Probing the B meson DA

with $B \to \ell\ell\ell'\nu$

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Based on "Probing the structure of the B meson with $B \to \ell\ell\ell'\nu$" , by AB, Bharti Kindra and Namit Mahajan, arXiv:2102.03193 [hep-ph]

Challenges in Semileptonic B Decays
Barolo, 19–23 Apr 2022

What do we know about the B meson's structure ?

Let's start with λ_B!

- Leading moment λ_B of the B-meson distribution amplitude ($\phi_B^+(k,\mu)$), $\lambda_B^{-1}(\mu) = \int_0^\infty \frac{dk}{k}\phi_B^+(k,\mu)$, not well known
- Theory uncertainty on λ_B large, from 200 MeV obtained using non-leptonic decays (Beneke et al. 2003, 2009) to 460 ± 110 MeV obtained from QCD sum rules (Braun et al. 2003).

$B^+ \to \ell^+\nu\gamma$ probes λ_B for $E_\gamma \gg \Lambda_{\rm QCD}$ (Beneke 2000, Grozin 1996, Bosch 2003)

Most stringent limits from BaBar collaboration (2009)

$$\mathcal{BR}(B^+ \to \ell^+\nu\gamma) < 15.6 \times 10^{-6} @ 90\%\,\mathrm{C.L.} \Rightarrow \lambda_B > 300\,\mathrm{MeV}$$

and from Belle (2015):

$$\mathcal{BR}(B^+ \to e^+/\mu^+\nu\gamma) < 4.3 \times 10^{-6}/3.4 \times 10^{-6} @ 90\%\,\mathrm{C.L.} \Rightarrow \lambda_B > 238\,\mathrm{MeV}$$

Updating this result is a priority at Belle II, and projections look very promising (Gelb et al. 2018, Kou et al. 2018).

What about $B \to \ell\ell\ell'\nu$?

- Measurement of $B \to \gamma\ell\nu$ @LHCb challenging
- However, if we replace the γ by $\gamma^* \to \ell^+\ell^-$, analysis feasible
- LHCb limit for $m_{\text{light}}(\mu^+\mu^-) < 980$ MeV (2018),

$$\mathcal{BR}(B^+ \to \mu^+\mu^-\mu^+\nu) < 1.6 \times 10^{-8}. \tag{1}$$

- Theory status: Vector meson dominance approach (Danilina et al. 2018, 2019), $\mathcal{BR} \sim 3 \times 10^{-7}$, no uncertainties.

Our aim: provide predictions for $B \to \ell\ell\ell'\nu$ to obtain complementary constraints on λ_B . Important cross-check for the results from Belle II.

Amplitude for $B^+(p_B) \to \ell^+(q_1)\ell^-(q_2)\ell'^+(p_1)\nu(p_2)$

(a): photon emission from u quark, (b): photon emission from b quark

$$iA = \frac{G_F V_{ub}}{\sqrt{2}} \frac{ie^2}{q^2} (\bar{u}_\ell \gamma^\mu v_\ell)[(\bar{u}_\nu \gamma^\rho P_L v_{\ell'}) T_{\mu\rho} - i(\bar{u}_\nu \gamma_\mu P_L v_{\ell'}) f_B]$$

- 1st term describes emission of γ^* from B meson, 2nd term from the lepton
- At L.O. 2nd term can trivially be written in terms of f_B, 1st term more complicated

On imposing EM current conservation and applying the equation of motion:

$$T_{\mu\rho} = iF_A(g_{\mu\rho}(v \cdot q) - v_\mu q_\rho) + F_V \epsilon_{\rho\mu\lambda\sigma} v^\lambda q^\sigma - iF_{\|} v_\mu q_\nu + if_B g_{\mu\rho},$$

where $p_B = m_B v$ and $q = q_1 + q_2$

Form factors at leading order

in $1/m_b$ and α_s

- In $B \to \gamma \ell \nu$, factorization holds for $E_\gamma \sim \mathcal{O}(m_b)$, in our case require hard-collinear virtual photon coupling to u quark. Coupling to b power suppressed.
- In light-cone coordinates ($l \equiv (l_+, l_-, l_\perp)$) where $l^\mu = \frac{l_+}{2} n_-^\mu + l_\perp + \frac{l_-}{2} n_+^\mu$. Spectator quark is soft: $k = (k_+, k_-, k_\perp) \sim (\lambda, \lambda, \lambda)$, virtual photon hard collinear: $q = (q_+, q_-, q_\perp) \sim (\lambda, 1, \lambda^{1/2})$
- u quark propagator can be expressed as

$$\frac{\not{q} - \not{k}}{(q-k)^2} = \frac{q_- \not{n}_+/2}{q^2 - q_- k_+} - \left(\frac{k_+ \not{n}_-/2}{q^2 - q_- k_+} + \frac{k_- \not{n}_+/2}{q^2 - q_- k_+} + \frac{\not{k}_\perp}{q^2 - q_- k_+} \right).$$

- Form factor at leading power ($F_V = F_A$)[1]:

$$F_{B \to \gamma^*}(q^2, p^2) = Q_u f_B \, m_B \int_0^\infty dk_+ \frac{\phi_B^+(k_+)}{q_- \, k_+ - q^2 - i\epsilon}.$$

[1] G. P. Korchemsky et al, hep-ph/9911427, Descotes-Genon and Sachrajda, hep-ph/0209216, Lunghi et al, hep-ph/0210091 and Bosch et al, hep-ph/0301123.

NLL corrections: $F_X = F_X^{\rm NLL} + \xi_X^{\rm NLL} + \Delta F_X^{(u)} + \Delta F_X^{(b)}$ $(X = V/A)$,

The factor $R(E_\gamma, \mu)$ (Beneke Rohrwild 2011) can be adapted to our case:

$$R(q_-, q^2, \mu, \mu_{h1}, \mu_{h2}) = C(q_-, \mu_{h1}) K^{-1}(\mu_{h2}) U(q_-, \mu_{h1}, \mu_{h2}, \mu) J(q_-, q^2, \mu).$$

Form factor @ LO in $1/m_b$, NLL in α_s expansion

$$F_{V/A}^{\rm NLL} = Q_u f_B\, m_B \int_0^\infty \frac{d\omega\, \phi_B^+(\omega, \mu)}{q_-\,\omega - q^2 - i\varepsilon} R(q_-, q^2, \mu, \mu_{h1}, \mu_{h2}),$$

- $C(q_-, \mu_{h1})$ obtained from matching QCD heavy-to-light current to SCET I current at μ_{h1} (Lunghi 2002,Bosch 2003, Bauer 2000).
- $K(\mu_{h2})$ accounts for conversion from static f_B in the SCET current to the standard definition in QCD, f_B
- $J(q_-, q^2, \mu)$ accounts for the hard-collinear radiative corrections, and was calculated for $B \to \gamma \ell \nu$ in (Lunghi 2002, Bosch 2003). For $B \to \ell\ell\ell'\nu$ where $q^2 \neq 0$ we can adopt the expression given in ALERT(Wang 2016).
- RGE factor $U(q_-, \mu_{h1}, \mu_{h2}, \mu)$ resums logarithms to all orders by solving a renormalization group equation (Bosch:2003fc), depends on three scales: the hard scales μ_{h1} and μ_{h2} are taken to be q_- and m_b respectively, while the hard-collinear scale μ is set to $(m_b \Lambda_{\rm QCD})^{1/2}$.

Soft@NLL: $F_X = F_X^{\mathrm{NLL}} + \xi_X^{\mathrm{NLL}} + \Delta F_X^{(u)} + \Delta F_X^{(b)}$ $(X = V/A)$, $F_\parallel = F_\parallel^{\mathrm{NLP}} + \xi_\parallel^{\mathrm{NLL}}$

Soft conitrbution at $q^2 \leq 0$ known at NLO (from Beneke et al. 2018,Wang 2016,18, based on Braun and Khodjamirian 2003)

Extend $\xi_{\mathrm{soft}}^{\mathrm{Eucl.}}$ to $q^2 > 0$ via once subtracted dispersion relation (following Guadagnoli et al. 2017, Kozachuk et al. 2017),

$$\xi_X^{\mathrm{NLL}}(q^2) = \xi_{X,\,\mathrm{soft}}^{\mathrm{Eucl.}}(q_0^2) + \sum_R \frac{q^2 - q_0^2}{m_R^2 - q_0^2} \frac{c_R f_R m_R e^{i\delta_R}}{m_R^2 - q^2 - i m_R \Gamma_R} F_X^{B\to R}$$

where $X = V, A$ and $\parallel$, $\xi_{V/A,\,\mathrm{soft}}^{\mathrm{Eucl.}}(q_0^2) = \xi_{\mathrm{soft}}^{\mathrm{Eucl.}}(q_0^2)$ and $\xi_{\parallel,\,\mathrm{soft}}^{\mathrm{Eucl.}}(0) = 0$, q_0^2 is the subtraction point, $R = \rho, \omega$, $c_\rho = \frac{1}{2}$, $c_\omega = \frac{1}{6}$ and

$$F_V^{B\to R} = \frac{2m_B}{m_B + m_R} V^{B\to R}(p^2)$$

$$F_A^{B\to R} = -\frac{m_B + m_R}{v.q} A_1^{B\to R}(p^2)$$

$$F_\parallel^{B\to R} = -\frac{q^2}{v.q} \frac{m_B + m_R}{m_B^2 + p^2} A_1^{B\to R}(p^2).$$

Uncertainties: q_0^2 in range [-5,-1] GeV^2, and δ_ρ and δ_ω $[-\pi/2, \pi/2]$.

The longitudinal form factor: $F_\parallel = F_\parallel^{\mathrm{NLP}} + \xi_\parallel^{\mathrm{NLL}}$,

Coming back to the expression for the propagator:

$$\frac{\not{q} - \not{k}}{(q-k)^2} = \frac{q_- \not{n}_+/2}{q^2 - q_- k_+} - \left(\frac{k_+ \not{n}_-/2}{q^2 - q_- k_+} + \frac{k_- \not{n}_+/2}{q^2 - q_- k_+} + \frac{\not{k}_\perp}{q^2 - q_- k_+} \right).$$

The third term matches onto a hadronic matrix element with a transverse derivative acting on the spectator-quark field, and contributes only to the longitudinal form factor. This can be computed within the Wandzura-Wilczek approximation, for hard collinear q^2 (Beneke et al. 2021):

$$F_\parallel^{\mathrm{NLP}} = -4 f_B m_B Q_u \frac{q_+}{q_-^2} \int_0^\infty dk_+ \frac{\phi_B^-(k_+)}{q_-\, k_+ - q^2 - i\epsilon}.$$

Symmetry-breaking corrections

$F_X = F_X^{\rm NLL} + \xi_X^{\rm NLL} + \Delta F_X^{(u)} + \Delta F_X^{(b)} \quad (X = V/A)$

Leading power suppressed term in the u-quark propagator[2], we find

$$\Delta F_V^{(u)} = -\Delta F_A^{(u)} = Q_u\, f_B\, m_B \int_0^\infty dk_+ \phi_B^+ \frac{1}{q_-^2} \left(\frac{q^2}{q_-\, k_+ - q^2} + 1 \right)$$
$$= \frac{Q_u}{q_-^2} \left(f_B\, m_B + q^2\, F_{B\to\gamma^*} \right).$$

Power-suppressed emission from the b-quark leg also yields symmetry breaking correction

$$\Delta F_V^{(b)} = -\Delta F_A^{(b)} = \frac{Q_b f_B m_B}{q^2 - 2m_b\, v.q}.$$

Note that in our analysis we adopt : $\phi_B^+(k,\mu) = \frac{k}{\lambda_B^2(\mu)} e^{-k/\lambda_B(\mu)}$

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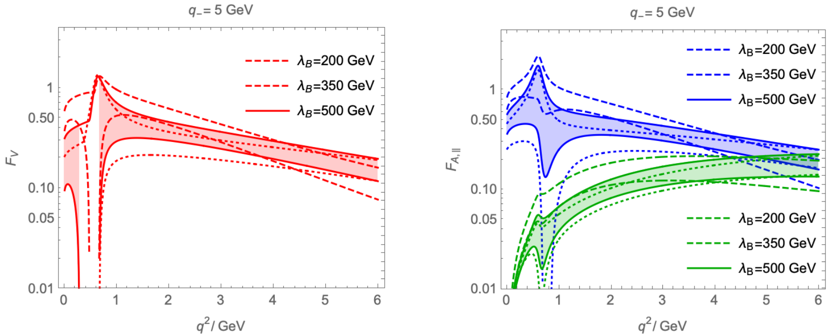
[2] symmetry breaking corrections at $\mathcal{O}(1/m_b, 1/E_\gamma)$ in QCD+soft corrections at twist-3 to 6 (Beneke et al. 2018,Wang 2016,18)

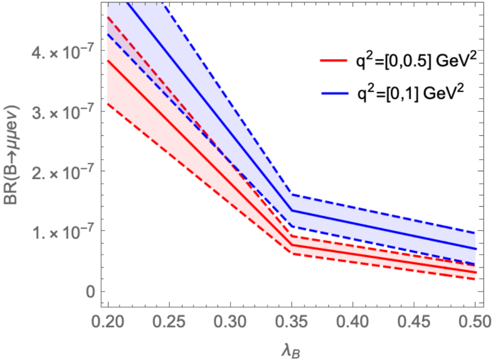
Results for the form factors

Preliminary!



Results for the Branching Ratio

Preliminary!



Note large dependence on λ_B:

Decrease in uncertainty when excluding resonance region

Thoughts

Problem

- Initial idea to provide theoretical prediction for experiment (principally LHCb which has measured $B \to 3\mu\nu$), to provide an alternative channel to determine λ_B, find that this idea has too potential issues:
  - Ambiguity in like-sign muons means different leptons easier to interpret theoretical [3]
  - In ideal kinematic region where largest rate and dependence on λ_B, resonances result in large uncertainties

Possible solutions

- Measurement of $B \to \ell\ell\ell'\nu$ with $\ell \neq \ell'$ (expect LHCb yied to dimish by a factor 3/4)
- Exclude resonances by choosing bin with $q^2 >$1 GeV^2. (But here there is less dependence on λ_B), or $q^2 = [0, 0.5]$ GeV^2
- Improve understanding of resonance region by experiment/double fit of resonances (including the associated phases) and λ_B? Angular analysis?

[3] It turns out that Beneke et al. find for small $q^2_{\rm low}$, cut on $n_- q_{\rm low} > 3$ GeV suffices to resolve this, as cases falling outside this cut (i.e. where true

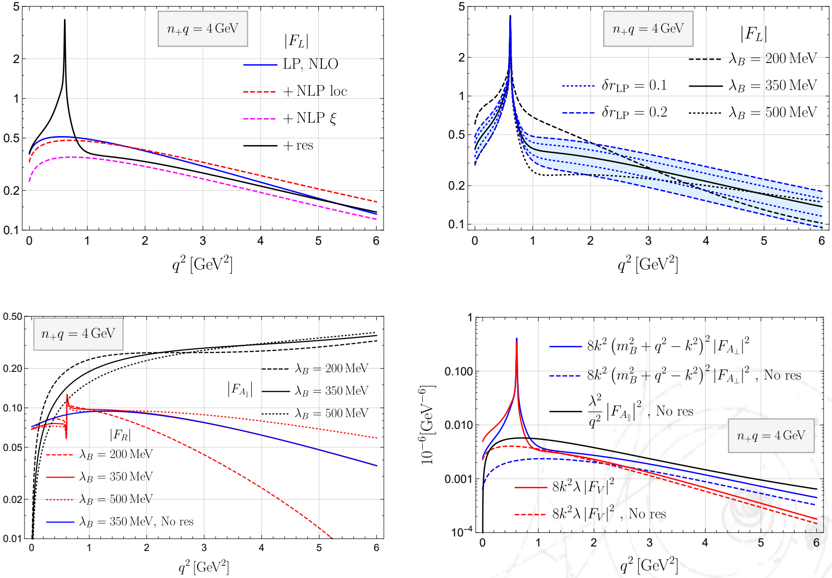
Summary

- We studied the form factors, and note the importance of calculating to NLL, of including the resonances and of the large size of the power suppressed longitudinal form factor
- Note that certain higher twist contributions which were calculated in the state-of-the-art $B \to \gamma \ell \nu$ analysis have been neglected here, i.e. the $1/m_b$ and $1/E_\gamma$ higher twist corrections and the twist 3 to 6 contributions to the soft correction. The effect of these missing pieces is conservatively estimated to add $\lesssim 5\%$ to the uncertainty, negligible compared to the uncertainty coming from $|V_{ub}|$.
- From the dependence of the partial branching ratio on λ_B we find that the dependence far outweighs the remaining uncertainties, suggesting that given a value of the partial branching ratio a measurement of λ_B should be feasible.
- While there are no official projections for these channels at LHCb and Belle II, naive estimates show that the prospects to measure the partial branching ratio is promising.
- We therefore look forward to these results and to the potential measurement of λ_B, complementary to that of $\mathcal{B}(B \to \gamma \ell \nu)$ at Belle II.

Parameters and Uncertainties

Parameter	Value	Ref.
m_B	$5.28\ \mathrm{GeV}$	PDG
f_B	192.0 ± 4.3 MeV	PDG
$\lvert V_{ub}\rvert^{\text{excl}}$	$(3.70 \pm 0.16) \times 10^{-3}$	PDG
G_F	$1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$	PDG
m_μ	$0.105\ \mathrm{GeV}$	PDG
τ_B	$(1.641 \pm 0.008) \times 10^{-12}$ s	PDG
m_ρ	0.775 GeV	PDG
m_e	$0.511 \times 10^{-3}\ \mathrm{GeV}$	PDG
α_{em}	$1/137$	PDG
λ_B	$[200 - 500]$ MeV	Beneke et al. 2018
s_0	1.5 ± 0.1 GeV^2	Beneke et al. 2018
M^2	1.25 ± 0.25 GeV^2	Beneke et al. 2018

Comparison with arXiv:2102.10060 [hep–ph]



Soft@NLL: $F_X = F_X^{\rm NLL} + \xi_X^{\rm NLL} + \Delta F_X^{(u)} + \Delta F_X^{(b)}$ $(X = V/A)$, $F_\parallel = F_\parallel^{\rm NLP} + \xi_\parallel^{\rm NLL}$

With $F^{\rm QCD}(E,s) = F_{V/A}^{\rm NLL}$, and $\omega' = s/q_-$ we find:

$$\xi_{\rm soft}^{\rm Eucl.} = Q_u f_B\, m_B\, C(q_-,\mu_{h1}) K^{-1}(\mu_{h2}) U(q_-,\mu_{h1},\mu_{h2},\mu) \int_0^{s_0/q_-} {\rm d}\omega' \left(\frac{e^{-(q_-\,\omega' - m_\rho^2)/M^2}}{m_\rho^2 - q^2 - i\varepsilon} - \frac{1}{q_-\,\omega' - q^2 - i\varepsilon} \right) \phi_B^{+,\rm eff}(\omega',\mu)$$

where $\phi_{B,\rm eff}^+(\omega,\mu) = \phi_B^+(\omega,\mu) + \frac{\alpha_s(\mu) C_F}{4\pi}\,\delta\phi_{B,\rm eff}^+(\omega',\mu)$ (Beneke et al. 2018,Wang 2016):

$$\delta\phi_{B,\rm eff}^+ = \int_0^{\omega'} d\omega \left(\frac{2}{\omega-\omega'} \ln\frac{\mu^2}{q_-(\omega'-\omega)} \right)_\oplus \phi_B^+(\omega,\mu) - \omega' \int_0^{\omega'} d\omega \left(\frac{1}{\omega-\omega'} \ln\frac{\omega'-\omega}{\omega'} \right)_\oplus \frac{\phi_B^+(\omega,\mu)}{\omega} +$$
$$\frac{\omega'}{2} \int_0^\infty d\omega \ln^2\left|\frac{\omega-\omega'}{\omega'}\right| \frac{d}{d\omega}\frac{\phi_B^+(\omega,\mu)}{\omega} - \int_{\omega'}^\infty d\omega \left(\ln^2\frac{\mu^2}{q_-\omega'} - \frac{\pi^2}{2} - 1 \right) \frac{d}{d\omega}\phi_B^+(\omega,\mu) +$$
$$\omega' \int_{\omega'}^\infty d\omega \left(\ln\frac{\mu^2}{q_-\omega'} \ln\frac{\omega-\omega'}{\omega'} - \frac{1}{2}\ln^2\frac{\mu^2}{q_-(\omega-\omega')} + \frac{1}{2}\ln^2\frac{\mu^2}{q_-\omega'} + 3\ln\frac{\omega-\omega'}{\omega'} - \frac{2\pi^2}{3} \right) \frac{d}{d\omega}\frac{\phi_B^+(\omega,\mu)}{\omega}$$