The Lattice Study of SU(2) with Many Fermions

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in various combinations

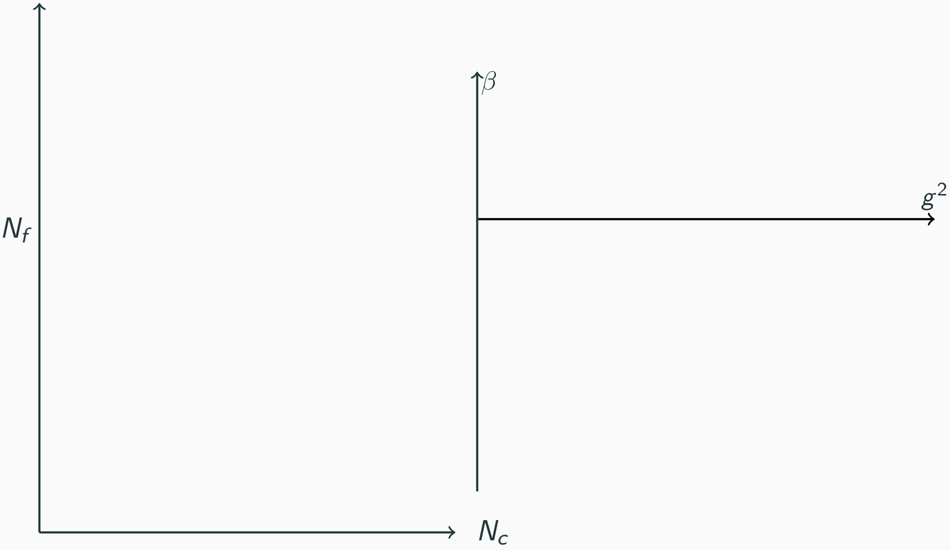
Based on: 1701.04666, 1707.04722, 1804.02319, 1806.07154, 1901.04605

IBS-Busan workshop
06.12.2019
Busan, Korea

Motivation

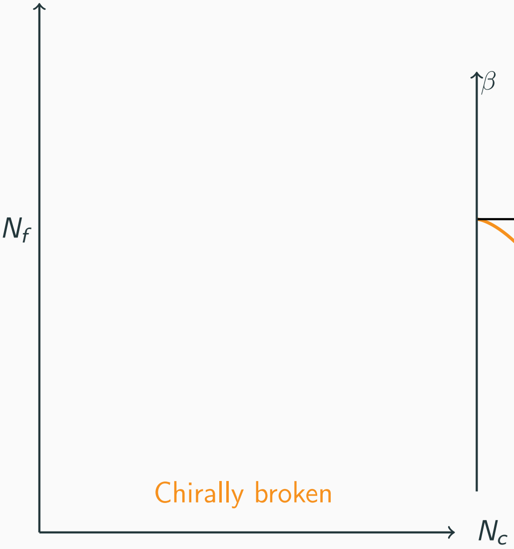
- Understanding strongly coupled theories
- Phases of gauge theories
  - Infrared behavior: χSB/Walking/IRFP/Trivial/Safe
  - IRFP properties i.e. anomalous dimensions
- Phenomenological motivation
  - Composite Higgs
  - (Extended) Technicolor
  - Understanding of asymptotically safe theories

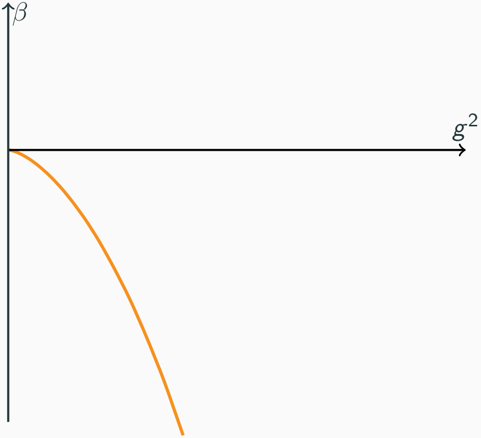
Vacuum structure



Vacuum phase of gauge theory depends on N_c and N_f

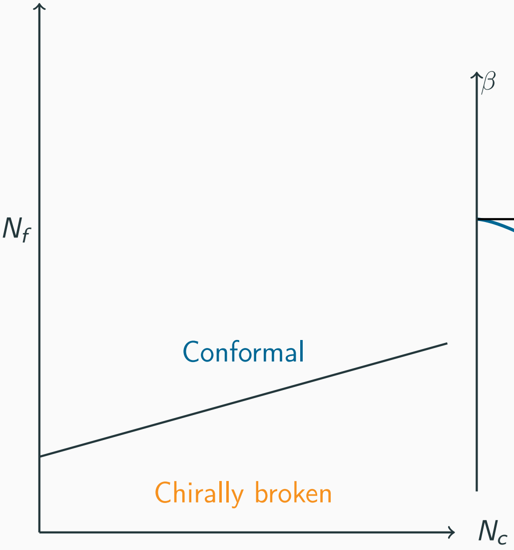
Vacuum structure

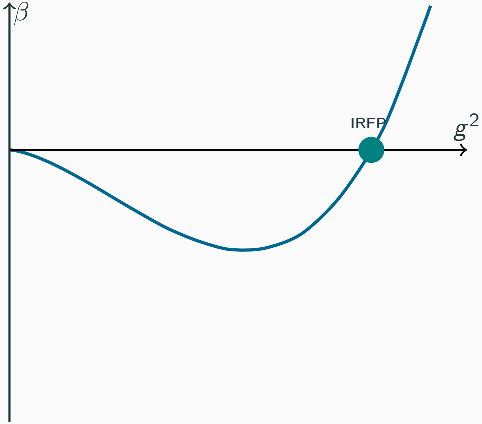




Small N_f, $\beta_0 > 0$,$\beta_1 > 0$. Confining,χSB

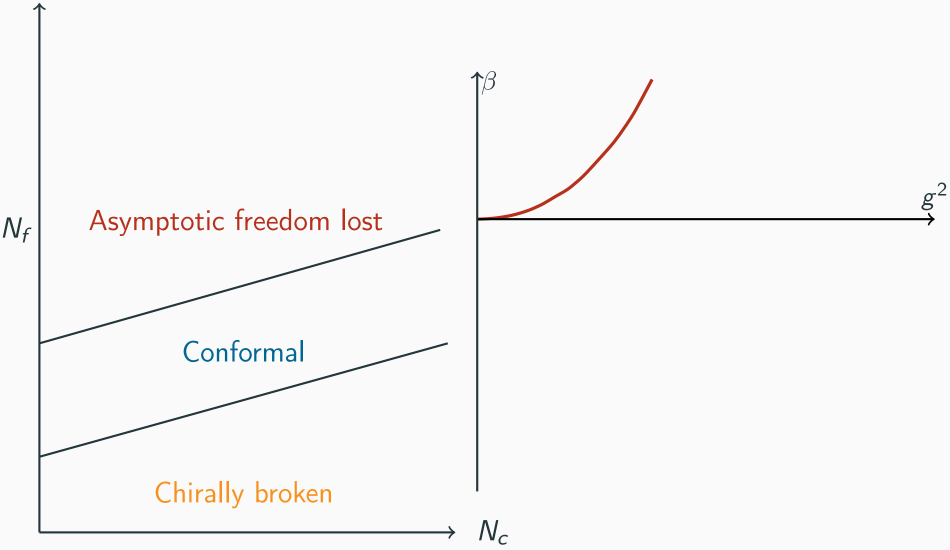
Vacuum structure





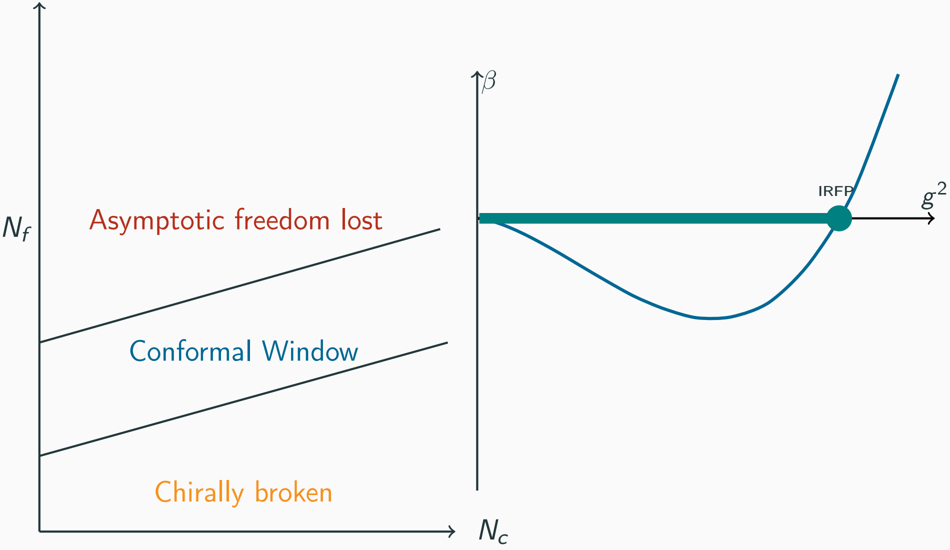
Intermediate N_f, $\beta_0 > 0$,$\beta_1 < 0$, Conformal, Scale Invariant

Vacuum structure



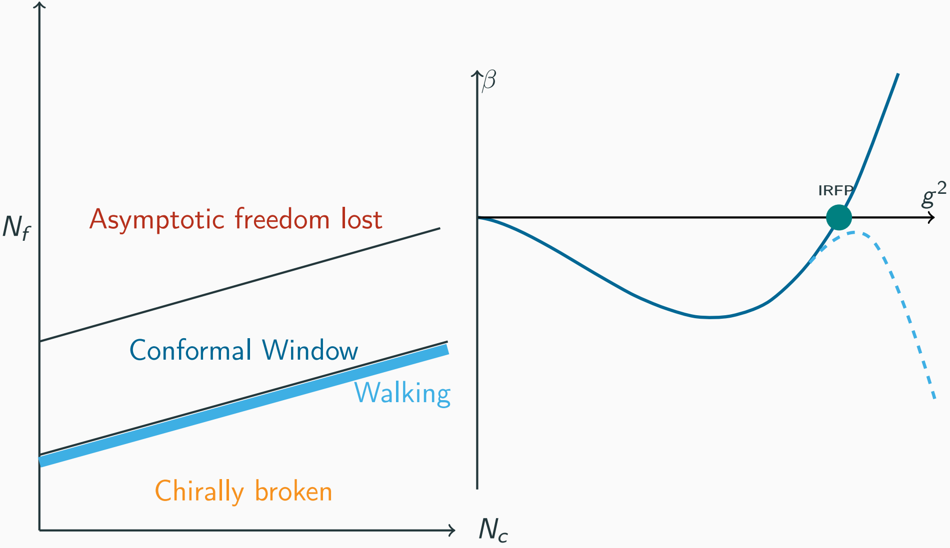
Large N_f, $\beta_0 < 0$,$\beta_1 < 0$, Asymptotic freedom lost, Landau Pole

Vacuum structure



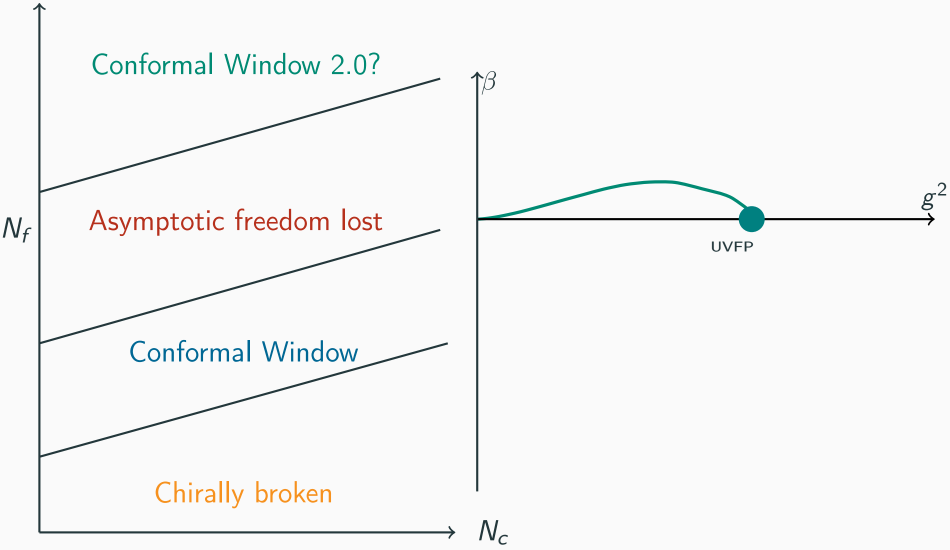
Conformal Window (1.0): the region where theory is conformal

Vacuum structure



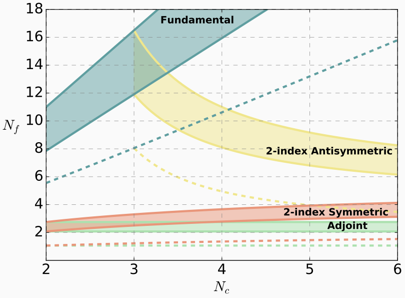
Lower boundary of window: almost conformal, Walking?

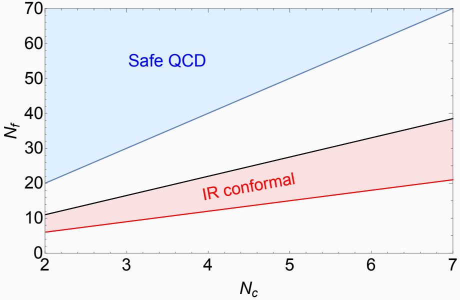
Vacuum structure



Very high N_f: possibility of UVFP, Asymptotic safety

Conformal Windows

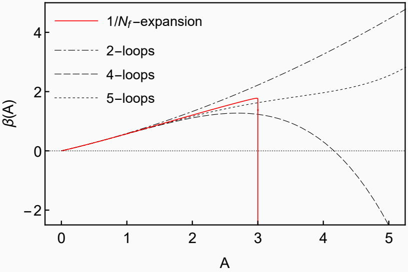




$$\beta(g) = \mu \frac{\mathrm{d}g}{\mathrm{d}\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2}\,, \quad \begin{cases} \beta_0 = \frac{11}{3} N_c - \frac{4}{3} T_r N_f \\ \beta_1 = \frac{34}{3} N_c^2 - \left(\frac{20}{3} N_c - 4 C_r\right) T_r N_f \end{cases}$$

- Behavior depends on group, representation and number of fermions

What happens at large N_f



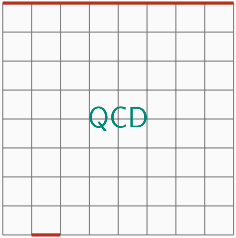
- Standard lore:
  - Asymptotic freedom lost, Landau Pole
  - Theory is trivial
- $N_f \to \infty$ suggest UVFP
  - Asymptotic safety

$$\frac{3}{2}\frac{\beta(A)}{A} = 1 + \frac{H_1(A)}{N_f} + \frac{H_2(A)}{N_f^2} + \cdots \,, \qquad A = \frac{\alpha}{2\pi} N_f$$

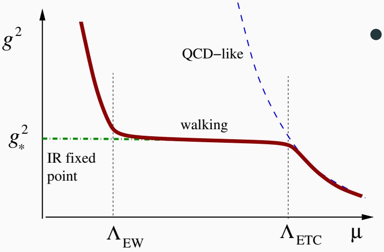
- $H_1(A)$ has a logarithmic singularity at A=3 $\Rightarrow \beta(A) = 0 \Rightarrow$ UVFP
- 4-loop MS β-function seems to suggest UVFP too
- UVFP expected to occur at smaller couplings when N_f is increased

We study SU(2) with N_f Wilson fermions

- Running of the coupling at $N_f = 6, 8$
  - Coupling has no unique definition on lattice, but existence of FP is universal (also anomalous dimensions at FP)
  - Probe the lower edge of conformal window
  - Measure mass and coupling anomalous dimensions at FP if exists
- Mass spectrum at $N_f = 2, 4, 6$
  - Another way to see the onset of conformality
- Large $N_f = 24, 48$ behavior
  - Do the lattice methods for IRFP work at non asymptotically free region with high fermion amounts
  - Can we distinguish asymptotic safety vs trivial theory (Landau pole)

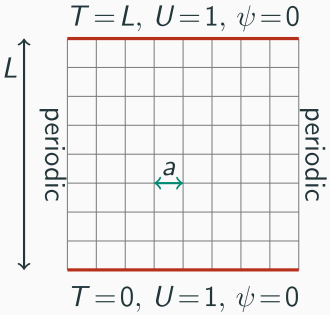


- In QCD
  - → Asymptotic freedom
  - → Clear cut between strong and weak coupling
  - → Coupling depends nicely on lattice scale



- On a walking/conformal theory, coupling large and mostly independent on scale
  - → Coupling equally strong almost everywhere
  - → Small β required, regardless of L

Lattice details: Boundary conditions



- Schrödinger functional boundaries $c_t = 1$
  - Periodic boundary conditions on spatial boundaries
  - Dirichlet boundary conditions on time boundaries
- Easier to tune fermion masses to zero with axial ward identity
- Enables measurement of mass anomalous dimension

Lattice details: Action

$$S = (1-c_g)S_G(U) + c_g S_G(V) + S_F(V)$$

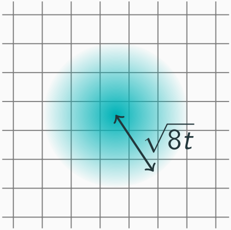
$$S_G = \beta \sum_{\square}(1 - \frac{1}{2}\mathrm{Tr}\square(x))$$

$$S_F = a^4 \sum_{N_f} \sum_{x} (\bar{\psi}(iD + m_0)\psi + \frac{ai}{4} c_{sw} \bar{\psi}\sigma_{\mu\nu}F_{\mu\nu}\psi)$$

- Mix HEX-smeared (V) and unsmeared (U) gauge fields with c_g
  - Moves bulk phase and allows reaching zero mass at higher couplings
  - Might destroy UV physics
- Clover improved Wilson action $c_{sw} = 1$
- Lattice parameters:
  - Conformal window study: $L \simeq 6^4 - 32^4$ and $\beta \simeq 8 - 0.4$
  - Large N_f: $L \simeq 12^4 - 30^4$, and $\beta \simeq 6 - -1$

Gradient flow

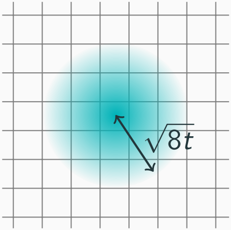
$$\partial_t B_{t,\mu} = -\frac{\delta S_{YM}}{\delta B} = D_{t,\mu} G_{t,\mu\nu}\,,$$
$$G_{t,\mu\nu} = \partial_\mu B_{t,\nu} - \partial_\nu B_{t,\mu} + [B_{t,\mu}, B_{t,\nu}]\,.$$
$$B_{0,\mu} = A_\mu \;\leftarrow \text{ the original gauge field}$$



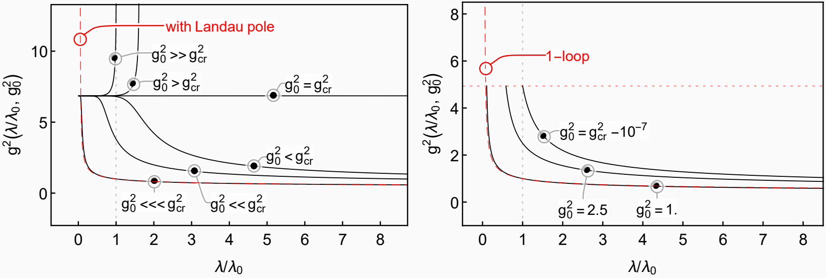
- Only gauge fields
- Evolve with fictitious time t
- Drives B_μ towards minima of S_{YM}
- Diffuses the initial gauge field with radius $\sqrt{8t} \equiv \lambda(t)$
    - $a < \lambda(t) < L$, smearing $\Rightarrow \lambda(t) \gtrsim 2 - 3$
- Many choices for S_{YM}

Gradient flow

$$\langle E(t)\rangle = \frac{1}{4}\langle G_{\mu\nu}(t)G_{\mu\nu}(t)\rangle$$
$$= \frac{3(N^2-1)g_0^2}{128\pi^2 t^2} + \mathcal{O}(g_0^4)\,,$$
$$g^2_{\mathrm{GF}}(\mu) = \mathcal{N}^{-1} t^2 \langle E(t)\rangle\,|_{x_0=L/2\,,\,t=1/8\mu^2}\,,$$

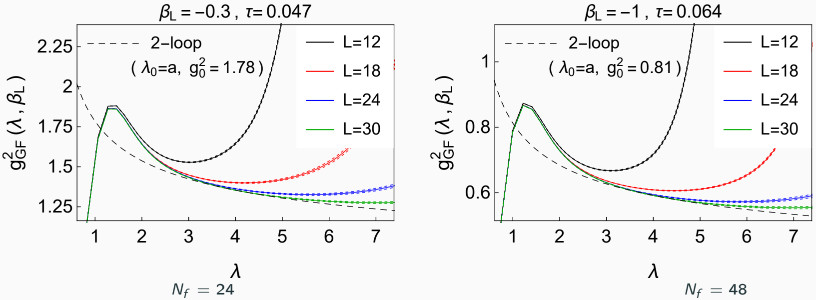


- Evolve the flow equation to time t
- Coupling at scale $\mu^{-1} = \sqrt{8t} = \lambda(t) = c_t L$
- Multiple possible discretization of E
- c_t defines the scheme
- Boundary conditions break time translation, only use $x_0 = L/2$

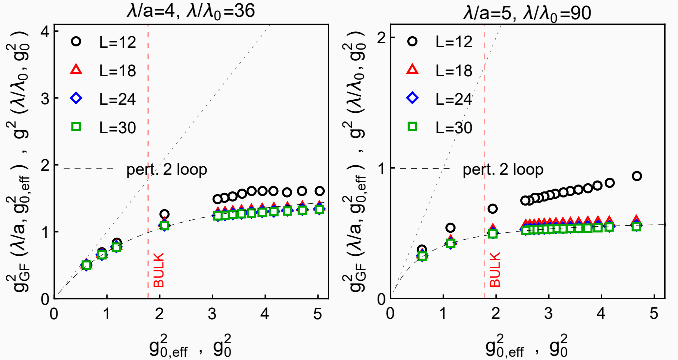


- Discerning UVFP from Landau pole hard
- Especially hard if slope of beta function steep
- If we could go past UVFP effect would be clear
- Set $\lambda_0 = a$

Running g^2 at large N_f: result



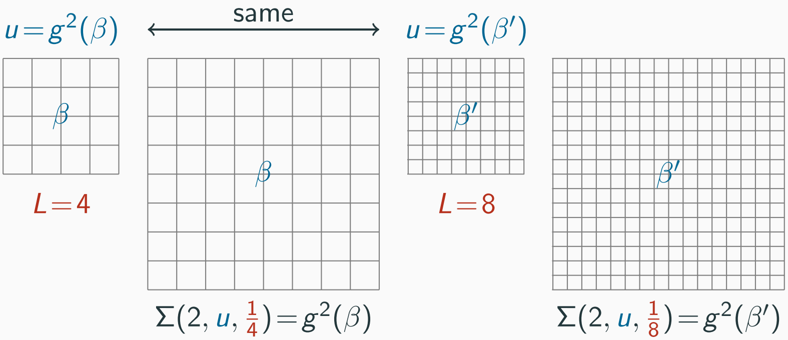
- Data replicates 2-loop running well at weak coupling
- Large finite lattice spacing and lattice size effects
- τ improvement offers some help
- Not enough data/range for continuum limit



$N_f = 24$ $N_f = 48$

- Negative β
  - Large numbers of fermions ruin the usual $\beta = 4/g_0^2$ relation
  - Match pure gauge SU(2) plaquette to plaquette to our model $g_{0,\text{eff}}^2 = 4/\beta_L^{\text{PG}}$
- See plateau
  - Indication of a cutoff
  - Cannot rule out UVFP at higher coupling

Step scaling idea



- Choose $\beta = 4/g_0^2$ and L, measure $u = g_{GF}^2(L)$
- Choose stepsize $s = 2$
- Double the lattice size, and measure $\Sigma(u, 1/L) = g_{GF}^2(sL)$
- Choose new bigger lattice size L'
- Tune β' such that $g_{GF}^2(L') = u$
- Double the lattice and measure $\Sigma(u, 1/L') = g_{GF}^2(sL')$
- Do for all lattice sizes, change u and repeat

Step scaling function

- Step scaling function in the lattice and continuum:

$$\Sigma(s,u,a/L) = g^2_{GF}(g_0, s\frac{L}{a})\Big|_{g^2_{GF}(g_0,\frac{L}{a})=u}, \quad \sigma(u,s) = \lim_{a/L\to 0}\Sigma(u,s,a/L)$$

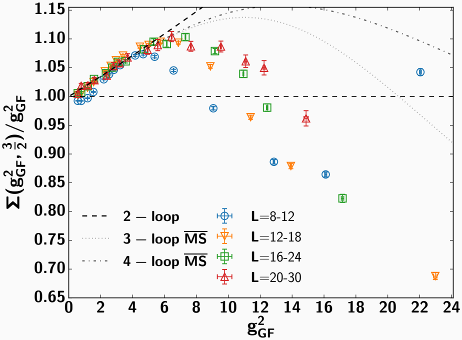
- Use interpolated couplings for consistent u and do the limit as:

$$\Sigma(u,s,a/L) = \sigma(u,s) + c(u)(\frac{L}{a})^{-2}$$

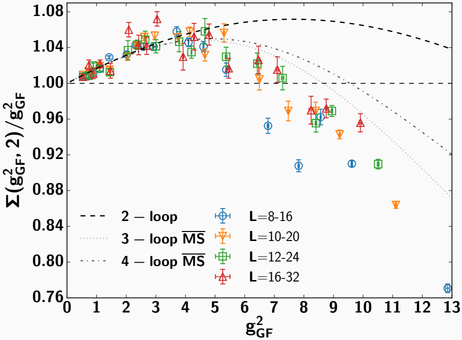
- At fixed point $\sigma(u)/u = 1$
- Related to beta function:

$$-2\ln(s) = \int_{\sqrt{u}}^{\sqrt{\sigma(u,s)}} \frac{dx}{\beta(x)}, \quad \beta(g) = \frac{g}{2\ln(s)}\left(1 - \frac{\sigma(g^2,s)}{g^2}\right)$$

Raw step scaling function

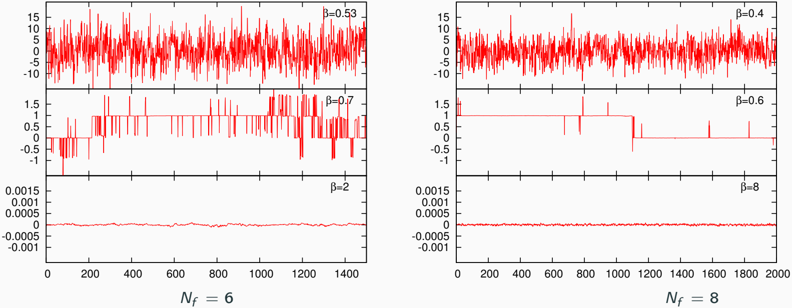


$N_f = 6$
$s = 3/2,\ c_t = 0.3$



$N_f = 8$
$s = 2,\ c_t = 0.4$

Topology

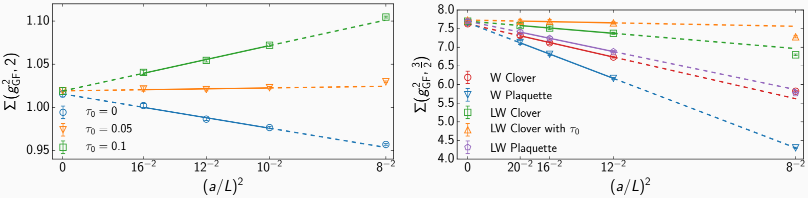


- Gradient flow allows measurement of topological charge

$$Q = \frac{1}{32\pi^2} \sum_x \epsilon_{\mu\nu\alpha\beta} G^a_{\mu\nu}(x;t) G^a_{\alpha\beta}(x;t)$$

- Frozen at small coupling, unfrozen at large couplings
- On intermediate couplings frozen to nonzero

Discretizations and τ-correction

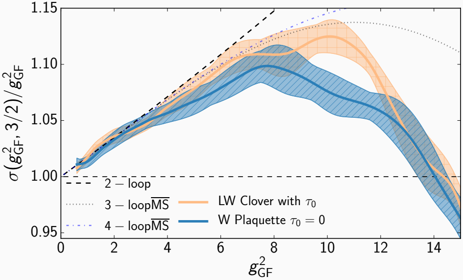


- Different actions of simulation, flow and energy density give different discretization effects
- Reduce effects by τ-shift:

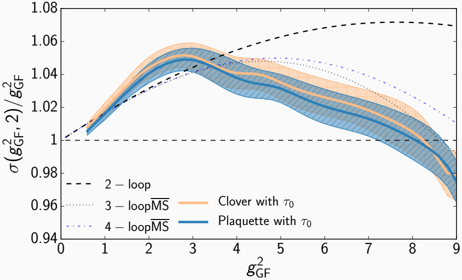
$$g^2_{GF} = \mathcal{N}^{-1} t^2 \langle E(t + \tau_0 a^2) \rangle = \mathcal{N}^{-1} t^2 \langle E(t) \rangle + \mathcal{N}^{-1} t^2 \langle \frac{\partial E(t)}{\partial t} \rangle \tau_0 a^2$$

- $N_f = 8$: $\tau = 0.06 \log(1 + g^2_{GF})$, $N_f = 6$: $\tau = 0.025 \log(1 + 2 * g^2_{GF})$
- Chosen discretization: LW evolved flow with Clover measurement

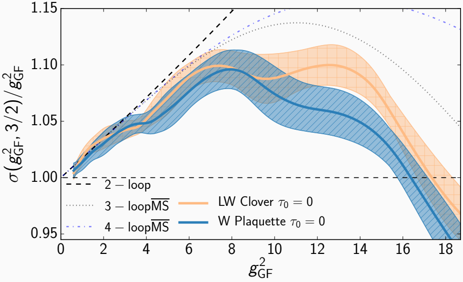
Continuum Step scaling function



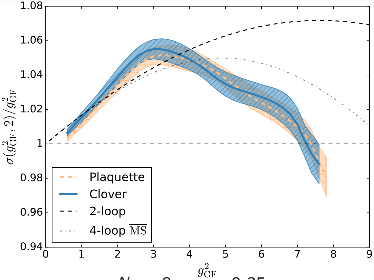
$N_f = 6\ ,\ c_t = 0.3$



$N_f = 8\ ,\ c_t = 0.4$



$N_f = 6\ ,\ c_t = 0.35$



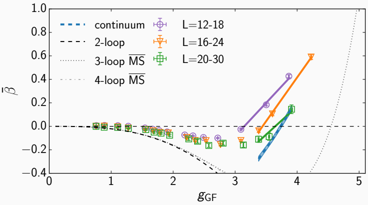
$N_f = 8\ ,\ c_t = 0.35$

Slope of β-function: γ_g^*

- Scheme independent observable at fixed point
- β-function related to step scaling function as:

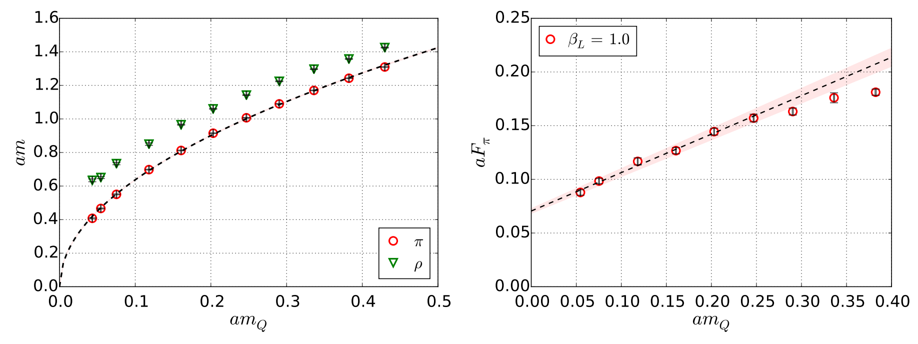
$$\beta(g) = \frac{g}{2\ln(s)}\left(1 - \frac{\sigma(g^2, s)}{g^2}\right)$$

- We can fit a line to points around IRFP



	$c_t = 0.3$	$c_t = 0.35$	R&S
$N_f = 6$	$0.66(4)^{+0.25}_{-0.13}$	$0.67(11)^{+0.21}_{-0.11}$	0.6515
$N_f = 8$	$0.19(8)^{+0.21}_{-0.09}$	0.2(1)	0.25

Mass scaling



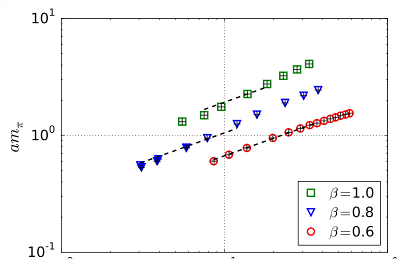
- Above: $N_f = 2$, Chiral symmetry breaking, $M_\pi \sim m_Q^{1/2}$
- Mass anomalous dimension defined by:

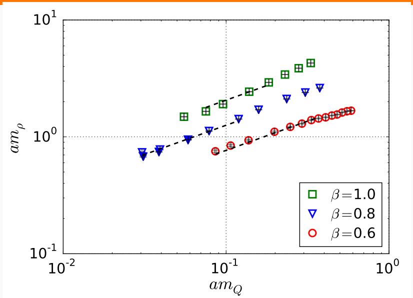
$$\mu \frac{dm(\mu)}{d\mu} = -\gamma(g^2) m(\mu)$$

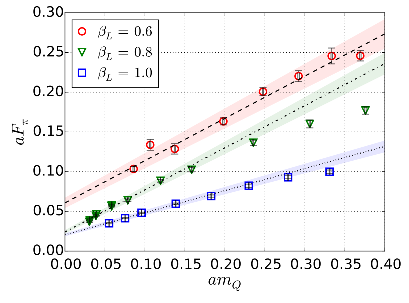
- In conformal theories masses should follow a power law:

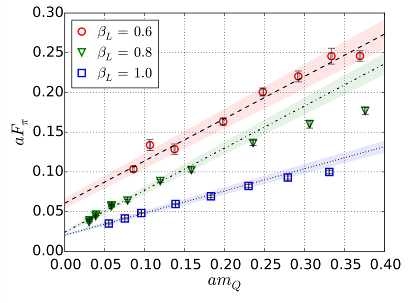
$$m_q^{1/(1+\gamma_m(g))}$$

$N_f = 4$ mass spectrum

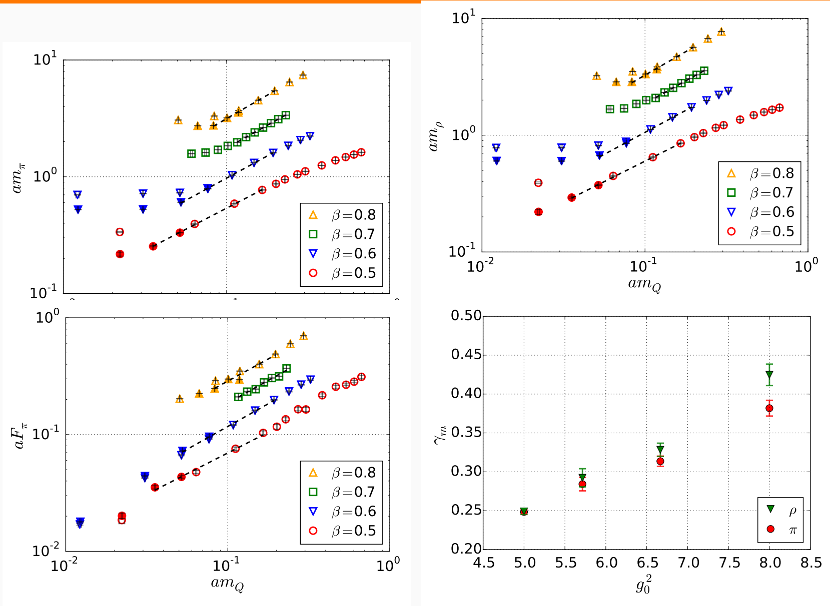








$N_f = 6$ mass spectrum



Mass anomalous dimension: Step scaling method

- Measure the pseudoscalar density renormalization constant:

$$Z_P(g_0, \frac{L}{a}) = \frac{\sqrt{Nf_1}}{f_p(\frac{1}{2}\frac{L}{a})}$$

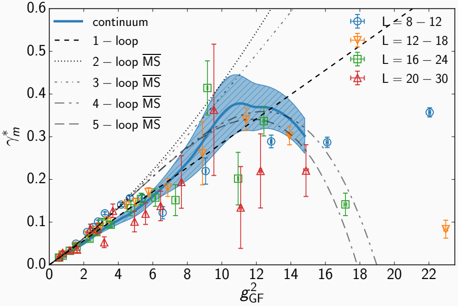
- Step scaling procedure similar to coupling

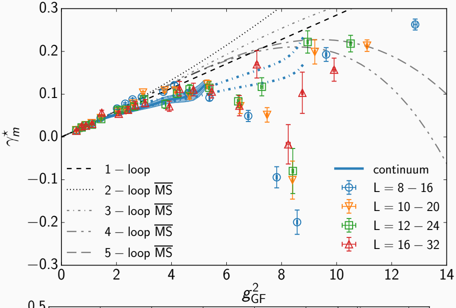
$$\Sigma_P(u, \frac{a}{L}) = \frac{Z_P(g_0, \frac{sL}{a})}{Z_P(g_0, \frac{L}{a})}\Bigg|_{u=g_{GF}^2}, \qquad \begin{array}{l} \Sigma_P(u) = \sigma_P(u,s) + c(u)(\frac{L}{a})^{-2} \\ \sigma_P(g^2) = \lim_{a\to 0}\Sigma_P(g^2, \frac{a}{L}) \end{array}$$

- Related to mass anomalous dimension γ by:

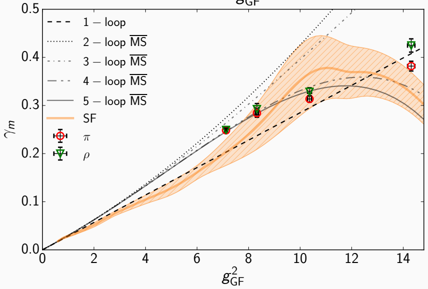
$$\gamma^* = -\frac{\log \sigma_P(g^2)}{\log s}$$

Mass anomalous dimension: Step scaling method





- Gives results comparable to perturbation theory
- Method breaks at large coupling



Mass anomalous dimension: Spectral method

- The mode number of Dirac operator defined from eigenvalue density:

$$\nu(\Lambda) \equiv 2 \int_0^{\sqrt{\Lambda^2 - m^2}} \rho(\lambda) d\lambda$$

- For massless theory at IRFP should follow the scaling:

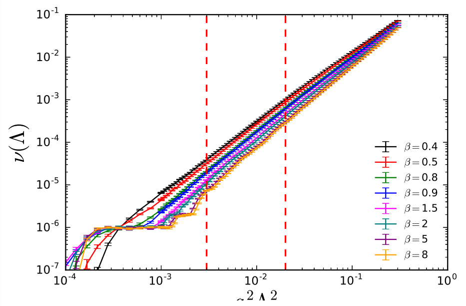
$$\nu(\Lambda) \simeq C \Lambda^{4/(1+\gamma_*)}$$

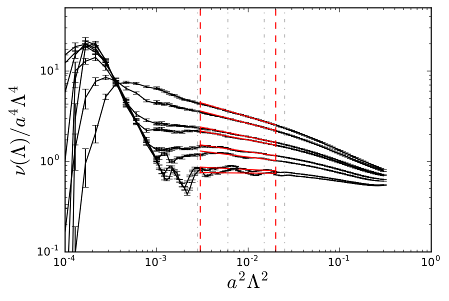
- Measure mode number from the lattice configurations:

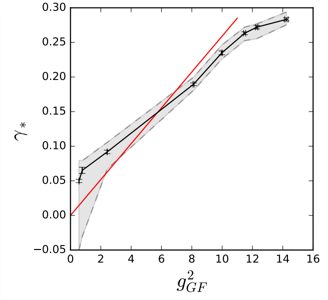
$$\nu(\Lambda) = \lim_{V \to \infty} \frac{1}{V} \left\langle \mathrm{tr}\, \mathbb{P}(\Lambda) \right\rangle ,$$

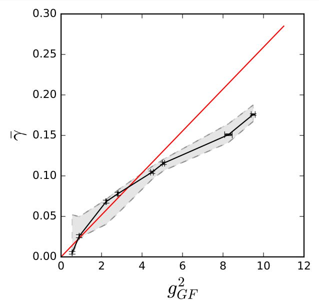
- $\mathbb{P}$ projects full eigenspace of $M = m^2 - \not{D}^2$ to the eigenspace of eigenvalues smaller than Λ^2; Stochastically do the trace

Mass anomalous dimension: Spectral method









Conclusions: Conformal Window

- We can accurately reach strong coupling with GF
- Clear indication for fixed point
  - $N_f = 6$: $g_*^2 = 14.5(3)^{+0.41}_{-1.38}$
  - $N_f = 8$: $g_*^2 = 8.24(59)^{+0.97}_{-1.64}$

  $\Rightarrow$ Conformal window edge between $N_f = 4 - 6$
- Mass anomalous dimension:
  - Mass step scaling works for small couplings
  - Spectral method works for large couplings
  - $\gamma^\star$ has relatively small value
    - $N_f = 6$: $\gamma_m^* = 0.283(2)^{+0.01}_{-0.01}$ (VS ~ 0.6 R&S)
    - $N_f = 8$: $\gamma_m^* = 0.15(2)$ (VS ~ 0.3 R&S)
- The slope of the beta function close to the Ryttov&Shrock result

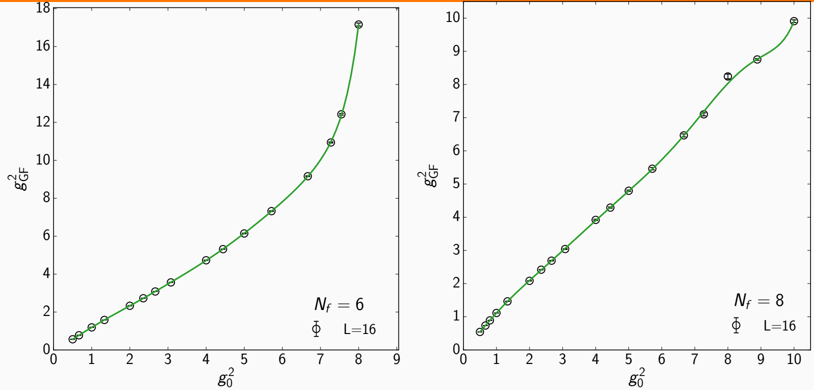
Conclusions: Large N_f

- Current data would indicate triviality
- UVFP cannot be ruled out
- Larger couplings needed
- Methods work in that we can match weak coupling physics correctly
- Larger coupling requires new methods

Thank you

Backup slides

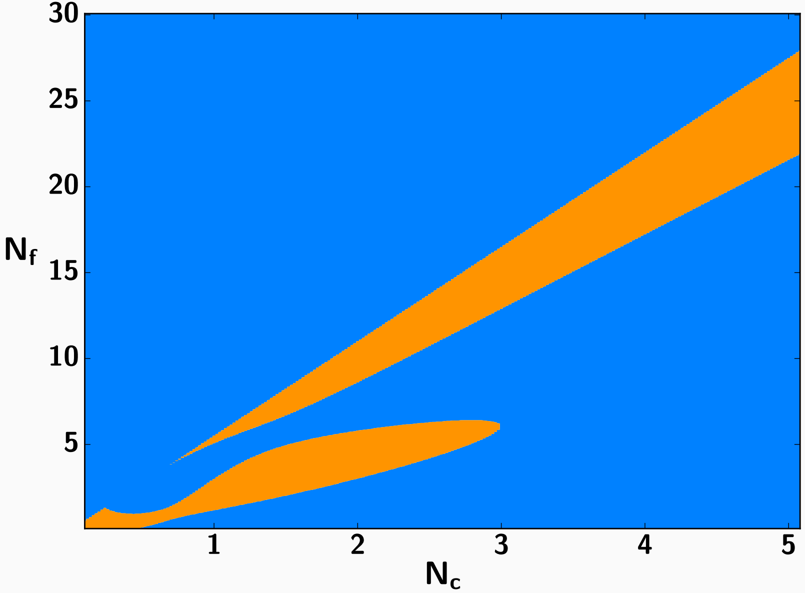
Interpolation



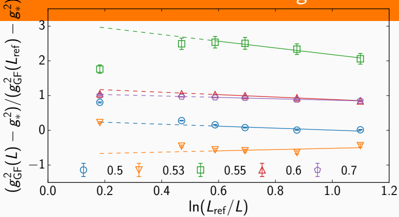
$$g^2_{GF} = g_0^2(1 + \sum_{i=1}^{n} a_i g_0^{2i}) \,, \qquad g^2_{GF} = g_0^2 \frac{1 + \sum_{i=1}^{n} a_i g_0^{2i}}{1 + \sum_{j=1}^{m} b_j g_0^{2j}}$$

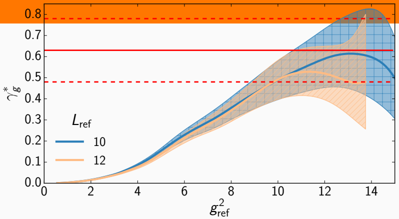
- For $N_f = 6$ $s = 1.5$, Interpolate using polynomial function $n = 9$
- For $N_f = 8$ $s = 2$, Interpolate using rational function $n = 7$ $m = 1$
- Estimate systematic errors by changing n, m by 1

5-loop problems



Slope of β-function: γ_g^*





- Previous slide relies on reliability of continuum limit at IRFP
- Alternative method:

$$\beta(g^2_{\rm GF}) = -\mu \frac{dg^2_{\rm GF}}{d\mu} = \gamma_g^*(g^2_{\rm GF} - g^2_*)$$

- Integrate from $L_{\rm ref}$ to L

$$g^2_{\rm GF}(\beta, L) - g^2_* = \left[g^2_{\rm GF}(\beta, L_{\rm ref}) - g^2_*\right]\left(\frac{L_{\rm ref}}{L}\right)^{\gamma_g^*} .$$