Four-form relaxation of Higgs mass and cosmological constant

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Ref. HML, 1908.04252, 1910.09171 (Higgs),

1908.05475 (Inflation)

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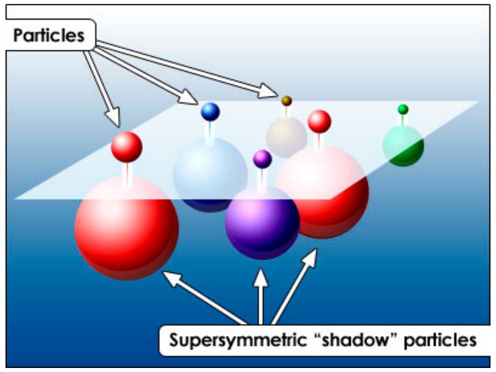
Outline

- Introduction on four-form flux
- Four-form flux and Higgs mass
- Reheating
- Chaotic inflation with four-form flux
- Conclusions

New physics for Higgs mass

- Supersymmetry

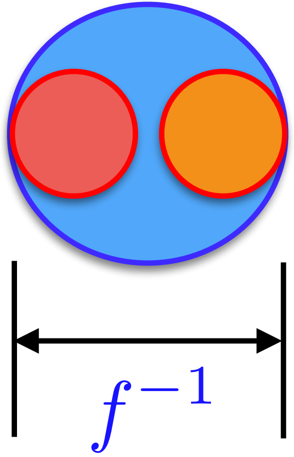
$$m_H^2 \sim \kappa M_{\rm SUSY}^2$$



- New composite dynamics

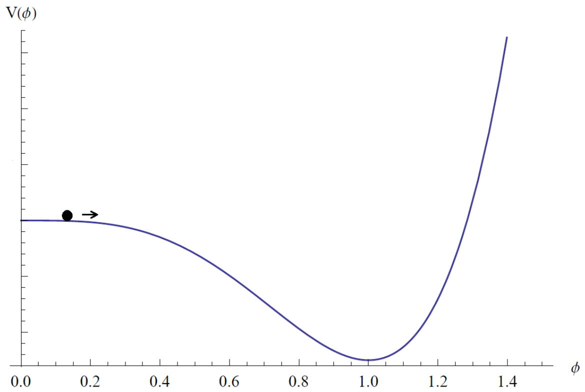
$$H = \bar{Q}'Q'$$

$$m_H^2 \sim \kappa f^2$$



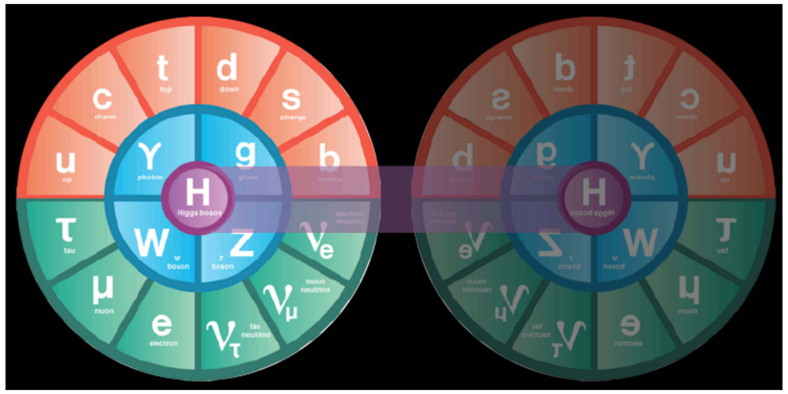
- Scale invariance

$$m_H^2 \sim \lambda' \langle \phi^2 \rangle \ll M_P^2 \sim \xi \langle \phi^2 \rangle$$



- Discrete symmetries

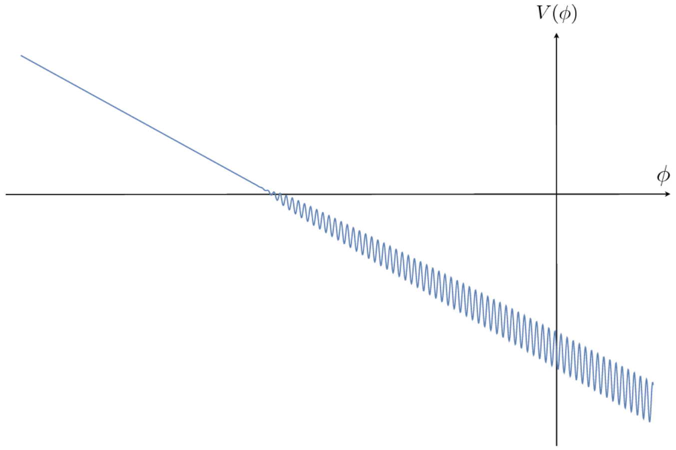
$$m_H^2 \sim \kappa M_{\rm SM'}^2$$



[N. Craig, CERN, 2018]

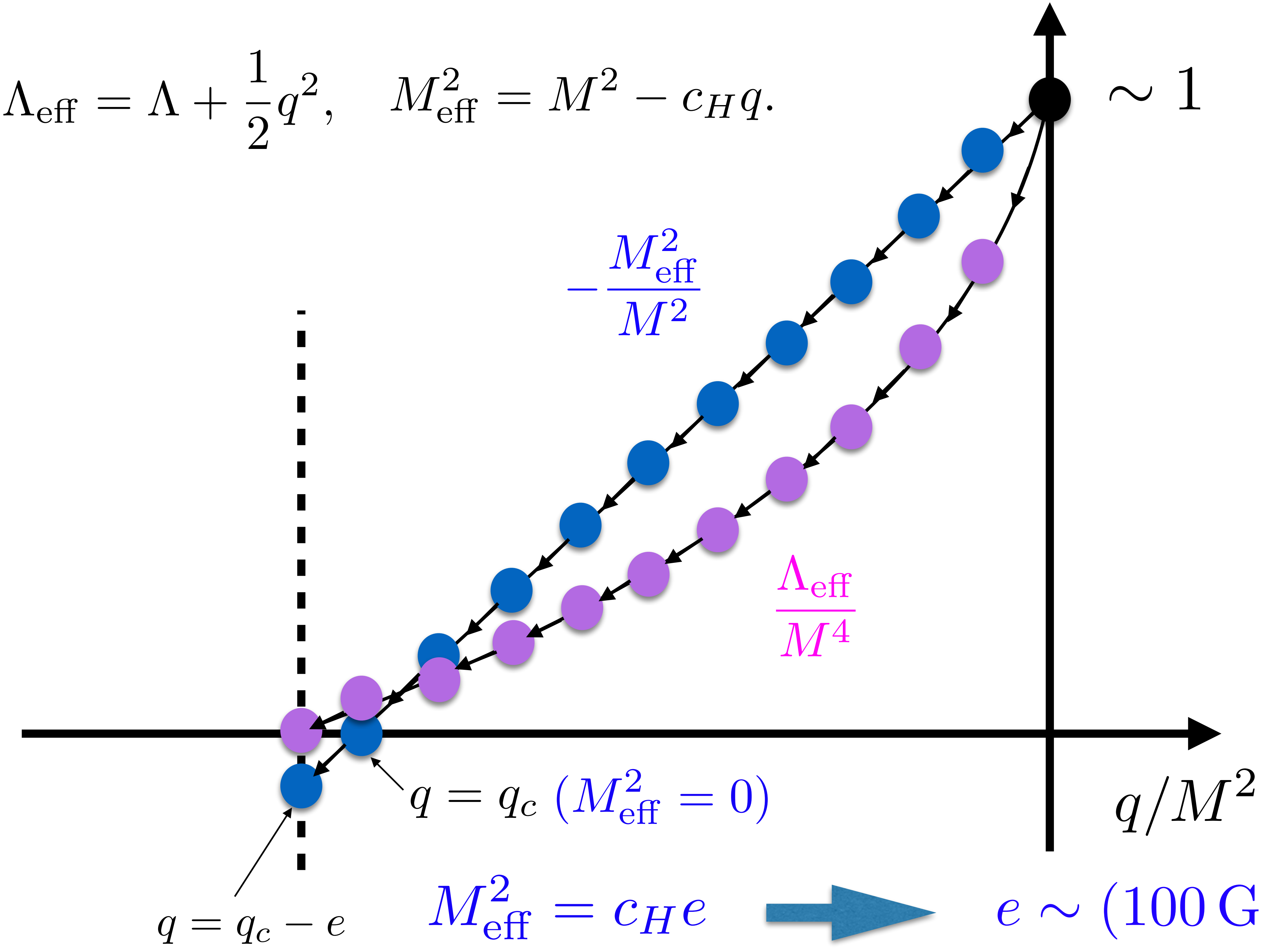
- Cosmological relaxation

$$m_{H,\rm eff}^2 = \pm m_H^2 + g\phi, \quad g \ll |m_H|$$



Four-form and Higgs mass

$$\Lambda_{\text{eff}} = \Lambda + \frac{1}{2}q^2, \quad M_{\text{eff}}^2 = M^2 - c_H q.$$



$$M_{\text{eff}}^2 = c_H e \quad \longrightarrow \quad e \sim (100\,\text{GeV})^2$$

$$\Lambda_{\text{eff}} \approx 0 \quad \longrightarrow \quad \Lambda = -\frac{1}{2}(q_c - e)^2 + \Delta\Lambda$$

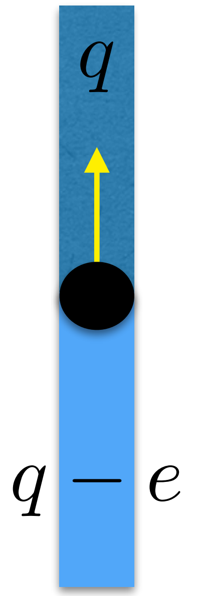
Particles from E-field

- The Maxwell field in 2D is non-dynamical (no polarizations), but electric field can reduce due to charge nucleation.

$$\mathcal{L}_{2D} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - m\int ds_E, \qquad \mathcal{L}_{\text{charge}} = e\int d\xi\, \delta^2(x - x(\xi))A_\mu \frac{\partial x^\mu}{\partial \xi}.$$

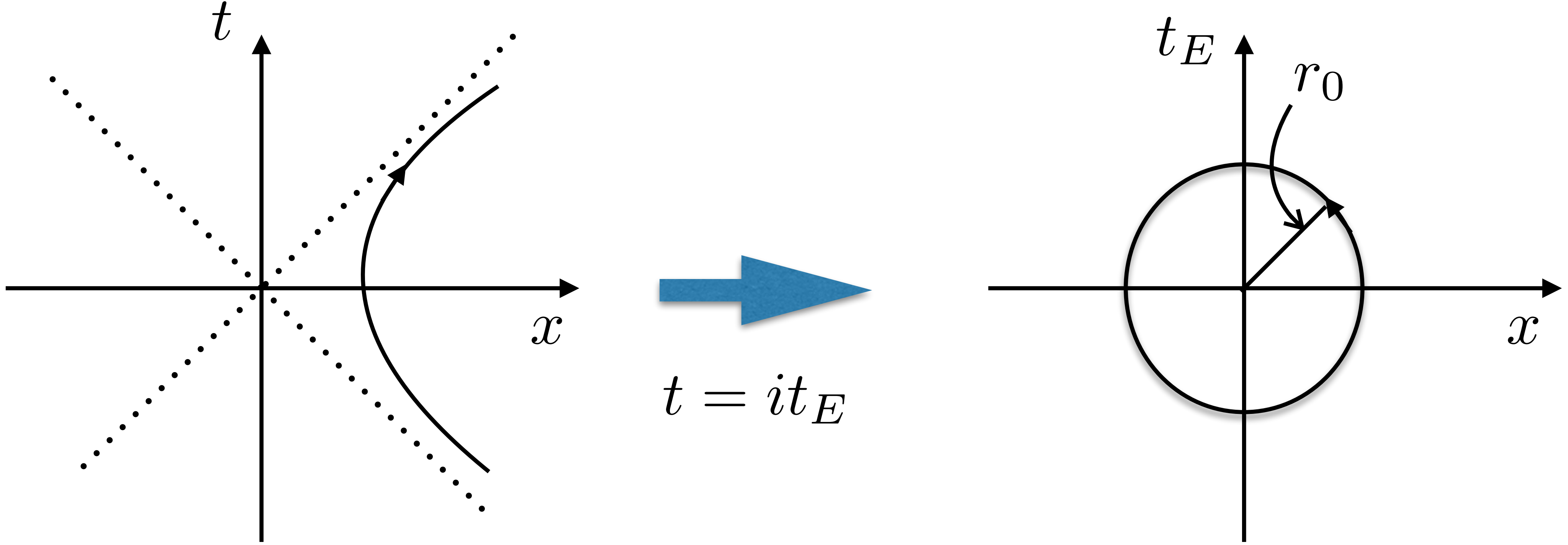
Eqs. of motion: $m\ddot{x}^\mu = eF^{\mu\nu}\dot{x}_\nu,$ [Brown, Teitelboim, 1987]

$$\partial_\mu F^{\mu\nu} = -e\int d\xi\, \delta^2(x - x(\xi))\frac{\partial x^\nu}{\partial \xi}.$$



2D E-field

$$F_{\mu\nu} = q\,\epsilon_{\mu\nu}$$



$$r_0 = \frac{m}{eE(q)}$$

Bounce action for particle

$$S_E = m\int ds_E + \frac{1}{4}\int d^2x_E F_{\mu\nu}^2;$$

$$B = S_E(\text{instanton}) - S_E(\text{background}),$$

$$B = m(2\pi r_0) + \frac{1}{2}[(q-e)^2 - q^2](\pi r_0^2)$$
$$\approx 2\pi m r_0 - \pi e E(q) r_0^2$$

$$\frac{\partial B}{\partial r_0} = 0 \quad \Longrightarrow \quad r_0 = \frac{m}{eE(q)} \quad \Longrightarrow \quad B = \frac{\pi m^2}{eE(q)}$$

1. Probability for charge production (= instanton tunneling to “E=q-e outside and E=q inside”):

$$P(q \to q-e) \approx \exp\Big(-\frac{1}{\hbar}B\Big) = \Big(-\frac{\pi m^2}{\hbar\, eE(q)}\Big)$$

2. Charge production continues, as far as E-field outside is positive, albeit exponentially suppressed.

Four-form flux and C.C.

- Three-form gauge field is not dynamical, but its field strength (four-form flux) contributes to cosmological constant: $F_{\mu\nu\rho\sigma} = 4\partial_{[\mu}A_{\nu\rho\sigma]}$

$$\mathcal{L} = \sqrt{-g}\Bigg(\frac{1}{2}R - \Lambda \underbrace{- \frac{1}{48}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}}_{\text{Kinetic term}}\Bigg) + \underbrace{\frac{1}{6}\partial_\mu\left[\sqrt{-g}F^{\mu\nu\rho\sigma}A_{\nu\rho\sigma}\right]}_{\text{Surface term}}.$$

Equation of motion: $\partial_\mu\left(\sqrt{-g}F^{\mu\nu\rho\sigma}\right) = 0$

→ $F^{\mu\nu\rho\sigma} = \dfrac{1}{\sqrt{-g}}\, q\, \epsilon^{\mu\nu\rho\sigma}$, or $F_{\mu\nu\rho\sigma} = q\, \epsilon_{\mu\nu\rho\sigma}.$ $q = \mathrm{const}$

Effective C.C: $\Lambda_{\text{eff}} = \Lambda + \dfrac{1}{2}q^2$

For $\Lambda < 0,$ Four-form flux cancels the bare cosmological constant to zero.

[Duff, van Nieuwenhuizen, 1980;
Witten, 1984;
Henneaux, Teitelboim, 1984;
Baum, 1983, Hawking, 1984]

Membranes from four-form

- The four-form flux can reduce due to the membrane nucleation.

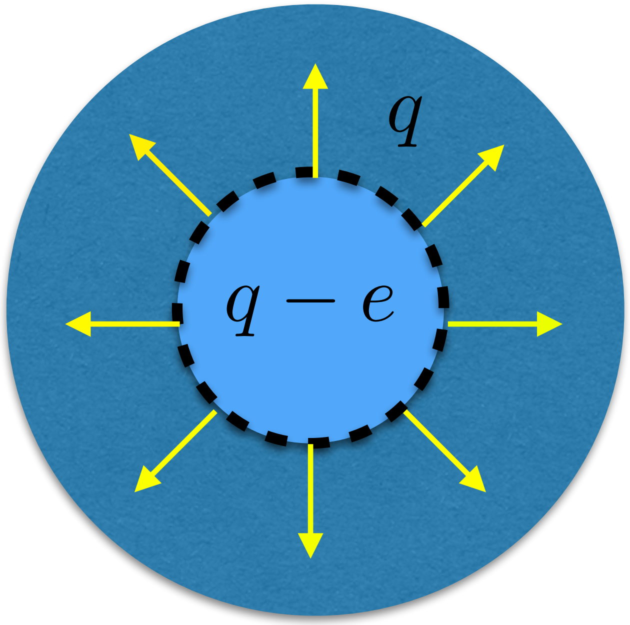
[Brown, Teitelboim, 1987]

$$\mathcal{L}_{\rm memb} = \frac{e}{6}\int d^3\xi\, \delta^4(x-x(\xi))\, A_{\nu\rho\sigma}\frac{\partial x^\nu}{\partial\xi^a}\frac{\partial x^\rho}{\partial\xi^b}\frac{\partial x^\sigma}{\partial\xi^c}\,\epsilon^{abc}$$

Eq. of motion for flux:

$$\epsilon^{\mu\nu\rho\sigma}\partial_\mu q = -e\int d^3\xi\, \delta^4(x-x(\xi))\,\frac{\partial x^\nu}{\partial\xi^a}\frac{\partial x^\rho}{\partial\xi^b}\frac{\partial x^\sigma}{\partial\xi^c}\epsilon^{abc}.$$

$q = en$: Flux quantization in microscopic theory.



4D 4-form

1. Closed membrane production (= instanton tunneling to "q-e inside and q outside"):

$$P(q\to q-e) \approx \exp\left(-\frac{27\pi^2T^2}{2(\Delta\Lambda)^3}\right), \quad \Delta\Lambda = \Lambda_{\rm eff}(q)-\Lambda_{\rm eff}(q-e).$$

[Coleman, 1977]

2. Membrane production would continue as far as the flux outside is nonzero, but dS curvature becomes important in the later stage.

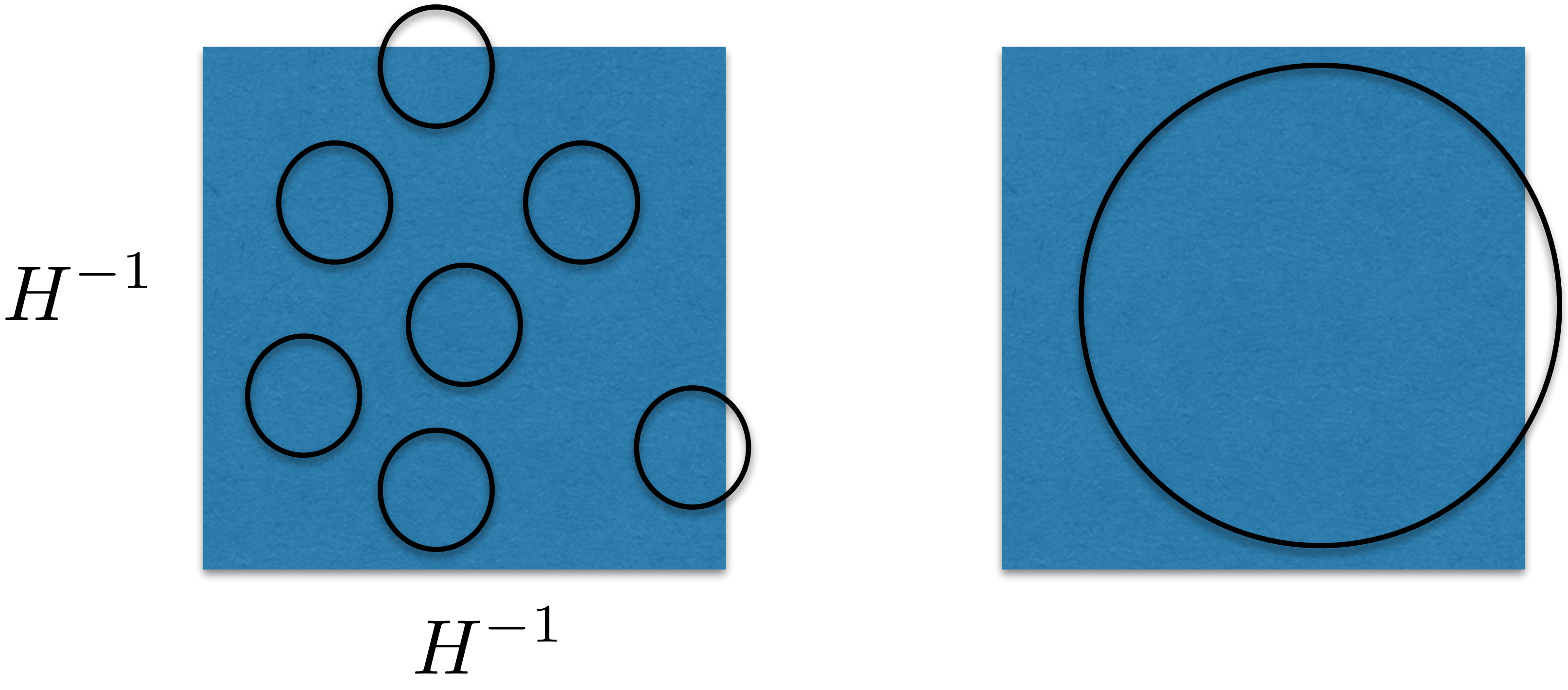
Gravity effects on bubbles

- The tunneling probability is corrected by dS curvature radius as [Coleman, de Luccia, 1980]

$$P(q \to q - e) = \exp\Big(- \frac{27\pi^2 T^2}{2(\Delta\Lambda)^3} \frac{1}{(1 + \frac{1}{4} r_0^2 H^2)^2} \Big) \approx \exp\Big(- \frac{24\pi^2 M_P^4}{\Lambda_{\rm eff}} \Big).$$

$$\Lambda_{\rm eff} \ll \frac{T^2}{M_P^2}$$

$r_0 = \frac{3T}{\Delta\Lambda}$: Bubble radius with no gravity.



- Membrane production stops when the cosmological constant is the smallest.

[Brown, Teitelboim, 1987]

Origin of four-forms

- M-theory contains M2-brane and M5-brane as sources for four-form fluxes. [Bousso, Polchinski, 2000]

➡ Membrane tensions and charges in 4D.

M5-brane wrapped on 3-cycle: $(M_P^2 = 2\pi M_{11}^9 V_7)$

$$\tau_i = 2\pi M_{11}^6 V_{3,i}, \quad e_i = \frac{\tau_i}{M_P}, \quad i \leq N_3.$$

M2-brane:

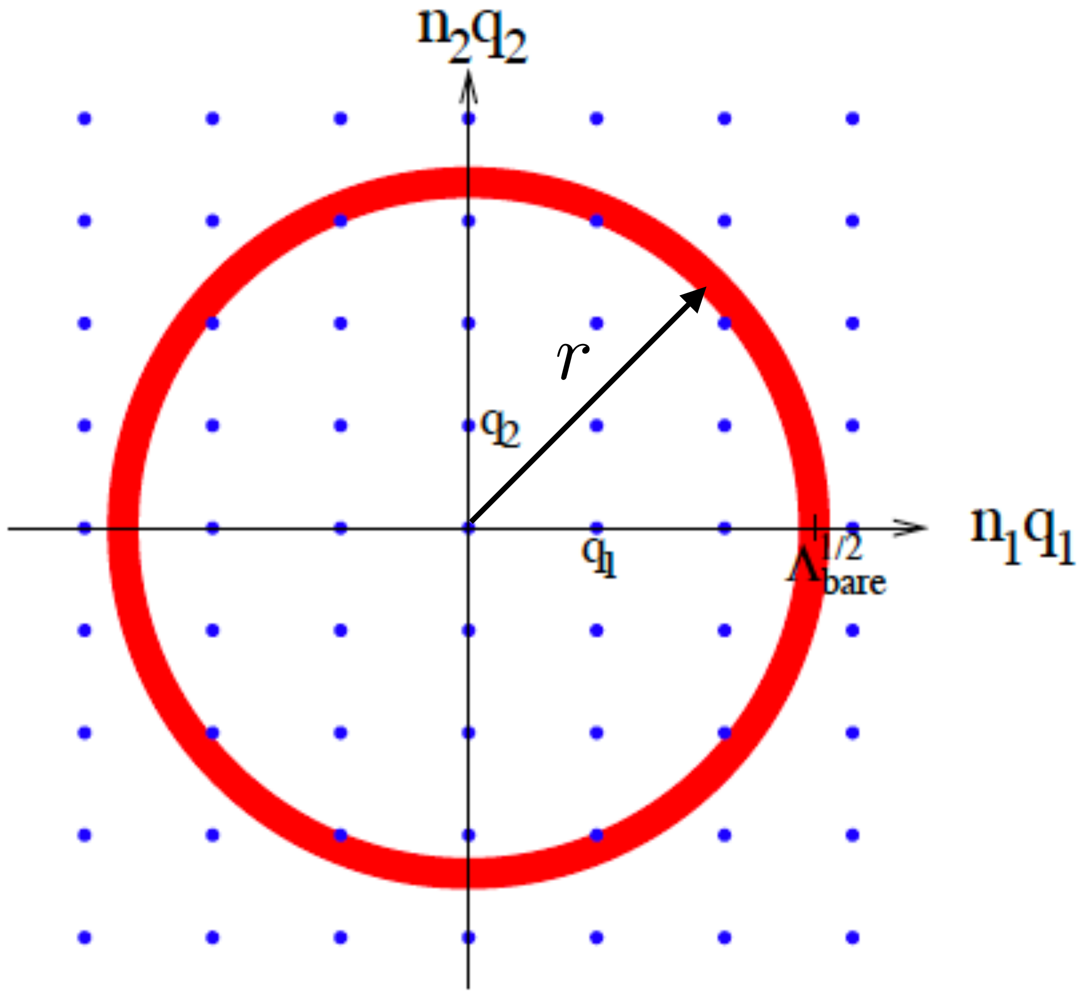
$$\tau_{N_3+1} = 2\pi M_{11}^3, \quad e_{N_3+1} = \frac{\tau_{N_3+1}}{M_P}.$$

M5-brane wrapped on 3-cycle with large volume can lead to a small membrane tension and charge.

Membrane tension and charge can be separate in a non-supersymmetric low energy theory: a small membrane charge is possible.

Multiple four-forms and C.C.

- M-theory compactified on the manifold with J 3-cycles leads to an enough number of four-form fluxes for the accurate cancellation of C. C. [Bousso, Polchinski, 2000]



$$\sqrt{2\Lambda} < r = \sqrt{\sum_i n_i^2 e_i^2} < \sqrt{2(\Lambda + \Delta\Lambda)}$$

Shell width: $$\Delta r = \frac{\Delta\Lambda}{\sqrt{2\Lambda}},$$

$$\Delta\Lambda \sim 10^{-120} M_P^4.$$

Sufficient grid points within shell:

$$\prod_i e_i \lesssim \omega_{J-1} r^{J-1} \Delta r$$

➡ $$\Delta\Lambda \sim \frac{\prod_i e_i}{|2\Lambda|^{J/2-1}}$$

$$\Lambda \sim M_P^4,\ J \sim 100:\ e_i \sim 10^{-1.6} M_P^2$$

Four-form flux and Higgs mass

Four-form and Higgs mass

- Four-form flux has a dimensionless coupling to the SM Higgs, scanning the Higgs mass parameter.

[Dvali, Vilenkin, 2004; Giudice, Kehagias, Riotto, 2019;
Kaloper, Westphal, 2019; HML, 2019]

$$\mathcal{L}_H = M^2|H|^2 - \lambda_H|H|^4 - \underline{\frac{c_H}{24}\,\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}\,|H|^2} - \frac{c_H}{6}\partial_\mu\Big(\epsilon^{\mu\nu\rho\sigma}|H|^2 A_{\nu\rho\sigma}\Big).$$

$$\Rightarrow \qquad \mathcal{L}_{\rm eff} = -\Lambda_{\rm eff} + M^2_{\rm eff}|H|^2 - \lambda_{H,{\rm eff}}|H|^4,$$

$$\Lambda_{\rm eff} = \Lambda + \frac{1}{2}q^2,$$

$$M^2_{\rm eff} = M^2 - c_H q,$$

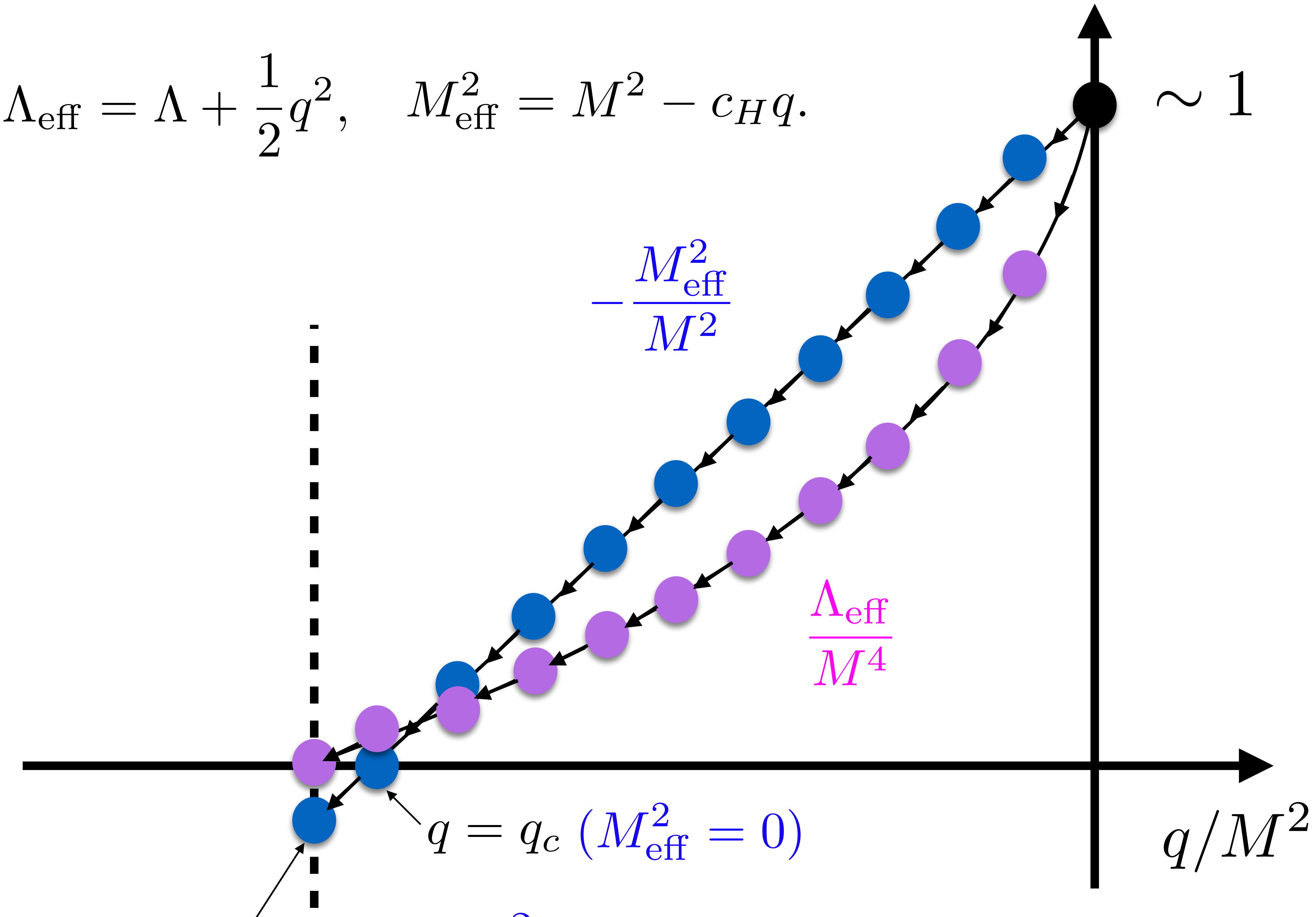
Four-form flux scans C.C. as well as Higgs mass.

$$\lambda_{H,{\rm eff}} = \lambda_H + \frac{1}{2}c_H^2.$$

Higgs quartic coupling has a constant shift.

Four-form and Higgs mass

$$\Lambda_{\text{eff}} = \Lambda + \frac{1}{2}q^2, \quad M_{\text{eff}}^2 = M^2 - c_H q.$$



$M_{\text{eff}}^2 = c_H e$ → $e \sim (100\,\text{GeV})^2$

$\Lambda_{\text{eff}} \approx 0$ → $\Lambda = -\frac{1}{2}(q_c - e)^2 + \Delta\Lambda$

★ $q = q_c$: **Last dS phase**

$$M_{\rm eff}^2 = M^2 - c_H q_c = 0 \quad \rightarrow \quad \boxed{q_c = \frac{M^2}{c_H} \sim M_P^2}$$

$$\Lambda_{\rm eff} = \Lambda + \frac{1}{2} q_c^2 \equiv \Lambda_{\rm last}$$

★ $q = q_c - e$: **Our vacuum**

$$M_{\rm eff}^2 = c_H e \sim (100\,{\rm GeV})^2, \qquad \Lambda_{\rm eff} = \Lambda + \frac{1}{2}(q_c - e)^2 \sim 0$$

$$\rightarrow \quad \boxed{\Lambda_{\rm last} \simeq e q_c \sim e\, M_P^2}$$

$$\rightarrow \quad H_{\rm last} \sim \frac{\Lambda_{\rm last}}{M_P} \sim \sqrt{e} \sim 100\,{\rm GeV}$$

Reheating constraint: $\rho_R = \frac{\pi^2}{30} g_* T_{\rm RH}^4 < 3 M_P^2 H_{\rm last}^2$

$$\rightarrow \quad \boxed{T_{\rm RH} < 8.5 \times 10^9\,{\rm GeV}}$$

Need of reheating

- During dS phases, inflaton is in either classical rolling or quantum fluctuation (random walk).

Classical:

$$\Delta\phi \sim \frac{1}{2}V'(\Delta t) = \frac{V'}{2H^2}$$

Quantum:

$$\delta\phi \sim \frac{H}{2\pi}$$

- Both Higgs mass & C.C. scan to observed values by a series of dS phases and the universe would be empty in the end.

→ Reheating mechanism needed.

- If inflaton rolls down after scans and decays, it can reheat the Universe.

But, inflaton potential is bounded by $\Lambda_{\rm last}$.

How to Reheat

- Kick the inflaton by quantum fluctuations.

$$\Delta\phi < \delta\phi \quad \Longrightarrow \quad H^3 > \frac{1}{\pi}V'$$

[Bousso, Polchinski, 2000]

$$\Lambda_{\rm last} \simeq e q_c = \frac{eM^2}{c_H} \quad \Longrightarrow \quad \frac{\pi e^{3/2} M^3}{c_H^{3/2} M_P^3} > V'$$

But, monomial inflation: $V = \alpha_n \phi^n$, $\left\{ \begin{array}{l} \alpha_n \sim \left(\frac{M_P}{\phi_*}\right)^{n+2} \left(\frac{M_{\rm GUT}}{M_P}\right)^4 M_P^{4-n}, \\ \phi_* \sim 10 M_P. \end{array} \right.$

$$\phi \sim \phi_*, \quad V' \sim 10^{-11} M_P^3$$

$$\Longrightarrow \quad \boxed{e^{1/2} > 10^{-4} c_H \left(\frac{M_P^2}{M}\right)} \; : \;\; M \sim M_P, c_H \sim 1$$

Classical rolling is dominant for $\sqrt{e} \sim 100\,{\rm GeV}$!

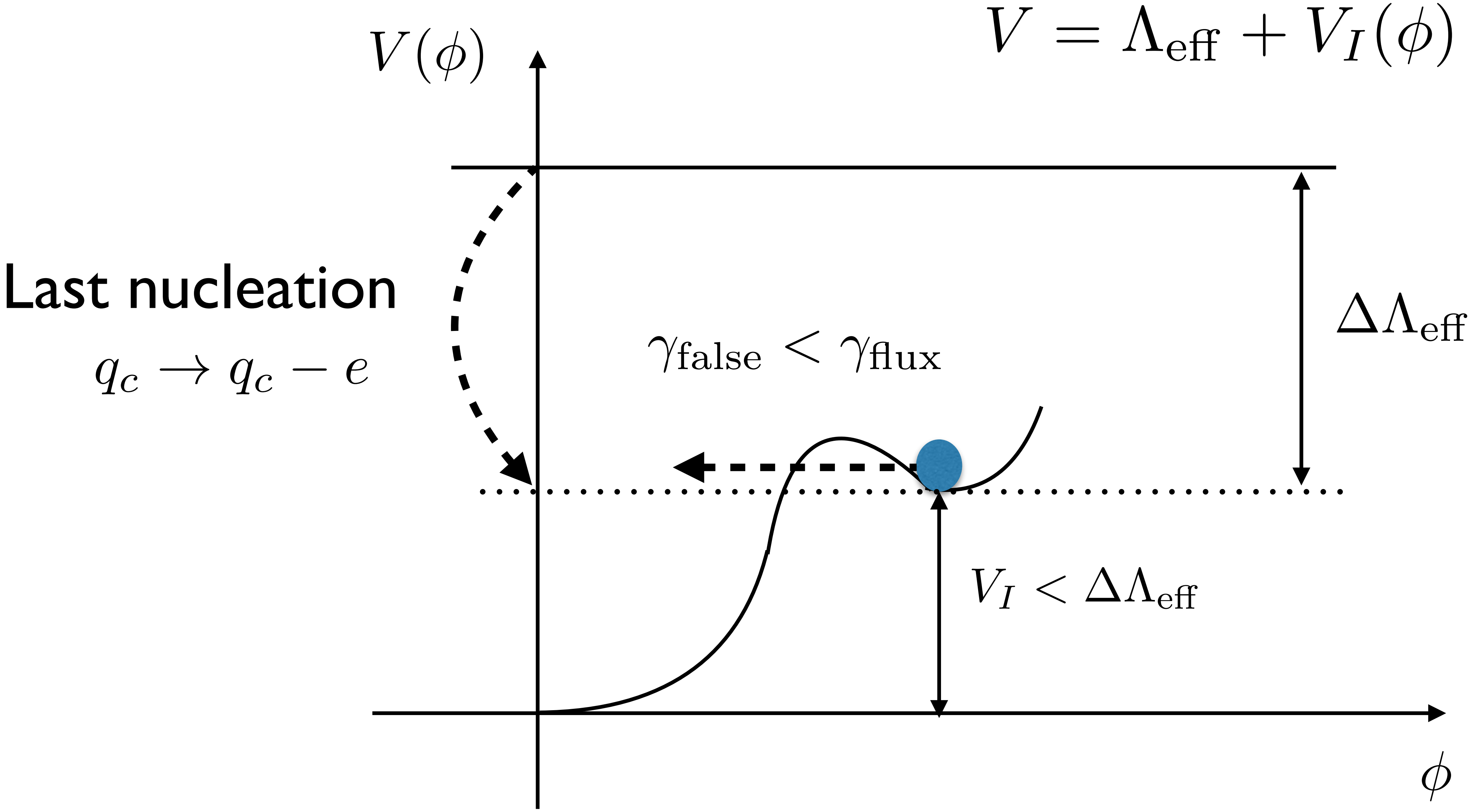
How to Reheat

- Inflaton stuck at the false vacuum in dS phases.

[Bousso, Polchinski, 2000]

Inflaton tunnels to the true vacuum after last nucleation.

$$V = \Lambda_{\text{eff}} + V_I(\phi)$$



How to Reheat

- Particle production due to flux-dependent Higgs VEV.

[Giudice, Kehagias, Riotto, 2019]

Higgs VEV becomes nonzero just after the last membrane nucleation.

Assume that the Hubble-dependent mass is absent.

→ frequencies of SM perturbations change due to the EW phase transition.

$$f_k'' + \omega^2 f_k = 0 \quad \omega^2 = \vec{k}^2 + m_P^2 a^2 + \left(\xi - \frac{1}{6}\right) a^2 R, \quad m_P = g_P\, v.$$

Efficient particle production only if $v \sim H_{\rm last} \sim \sqrt{e}$:

$$n_P = N_P \left(\frac{g_P^2\, v\sqrt{e}}{8\pi c}\right)^{3/2} \exp\left(-c\frac{v}{\sqrt{e}}\right), \qquad c \equiv \frac{\sqrt{3}\,\pi\, g_P M_P}{\kappa\,(2\Lambda_0)^{1/4}}.$$

κ : speed of transition.

How to Reheat

- Flux-dependent minimum of inflation potential.

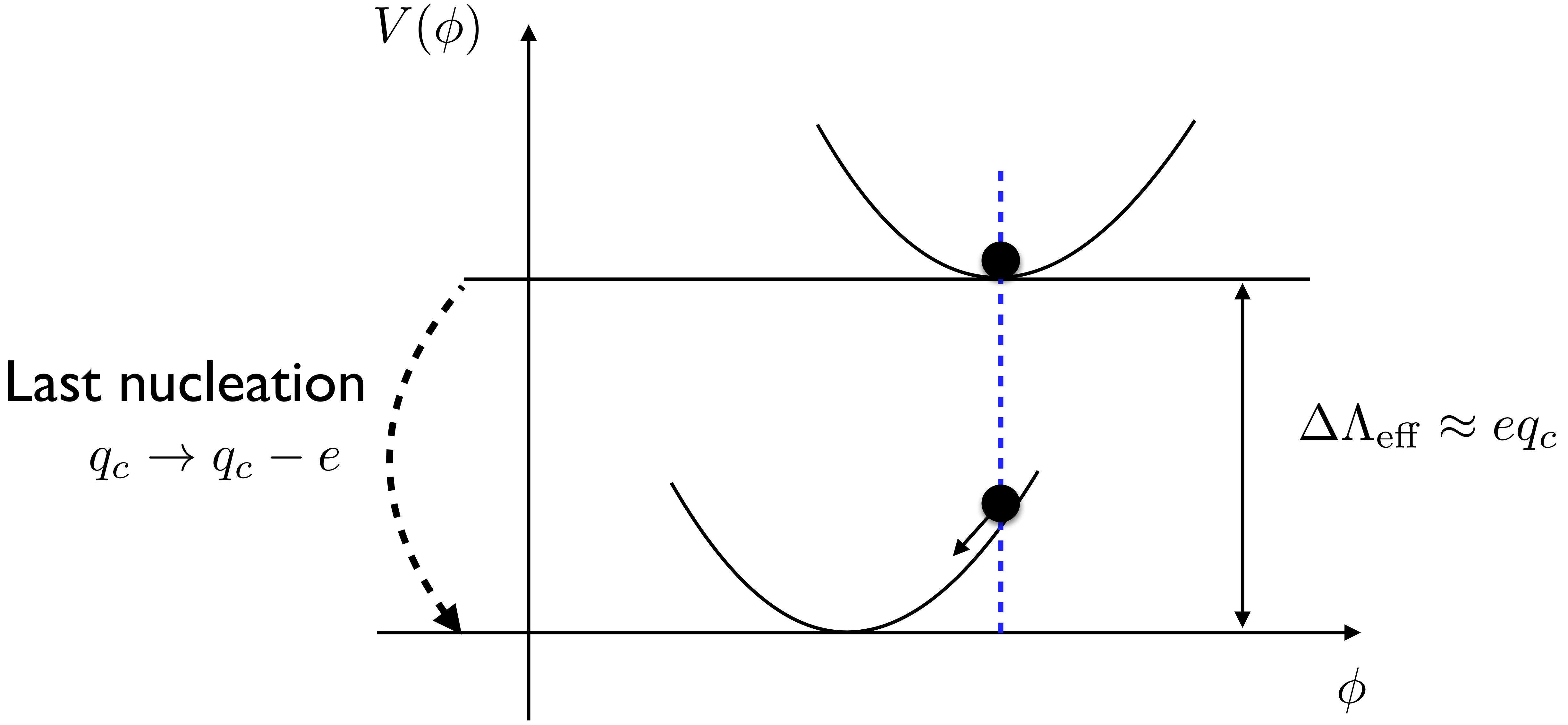
[Bousso, Polchinski, 2000; HML 2019]

$$V(H,\phi) = V_{\rm eff}(H) + \underset{\text{Flux-dependent minimum}}{\underline{(k_1\phi^n + q + k_2)^2}} + \underset{\text{Reheating}}{\underline{V_{\rm int}(\phi, H)}}$$

→ The minimum of the inflaton potential shifts after the last nucleation.

General mechanism to kick the inflaton away from the true minimum.

Flux-dependent minimum



Reheating from general four-form couplings

General four-form couplings

- The general four-form Lagrangian contains a dimensionless non-minimal coupling, which gives rise to a dynamical scalar field. [HML, 2019]

$$\mathcal{L}_{\rm non-minimal} = \underbrace{-\frac{c_1}{24}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}R}_{\text{Non-minimal coupling}} + \underbrace{\sqrt{-g}\left(\frac{1}{2}\zeta^2 R^2\right)}_{\text{Higher curvature}} + \frac{c_1}{24}\partial_\mu(\epsilon^{\mu\nu\rho\sigma}RA_{\nu\rho\sigma}).$$

→ Gauge field for four-form flux becomes dynamical due to graviton mixing.

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\Omega(H,q)R + \frac{1}{2}(\zeta^2 - c_1^2)R^2 + M^2_{\rm eff}|H|^2 - \lambda_{H,{\rm eff}}|H|^4 - \Lambda_{\rm eff},$$

$\Omega(H,q) = 1 + c_1(c_H|H|^2 + q)$: Four-form flux also scans Planck mass!

Dual-scalar theory

- Inflaton potential from scalar dual theory.

$$\frac{1}{2}(\zeta^2 - c_1^2)R^2 \longrightarrow \sqrt{\zeta^2 - c_1^2}\,\chi R - \frac{1}{2}\chi^2.$$: Hubbard-Stratonovich transf.

$$\sigma = c_H|H|^2 + q + \frac{\sqrt{\zeta^2 - c_1^2}}{c_1}\chi$$: Field redefinition

➡ $$\mathcal{L}_1 = \sqrt{-g}\Big[\frac{1}{2}(1 + c_1\sigma)R - |D_\mu H|^2 - V(H, \sigma, q)\Big]$$

$$V(H, \sigma, q) = -M_{\rm eff}^2|H|^2 + \lambda_{H,\rm eff}|H|^4 + \Lambda_{\rm eff} + \boxed{\frac{1}{2}\frac{c_1^2}{\zeta^2 - c_1^2}\Big(\sigma - c_2|H|^2 - q\Big)^2.}$$

Inflaton potential has a flux-dependent minimum.

R^2 term is necessary for the inflaton potential to be bounded from below.

Dual-scalar theory

- Einstein frame Lagrangian.

$$\mathcal{L}_E = \sqrt{-g_E}\left[\frac{1}{2}R(g_E) - \frac{1}{2}(\partial_\mu\bar{\sigma})^2 - e^{-\sqrt{\frac{2}{3}}\bar{\sigma}}\,|D_\mu H|^2 - V_E(H,\bar{\sigma})\right]$$

Canonical inflaton: $\sigma = \frac{1}{c_1}\left(e^{\sqrt{\frac{2}{3}}\bar{\sigma}} - 1\right)$

$$\begin{aligned} V_E(H,\bar{\sigma}) \;&=\; \Lambda_{\rm eff}\, e^{-2\sqrt{\frac{2}{3}}\bar{\sigma}} + \frac{3}{4}m_{\bar{\sigma}}^2\left(1-(1+c_1 q)e^{-\sqrt{\frac{2}{3}}\bar{\sigma}} - c_1 c_2\, e^{-\sqrt{\frac{2}{3}}\bar{\sigma}}|H|^2\right)^2 \\ &\quad + e^{-2\sqrt{\frac{2}{3}}\bar{\sigma}}\left(-M_{\rm eff}^2|H|^2 + \lambda_{H,{\rm eff}}|H|^4\right). \end{aligned}$$

$$m_{\bar{\sigma}} = \sqrt{\frac{2}{3}}\frac{M_P}{\sqrt{\zeta^2 - c_1^2}}.$$

Assume that the Higgs is stabilized during dS phases:

$$V_E(\sigma) = V_0(q) + \left[\frac{3}{4}m_{\bar{\sigma}}^2\left(1+c_1\left(q+\frac{1}{2}c_H v^2\right)\right)^2 + \Lambda_{\rm eff}\right]\left(e^{-\sqrt{\frac{2}{3}}\bar{\sigma}} - e^{-\sqrt{\frac{2}{3}}\bar{\sigma}_{\rm m}(q)}\right)^2$$

$$e^{-\sqrt{\frac{2}{3}}\bar{\sigma}_{\rm m}(q)} \;=\; \frac{3m_{\bar{\sigma}}^2(1+c_1(q+\frac{1}{2}c_H v^2))}{3m_{\bar{\sigma}}^2(1+c_1(q+\frac{1}{2}c_H v^2))^2 + 4\Lambda_{\rm eff}},$$

Flux-dep. inflaton minimum.

$$V_0(q) \;=\; \frac{3m_{\bar{\sigma}}^2\Lambda_{\rm eff}}{3m_{\bar{\sigma}}^2(1+c_1(q+\frac{1}{2}c_H v^2))^2 + 4\Lambda_{\rm eff}}.$$

Flux-dependent C.C.

Reheating after scanning

- Just before the last nucleation:

$$q = M^2/c_2 \equiv q_c \text{ and } v = 0,$$

$$e^{-\sqrt{\frac{2}{3}}\bar{\sigma}_{\rm m}(q_c)} \approx \frac{1}{1+c_1 q_c}\left(1+\frac{4eq_c}{3m_{\bar{\sigma}}^2(1+c_1q_c)^2}\right)^{-1}, \quad V_0(q_c) \approx \frac{3m_{\bar{\sigma}}^2 e q_c}{3m_{\bar{\sigma}}^2(1+c_1q_c)^2+4eq_c}$$

- Just after the last nucleation:

$$q = q_c - e, \quad v \neq 0, \qquad e^{-\sqrt{\frac{2}{3}}\bar{\sigma}_m(q_c-e)} \approx \frac{1}{1+c_1q_c}, \quad V_0(q_c-e) \approx 0.$$

→ Initial potential energy: $V_i \equiv V_E(\bar{\sigma}_i) = \dfrac{12(eq_c)^2 m_{\bar{\sigma}}^2}{(3m_{\bar{\sigma}}^2(1+c_1q_c)^2+4eq_c)^2}$

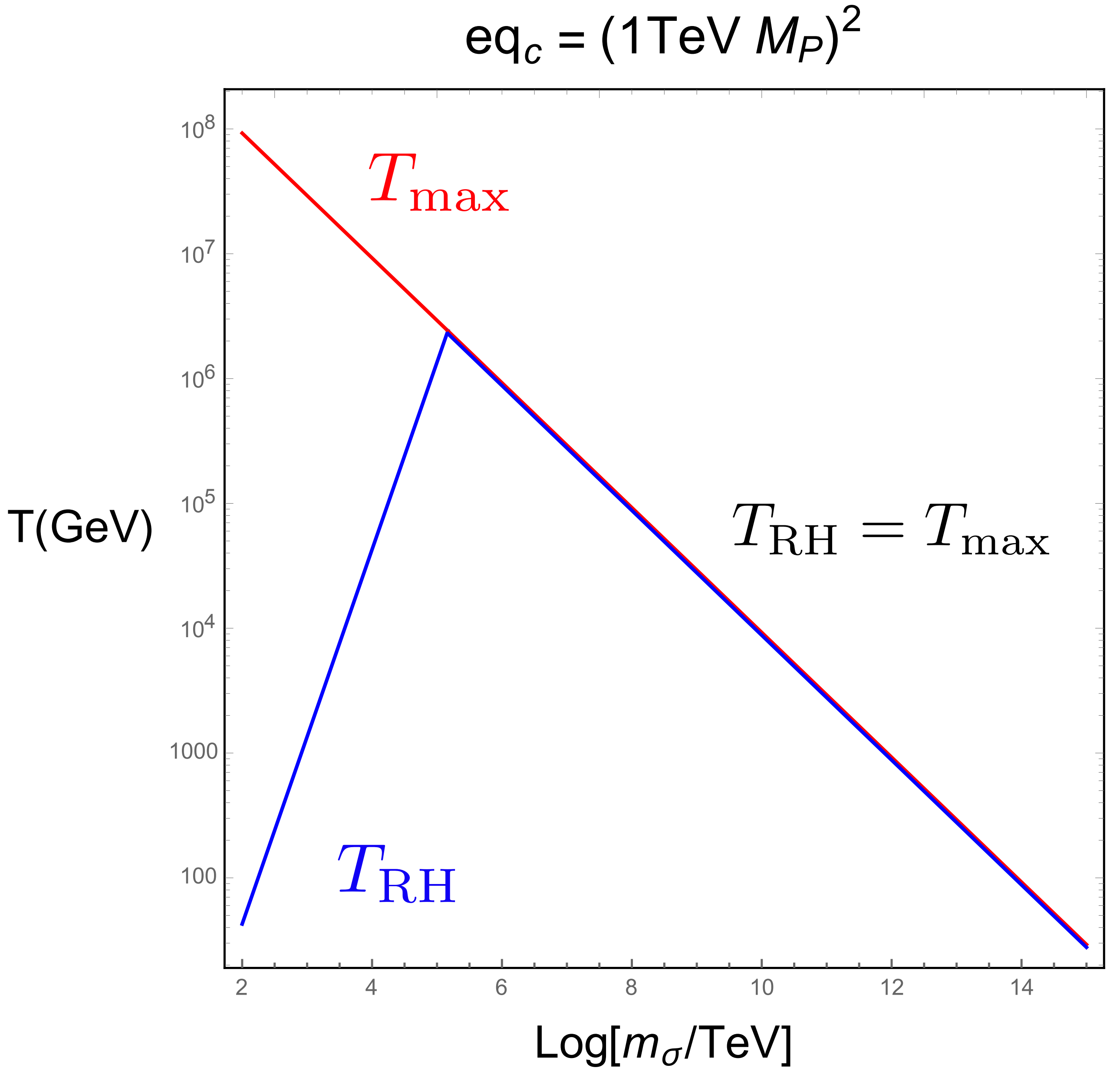
Maximum reheating temperature:

$$T_{\rm max} \simeq 1.5\times 10^9\,{\rm GeV}\left(\frac{100}{g_*}\right)^{1/4}\left(\frac{eq_c}{(1\,{\rm TeV}\cdot M_P)^2}\right)^{1/2}\left(\frac{380\,{\rm TeV}}{m_{\bar{\sigma}}}\right)^{1/2}$$

But, inflaton decay rate is small: $\Gamma_{\bar{\sigma}} = \dfrac{3c_1^2c_2^2}{64\pi}\dfrac{m_{\bar{\sigma}}^3}{M_P^2}$

$$T_{\rm RH} = \left(\frac{90}{\pi^2 g_*}\right)^{1/4}(\Gamma_{\bar{\sigma}}M_P)^{1/2} = 10\,{\rm MeV}\left(\frac{100}{g_*}\right)^{1/4}\left(\frac{c_1}{1}\right)\left(\frac{c_2}{1}\right)\left(\frac{m_{\bar{\sigma}}}{380{\rm TeV}}\right)^{3/2}.$$

Successful reheating



$T_{\rm RH} > 10\,\text{MeV}$: $\quad m_\sigma > 380\,\text{TeV}$ (or $\zeta < 5.2\times10^{12}$)

Slow-roll inflation: $m_\sigma \ll H_I = 8\times10^{13}\,\text{GeV}(r/0.1)^{1/2}$

Flux coupling to pseudo-scalar

- The four-form coupling to a pseudo-scalar with approximate shift symmetry. [HML, 2019]

$$\mathcal{L}_{\rm pseudo-scalar} = -\frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 + \underbrace{\frac{\mu}{24}\,\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}\,\phi}_{\text{Four-form coupling}} - \frac{\mu}{6}\,\partial_\mu\Big(\epsilon^{\mu\nu\rho\sigma}\phi A_{\nu\rho\sigma}\Big).$$

→

$$\mathcal{L}_{\rm II} = M^2|H|^2 - \lambda_H|H|^4 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{2}(\mu\phi + c_2|H|^2 + q)^2.$$

Flux-dependent VEVs:

$$v_H(q) = \sqrt{\frac{M^2 - c_2(q + \mu v_\phi)}{\lambda_H + \frac{1}{2}c_2^2}},$$

$$v_\phi(q) = -\frac{\mu}{\mu^2 + m_\phi^2}\cdot\left(\frac{1}{2}c_2 v_H^2 + q\right).$$

Flux-dependent masses & mixing:

$$m_{h_{1,2}}^2 = \frac{1}{2}(m_\varphi^2 + m_h^2) \mp \frac{1}{2}\sqrt{(m_\varphi^2 - m_h^2)^2 + 4c_2^2\mu^2 v_H^2(q)}, \qquad \tan 2\theta(q) = \frac{2c_2\mu v_H(q)}{m_\varphi^2 - m_h^2}.$$

$$m_\varphi^2 = m_\phi^2 + \mu^2, \qquad m_h^2 = 2\lambda_{H,\rm eff}v_H^2(q).$$

Reheat with pseudo-scalar

- Just before the last nucleation:

$$q_c = \frac{\mu^2 + m_\phi^2}{m_\phi^2}\frac{M^2}{c_2}, \qquad v_H = 0, \qquad v_\phi(q_c) = -\frac{\mu}{m_\phi^2}\frac{M^2}{c_2} \equiv v_{\phi,c},$$

- Just after the last nucleation:

$$q = q_c - e, \quad v_H^2 = \frac{m_\phi^2}{\mu^2 + m_\phi^2}\left(\frac{c_2 e}{\lambda_{H,\mathrm{eff}} - \frac{1}{2}\frac{c_2^2\mu^2}{\mu^2+m_\phi^2}}\right), \qquad V_0(q_c - e) \approx 0.$$

$$v_{\phi,0} = v_{\phi,c} + \boxed{\frac{\mu}{\mu^2 + m_\phi^2}\left(\frac{1}{2}c_2 v_H^2 - e\right)}.$$

"shifted singlet VEV"

➡ EW scale: $v_H \sim \sqrt{e}$ for $\mu \sim m_\phi$.

Initial potential energy: $V_i = \frac{1}{2}\frac{\mu^2}{\mu^2 + m_\phi^2}\left(e - \frac{1}{2}c_2 v^2\right)^2.$

$\boxed{V_i \sim e^2 \text{ for } \mu \sim m_\phi.}$ independent of pseudo-scalar mass.

Inflaton decays into a Higgs
pair & reheats instantaneously.

$$T_{\max} = \left(\frac{90 V_i}{\pi^2 g_*}\right)^{1/4} \simeq 55\,\mathrm{GeV}\left(\frac{V_i^{1/4}}{100\,\mathrm{GeV}}\right)\left(\frac{100}{g_*}\right)^{1/4}$$

Flux coupling to c-scalar

[HML, 2019]

- The four-form coupling to a complex scalar with U(1) symmetry.

$$\mathcal{L}_{\rm c-scalar} = -|\partial_\mu\Phi|^2 - m_\Phi^2|\Phi|^2 - \lambda_\Phi|\Phi|^4 + \underbrace{\frac{\alpha}{24}\,\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}\,|\Phi|^2}_{\text{Four-form coupling}} - \frac{\alpha}{6}\partial_\mu\Big(\epsilon^{\mu\nu\rho\sigma}|\Phi|^2 A_{\nu\rho\sigma}\Big).$$

➡ $$\mathcal{L}_{\rm III} = M^2|H|^2 - \lambda_H|H|^4 - m_\Phi^2|\Phi|^2 - \lambda_\Phi|\Phi|^4 - \frac{1}{2}(\alpha|\Phi|^2 + c_2|H|^2 + q)^2.$$

Higgs and singlet masses are scanned at the same time.

Flux-dependent VEVs:

$$v_\phi(q) = \sqrt{\frac{-m_\phi^2 - \boxed{\alpha q} - \frac{1}{2}\alpha c_2 v_H^2(q)}{\lambda_{\phi,{\rm eff}}}},$$

$\alpha > 0$ and $m_\phi^2 < 0$,

$$v_H(q) = \sqrt{\frac{M^2 - \boxed{c_2 q} - \frac{1}{2}\alpha c_2 v_\phi^2(q)}{\lambda_{H,{\rm eff}}}}.$$

Similar flux-dependent masses & mixing arise!

Reheat with pseudo-scalar

- Just before the last nucleation:

$$q_c = \frac{1}{\lambda_\phi}\left(\frac{\lambda_{\phi,\rm eff}}{c_2}M^2 + \frac{\alpha}{2}m_\phi^2\right), \qquad v_H = 0, \qquad v_\phi^2(q_c) = -\frac{1}{\lambda_\phi}\left(m_\phi^2 + \frac{\alpha}{c_2}M^2\right) \equiv v_{\phi,c}^2.$$

- Just after the last nucleation:

$$q = q_c - e, \quad v^2 = \frac{\lambda_\Phi c_2 e}{\lambda_{\Phi,\rm eff}\lambda_{H,\rm eff} - \frac{1}{4}(\alpha c_2)^2}, \qquad \lambda_{\Phi,\rm eff} = \lambda_\Phi + \frac{1}{2}\alpha^2,$$

$$v_{\phi,0}^2 = v_{\phi,c}^2 + \boxed{\frac{\alpha}{\lambda_{\Phi,\rm eff}}\left(e - \frac{1}{2}c_2 v^2\right)}.$$

"shifted singlet VEV"

→ EW scale: $v_H \sim \sqrt{e}$ for $\mu \sim m_\phi$.

Initial potential energy: $V_i = \frac{1}{2}\frac{\alpha^2}{\lambda_{\phi,\rm eff}}\left(e - \frac{1}{2}c_2 v^2\right)^2.$

$V_i \sim e^2$, independent of singlet mass.

Reheating is similar as in pseudo-scalar case with $\mu \sim m_\phi$.

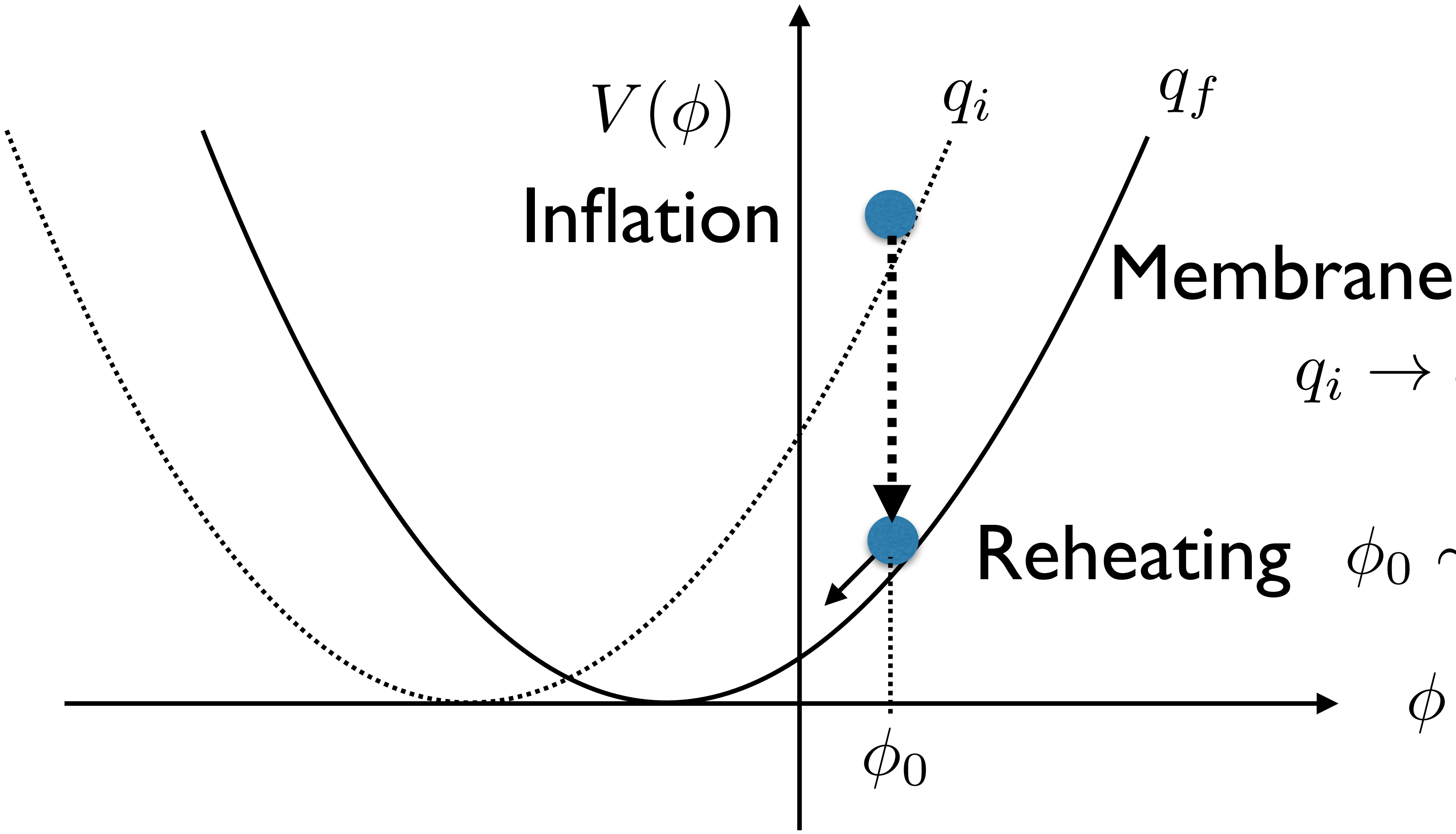
Chaotic inflation with four-form flux

Pseudo-scalar inflation

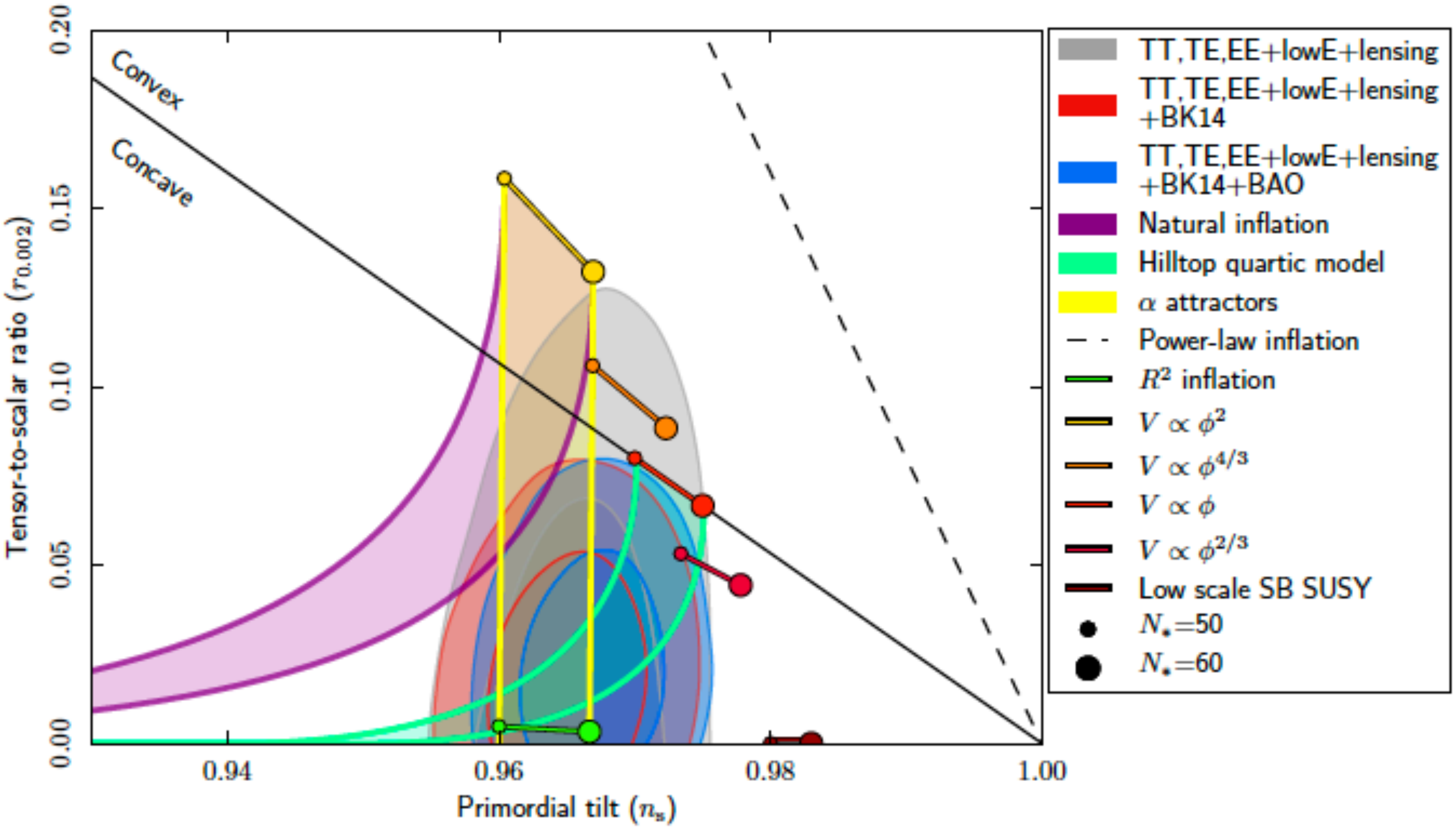
- Consider a massless pseudo-scalar with four-form coupling. [Kaloper, Sorbo, 2009]

$$\mathcal{L}_{\rm inf} = -\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}(\mu\phi + q)^2.$$

Shift symmetry: $\phi \to \phi + c, \quad q \to q - \mu c.$



Planck and inflation



Single-field & canonical inflation is favored.

Monomial-type inflation models are in tension with Planck tensor-to-scalar ratio.

Non-minimal four-form

- Maintain the shift symmetry by the non-minimal four-form coupling.

[HML, 2019]

$$\mathcal{L}_{nm} = -\frac{\alpha}{24}\,\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}\,R.$$

$$\Longrightarrow \quad \mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\Big(1+\underline{\alpha(\mu\phi+q)}\Big)R + \underline{\frac{1}{2}(\zeta^2-\alpha^2)R^2} -\frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}(\mu\phi+q)^2\right]$$

For $\zeta \gtrsim \alpha$ and $\mu \lesssim M_P$,

dual-scalar from R^2 can be decoupled.

Einstein frame: $$\mathcal{L}_E = \sqrt{-g_E}\left[\frac{1}{2}R(g_E) - \frac{1}{2}K(\phi)(\partial_\mu\phi)^2 - V_I(\phi)\right]$$

$$K(\phi) = \frac{1+\frac{3}{2}\alpha^2\mu^2+\alpha(\mu\phi+q)}{(1+\alpha(\mu\phi+q))^2}, \qquad V_I(\phi) = \frac{1}{2}\frac{(\mu\phi+q)^2}{(1+\alpha(\mu\phi+q))^2}.$$

Rescaled potential!

Inflationary predictions

- Non-minimal coupling makes the potential flat for $\alpha(\mu\phi+q)\gtrsim 1.$

Canonical field: $\mu\phi+q=\frac{1}{4}\alpha\mu^2\varphi^2$

→ $$\boxed{V_I(\varphi)=\frac{1}{2\alpha^2}\left(1+\frac{4}{\alpha^2\mu^2\varphi^2}\right)^{-2}.}$$

Inflationary predictions: $N=\int_{\varphi_f}^{\varphi_*}\frac{d\varphi}{\sqrt{2\varepsilon}}\simeq\frac{\alpha^2\mu^2}{64}\varphi_*^4$

$$\begin{aligned}n_s&=1-6\varepsilon_*+2\eta_*\\&=1-\frac{3}{2\alpha\mu}\frac{1}{N^{3/2}}-\frac{3}{2N}\end{aligned}$$

$$r=16\epsilon_*=\frac{4}{\alpha\mu}\frac{1}{N^{3/2}}.$$

cf. CMB: $\alpha=38000(\alpha\mu)^{1/2}\left(\frac{N}{50}\right)^{3/4}.$

$\alpha\mu=1$ and $N=50(60)$, → $\boxed{n_s=0.966(0.972),\ r=0.011(0.0086)}$

$\alpha=3.8(4.4)\times10^4$ and $\mu=6.3(5.5)\times10^{13}$ GeV.

in perfect agreement with Planck.

Robustness of predictions

- Unitarity scale is Planck scale or higher.

$$\frac{\phi^n}{(\Lambda_n)^{n-4}}, \quad \Lambda_n = M_P\left[\frac{\alpha\mu}{(1+\frac{3}{2}(\alpha\mu)^2)^{1/2}}\right]^{\frac{n}{n-4}}$$

: insensitive to $\alpha\mu$.

- Higher order corrections appear in the form,

$$\frac{c_n}{\Lambda^{4(n-1)}}\left(-\frac{1}{24}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}\right)^n$$

$$= \frac{c_n}{\Lambda^{4(n-1)}}\left[(\mu\phi+q)^{2n} - 2n\alpha(\mu\phi+q)^{2n-1}R + \cdots\right]$$

$$F_{\mu\nu\rho\sigma} = (\mu\phi+q)\epsilon_{\mu\nu\rho\sigma}$$

$$\Lambda = M_P, \qquad c_n\left(\frac{M_P}{\sqrt{\alpha}}\right)^{4(n-1)} \lesssim c_n(\mu\phi+q)^{2(n-1)} \lesssim M_P^{4(n-1)}.$$

→ Inflationary predictions are untouched for a wide range of parameter space.

$$\frac{M_P}{\sqrt{\alpha}} \ll M_P$$

Model-indep. reheating

- Inflaton couples to the trace of energy-momentum tensor like dilaton:

$$\mathcal{L}_{\rm int} = \frac{1}{2M_P^2}\,\alpha(\mu\phi + q)\,T^\mu_\mu$$

$$\longrightarrow \quad \Gamma_\phi = \frac{m_\phi^3}{32\pi M_P^2}\,\frac{(\alpha\mu)^2}{1+\frac{3}{2}(\alpha\mu)^2}. \qquad (W, Z, h)$$

$$\alpha\mu = 1: \quad T_{\rm RH} = 3.5\times 10^{11}\,{\rm GeV}\left(\frac{100}{g_*}\right)^{1/4}\left(\frac{m_\phi}{10^{14}\,{\rm GeV}}\right)^{3/2}$$

"Robust prediction for reheating"

cf. But, reheating temperature can be higher.

$$\mathcal{L}_{\rm int} = \frac{\phi}{f_\phi}\,G_{\mu\nu}\tilde{G}^{\mu\nu}, \quad f_\phi < M_P.$$

Conclusions

- Four-form flux can scan the cosmological constant, the Higgs mass as well as the Planck mass.
- Non-minimal four-form coupling is the minimal setup for the relaxation of Higgs mass & the successful reheating.
- Minimal four-form couplings to extra scalars lead to the reheating temperature of order weak scale, insensitive to scalar masses.
- Non-minimal four-form coupling can cure the pseudo-scalar chaotic inflation models with successful and robust predictions for inflation.