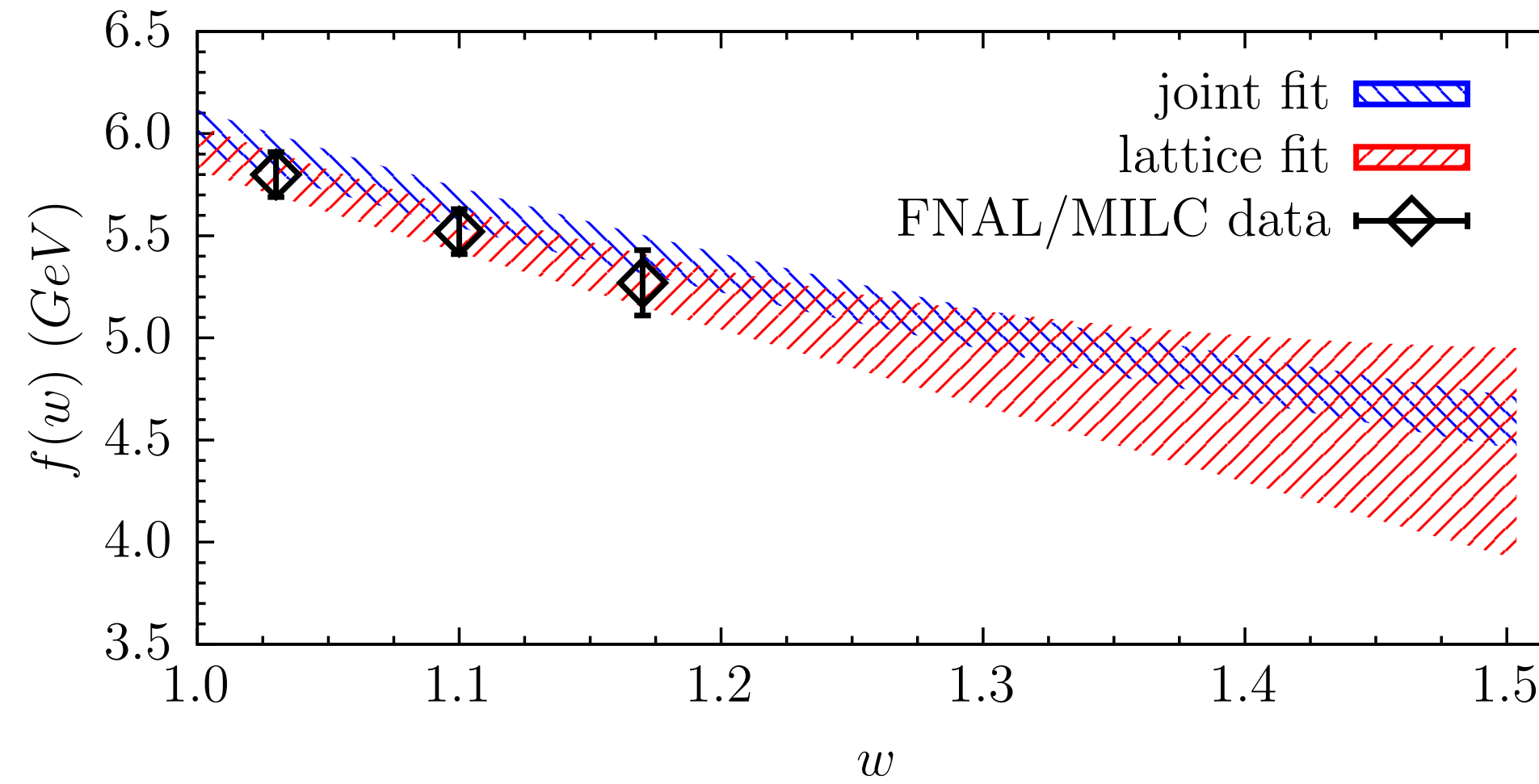
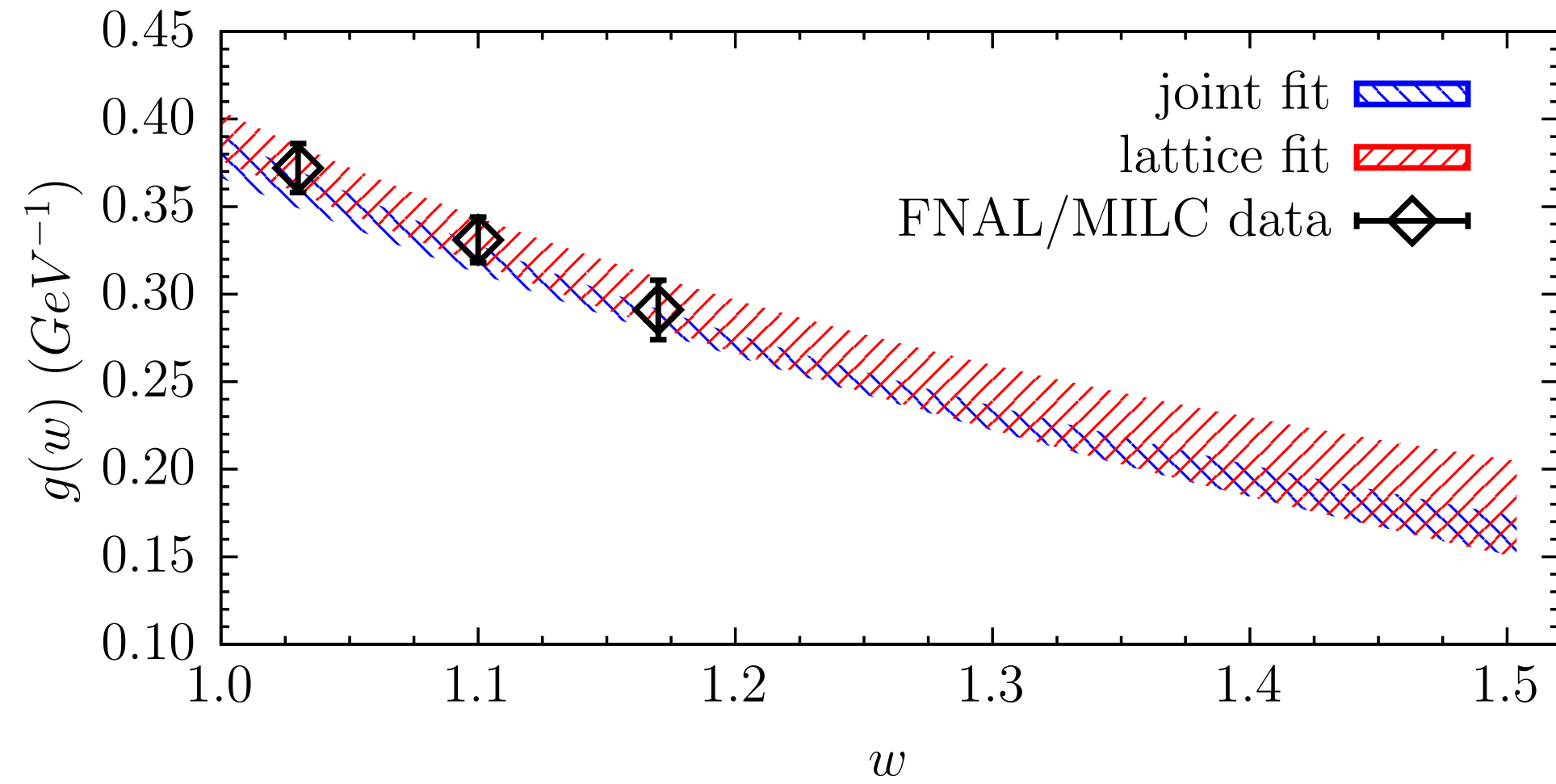
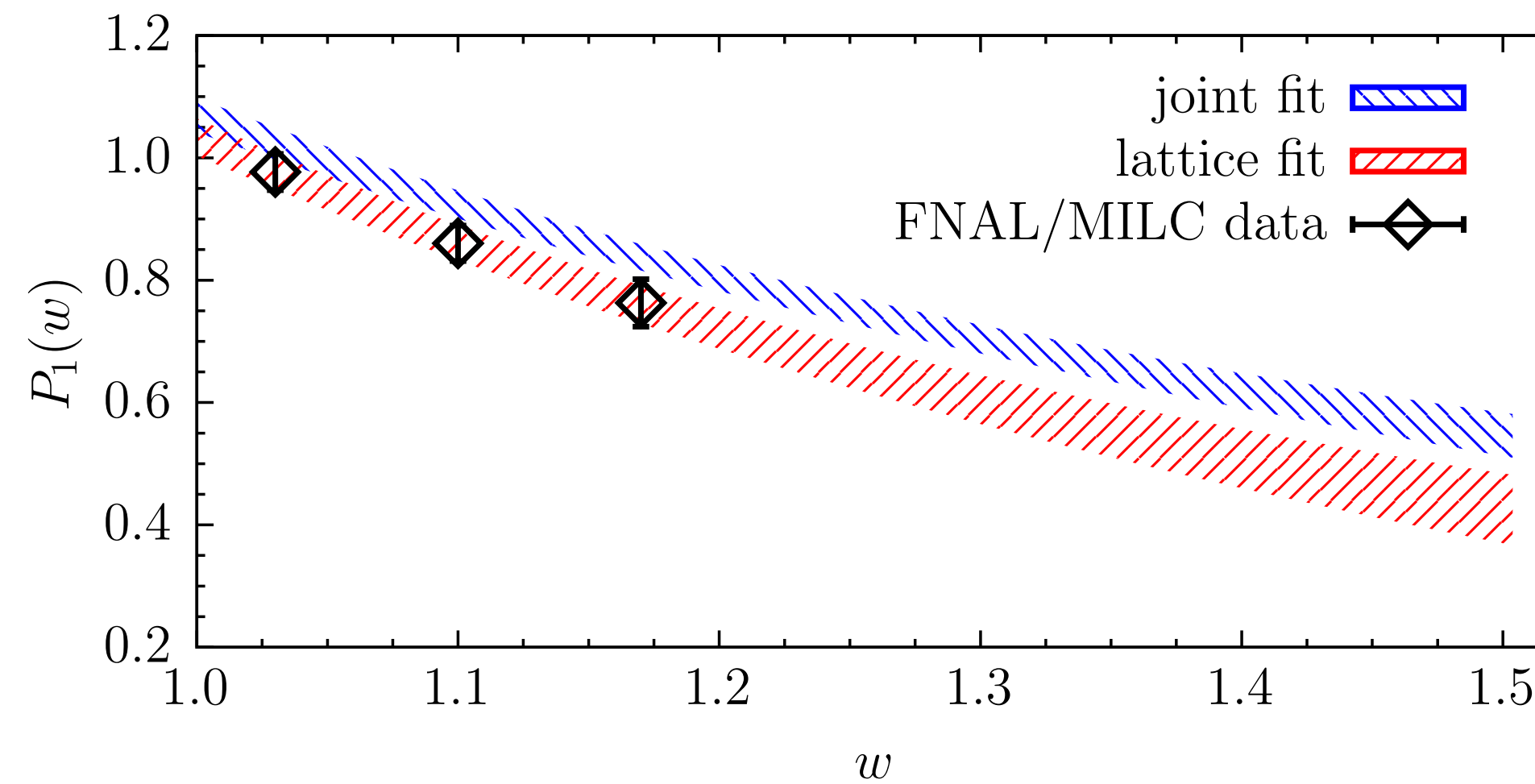
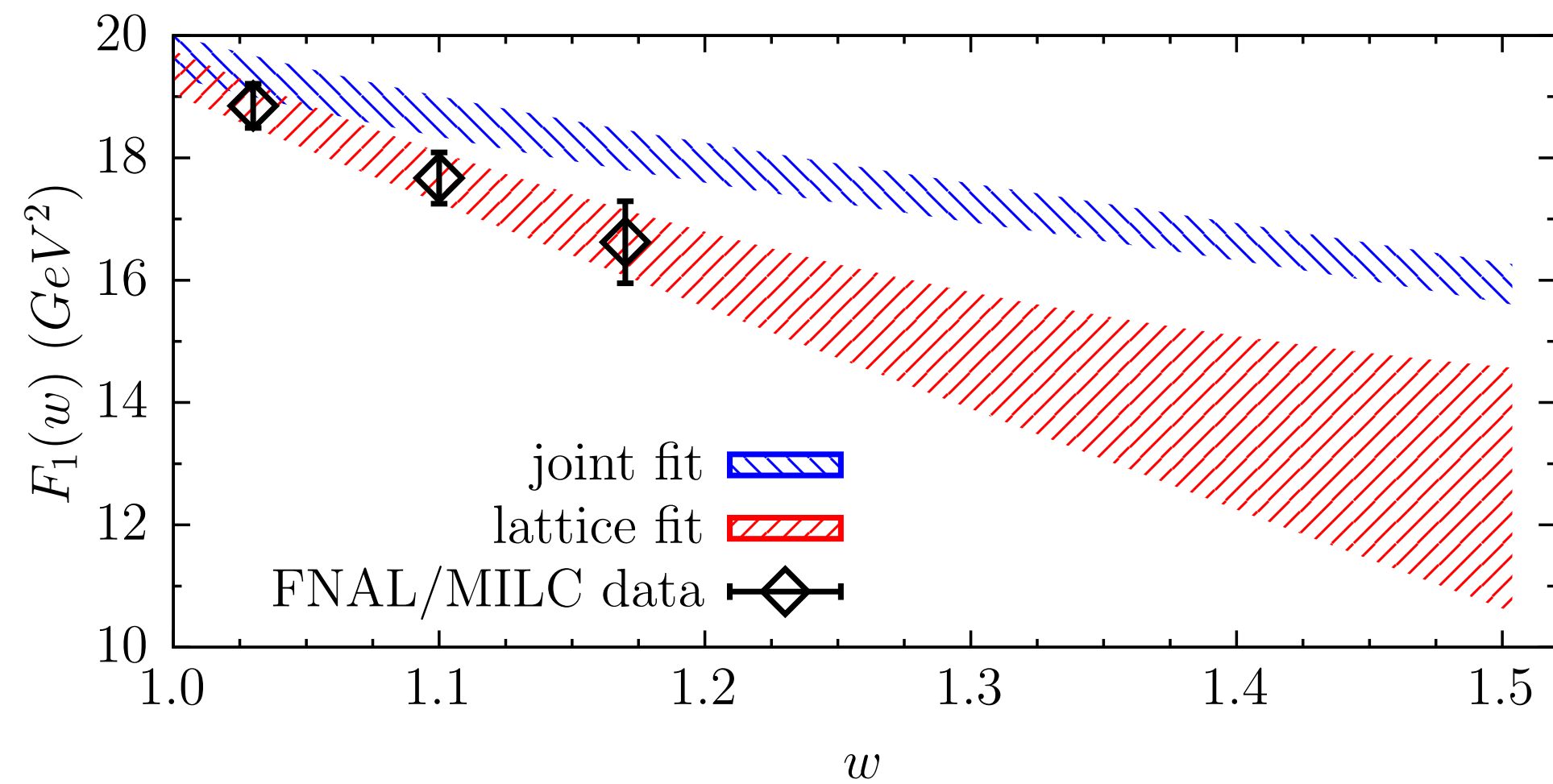


important news: LQCD form factors for $B \rightarrow D^* \ell \nu_\ell$ decays from FNAL/MILC (arXiv:2105.14019)

synthetic data points at 3 non-zero values of the recoil ($w - 1$)



joint fit:
BGL fit of LQCD points +
Belle + BaBar exp. data
 $|V_{cb}| \cdot 10^3 = 38.40 \pm 0.74$
 $\simeq 1.9\%$
 $R(D^*) = 0.2483 \pm 0.0013$



lattice fit:
quadratic BGL fit of LQCD
points only
 $R(D^*) = 0.265 \pm 0.013$

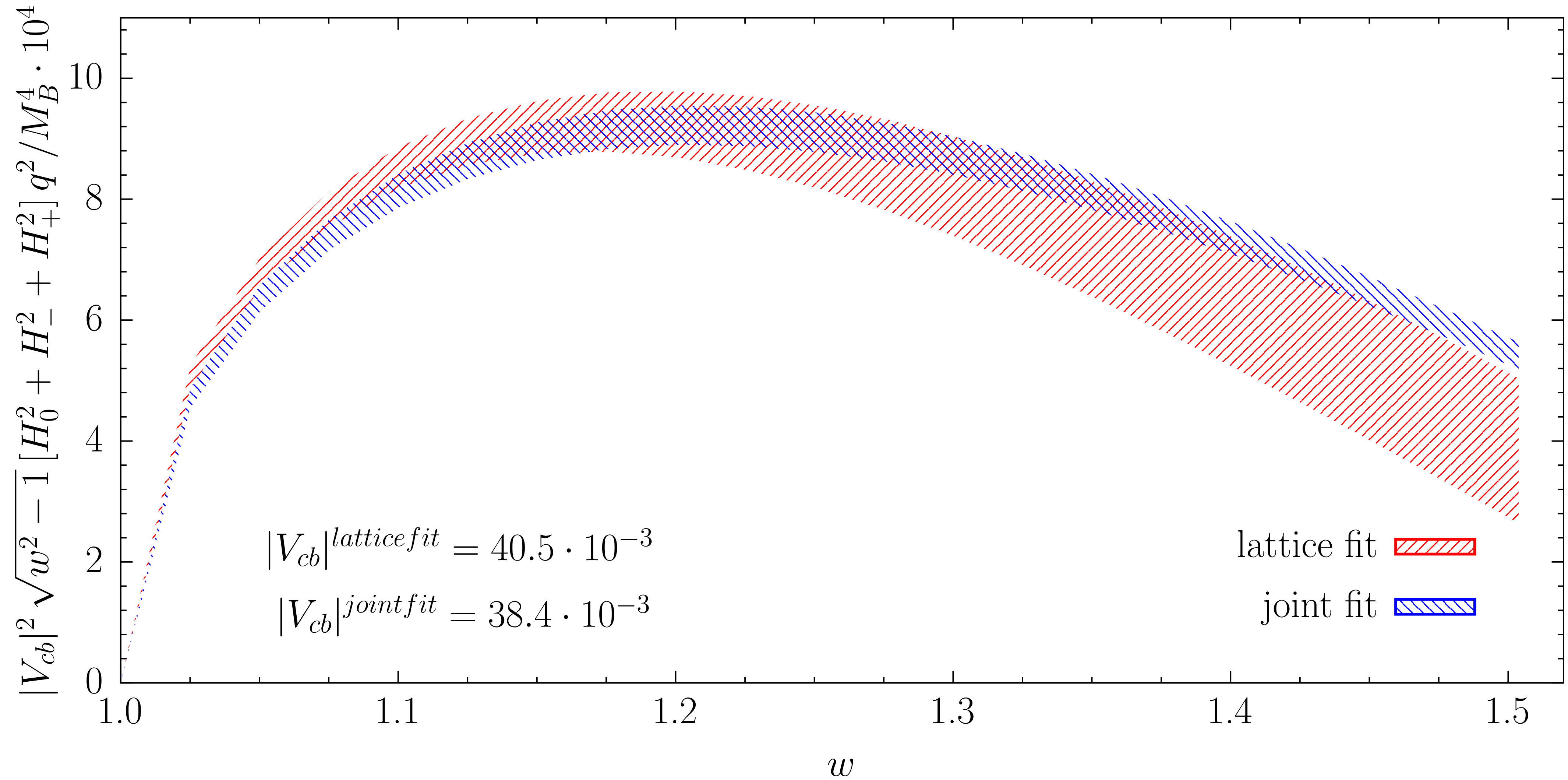
simultaneous fit of the lattice points and experimental data

FFs treated as pure theoretical quantities

the extracted FFs are hybrid quantities: their shape and uncertainties are affected by the experimental data

including their uncertainties

$$\frac{d\Gamma}{dw} \propto |V_{cb}|^2 \sqrt{w^2 - 1} \frac{q^2}{M_B^4} [H_0^2(w) + H_-^2(w) + H_+^2(w)] = |V_{cb}|^2 \sqrt{w^2 - 1} \left\{ \left(\frac{\mathcal{F}_1(w)}{M_B^2} \right)^2 + 2 \frac{q^2}{M_B^2} \left[\left(\frac{f(w)}{M_B} \right)^2 + r^2 (w^2 - 1) m_B^2 g^2(w) \right] \right\} \quad m_\ell = 0$$



extraction of $|V_{cb}|$ from $B \rightarrow D^* \ell \nu_\ell$ decays

[arXiv:2109.15248]

*** we do not mix theoretical calculations with experimental data to describe the shape of the FFs ***

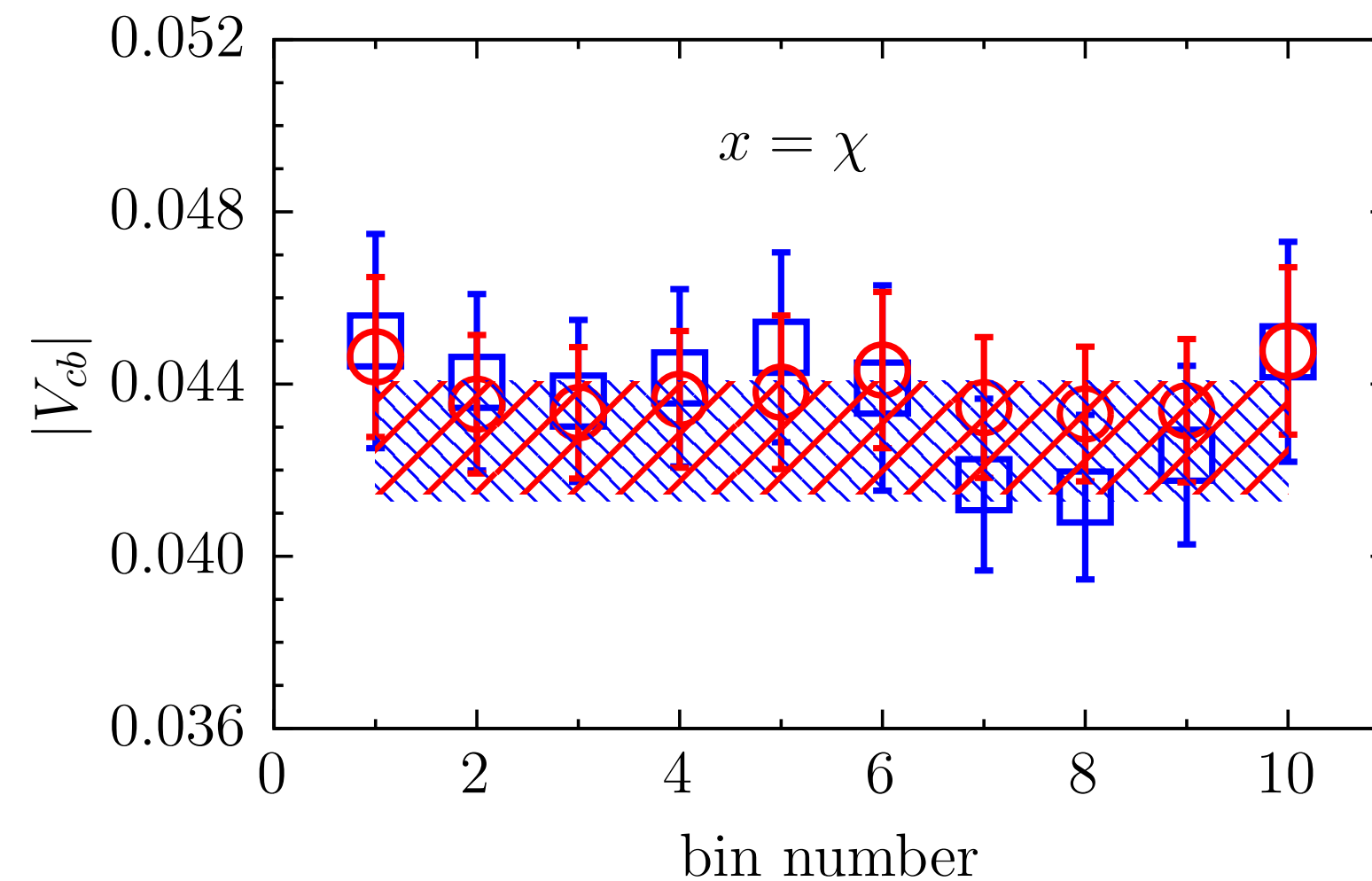
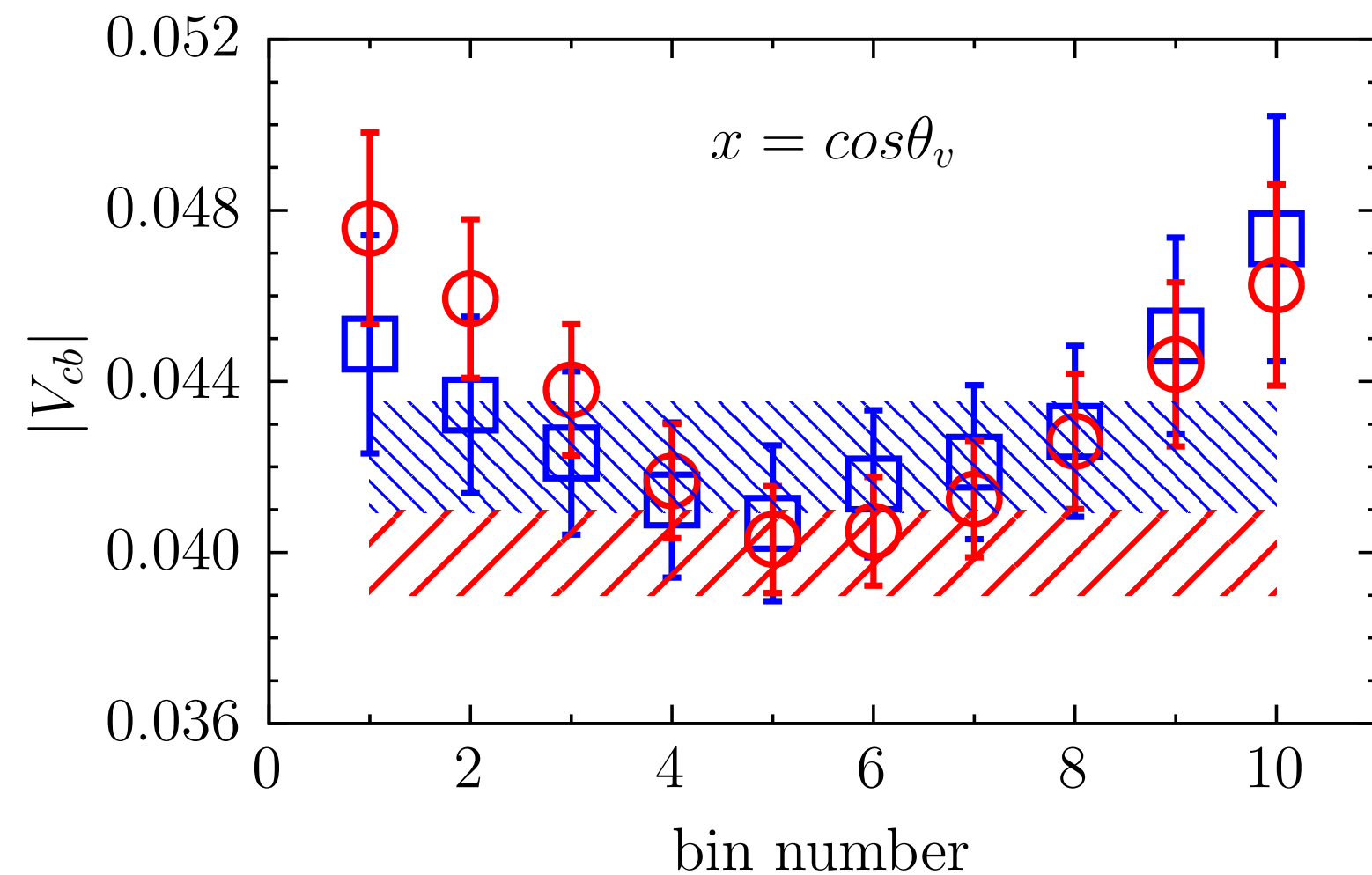
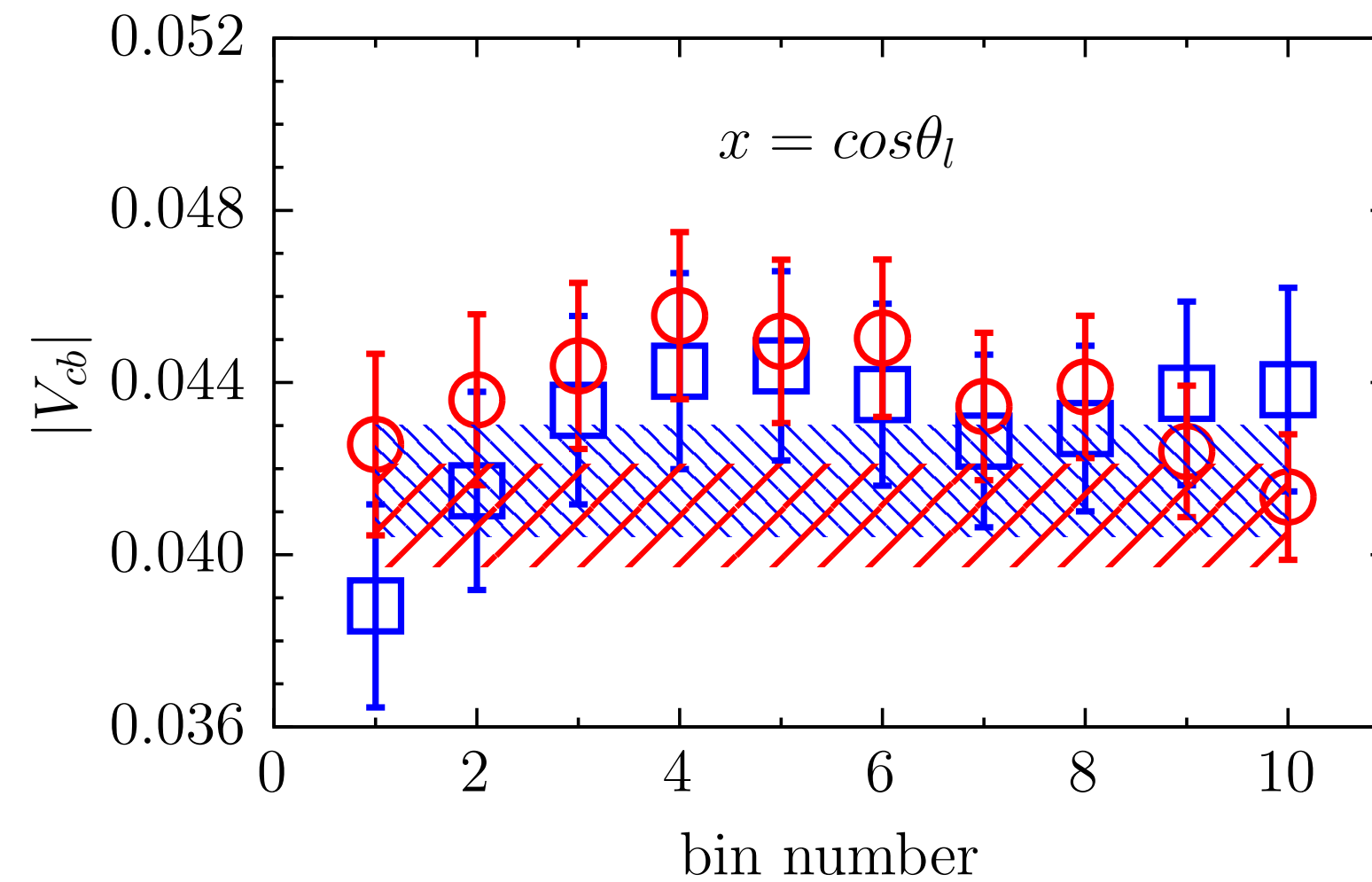
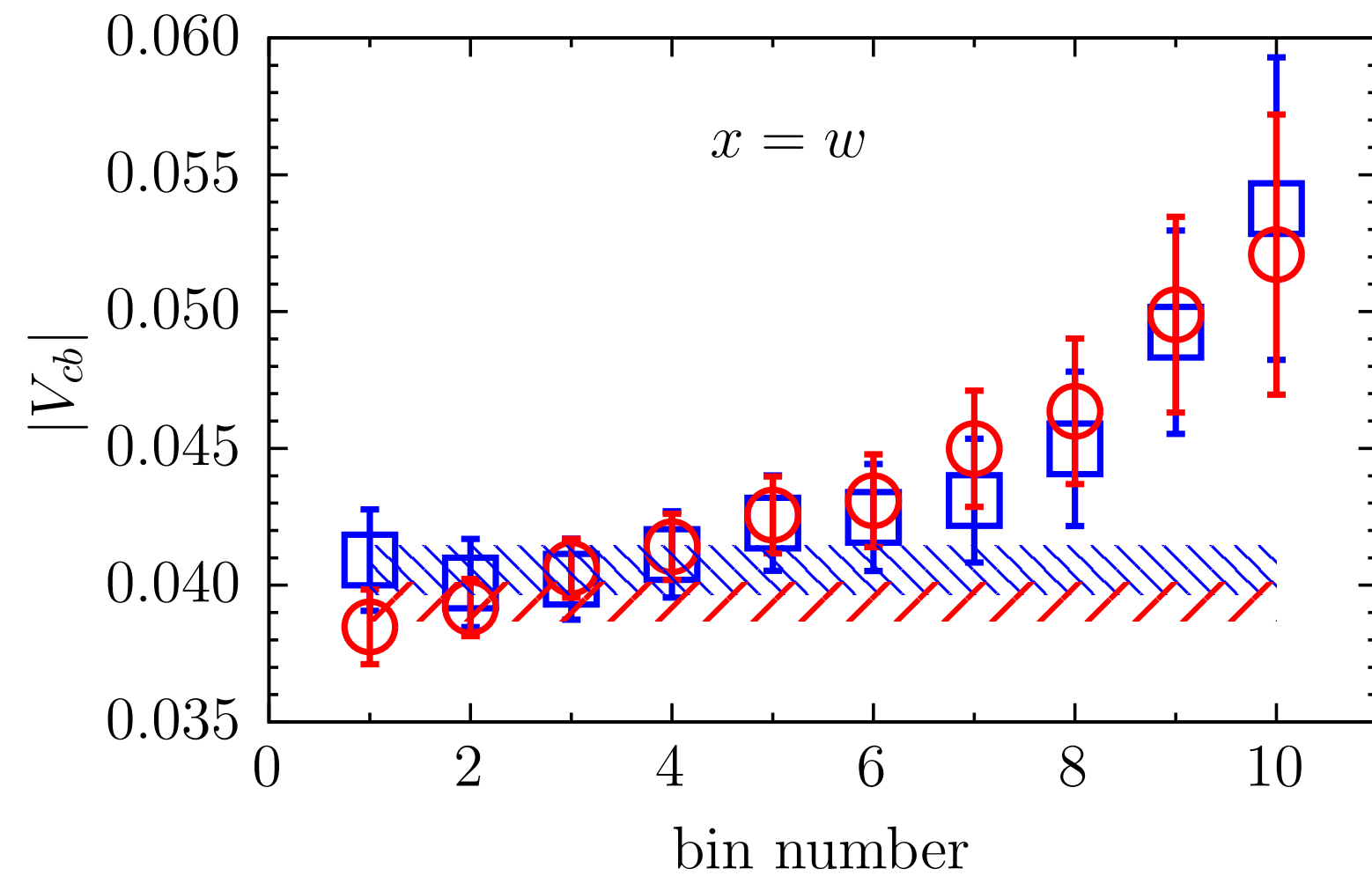
$$|V_{cb}|_i \equiv \sqrt{\frac{(d\Gamma/dx)_i^{exp}}{(d\Gamma/dx)_i^{th}}} \quad i = 1, \dots, N_{bins}$$

four different differential decay rates
 $d\Gamma/dx$ where $x = \{w, \cos\theta_\nu, \cos\theta_\ell, \chi\}$:

- 10 bins for each variable
- total of 80 data points

blue data: Belle 1702.01521

red data: Belle 1809.03290



bands are (correlated) weighted averages

$$|V_{cb}| = \frac{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij} |V_{cb}|_j}{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij}},$$

$$\sigma_{|V_{cb}|}^2 = \frac{1}{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij}},$$

Belle 1702.01521

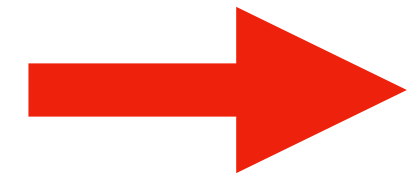
Belle 1809.03290

experiment	$ V_{cb} (x = w)$	$ V_{cb} (x = \cos\theta_l)$	$ V_{cb} (x = \cos\theta_v)$	$ V_{cb} (x = \chi)$
Ref. [11]	0.0398 (9)	0.0422 (13)	0.0421 (13)	0.0426 (14)
Ref. [12]	0.0395 (7)	0.0405 (11)	0.0402 (10)	0.0430 (13)

averaging procedure

$$\mu_x = \frac{1}{N} \sum_{k=1}^N x_k ,$$

$$\sigma_x^2 = \frac{1}{N} \sum_{k=1}^N \sigma_k^2 + \frac{1}{N} \sum_{k=1}^N (x_k - \mu_x)^2 ,$$

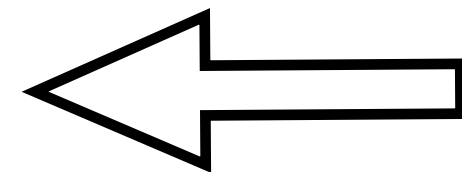


$$|V_{cb}|_{excl.} \cdot 10^3 = 41.3 \pm 1.7$$

$$|V_{cb}|_{incl.} \cdot 10^3 = 42.16 \pm 0.50 \quad (\text{Bordone et al: arXiv:2107.00604})$$

exclusive/inclusive tension reduced to less than 1σ

the use of exp. data to describe the shape of the FFs leads to smaller errors, but it produces a bias on the extracted value of $|V_{cb}|$



$$|V_{cb}|_{excl.} \cdot 10^3 = 39.6_{-1.0}^{+1.1} \quad \text{Gambino et al., arXiv:1905.08209}$$

$$|V_{cb}|_{excl.} \cdot 10^3 = 39.56_{-1.06}^{+1.04} \quad \text{Jaiswal et al., arXiv:2002.05726}$$

$$|V_{cb}|_{excl.} \cdot 10^3 = 38.86 \pm 0.88 \quad \text{FLAG '21, arXiv:2111.09849}$$