Preliminary result on $\frac{\Delta\mathcal{B}(B\to X_u \ell\nu)}{\Delta\mathcal{B}(B\to X_c \ell\nu)}$ at Belle
Towards inclusive $|V_{ub}|/|V_{cb}|$

Marcel Hohmann

The University of Melbourne

mhohmann@student.unimelb.edu.au

Challenges in Semileptonic B Decays

Barolo, Italy

22/04/22

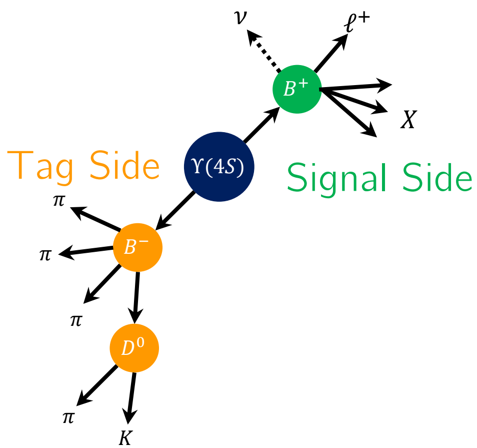
Inclusive $\frac{|V_{ub}|}{|V_{cb}|}$ via $\frac{\Delta\mathcal{B}(B\to X_u\ell\nu)}{\Delta\mathcal{B}(B\to X_c\ell\nu)}$

- Ratio measurement cancels several sources of uncertainty:
  - Experiment:
    - Cancel tagging and lepton identification uncertainties.
  - Theory:
    - Partial cancellation of m_b dependence.
      - m_b uncertainty usually large in inclusive $|V_{ub}|$ determinations.
    - Other cancellations in the OPE?

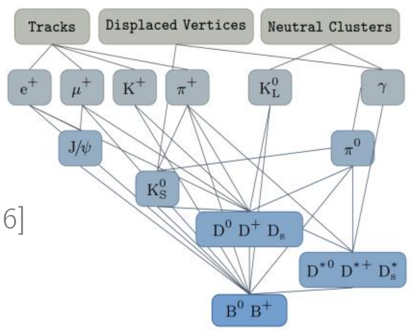
World Averages Precision:
BLNP: $\pm 5.7\%$ ($\pm 2.8\%$ m_b, μ_π^2)
DGE: $\pm 3.4\%$ ($^{+3.2}_{-2.9}\%$ m_b)
GGOU: $\pm 4.0\%$ ($\pm 1.9\%$ $m_b, \alpha_s, \ldots$)
[HFLAV, Eur. Phys. J. C (2021) 81:226]

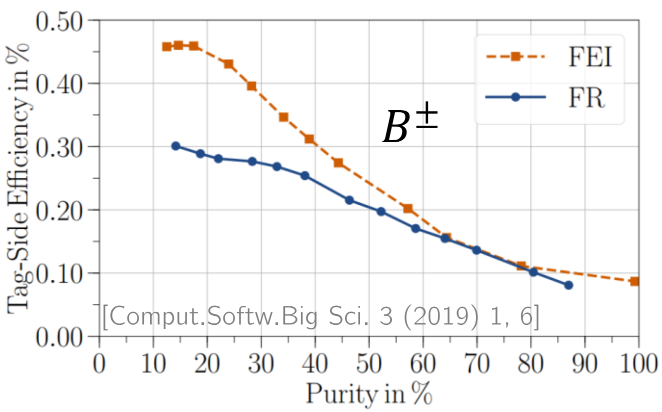
- Directly constrain Unitarity Triangle side $|V_{ub}|/|V_{cb}|$ with consistent error treatment.

$B \to X\ell\nu$ Reconstruction at Belle

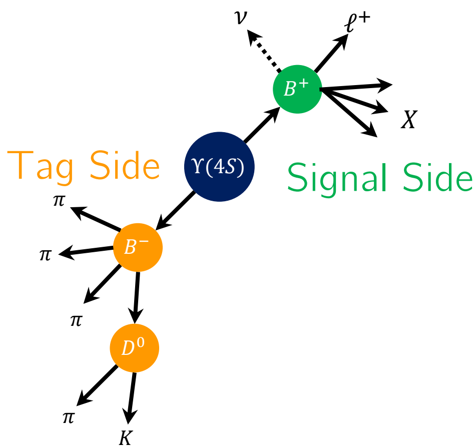


- Analyse full Belle sample, $711\ fb^{-1}$, in Belle II software.
- Belle II tagging algorithm - Full Event Interpretation
- Hierarchically reconstruct $\mathcal{O}(10\,000)$ hadronic channels.
- $\mathcal{O}(200)$ Boosted Decision Trees to select good candidates.
- Up-to 50% higher efficiency than previous Belle tagging algorithm, Full Reconstruction - 1104 channels.
[Comput.Softw.Big Sci. 3 (2019) 1, 6]



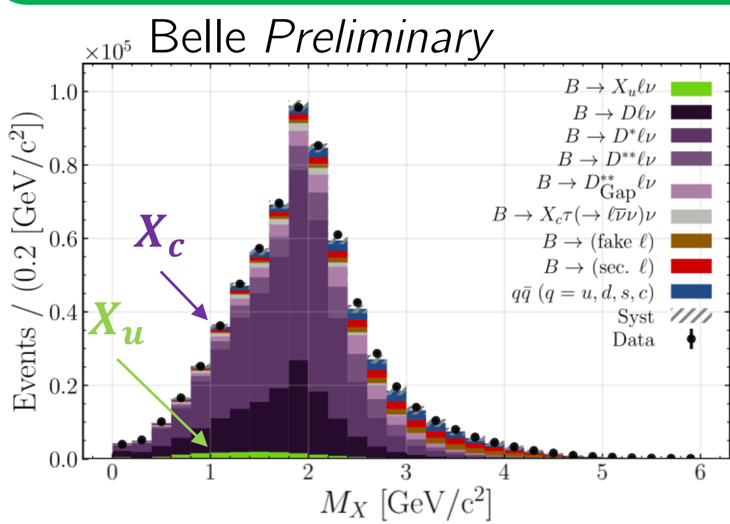


$B \to X\ell\nu$ Reconstruction at Belle

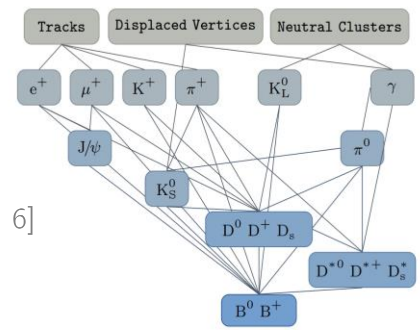


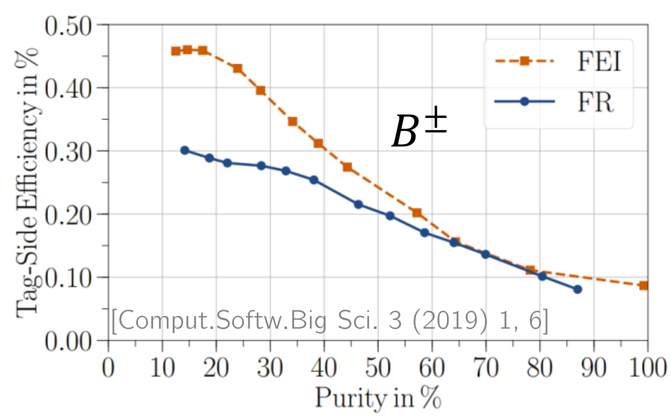
- Search for well identified lepton
- Reconstruct X system:

$$p_X = \sum_{i\in\{tracks\}} \left(\sqrt{m_{h,i}^2 + |\boldsymbol{p}_i|^2}, \boldsymbol{p}_i\right) + \sum_{j\in\{clusters\}} (E_j, \boldsymbol{p}_j)$$



- Analyse full Belle sample, 711 fb^{-1}, in Belle II software.
- Belle II tagging algorithm - Full Event Interpretation
- Hierarchically reconstruct $\mathcal{O}(10\,000)$ hadronic channels.
- $\mathcal{O}(200)$ Boosted Decision Trees to select good candidates.
- Up-to 50% higher efficiency than previous Belle tagging algorithm, Full Reconstruction - 1104 channels.
  [Comput.Softw.Big Sci. 3 (2019) 1, 6]



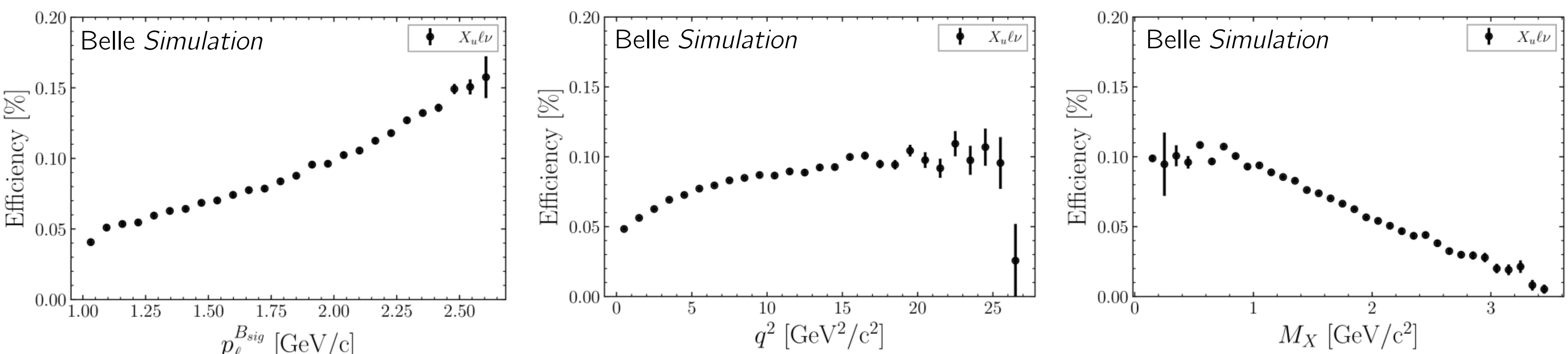


$B \to X_u \ell \nu$ Selection

- Precise $B \to X_u \ell \nu$ extraction complicated by large $B \to X_c \ell \nu$ background.
  - $\frac{\mathcal{B}(B \to X_c \ell \nu)}{\mathcal{B}(B \to X_u \ell \nu)} \approx 50$ with similar experimental signature.
- Focus on high inclusivity $B \to X_u \ell \nu$ extraction, $p_\ell^B > 1.0$ GeV/c (f_u~86%).
- Simple cut-based selection to suppress $B \to X_c \ell \nu$ background
  - loose to minimise bias towards resonances and $B \to X_u \ell \nu$ sculpting.
  - $|m_\nu^2| \approx |m_{Miss}^2| < 0.43\ GeV^2/c^4$
  - Charged slow pion veto.
  - Kaon veto: even $N_{K^\pm} + N_{K_S^0}$

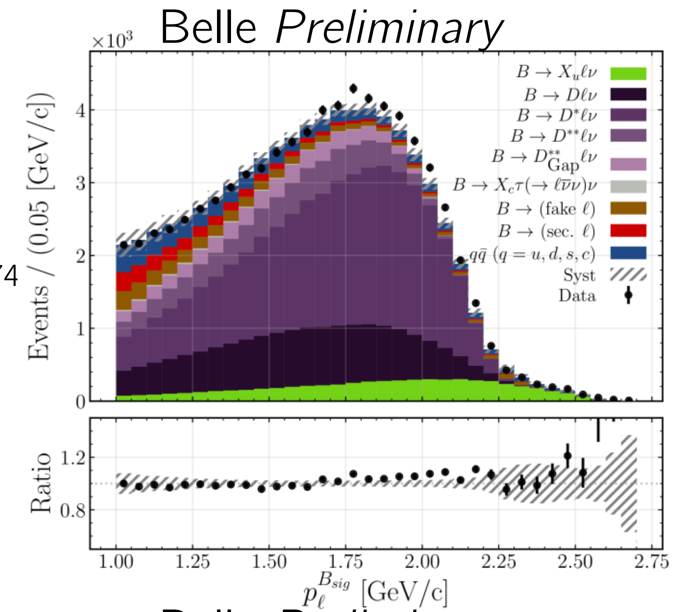
Decay Channel	B^+ [%]	B^0 [%]
$B \to \pi \ell \nu$	0.152(3)	0.082(2)
$B \to \rho \ell \nu$	0.147(2)	0.082(1)
$B \to \omega \ell \nu$	0.127(2)	-
$B \to \eta \ell \nu$	0.127(4)	-
$B \to \eta' \ell \nu$	0.097(4)	-
$B \to x_u \ell \nu$	0.1030(5)	0.0540(4)

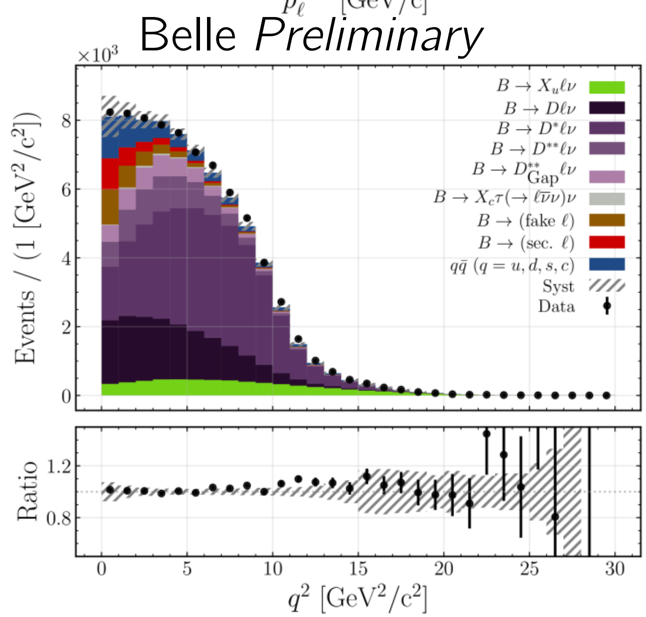
- $B \to X_u \ell \nu$ Efficiency:



$B \to X_u \ell \nu$ Sample

- Data excess at high p_ℓ^B, q^2.
  - Repeated indications seen by Belle, BaBar, and $B \to X_c \ell \nu$ moments analysis.
    Bernlocher, et al. 2014 [Eur.Phys.J.C 74 6, 2914],
    BaBar 2012 [PRD 86, 032004],
    Belle 2021 [PRD 104, 012008],
    Belle 2021 [PRD 104, 112011])
  - Reason unclear, $B \to D^{**} \ell \nu$ modelling?
  - Mismodelling might cause bias in inclusive $|V_{ub}|$ determinations.





$$q^2 = ((|\boldsymbol{p}_{Miss}|, \boldsymbol{p}_{Miss}) + p_\ell)^2$$

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  - Repeated indications seen by Belle, BaBar, and $B \to X_c \ell \nu$ moments analysis.
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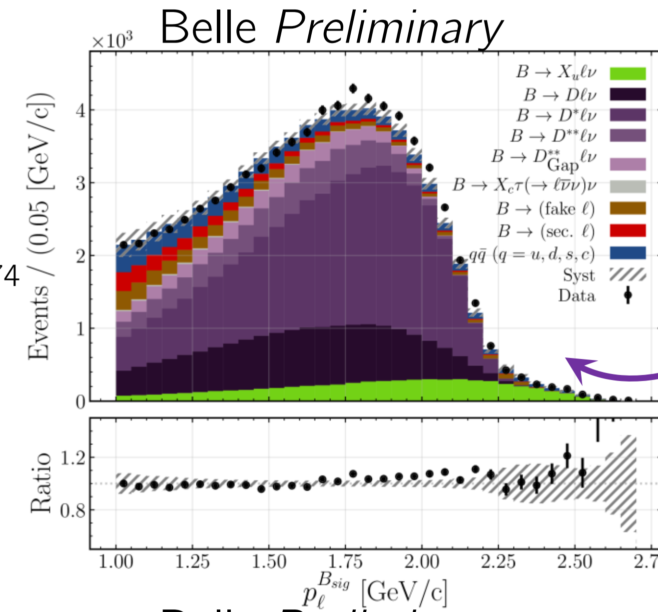
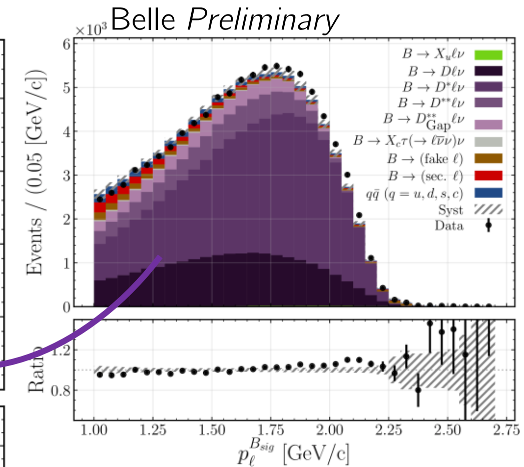
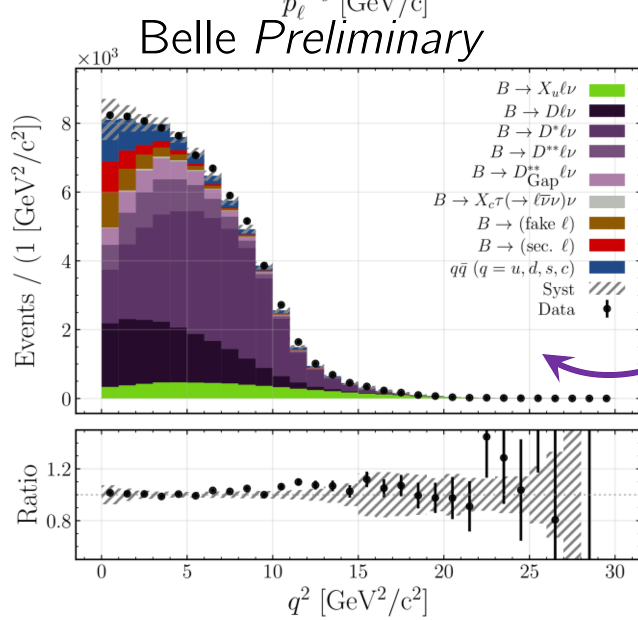
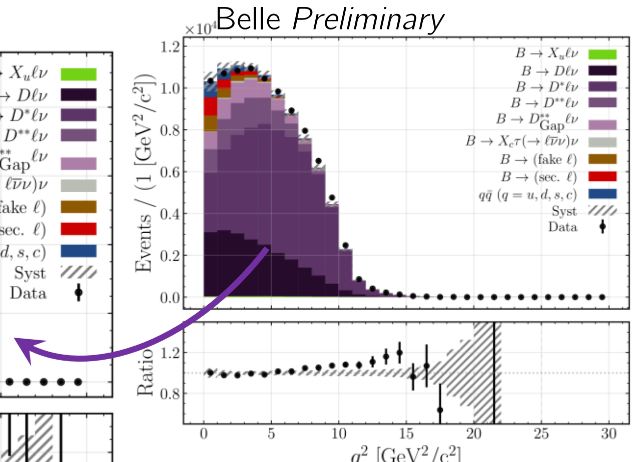
    Belle 2021 [PRD 104, 112011])
  - Reason unclear, $B \to D^{**} \ell \nu$ modelling?
  - Mismodelling might cause bias in inclusive $|V_{ub}|$ determinations.
- Kaon vetoed sample → Consistent Data-MC disagreement.
  - Expected if issue is semi-leptonic B decay modelling.
  - Take data-driven templates..









$$q^2 = ((|\boldsymbol{p}_{Miss}|, \boldsymbol{p}_{Miss}) + p_\ell)^2$$

$B \to X_u \ell \nu$ Extraction

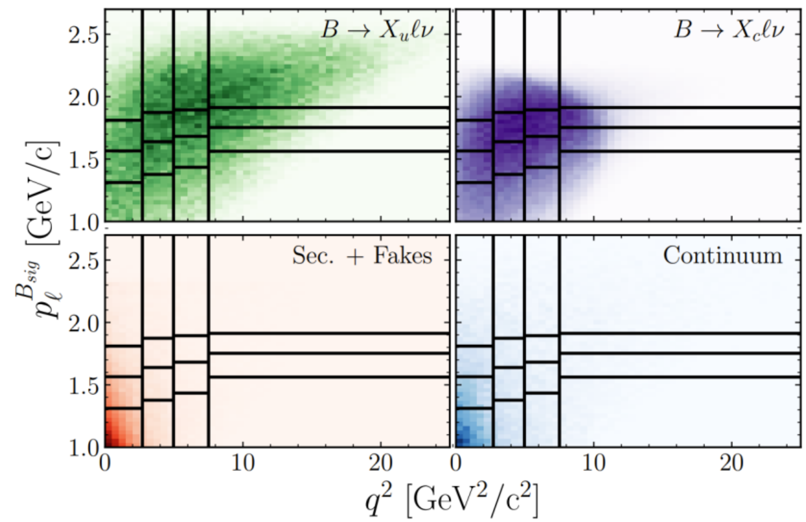
- Extract $B \to X_u \ell \nu$ yield from 2D fit to $q^2 : p_\ell^B$.
  - Equal frequency $B \to X_c \ell \nu$.
  - Final broad bins average over shape function region - reduce dependence on $\mathrm{B} \to X_u \ell \nu$ modelling.
- Continuum MC calibrated to off-resonance sample.
- Secondary and fake lepton MC calibrated to high M_X, low p_ℓ^B control region.
- Take $B \to X_c \ell \nu$ template from kaon vetoed sample as:

Data and MC yields in kaon vetoed sample

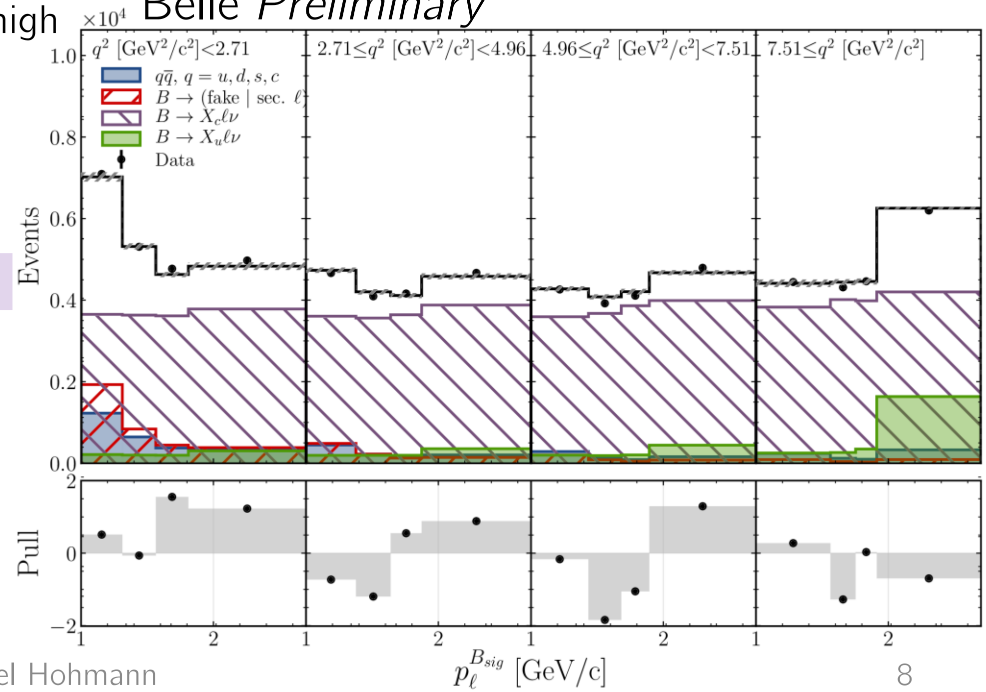
$$T_i = \tau_i (N_{i,K}^{Data} - \eta_{i,K}^{B \to X_u \ell \nu} - \eta_{i,K}^{q\bar{q}} - \eta_{i,K}^{Sec.Fakes})$$

Transfer factor taken from MC

- Maximum likelihood fit floating proportion of $B \to X_u \ell \nu / B \to X_c \ell \nu$.
  - $\chi^2 / dof = 16.8/15$

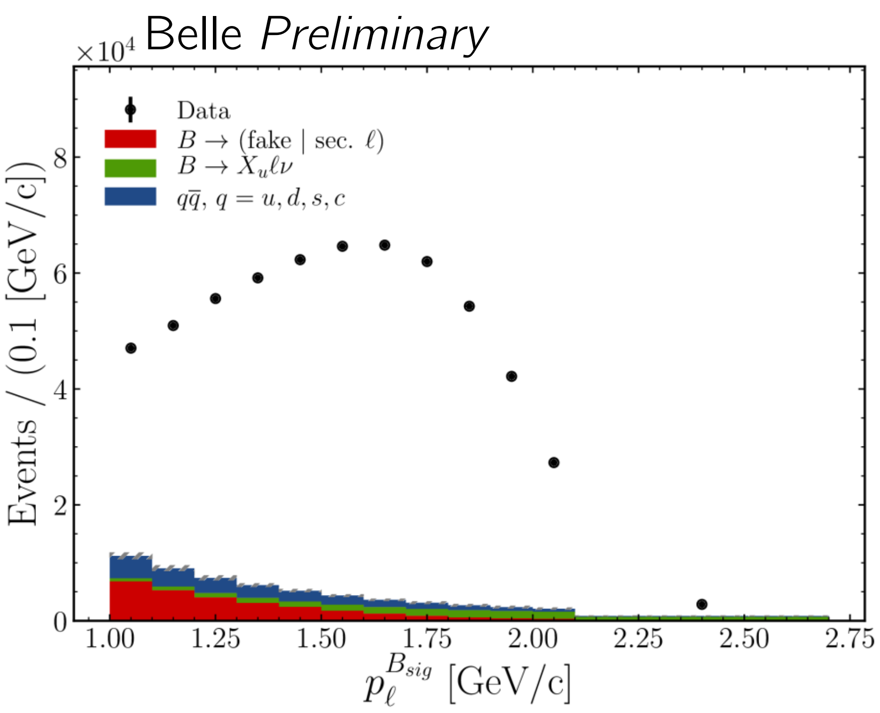


Belle *Preliminary*



$B \to X_c \ell \nu$ Extraction

- Extract $B \to X_c \ell \nu$ yield via simple background subtraction in total $\mathrm{B} \to \mathrm{X}\ell\nu$ sample.
- Normalize $B \to X_u \ell \nu$ by fit result.
- Continuum scaled by calibration in off-resonance sample.
- Secondary and fake lepton contribution fixed after calibration to high M_X, low p_ℓ^B control region.



Ratio of Partial Branching Fractions

- Take ratio as:

$1.98 \pm 0.0 \pm 0.04$ $5390 \pm 440 \pm 310$

$$\frac{\Delta\mathcal{B}\left(B \to X_u \ell\nu\colon\, p_\ell^B > 1.0\ GeV/c\right)}{\Delta\mathcal{B}\left(B \to X_c \ell\nu\colon\, p_\ell^B > 1.0\ GeV/c\right)} = \frac{\epsilon^{B \to X_c \ell\nu}}{\epsilon^{B \to X_u \ell\nu}} \frac{N^{B \to X_u \ell\nu}}{N^{B \to X_c \ell\nu}}$$

$545100 \pm 1400 \pm 2300$

Belle *Preliminary*

$$\frac{\Delta\mathcal{B}\left(B \to X_u \ell\nu\colon\, p_\ell^B > 1.0\ GeV/c\right)}{\Delta\mathcal{B}\left(B \to X_c \ell\nu\colon\, p_\ell^B > 1.0\ GeV/c\right)} = 1.95\left(1 \pm 8.4\%_{stat} \pm 7.2\%_{syst}\right) \times 10^{-2}$$

- Final Step: Extract $\frac{|V_{ub}|}{|V_{cb}|} = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u \ell\nu)}{\Delta\mathcal{B}(B \to X_c \ell\nu)} \frac{\Delta\Gamma(B \to X_c \ell\nu)}{\Delta\Gamma(B \to X_u \ell\nu)}}$

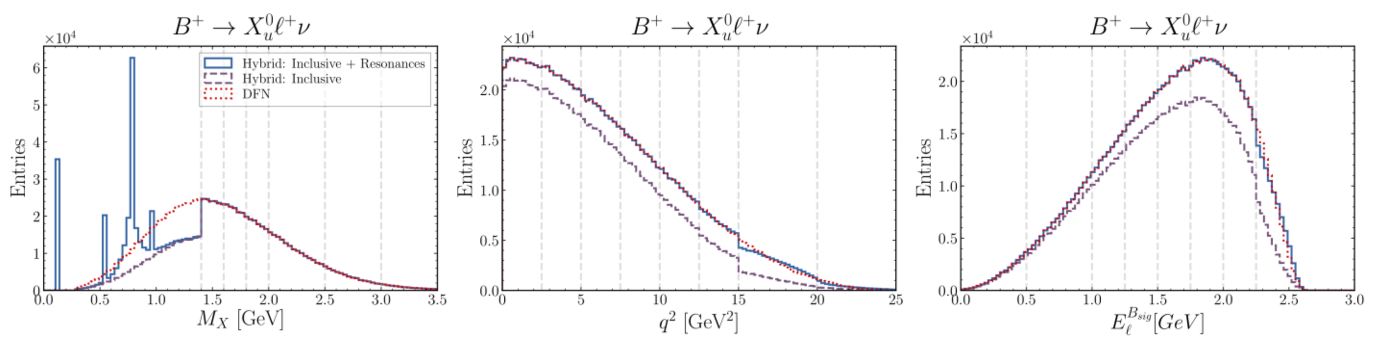
Need theory predictions for ratio of partial rates!

Systematics - $B \to X_u \ell \nu$ Modelling

Belle *Preliminary*

	$\Delta R/R$ [%]
Data Stat.	8.4
$\mathcal{B}(B \to \pi/\rho/\omega/\eta/\eta'\ell\nu)$	0.2
$\mathcal{FF}(B \to \pi/\rho/\omega/\eta/\eta'\ell\nu)$	0.3
$\mathcal{B}(B \to x_u \ell\nu)$	0.6
DFN(m_b, a)	5.0
Hybrid Model (BLNP)	0.6
$N_{s\bar{s}}$	1.3
$\mathcal{B}(B \to D\ell\nu)$	0.1
$\mathcal{B}(B \to D^*\ell\nu)$	0.8
$\mathcal{B}(B \to D^{**}\ell\nu)$	0.3
$\mathcal{B}(B \to D^{(*)}\eta\ell\nu)$	0.2
$\mathcal{B}(B \to D^{(*)}\pi\pi\ell\nu)$	0.2
$\mathcal{FF}(B \to D\ell\nu)$	0.2
$\mathcal{FF}(B \to D^*\ell\nu)$	0.9
$\mathcal{FF}(B \to D^{**}\ell\nu)$	0.4
Sec.Fakes. Composition	3.8
ℓID Eff.	0.1
ℓID Fake.	0.3
$K\pi$ID Eff.	1.1
$K\pi$ID Fake.	0.6
K_S^0 Eff.	0.2
π_{slow}	< 0.1
Tracking	0.1
Continuum Calibration	0.4
N_{BB}	< 0.1
$f_{+/0}$	< 0.1
MC Stat.	2.6
Total Syst.	7.2

- Nominal $B \to X_u \ell \nu$ MC: DFN [JHEP 06(1999), 017] NLO calculation + non-perturbative QCD inputs.
  - Hadronised with JETSET/Pythia ($m_X > 2m_\pi$)
- Resonances ($B \to (\pi, \rho, \omega, \eta, \eta')\ell\nu$) added ad-hoc via hybrid approach [PRD 41, 1496].
  PDG 2020 branching fractions and up-to-date models.
  $\Delta\mathcal{B}_{ijk}^{inc.} = \Delta\mathcal{B}_{ijk}^{exc.} + \mathrm{w_{ijk}} \cdot \Delta\mathcal{B}_{ijk}^{inc.}$



Decay Channel	B^+ [$\times 10^{-3}$]	B^0 [$\times 10^{-3}$]
$B \to X_u \ell \nu$	2.21 ± 0.31	2.05 ± 0.29
$B \to \pi \ell \nu$	0.078 ± 0.003	0.150 ± 0.006
$B \to \rho \ell \nu$	0.158 ± 0.011	0.294 ± 0.021
$B \to \omega \ell \nu$	0.119 ± 0.009	-
$B \to \eta \ell \nu$	0.039 ± 0.005	-
$B \to \eta' \ell \nu$	0.023 ± 0.008	-
$B \to x_u \ell \nu$	1.79 ± 0.32	1.60 ± 0.30

Systematics - $B \to X_u \ell \nu$ Modelling

Belle *Preliminary*

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$\mathcal{B}(B \to D^{**}\ell\nu)$	0.3
$\mathcal{B}(B \to D^{(*)}\eta\ell\nu)$	0.2
$\mathcal{B}(B \to D^{(*)}\pi\pi\ell\nu)$	0.2
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N_{BB}	< 0.1
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MC Stat.	2.6
Total Syst.	7.2

- Small exposure to resonances in hybrid modelling composition.
- Dominant systematic due to $m_b^{KN} = 4.66 \pm 0.04$ GeV , $a^{KN} = 1.3 \pm 0.5$ uncertainty [PRD 73, 073008].
- Switch DFN -> BLNP [PRD 72, 073006] for inclusive. Shape difference mostly at endpoint – averaged over in broad bins.
- $\boldsymbol{K}$ production in X_u via gluon splitting – vary relative contribution $\pm 25\%$.

Systematics - $B \to X_c \ell \nu$ Modelling

See Ray's talk for details!

Belle *Preliminary*

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- $B \to D^{(*)}\ell\nu$: latest BGL form factors [PRD 93, 0.32006], [PRD 103, 073005] and PDG2020 branching fractions
- $B \to D^{**}\ell\nu,\ D^{**} \in (D_1, D_2, D_0, D_1')$: D^{**} masses and widths updated to PDG2020. LLSW form factors.
- $\sim$10% gap between inclusive and sum of exclusive measurements.
  - Filled with "best guess" $B \to D^{(*)}\eta\ell\nu, B \to D^{(*)}\pi\pi\ell\nu$

Decay Channel	B^+ [$\times 10^{-3}$]	B^0 [$\times 10^{-3}$]
$B \to X_c\ell\nu$	108 ± 4	101 ± 4
$B \to D\ell\nu$	23.5 ± 1	23.1 ± 1
$B \to D^*\ell\nu$	56.6 ± 2	50.5 ± 1
$B \to D_0(\to D\pi)\ell\nu$	4.2 ± 0.8	3.9 ± 0.7
$B \to D_1'(\to D^*\pi)\ell\nu$	4.2 ± 0.8	3.9 ± 0.8
$B \to D_1(\to D^*\pi)\ell\nu$	4.2 ± 0.3	3.9 ± 0.3
$B \to D_1(\to D\pi\pi)\ell\nu$	2.4 ± 0.1	2.3 ± 0.9
$B \to D_2(\to D^*\pi)\ell\nu$	1.2 ± 0.1	1.1 ± 0.1
$B \to D_2(\to D\pi)\ell\nu$	1.8 ± 0.2	1.7 ± 0.2
$B \to D\pi\pi\ell\nu$	0.6 ± 0.6	0.6 ± 0.6
$B \to D^*\pi\pi\ell\nu$	2.2 ± 1	2.0 ± 1
$B \to D\eta\ell\nu$	3.6 ± 2	4.0 ± 2
$B \to D^*\eta\ell\nu$	3.6 ± 2	4.0 ± 2

Via intermediate broad D^{**} → Better description than 4/5 body phase-space model.

Systematics - Other

Belle *Preliminary*

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Continuum Calibration	0.4
N_{BB}	< 0.1
$f_{+/0}$	< 0.1
MC Stat.	2.6
Total Syst.	7.2

- Vary secondary lepton, fake lepton, and $B \to X\tau(\to \ell\nu\bar{\nu})\nu$ relative contributions by **30%.**
  - Combined normalisation constrained by fits to high mass, low lepton momentum control regions.

- Detector effects and particle identification well understood from control mode samples.

Naïve $|V_{ub}|$ Extraction ($p_\ell^B > 1.0\ GeV/c$)

Just for this workshop!

$$|V_{ub}| = \sqrt{\frac{1}{\tau_B \Delta\Gamma} \frac{\Delta\mathcal{B}(B \to X_u \ell\nu)}{\Delta\mathcal{B}(B \to X_u \ell\nu)} \Delta\mathcal{B}(B \to X_c \ell\nu)}$$

Belle, 2007 [PRD 75, 032001]: $(8.41 \pm 0.15 \pm 0.17)\%$
Babar, 2010 [PRD 81, 0032003]: $(8.63 \pm 0.17)\%$

Naïve average: $(8.55 \pm 0.13)\%$ - Assume uncorrelated.

$\tau_B = 1.579 \pm 0.004$ ps

$1.95(1 \pm 0.084 \pm 0.072) \times 10^{-2}$

BLNP: $61.5^{+6.4}_{-5.1}\ ps^{-1}$ [PRD 72, 073006]
GGOU: $58.5^{+2.7}_{-2.3}\ ps^{-1}$ [JHEP 0710:058]
DGE: $58.2^{+3.6}_{-3.0}\ ps^{-1}$ [JHEP 0601:097]
Consistent with Belle, 2021 [PRD 104, 012008]

Just for this workshop!

Naïve $|V_{ub}|$ Extraction ($p_\ell^B > 1.0\ GeV/c$)

$$|V_{ub}| = \sqrt{\frac{1}{\tau_B \Delta\Gamma} \frac{\Delta\mathcal{B}(B \to X_u \ell \nu)}{\Delta\mathcal{B}(B \to X_u \ell \nu)} \Delta\mathcal{B}(B \to X_c \ell \nu)}$$

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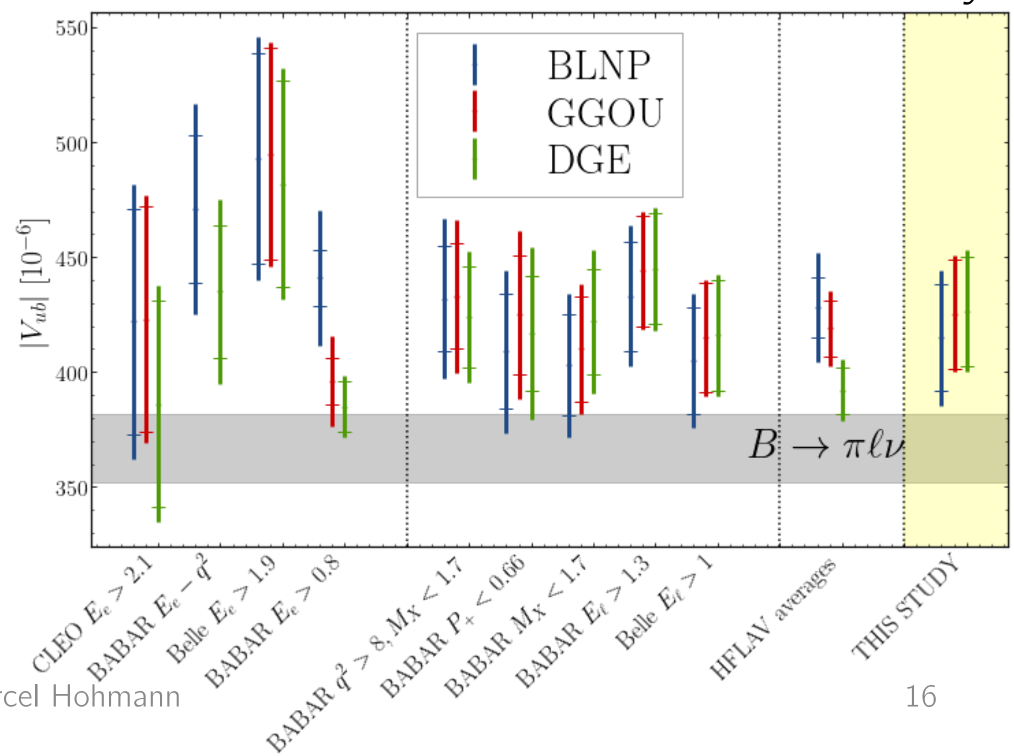
BLNP: $61.5^{+6.4}_{-5.1}\ ps^{-1}$ [PRD 72, 073006]
GGOU: $58.5^{+2.7}_{-2.3}\ ps^{-1}$ [JHEP 0710:058]
DGE: $58.2^{+3.6}_{-3.0}\ ps^{-1}$ [JHEP 0601:097]
Consistent with Belle, 2021 [PRD 104, 012008]

$$|V_{ub}|^{GGOU} = (4.25 \pm 0.18 \pm 0.16\ ^{+0.09}_{-0.09}) \times 10^{-3}$$
$$|V_{ub}|^{BLNP} = (4.15 \pm 0.17 \pm 0.15\ ^{+0.18}_{-0.20}) \times 10^{-3}$$
$$|V_{ub}|^{DGE} = (4.26 \pm 0.18 \pm 0.16\ ^{+0.11}_{-0.13}) \times 10^{-3}$$

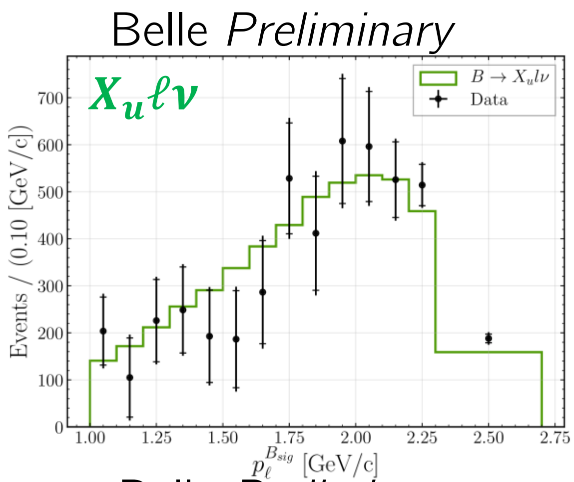
Good agreement with HFLAV averages!

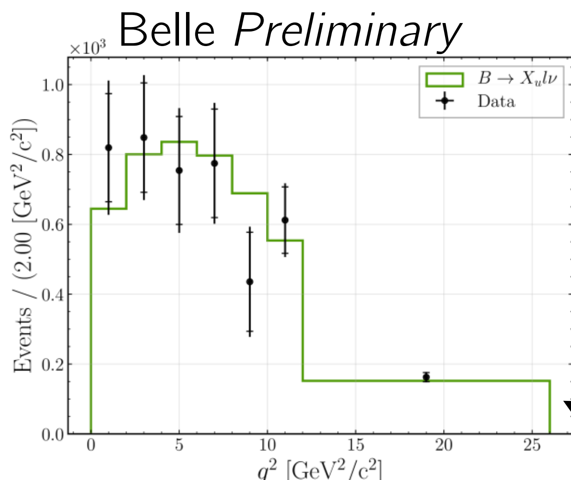
Belle *Preliminary*

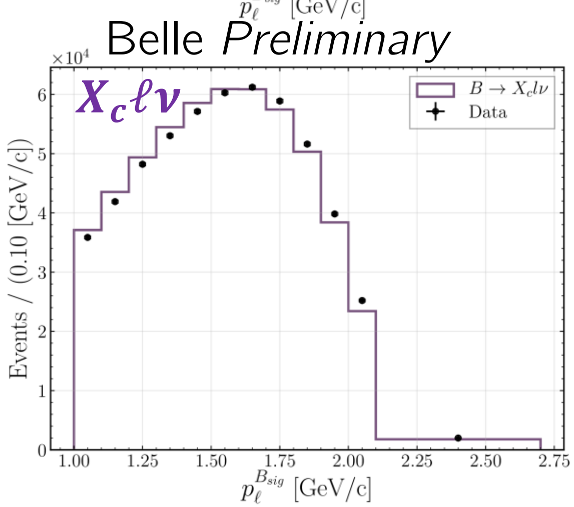


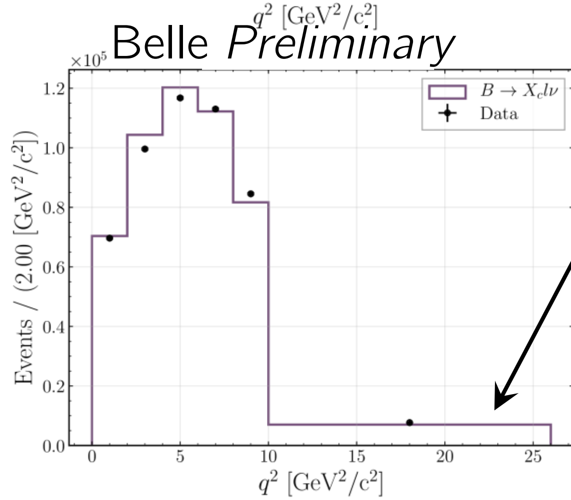
Background Subtracted Spectra

- Normalise to nominal fit results and project onto p_ℓ^B, q^2 - no additional selections.
- Subtract backgrounds, correct $B \to X_c \ell \nu$ shape from kaon vetoed sample.





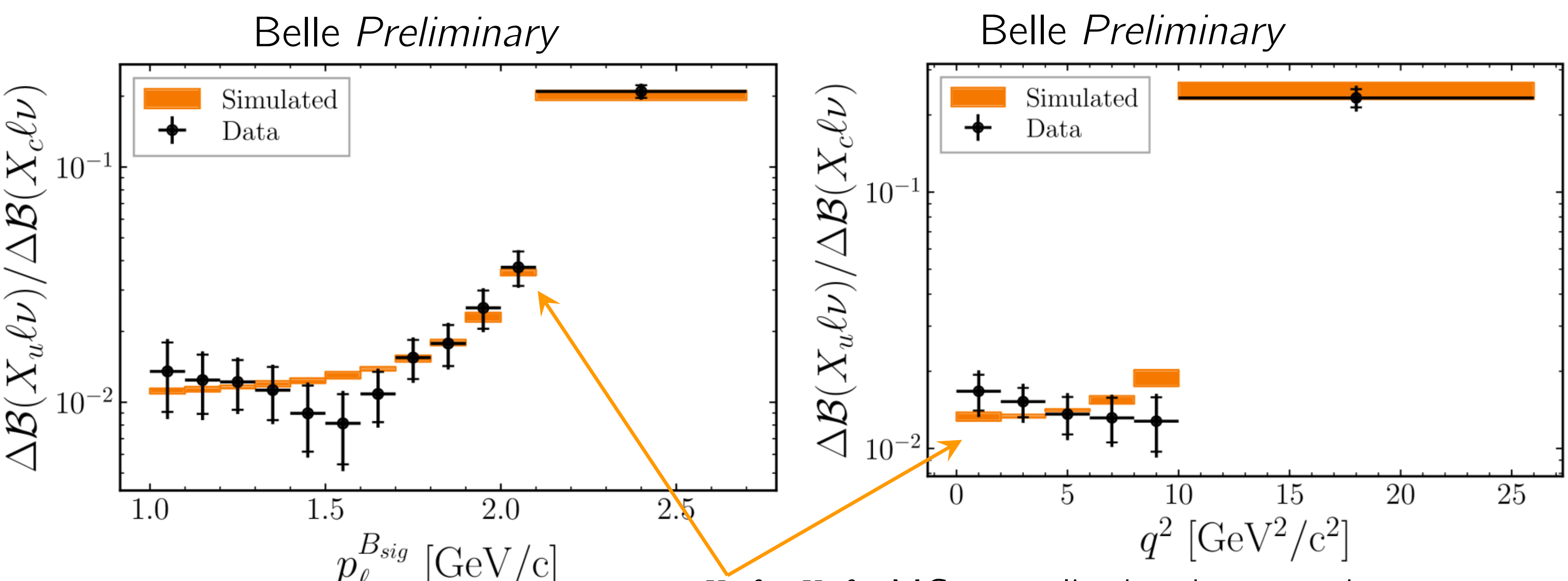




Nominal MC normalised to background subtracted yields.

Unfolded Ratio

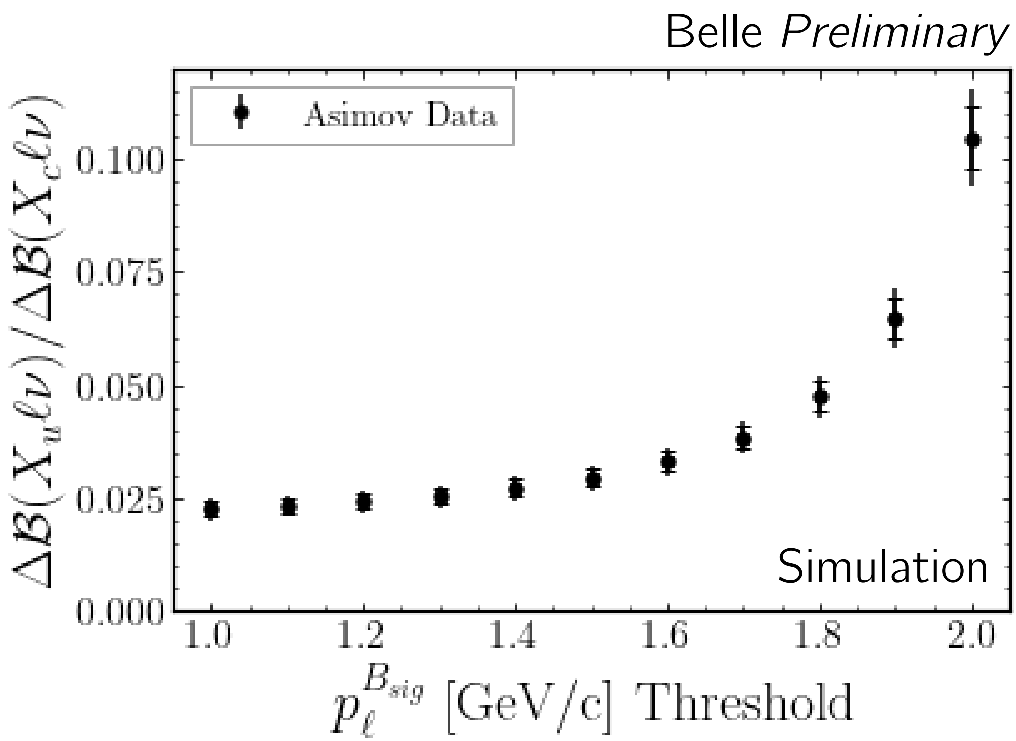
- Unfold $B \to X_u \ell \nu, B \to X_c \ell \nu$ yields via Singular Value Decomposition method of [NIMA 372:469(1996)]
- Tune regularisation parameter to minimise model bias.
- Take ratio and correct for efficiency to form differential ratios.



$X_u \ell \nu, X_c \ell \nu$ MC normalised to integrated efficiency corrected data yields.

Work in Progress: Ratio as function of p_ℓ^B Threshold

- Repeat measurement tightening p_ℓ^B selection.
- Probes stability near endpoint.
- Highly correlated across all thresholds.



Summary

- Preliminary result on $\frac{\Delta\mathcal{B}(B\to X_u\ell\nu)}{\Delta\mathcal{B}(B\to X_c\ell\nu)}$ at Belle.

- Theory predictions of ratio of partial rates needed to extract $|V_{ub}|/|V_{cb}|$.

- Naïve $|V_{ub}|$ extraction in good agreement with world averages.

- Data-driven $X_c\ell\nu$ modelling corrections will be beneficial for Belle II measurements.

- What can be extracted from the unfolded ratios?

Belle *Preliminary*