Wayne State University

# $\bar{B} \to X_u \, l \, \bar{\nu}$ theory


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# Outline

- Theoretical framework
- Recent progress
- Critical review of current approaches
- Open problems and future progress

# Theoretical framework

# Reminders

- For $B$ decays: $5\,\text{GeV} \sim m_b \gg \Lambda_{\text{QCD}} \sim 0.5\ \text{GeV}$
  Observables expandable in $\Lambda_{\text{QCD}}/m_b \sim 0.1$

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$$d\Gamma \sim \sum_n c_n \frac{\langle O_n \rangle}{m_b^n}$$

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| | | |
|---|---|---|
| $M_X^2 \sim m_b^2$ | local OPE | ("OPE region") |
| $M_X^2 \sim m_b \Lambda_{\mathrm{QCD}}$ | Non local OPE | ("end point region") |
| $M_X^2 \sim \Lambda_{\mathrm{QCD}}^2$ | No inclusive description | ("resonance region") |

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- The same shape function appears at leading power for $\bar{B} \to X_s \gamma$

# Lessons from hard QCD

- Situation familiar from hard QCD

$$d\sigma = c \otimes f + \mathcal{O}\left(\Lambda^2_{\mathrm{QCD}}/Q^2\right)$$

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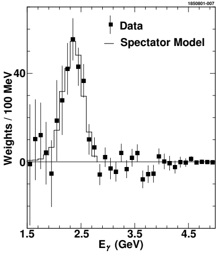
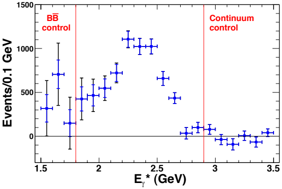
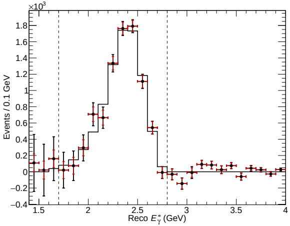
CLEO (2001)




BaBar (2012)




Belle (2016)



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- Unlike hadron colliders where $\Lambda^2_{\rm QCD} \ll Q^2$, we have $Q^2 = m_b^2$
- At subleading power in $\Lambda_{\rm QCD}/m_b$:
- Several subleading shape functions (SSF) appear
- Different linear combinations for $\bar{B} \to X_u\, l\, \bar{\nu}$ and $\bar{B} \to X_s\,\gamma$
- $\bar{B} \to X_s\,\gamma$ has unique SSF ("resolved photon contributions")

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    What are these unique SSF?
- For $\bar{B} \to X_s \gamma$ need Effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left( C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3,\ldots,10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}$$

  - At leading power only $Q_{7\gamma} - Q_{7\gamma}$ contribute
  - At higher orders need other $Q_i - Q_j$ contributions
- Most important: $Q_{7\gamma}$, $Q_{8g}$, and $Q_1$

$$\begin{aligned} Q_{7\gamma} &= \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu}(1+\gamma_5) F^{\mu\nu} b \\ Q_{8g} &= \frac{-g_s}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu}(1+\gamma_5) G^{\mu\nu} b \\ Q_1^q &= (\bar{q} b)_{V-A} (\bar{s} q)_{V-A} \quad (q = u, c) \end{aligned}$$

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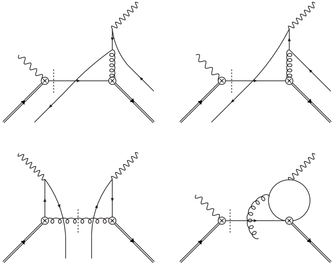
- Systematic analysis at $\Lambda_{\rm QCD}/m_b$ [Benzke, Lee, Neubert, GP '10]:



| | |
|---|---|
| Top line: | $Q_{7\gamma} - Q_{8g}$ |
| Bottom left: | $Q_{8g} - Q_{8g}$ |
| Bottom right: | $Q_1 - Q_{7\gamma}$ |

- $Q_1 - Q_{8g}$ and $Q_1 - Q_1$ give $\Lambda^2_{\rm QCD}/m_b^2$ effects

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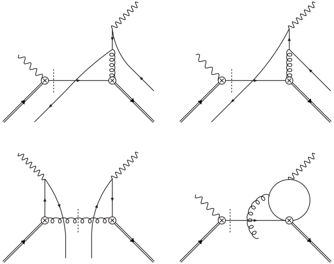

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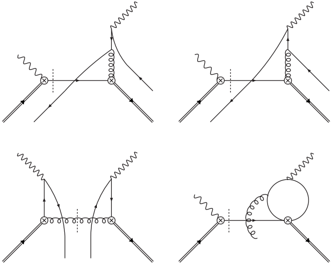

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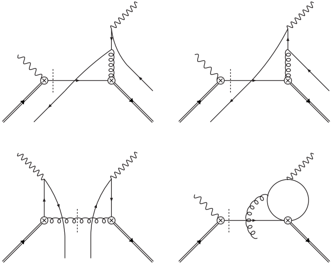

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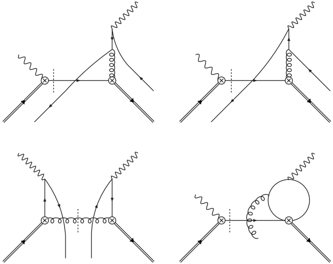

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- Horizontal: $E_\gamma$ Vertical: unobservable
- 2010 analyses focused on integrated rate and CP asymmetry
- Moments of $g_{17} \leftrightarrow$ HQET parameters. Can we improve those?

# Recent progress

# Power corrections

- Dimension 7 and 8 HQET operators contribution to $\bar{B} \to X_c \ell \bar{\nu}_\ell$
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

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**Table 2**
Default fit results: the second and third columns give the central values and standard deviations.

| | | | | | |
|---|---|---|---|---|---|
| $m_b^{kin}$ | 4.546 | 0.021 | $r_1$ | 0.032 | 0.024 |
| $\overline{m}_c(3\text{ GeV})$ | 0.987 | 0.013 | $r_2$ | −0.063 | 0.037 |
| $\mu_\pi^2$ | 0.432 | 0.068 | $r_3$ | −0.017 | 0.025 |
| $\mu_G^2$ | 0.355 | 0.060 | $r_4$ | −0.002 | 0.025 |
| $\rho_D^3$ | 0.145 | 0.061 | $r_5$ | 0.001 | 0.025 |
| $\rho_{LS}^3$ | −0.169 | 0.097 | $r_6$ | 0.016 | 0.025 |
| $\overline{m}_1$ | 0.084 | 0.059 | $r_7$ | 0.002 | 0.025 |
| $\overline{m}_2$ | −0.019 | 0.036 | $r_8$ | −0.026 | 0.025 |
| $\overline{m}_3$ | −0.011 | 0.045 | $r_9$ | 0.072 | 0.044 |
| $\overline{m}_4$ | 0.048 | 0.043 | $r_{10}$ | 0.043 | 0.030 |
| $\overline{m}_5$ | 0.072 | 0.045 | $r_{11}$ | 0.003 | 0.025 |
| $\overline{m}_6$ | 0.015 | 0.041 | $r_{12}$ | 0.018 | 0.025 |
| $\overline{m}_7$ | −0.059 | 0.043 | $r_{13}$ | −0.052 | 0.031 |
| $\overline{m}_8$ | −0.178 | 0.073 | $r_{14}$ | 0.003 | 0.025 |
| $\overline{m}_9$ | −0.035 | 0.044 | $r_{15}$ | 0.001 | 0.025 |
| $\chi^2/dof$ | 0.46 | | $r_{16}$ | 0.001 | 0.025 |
| $BR(\%)$ | 10.652 | 0.156 | $r_{17}$ | −0.028 | 0.025 |
| $\mathbf{10^3 \lvert V_{cb} \rvert}$ | **42.11** | **0.74** | $r_{18}$ | −0.001 | 0.025 |

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- Answer given in [Gunawardana, GP, JHEP **1707** 137 (2017)] (See also appendix A of [Heinonen, Mannel, arXiv:1609.01334])
- Method of [Gunawardana, GP, JHEP **1707** 137 (2017)] allows to

1) Find such relations
2) List HQET parameters, in principle, to *arbitrary* dimension
3) Construct NRQED and NRQCD bilinear operators, in principle, to *arbitrary* dimension

- Example: Spin-dependent dimension 8 HQET operators

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$$\frac{1}{2M_H}\langle H|\bar h\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, iD^{\mu_5}\, s^\lambda\, h|H\rangle = i\tilde c^{(8)} v^{\mu_2} v^{\mu_3} v^{\mu_4} \epsilon^{\rho\mu_1\mu_5\lambda} v_\rho$$

$$i\tilde a_{12}^{(8)}\left(v^{\mu_3}\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde a_{14}^{(8)}\left(v^{\mu_3}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_4\mu_1\lambda}v_\rho\right) +$$

$$+ i\tilde a_{15}^{(8)} v^{\mu_3}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_2\mu_4\lambda}v_\rho + i\tilde a_{24}^{(8)} v^{\mu_3}\Pi^{\mu_2\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho +$$

$$+ i\tilde b_{13}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_3}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_3}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde b_{14}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_3\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_3\mu_1\lambda}v_\rho\right) +$$

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$$+ i\tilde b_{35}^{(8)}\left(v^{\mu_2}\Pi^{\mu_3\mu_5}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_1}\epsilon^{\rho\mu_5\mu_2\lambda}v_\rho\right) + i\tilde b_{45}^{(8)}\left(v^{\mu_2}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_1\mu_3\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_2\mu_1}\epsilon^{\rho\mu_5\mu_3\lambda}v_\rho\right).$$

- Example: Spin-dependent dimension 8 HQET operators

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$$i\tilde a^{(8)}_{12}\left(v^{\mu_3}\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde a^{(8)}_{14}\left(v^{\mu_3}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_4\mu_1\lambda}v_\rho\right) +$$
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$$+ i\tilde b^{(8)}_{35}\left(v^{\mu_2}\Pi^{\mu_3\mu_5}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_1}\epsilon^{\rho\mu_5\mu_2\lambda}v_\rho\right) + i\tilde b^{(8)}_{45}\left(v^{\mu_2}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_1\mu_3\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_2\mu_1}\epsilon^{\rho\mu_5\mu_3\lambda}v_\rho\right).$$

- Relation to $r_8 - r_{18}$

$$r_8 = 6\tilde c^{(8)},\ r_9 = -6\left[\tilde b^{(8)}_{14} + \tilde b^{(8)}_{15} - \tilde b^{(8)}_{34} - \tilde b^{(8)}_{35} - 3\tilde b^{(8)}_{45}\right],\ r_{10} = 6\left[3\tilde b^{(8)}_{13} + \tilde b^{(8)}_{14} - \tilde b^{(8)}_{15} + \tilde b^{(8)}_{34} - \tilde b^{(8)}_{35}\right],$$
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- Example: Spin-dependent dimension 8 HQET operators

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, iD^{\mu_5}\, s^\lambda\, h|H\rangle = i\tilde{c}^{(8)} v^{\mu_2} v^{\mu_3} v^{\mu_4} \epsilon^{\rho\mu_1\mu_5\lambda} v_\rho$$

$$i\tilde{a}^{(8)}_{12}\left(v^{\mu_3}\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{a}^{(8)}_{14}\left(v^{\mu_3}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_4\mu_1\lambda}v_\rho\right) +$$

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  [Benzke, Hurth PRD **102** 114024 (2020)]

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- Moments of $g_{78}$ and $g_{88}$ are *not* related to to these HQET parameters. Very little is known about them, in particular, their $E_\gamma$ dependence
- It is not clear if such modeling of $g_{78}$ and $g_{88}$ is better than assuming a $\Lambda_{\rm QCD}/m_b$ uncertainty on the extraction of the leading shape function from $\bar{B} \to X_s\,\gamma$

# Critical review of current approaches

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- I'll try to distinguish the two in the following

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- Leading power $H \cdot J \otimes S$ at $\mathcal{O}(\alpha_s)$
- Subleading shape functions: $H \cdot J \otimes s_i$ at $\mathcal{O}(\alpha_s^0)$
- $\alpha_s/m_b^{\{1,2\}}$ and $1/m_b^2$ incorporated as naive convolution with LO SF
- $S$ extracted from $\bar{B} \to X_s \gamma$, $s_i$ modeled ($\sim$ 700 models)
- Smoothly merges to local OPE when integrated over phase space
- Hard, Jet, and Soft scale separated with NLO resummation

# BLNP: possible issues

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + ...$$

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2) Resummation: 2009 Study of implication of $\mathcal{O}(\alpha_s^2)$ on $|V_{ub}|$
   [Greub, Neubert, Pecjak, EPJC **65** 501 (2010)]
   "factorization ... perturbative coefficient...into jet and hard functions is not strictly necessary: using ... fixed-order... does not lead to large scale uncertainties ... nor to a poor convergence ..."

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4) Uses the shape function mass scheme, not easy to switch to other schemes. e.g. kinetic

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$$S(\omega, \mu_\Lambda) = \int dk\, C_0(\omega - k, \mu_\Lambda) F(k)$$

"where $C_0(\omega, \mu_\Lambda)$ is the $b$ quark matrix element of the shape function operator calculated in perturbation theory, and $F(k)$ is a nonperturbative function that can be extracted from data."
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- "The advantage of our construction [...] is that the tail automatically turns on in a smooth manner when it dominates over the nonperturbative function $F(k)$ and provides the proper $\mu$ dependence for $S(\omega, \mu)$ at any $\omega$."

  [Ligeti, Stewart, Tackmann PRD **78** 114014 (2008)]

- The same paper also suggested to use orthonormal basis functions

  Among other things it allows to better fit $\bar{B} \to X_s \gamma$ photon spectrum

# SIMBA: Possible issues: Positivity

1) The same paper also introduced

$$S(\omega) = \int dk\, C_0(\omega - k) F(k) = \int dk\, \widehat{C}_0(\omega - k)\, \widehat{F}(k)$$

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- "We expect on physical grounds that $\widehat{F}(k)$ is positive..."

- From [Gunawardana, GP '17] and [Gambino, Healey, Turczyk '16)]

$$\int d\omega\, S(\omega) = 1,\ \int d\omega\, \omega\, S(\omega) = 0,\ \int d\omega\, \omega^2\, S(\omega) = \mu_\pi^2/3 = 0.144 \pm 0.023\ \text{GeV}^2,$$

$$\int d\omega\, \omega^3\, S(\omega) = -\rho_D^3/3 = -0.048 \pm 0.020\ \text{GeV}^3,$$

$$\int d\omega\, \omega^4\, S(\omega) = m_1/5 - m_2/3 = 0.023 \pm 0.017\ \text{GeV}^4,$$

$$\int d\omega\, \omega^5\, S(\omega) = \left(-8r_1 + 2r_2 + 2r_3 + 2r_4 + r_5 + r_6 + r_7\right)/15 = -0.027 \pm 0.015\ \text{GeV}^5$$

- Starting with the third, the moments are not positive

# SIMBA: Possible issues: Multiplicity

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$$\widehat{F}^{\rm mod}(k) = \frac{1}{\lambda}\left[\sum_{n=0}^{2} c_n\, f_n\Big(\frac{k}{\lambda}\Big)\right]^2$$

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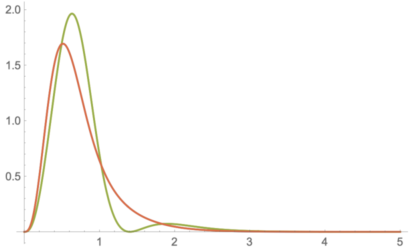


Figure 1: The variation of $F$ against $k$; red curve represent combination $\{c0 \to 0.949031, c1 \to -0.308648, c2 \to 0.0638516\}$

[Image by A. Gunawardana, private communication]

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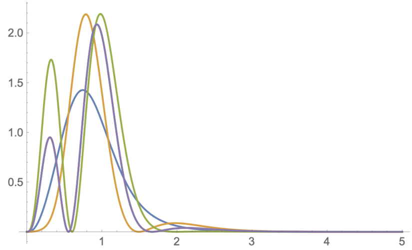


Figure 2: The variation of $F$ against $k$; blue curve represent combination $\{c0 \to 0.988539, c1 \to -0.115211, c2 \to -0.0975604\}$

[Image by A. Gunawardana, private communication]

# SIMBA: Possible issues: Small momentum behavior

3) "Due to the short distance subtractions ... to ensure that $S(\omega, \mu)$ goes to zero at $\omega = 0$, we need $\widehat{F}(k)$ to go to zero at least as $k^3$ for $k \to 0$"

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- If we want to include higher moments, we might need to increase the power of $k$ [B. Lange, private communication]

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- Based on

$$W_i \sim F_i \otimes W_i^{pert}$$

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  About 100 forms considered in GGOU

  Each parameterized by simple 2-parameter functional forms

  [Gambino, CKM 2016 talk]

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- I am not familiar with factorization formula that shows such a symmetry for all power corrections

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The future looks promising for $\bar{B} \to X_u \, l \, \bar{\nu}$ and inclusive $|V_{ub}|$!