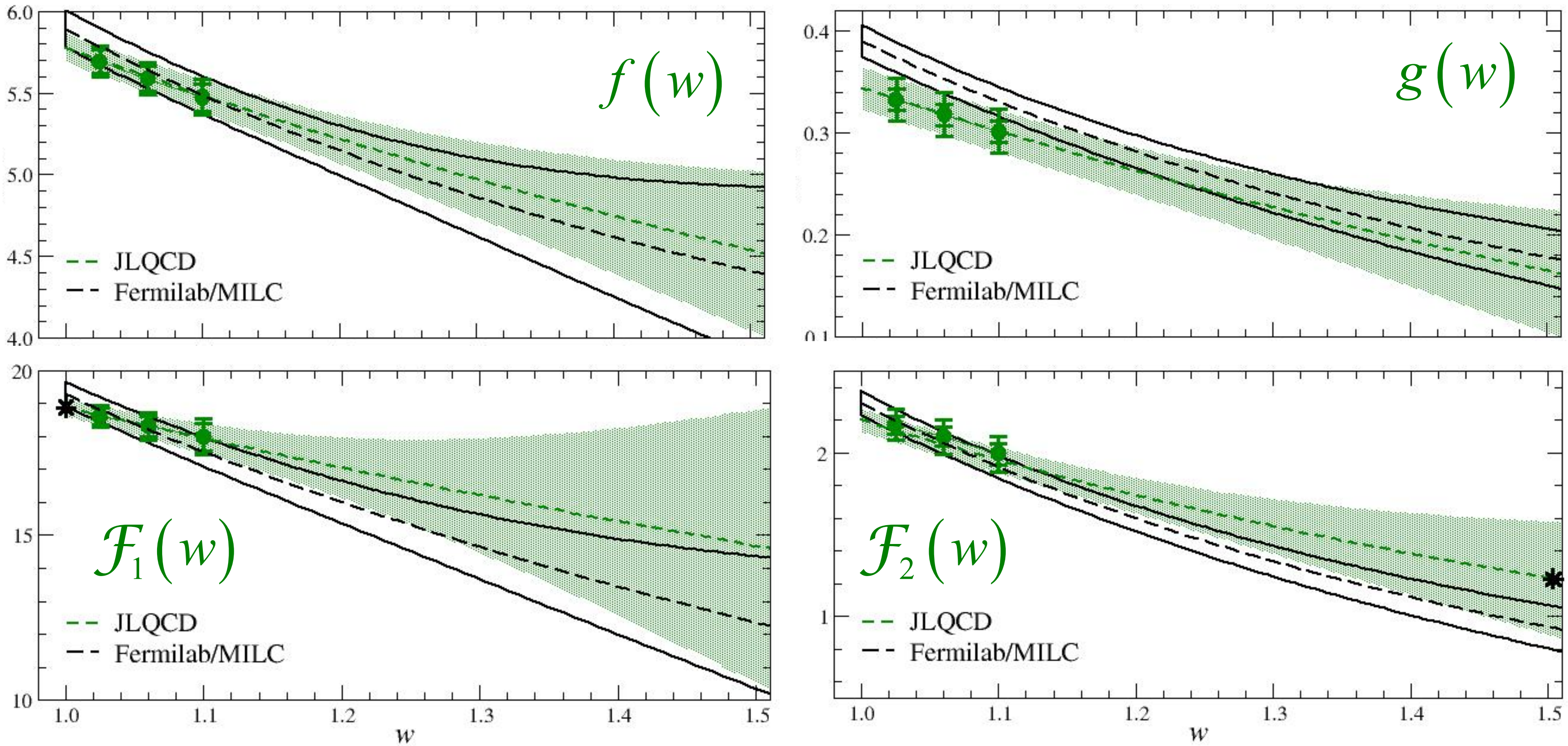
- synthetic data @ reference w = 1.025, 1.060, 1.100 (z=0.003, 0.007, 0.012)
- quadratic expansion n_{g}, n_{f}, $n_{\mathcal{F}1}$, $n_{\mathcal{F}2}$ = 2 : sufficient for mild z-dep. (χ^2/dof ~ 0.5)
- w/ kinematical constraints @ $w = 1$, $w_{\max}$ (q^2=0)

  $\mathcal{F}_1(1) = (M_B\text{-}M_{D*})\,f(1)$ $\Rightarrow$ fix $a^{\mathcal{F}1}{}_0$

  $\mathcal{F}_2(w_{\max}) = (1+r)\,\mathcal{F}_1(w_{\max})\,/\,M_B{}^2\,r(1+r)(1+w_{\max})$ $\Rightarrow$ fix $a^{\mathcal{F}2}{}_2$

comparison of FFs

JLQCD vs Fermilab/MILC

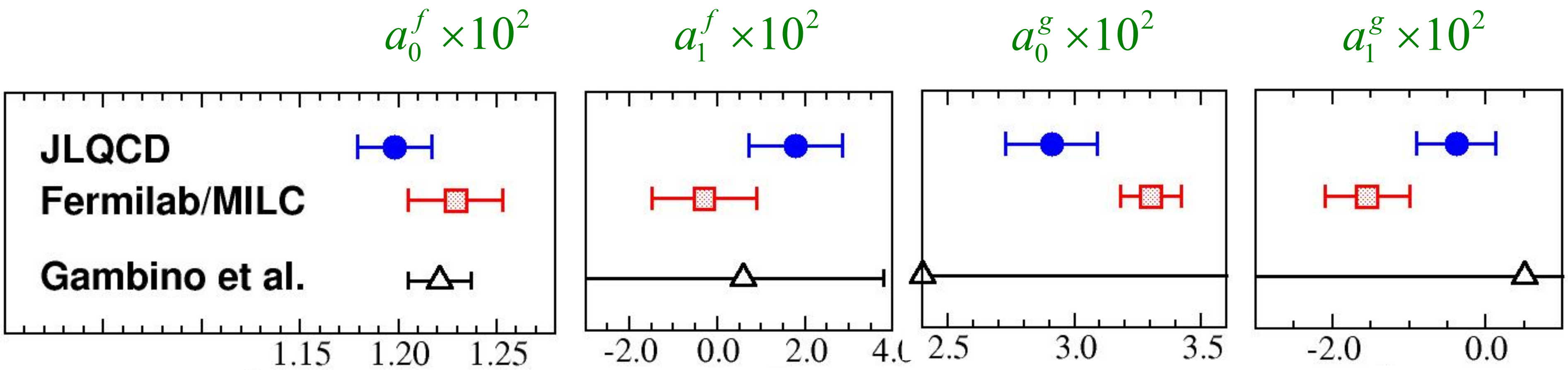


- reasonably consistent

  ⇔ g @ w ~ 1

- larger error @ larger w ⇔ narrower region of w = [1.00,1,10] ⇔ [1.00,1.17]

coefficients for f, g

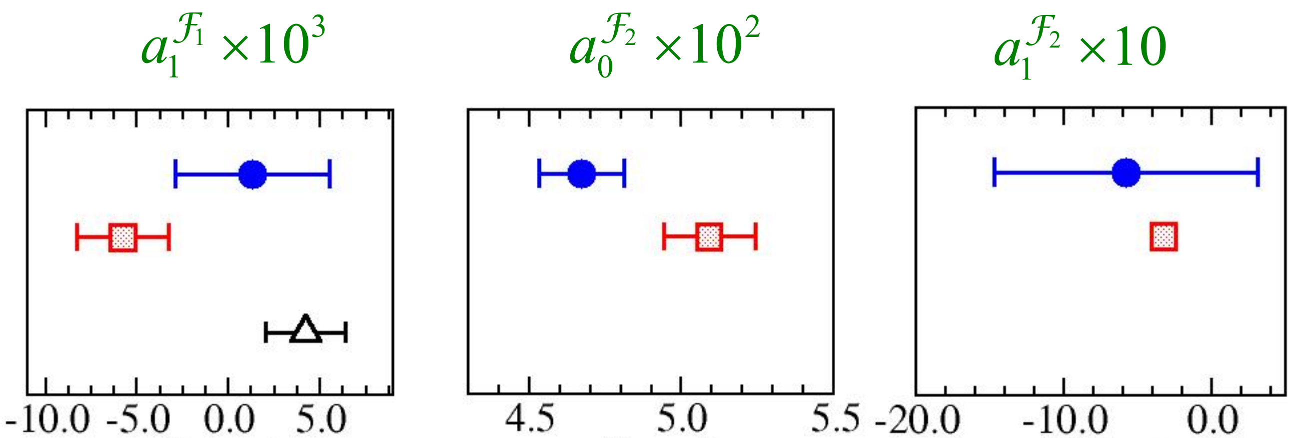


Gambino et al. '19: BGL analysis of Belle data + lattice @ w=1, w/ weak unitarity bounds

$f \sim \langle D^* | A_\mu | B \rangle$ @ $w=1$, $\quad g \sim \langle D^* | V_\mu | B \rangle$

- mild z-dependence $\Rightarrow$ ±100% uncertainty for quadratic coefficients
- two independent lattice studies consistent $\Leftrightarrow$ very different systematics
  - JLQCD : favor smaller constants, slopes (FF vs w)
- lattice @ $w \neq 1$ helps constrain coefficients
  - $a^f{}_0$: constrained by lattice @ w=1

coefficients for $\mathcal{F}_1$, $\mathcal{F}_2$



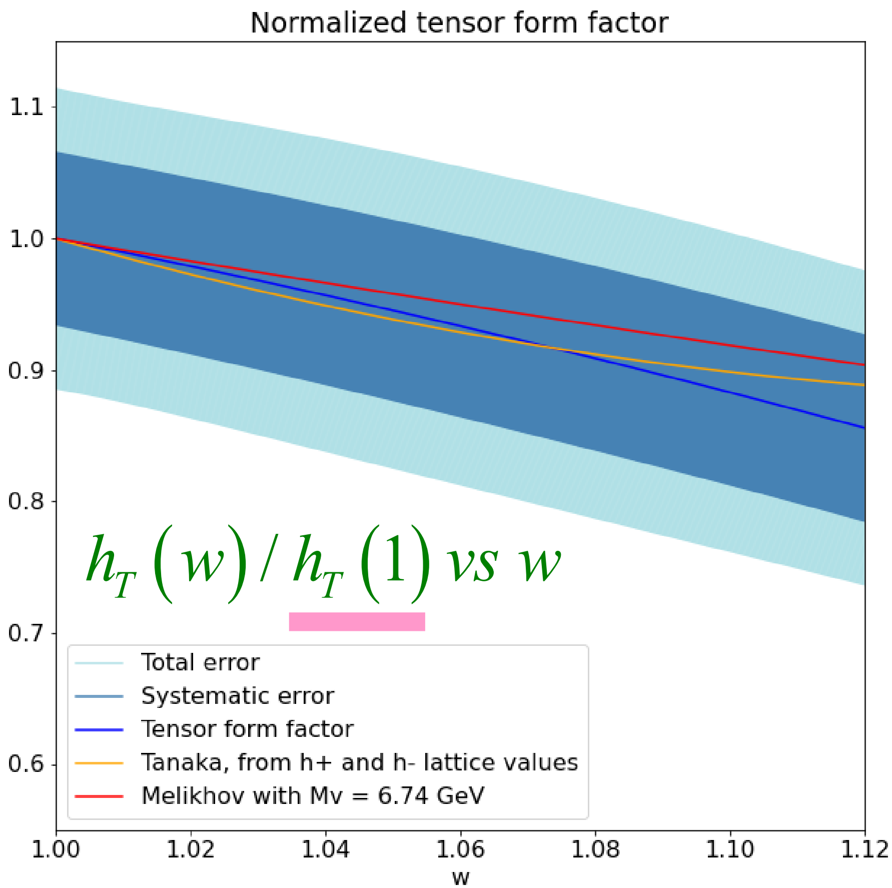
- ±100% uncertainty for quadratic coefficients
- kinematical constraints @ w=1 and w_{max} fix $a^{\mathcal{F}1}{}_0$, $a^{\mathcal{F}2}{}_2$

- $\mathcal{F}_1 \sim \langle D^* | A_\mu | B \rangle$ @ $w \neq 1$
  - well constrained by exp'tal data, consistent w/ lattice

- $\mathcal{F}_2 \sim O(m_l{}^2)$ to Γ
  - poorly constrained by exp't (e,μ) / input to new physics search by τ mode
  - $a^{\mathcal{F}2}{}_2$ fixed $\Rightarrow$ describes z-dependence $\Rightarrow$ poorly constrained $a^{\mathcal{F}2}{}_1$

beyond SM

$B \to D\ell\nu$ tensor FF (M. Faur [Paris ENS, internship] + Kou + JLQCD)



$$\langle D(p')|T_{\mu\nu}|B(p)\rangle = i\left(v'^{\mu}v^{\nu} - v'^{\nu}v^{\mu}\right)h_T(w)$$

- ✓ extraction of tensor FF
- ✓ continuum-chiral extrapolation
- ✓ systematic uncertainties
- renormalization in progress

T. Ishikawa @ Lattice 2021

- 10% stat. and 10% sys. errors
- consistent w/ phenomenology

useful input for BSM interpretation of B anomalies

Summary

$B \to D^{(*)} \ell \nu$ semileptonic FFs from JLQCD

- relativistic lattice QCD w/ chiral symmetry
  - ~~matching of EFT~~, ~~renormalization for SM FFs~~, ChPT for extrap., ...
- $B \to D^* \ell \nu$
  - mild a, m_q dependence $\Rightarrow$ controlled continuum – chiral extrap.
  - reasonable consistency w/ Fermilab/MILC '21
  - (hopefully) finalized results $\Rightarrow$ appeared in talks @ this workshop

    !!! any feedback / requests are very welcome (choice of $w_{\rm ref}$, ...) !!!
- future directions
  - BSM FFs: $B \to D \ell \nu$ tensor FF / expect more in the future
  - extension to smaller a, larger w, changing m_c, ... $\Leftrightarrow$ Fugaku